

A Unified Equity Valuation Framework under Alternative Dividend Policies

Abstract

The study establishes a Dividend Policy Invariance Theorem that is conceptually distinct from the dividend irrelevance proposition of Miller and Modigliani (1961). Whereas the latter is an equilibrium proposition concerning market value under perfect capital markets, the proposed theorem demonstrates that alternative earnings-allocation mechanisms generate identical terminal shareholder wealth under a common period-by-period return sequence.

Building upon a common mean-reverting return-on-equity (ROE) process, the study develops separate policy-specific derivations for three earnings-allocation mechanisms: complete cash-dividend distribution, complete earnings retention, and partial earnings retention.

Rather than assuming the equivalence of these dividend policies, each wealth-accumulation expression is derived separately under the common return structure. The mathematical analysis demonstrates that, although shareholder wealth accumulates through different internal and external capital accumulation mechanisms, all three earnings-allocation systems generate identical terminal shareholder wealth. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression as the horizon approaches infinity.

Based on these separate policy-specific derivations under a common return structure, the study establishes the Dividend Policy Invariance Theorem, which states that, under a common period-by-period return sequence for internal investment and external shareholder reinvestment, terminal shareholder wealth is invariant to the earnings-retention ratio for any finite investment horizon. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression as the horizon approaches infinity.

The proposed framework provides a common capital-accumulation perspective for examining the relationships among dividend policy, profitability dynamics, and shareholder wealth accumulation. It demonstrates that, under the model's common return dynamics, alternative earnings-allocation mechanisms generate identical terminal shareholder wealth. The framework also provides a conceptual bridge among dividend-based valuation, accounting-based valuation, and mean-reverting valuation approaches.

Keywords: Equity valuation; Capital accumulation; Mean reversion; Return on equity; Dividend policy invariance; Closed-form valuation; Corporate finance; Intrinsic value

1. Introduction

1.1 Background and Motivation

Equity valuation has long been one of the fundamental topics in finance, accounting, and investment analysis. Determining the intrinsic value of a firm's equity is essential for investment decision making, corporate financial management, securities analysis, and capital allocation. Consequently, a wide variety of valuation models have been developed over the past century, ranging from traditional dividend discount models and accounting-based valuation models to more recent dynamic valuation frameworks based on profitability and mean reversion.

Among the many issues related to equity valuation, the role of dividend policy has remained one of the most extensively debated questions in corporate finance. Since the pioneering work of Lintner (1956) and the dividend irrelevance proposition of Miller and Modigliani (1961), researchers have devoted considerable effort to understanding whether dividend policy influences firm value. Numerous theories—including the bird-in-the-hand theory, signaling theory, agency theory, and life-cycle theory—have proposed different economic mechanisms through which dividend policy may affect investors' perceptions or corporate valuation.

At the same time, valuation theory has evolved along a largely independent path. Traditional dividend discount models value equity as the present value of expected future dividends, whereas accounting-based valuation models emphasize book value, residual income, and profitability. More recently, mean-reverting valuation models have incorporated the dynamic evolution of return on equity into closed-form valuation equations, thereby providing a richer theoretical description of the valuation process.

Although these research streams have achieved substantial progress, they have generally evolved independently. Dividend policy research focuses primarily on whether payout decisions influence firm value, whereas valuation research seeks analytical models for estimating intrinsic value. As a result, the interaction between dividend policy and valuation has not been fully integrated within a unified theoretical framework.

If the same accounting earnings are allocated differently between retained earnings and cash dividends, how does the resulting path of shareholder wealth accumulation change? More specifically, do alternative dividend policies generate different terminal shareholder wealth, or are they alternative capital-accumulation mechanisms that produce the same terminal wealth under a common period-by-period return sequence?

The present study is motivated by this unresolved question. Rather than examining the informational or behavioral effects of dividend policy, it investigates the mathematical structure of equity valuation itself. The objective is to determine whether

complete cash-dividend distribution, complete earnings retention, and partial earnings retention produce different valuation equations or whether they converge to a common analytical solution under a shared mean-reverting profitability process.

Answering this question is important not only for dividend policy theory but also for the broader development of equity valuation. If alternative earnings-allocation mechanisms generate the same terminal shareholder wealth under common return dynamics, dividend policy can be interpreted as an alternative path of capital accumulation within the model. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression.

1.2 Research Gap

Despite the extensive literature on equity valuation and dividend policy, an important theoretical gap remains unresolved. Existing studies have developed sophisticated valuation models and numerous explanations for corporate payout decisions, yet these two research streams have largely evolved independently. As a result, the relationship between dividend policy and the mathematical structure of equity valuation remains insufficiently understood.

Traditional dividend discount models assume that dividends constitute the fundamental source of shareholder value. Consequently, the firm's dividend policy is embedded directly within the valuation framework. Accounting-based valuation models reduce this dependence by emphasizing book value, earnings, and residual income, but dividend policy generally remains an implicit assumption rather than an explicit analytical variable. Mean-reverting valuation models further incorporate dynamic profitability into valuation, yet they likewise assume a given earnings-allocation mechanism throughout the derivation.

Research on dividend policy approaches the problem from a different perspective. The dividend irrelevance proposition of Miller and Modigliani (1961) demonstrates that payout policy does not affect firm value under perfect capital markets, whereas subsequent theories explain how information asymmetry, agency conflicts, taxes, or investor preferences may cause dividend policy to influence market valuation. These theories therefore investigate the economic consequences of dividend policy rather than the mathematical structure of valuation equations.

Existing accounting-based valuation research has already established important forms of dividend irrelevance. Under clean-surplus accounting, residual-income valuation permits dividends to be displaced by corresponding changes in book value and future earnings, and the dividend-displacement literature further shows conditions under which valuation can be insensitive to future dividend policy. The present study

does not claim that payout invariance is absent from prior valuation theory. Rather, it addresses a narrower analytical question: whether complete payout, complete retention, and partial retention, modeled explicitly as alternative internal and external capital-accumulation mechanisms under common profitability dynamics, converge to the same terminal shareholder wealth and closed-form valuation equation.

Consequently, an important theoretical question has received relatively little attention. This question approaches dividend-policy invariance from a different analytical perspective than the classical dividend irrelevance proposition of Miller and Modigliani (1961). Their proposition concerns whether payout policy affects firm value under perfect capital markets and is established through equilibrium and arbitrage arguments. In contrast, the present study asks whether alternative earnings-allocation mechanisms generate identical terminal shareholder wealth under a common period-by-period return sequence. The issue addressed here therefore concerns the mathematical structure of equity valuation rather than the equilibrium determination of market value.

More specifically, three fundamental earnings-allocation mechanisms may be distinguished. Under complete earnings retention, all accounting earnings remain within the firm and generate future profitability through internal investment. Under complete cash-dividend distribution, all earnings are transferred to shareholders and accumulate through external reinvestment. Under partial earnings retention, both accumulation mechanisms operate simultaneously. Although these mechanisms represent different organizational paths of capital accumulation, it remains unclear whether they generate different terminal shareholder wealth under otherwise common return dynamics.

Accordingly, the principal theoretical gap addressed in this study is the limited direct analysis of complete payout, complete retention, and partial retention as alternative internal and external capital-accumulation mechanisms within a common dynamic profitability framework. Resolving this issue clarifies whether, under the assumptions of the present model, alternative earnings-allocation policies change terminal shareholder wealth or merely alter the organizational path through which that wealth accumulates.

Accordingly, the present study develops separate policy-specific derivations for the three earnings-allocation mechanisms under a common period-by-period return sequence. Their analytical comparison provides the basis for establishing the Dividend Policy Invariance Theorem developed in the subsequent sections.

Figure 1 illustrates the theoretical positioning of the present study within the existing literature. Rather than belonging exclusively to equity valuation, dividend policy, or profitability research, the proposed framework provides a common capital-accumulation perspective for relating these research streams and investigates whether

alternative dividend policies generate identical terminal shareholder wealth under common return dynamics.

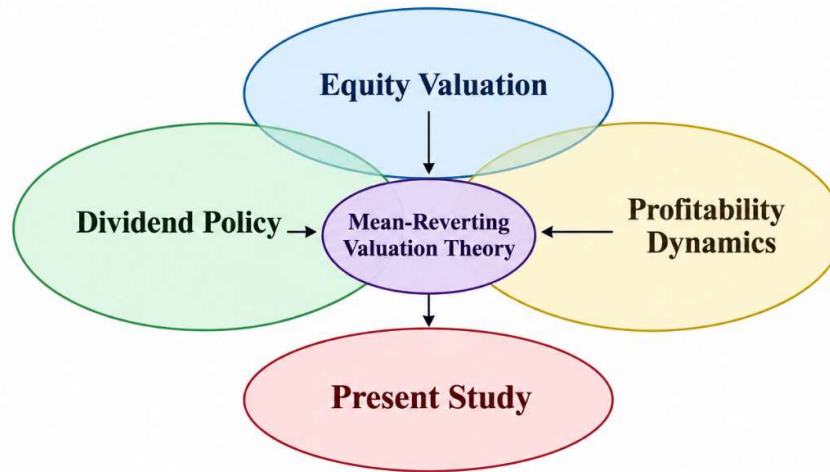


Figure 1. Theoretical positioning of the present study.

1.3 Research Framework and Objectives

To address the theoretical gap identified in the preceding section, this study develops a common capital-accumulation framework for jointly examining equity valuation, dividend policy, and profitability dynamics. Rather than assuming a particular dividend policy, the framework develops separate policy-specific derivations under alternative earnings-allocation mechanisms and subsequently compares their analytical properties.

The analysis begins with a common assumption that return on equity follows a mean-reverting profitability process. This dynamic process serves as the shared economic foundation for all subsequent derivations. Based on this common profitability framework, three distinct capital accumulation mechanisms are considered.

The first mechanism assumes complete cash-dividend distribution, in which all earnings are distributed to shareholders and subsequently accumulate through external reinvestment. The second mechanism assumes complete earnings retention, in which all accounting earnings remain within the firm and accumulate through internal corporate investment. The third mechanism considers partial earnings retention, where retained earnings and distributed dividends coexist and jointly contribute to shareholder wealth.

Separate policy-specific derivations are developed for each of these three earnings-allocation mechanisms under a common return structure. Rather than assuming their equivalence, the study derives each wealth-accumulation expression separately and subsequently compares the resulting terminal shareholder wealth. The resulting analytical expressions are then compared to determine whether alternative

dividend policies generate different terminal shareholder wealth under the common return structure.

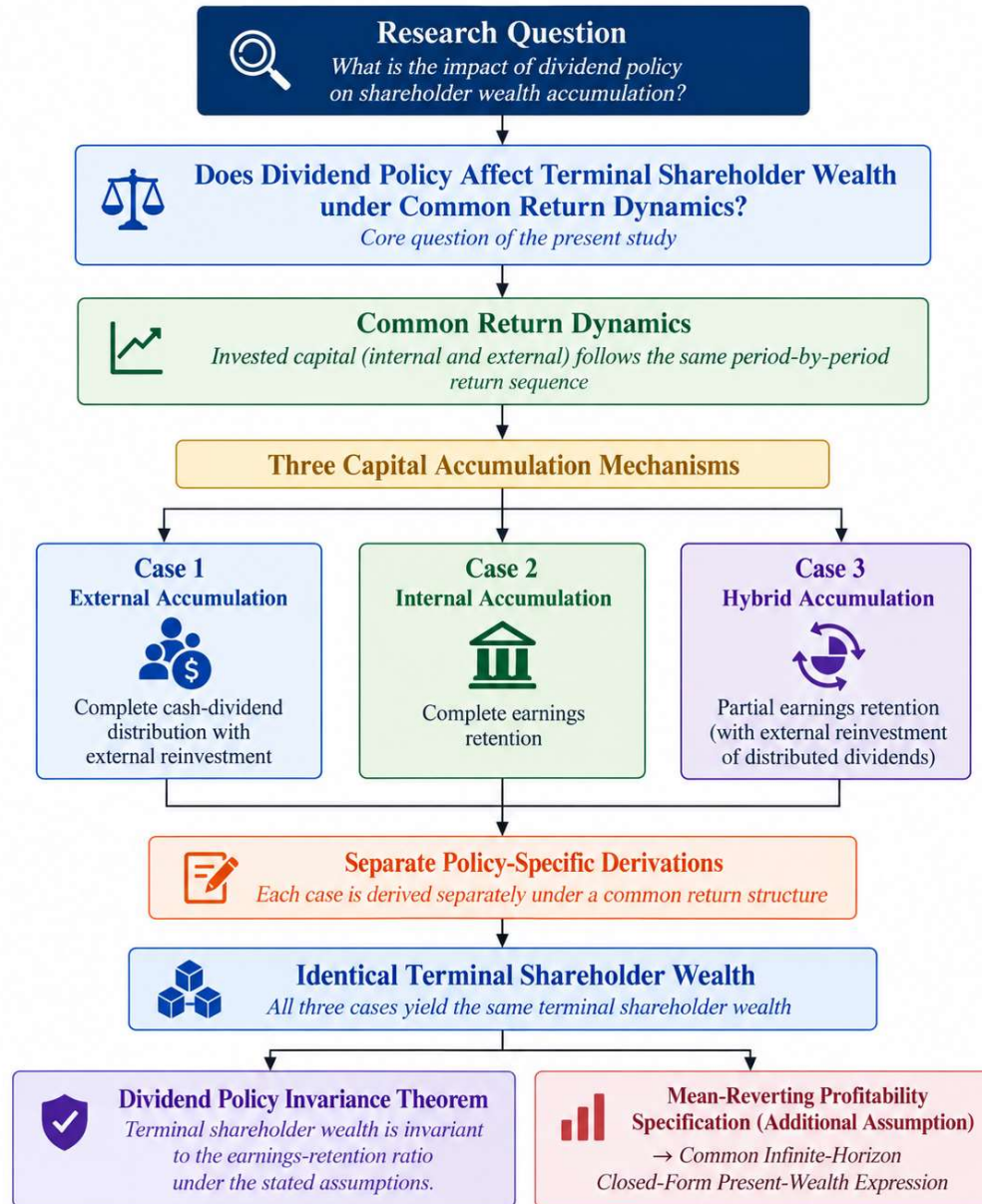


Figure 2. Overall Research Framework

The primary objective of this study is therefore to determine whether alternative earnings-allocation policies affect terminal shareholder wealth under the common return structure specified in the model. More specifically, the study investigates whether complete cash-dividend distribution, complete earnings retention, and partial earnings retention generate identical terminal shareholder wealth under a common period-by-period return sequence. Under the additional mean-reverting profitability specification, the study further examines whether the present value of this common

terminal wealth converges to a closed-form expression as the investment horizon approaches infinity.

A second objective is to interpret shareholder wealth accumulation from the perspective of alternative capital-allocation paths. Instead of viewing dividend policy as the source of intrinsic value, the proposed framework interprets retained earnings and distributed dividends as alternative organizational channels through which productive capital continues to generate future shareholder wealth.

The overall research framework is illustrated in Figure 2. Beginning with the central research question, the analysis develops three separate policy-specific derivations under a common period-by-period return structure. Their convergence to identical terminal shareholder wealth establishes the Dividend Policy Invariance Theorem. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to a common closed-form expression as the investment horizon approaches infinity.

1.4 Theoretical Contributions

The present study makes several theoretical contributions to the literature on equity valuation and corporate finance.

First, it develops a common capital-accumulation framework for examining the relationships among dividend policy, profitability dynamics, and shareholder wealth accumulation. Previous studies have generally examined these topics independently, whereas the present framework establishes explicit theoretical connections among them.

Second, the study develops separate policy-specific derivations for three alternative earnings-allocation mechanisms under a common period-by-period return sequence: complete cash-dividend distribution, complete earnings retention, and partial earnings retention. Rather than imposing equality of the resulting wealth expressions, the three earnings-allocation mechanisms are derived separately under a common period-by-period return sequence.

Third, the study establishes a Dividend Policy Invariance Theorem that provides an alternative analytical formulation to the classical dividend irrelevance proposition of Miller and Modigliani (1961). Rather than proving that dividend policy does not affect firm value under market equilibrium, the theorem demonstrates that complete retention, complete distribution, and partial retention generate identical terminal shareholder wealth under a common period-by-period return sequence.

Fourth, the study introduces a capital-accumulation perspective on shareholder wealth. Under the common return dynamics assumed in the model, retained earnings and externally reinvested dividends represent alternative channels through which shareholder wealth accumulates. Dividend policy therefore determines the allocation

path of capital, while terminal shareholder wealth remains invariant to the earnings-retention ratio under the stated assumptions.

Finally, the proposed framework provides a theoretical bridge between classical dividend-based valuation models, accounting-based valuation models, and modern mean-reverting valuation theory. By providing a common capital-accumulation perspective for interpreting these different approaches, the study offers a more unified interpretation of intrinsic equity valuation and extends the theoretical scope of existing valuation models.

Taken together, these contributions establish a capital-accumulation framework in which alternative dividend policies determine the organizational path of wealth accumulation. Under a common period-by-period return sequence, these alternative paths generate identical terminal shareholder wealth; under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression.

1.5 Organization of the Paper

The remainder of this paper is organized as follows.

Section 2 reviews the theoretical foundations of the study, including the evolution of equity valuation theory, dividend policy theory, profitability dynamics, and the capital accumulation perspective. It concludes by positioning the present study within the existing literature.

Section 3 derives the valuation model under complete cash-dividend distribution. Shareholder wealth accumulates through the external reinvestment of distributed dividends, providing the first policy-specific closed-form valuation benchmark.

Section 4 develops the valuation model under complete earnings retention. It demonstrates that internal book-equity accumulation produces the same terminal shareholder wealth and the same closed-form valuation equation as the complete cash-dividend distribution policy examined in Section 3.

Section 5 considers the general case of partial earnings retention. Internal capital accumulation and external shareholder reinvestment are analyzed simultaneously, leading to a unified closed-form valuation equation and the establishment of the Dividend Policy Invariance Theorem.

Section 6 discusses the theoretical implications of the proposed framework by comparing it with existing valuation theories, interpreting its economic meaning, examining its assumptions and limitations, and identifying directions for future research.

Finally, Section 7 summarizes the principal findings and theoretical contributions of the study and concludes with the broader implications of the proposed valuation framework for equity valuation and corporate finance.

2. Theoretical Foundations

2.1 Evolution of Equity Valuation Theory

Equity valuation has been one of the central topics in finance for nearly a century. The fundamental objective of valuation theory is to determine the intrinsic value of equity by relating current accounting or cash-flow information to future economic benefits. Although numerous valuation models have been proposed, they differ substantially in their theoretical foundations, the information variables employed, and the mechanisms through which firm value is generated.

The earliest modern valuation framework is the Dividend Discount Model (DDM) developed by Williams (1938), which states that the intrinsic value of equity equals the present value of all expected future dividends. Gordon (1959, 1962) later introduced the constant-growth dividend model, providing a simple closed-form solution under perpetual dividend growth. Owing to its analytical simplicity, the Gordon model has become one of the most widely taught valuation models in finance. Nevertheless, dividend-based valuation requires explicit assumptions regarding future dividend growth and payout policies, which may not adequately represent firms with unstable or changing dividend distributions.

To reduce the dependence on dividend assumptions, subsequent research increasingly incorporated accounting information into valuation models. The Residual Income Model (RIM), formalized by Ohlson (1995) and Feltham and Ohlson (1995), expresses firm value as the sum of current book value and the present value of expected future residual income. By linking accounting variables with valuation, the residual income framework substantially broadened the theoretical foundation of equity valuation and established profitability as a central determinant of intrinsic value.

More recent studies have emphasized the dynamic behavior of corporate profitability. Rather than assuming constant long-run growth, profitability is viewed as a stochastic process that gradually converges toward an equilibrium level. This perspective is consistent with extensive accounting and finance evidence documenting the persistence and eventual mean reversion of return on equity (Fama & French, 2000; Penman, 2013; Nissim & Penman, 2001). Building upon this idea, recent mean-reverting valuation models derive closed-form valuation equations by explicitly modeling the evolution of profitability over time (Yeh, 2024, 2025; Yeh & Hsu, 2011; Yeh & Chi, 2025).

Despite these important developments, existing valuation theories differ considerably in the mechanisms through which intrinsic value is generated. Dividend-based models emphasize future cash distributions, accounting-based models emphasize book value and residual income, whereas mean-reverting valuation models emphasize the dynamic evolution of profitability. Consequently, the relationship between dividend policy and intrinsic valuation remains theoretically fragmented. Existing studies generally assume a particular dividend policy rather than examining whether alternative earnings-allocation mechanisms produce the same valuation equation.

The present study extends the evolution of valuation theory from a different perspective. Instead of treating dividend policy as a primitive assumption, it examines how alternative earnings-allocation mechanisms influence the process of capital accumulation. Specifically, the study examines alternative capital-accumulation paths under a common return structure.

Separate policy-specific derivations are developed for complete cash-dividend distribution, complete earnings retention, and partial earnings retention. The analysis examines whether these mechanisms generate identical terminal shareholder wealth under a common period-by-period return sequence. The additional mean-reverting profitability specification is then used to derive the corresponding infinite-horizon closed-form present-wealth expression.

2.2 Dividend Policy Theory

Dividend policy has long been regarded as one of the fundamental issues in corporate finance. Since the pioneering work of Lintner (1956), researchers have sought to understand why firms distribute dividends, how payout decisions are made, and whether dividend policy influences firm value. Although numerous theories have been proposed over the past several decades, they differ primarily in the economic mechanisms through which dividends affect corporate valuation rather than in the mathematical structure of valuation itself.

One of the earliest explanations is the bird-in-the-hand theory, which argues that investors prefer certain current dividends to uncertain future capital gains (Gordon, 1959, 1962). Under this view, firms with higher dividend payouts may command higher valuations because current cash distributions reduce perceived investment risk. Although this theory provides an intuitive explanation for investor preferences, its conclusions depend critically on assumptions regarding investor behavior and risk perception.

In contrast, the dividend irrelevance proposition developed by Miller and Modigliani (1961) demonstrates that, under perfect capital markets, dividend policy has no effect on firm value. Their argument is based on market equilibrium: investors can

replicate any desired dividend stream through homemade dividends by buying or selling shares. Consequently, firm value depends on the firm's investment policy rather than on its payout policy. This proposition has become one of the cornerstones of modern corporate finance. Although the present study reaches an invariance result similar in spirit, it approaches dividend-policy invariance through a different analytical formulation from that of Miller and Modigliani (1961).

Subsequent research has relaxed the assumptions of perfect capital markets by introducing various market imperfections. Signaling theory proposes that dividend changes convey managers' private information about future profitability, thereby influencing investors' expectations and market prices (Bhattacharya, 1979; Miller & Rock, 1985). Agency theory argues that dividend payments reduce free cash flow available for managerial discretion, thereby mitigating agency conflicts between managers and shareholders (Jensen, 1986; Easterbrook, 1984). More recent studies, including the life-cycle theory of dividends, suggest that payout policies evolve systematically as firms progress through different stages of corporate development (DeAngelo, DeAngelo, & Stulz, 2006).

Although these theories provide important economic explanations for corporate payout decisions, they share a common characteristic: they investigate whether dividend policy influences firm value, but they do not derive a unified valuation equation that remains valid across alternative dividend policies. Dividend policy is generally treated as an exogenous assumption or as an economic determinant of value rather than as a variable within the mathematical structure of the valuation model itself.

The present study addresses this unresolved theoretical issue from a different perspective. Instead of asking whether dividends convey information or alleviate agency conflicts, the present study asks whether alternative earnings-allocation mechanisms generate different terminal shareholder wealth under a common return structure. By deriving the three policy cases separately, the study shows that they generate identical terminal shareholder wealth when retained capital and externally reinvested dividends follow the same period-by-period return sequence. Dividend policy is therefore interpreted, within the stated assumptions, as an alternative path of shareholder wealth accumulation.

Table 1 summarizes the major theories of dividend policy discussed above. Although these theories provide different explanations for the role of dividends in corporate finance, they primarily focus on the economic consequences of payout decisions rather than on the mathematical structure of equity valuation. In contrast, the present study investigates whether alternative dividend policies generate different terminal shareholder wealth under a common return structure and demonstrates that the three earnings-allocation mechanisms generate identical terminal shareholder wealth under the stated assumptions.

Table 1. Comparison of Major Dividend Policy Theories and the Present Study

Theory	Primary Question	Main Conclusion	Remaining Limitation
Dividend Discount Model	How do dividends determine value?	Value equals the present value of future dividends.	Assumes dividends as the valuation basis.
Dividend Irrelevance Theory	Does payout policy affect firm value?	Dividend policy is irrelevant under perfect markets.	Does not derive a common valuation equation.
Signaling Theory	Why do investors respond to dividends?	Dividends convey private information.	Focuses on information effects rather than valuation equations.
Agency Theory	Why do firms pay dividends?	Dividends mitigate agency conflicts.	Does not address intrinsic valuation.
Present Study	Do alternative payout policies generate the same terminal shareholder wealth under common return dynamics?	Terminal shareholder wealth is invariant to the retention ratio under the stated assumptions.	—

2.3 Profitability Dynamics and Mean-Reverting Valuation

The long-run behavior of corporate profitability has become one of the central issues in both accounting and finance research. While profitability is widely recognized as a major determinant of firm value, empirical evidence consistently shows that abnormal profitability is neither perfectly persistent nor completely transitory. Instead, return on equity (ROE) generally exhibits persistence over short horizons while gradually converging toward a long-run equilibrium level. This dynamic adjustment process is commonly referred to as profitability mean reversion.

Numerous empirical studies document that firms with exceptionally high profitability tend to experience declining future profitability, whereas firms with unusually low profitability often recover over time. Fairfield et al. (1996) further show that abnormal profitability gradually declines over time, providing evidence for long-

run profitability mean reversion. Fama and French (2000) report that profitability exhibits significant mean-reverting behavior across firms, implying that competitive forces gradually eliminate abnormal returns. Similar evidence is documented by Nissim and Penman (2001), who demonstrate that accounting profitability evolves systematically through time and contains substantial information about future firm performance. Penman (2013) further emphasizes that profitability persistence is one of the primary determinants of intrinsic equity value because current accounting performance provides information about future earnings generation.

These findings have motivated the development of dynamic valuation models in which profitability is treated explicitly as an evolving dynamic process rather than as a constant growth rate. Unlike the traditional Gordon model, which assumes perpetual dividend growth, mean-reverting valuation models describe firm value through the dynamic evolution of return on equity. The resulting valuation equations therefore depend on the speed with which profitability converges toward its long-run equilibrium rather than on a fixed perpetual growth assumption.

Recent theoretical studies have shown that mean-reverting profitability permits closed-form valuation solutions under relatively general conditions. By modeling the evolution of return on equity as a dynamic process, these studies derive analytical valuation equations that connect current profitability with future intrinsic value while avoiding the restrictive assumptions of constant-growth dividend models (Yeh, 2024, 2025; Yeh & Hsu, 2011; Yeh & Chi 2025).

The present study adopts this mean-reverting profitability framework as the common analytical foundation for all subsequent derivations. Unlike previous studies, however, the objective is not to develop another profitability-based valuation model. Instead, the same profitability dynamics are imposed on three alternative earnings-allocation mechanisms: complete cash-dividend distribution, complete earnings retention, and partial earnings retention. The central question is whether these different dividend policies alter the resulting valuation equation when the underlying profitability dynamics remain unchanged.

The common period-by-period return sequence provides the sufficient condition for the finite-horizon wealth-equivalence result examined in Sections 3–5. The mean-reverting profitability specification serves a separate analytical role: it permits the common return sequence to be evaluated in closed form as the investment horizon approaches infinity.

The empirical literature summarized in Table 2 consistently supports the view that corporate profitability exhibits persistence together with gradual mean reversion. These findings provide the economic justification for adopting a mean-reverting profitability process as the common analytical foundation of the present valuation framework.

Table 2. Empirical Evidence Supporting Profitability Mean Reversion

Literature	Main Finding	Implication for This Study
Freeman et al. (1982)	ROE predicts future earnings	Profitability exhibits persistence
Penman (1991)	Accounting profitability contains valuation information	ROE is a fundamental valuation variable
Fairfield et al. (1996)	Abnormal profitability gradually declines	Supports long-run mean reversion
Fama & French (2000)	Profitability converges toward equilibrium	Supports dynamic profitability modeling
Nissim & Penman (2001)	Abnormal profitability dissipates over time	Consistent with competitive adjustment
Mueller (1977)	Excess profits are not permanent	Provides economic justification
Porter (1980)	Competition erodes abnormal returns	Explains the mechanism of mean reversion

2.4 Capital Accumulation as the Foundation of Equity Valuation

The valuation models reviewed in the preceding sections differ substantially in their analytical formulations. Dividend discount models emphasize future cash distributions, residual income models focus on accounting profitability and book value, and mean-reverting valuation models describe the dynamic evolution of return on equity. Despite these differences, all valuation theories seek to explain a common economic question: how productive capital generates future shareholder wealth.

The present study adopts a capital accumulation perspective that differs fundamentally from previous valuation frameworks. Rather than treating dividends, accounting variables, or profitability as alternative valuation inputs, this study regards them as different manifestations of the same underlying capital accumulation process. Productive capital may remain inside the firm through retained earnings or be transferred to shareholders through cash dividends. Regardless of its organizational location, however, capital continues to generate future economic returns through investment.

From this perspective, the present framework focuses on how shareholder wealth accumulates through productive capital rather than on the organizational location of that capital. Dividend policy merely specifies how capital is allocated between corporate investment and shareholder reinvestment. If both channels are governed by

the same profitability dynamics, capital continues to compound at the same economic rate even though the accounting representation of wealth differs.

This interpretation shifts the analytical focus from dividend payments to wealth accumulation. Under complete earnings retention, shareholder wealth grows through the expansion of corporate book value. Under complete cash-dividend distribution, shareholder wealth grows through the reinvestment of distributed dividends. Under partial earnings retention, both mechanisms operate simultaneously. Although these accumulation paths appear different from an accounting perspective, they all describe the evolution of the same underlying productive capital.

Consequently, the valuation problem may be reformulated as a capital accumulation problem. Once the accumulation process is specified, the valuation equation follows naturally from the discounted value of future shareholder wealth. Dividend policy therefore determines the allocation path of capital, while terminal shareholder wealth remains invariant under the common return dynamics specified in the model.

This perspective provides the conceptual bridge connecting the literature reviewed in the previous sections. Dividend policy determines how capital is allocated, profitability determines how efficiently capital generates future returns, and valuation translates the resulting wealth accumulation process into present value. The central contribution of the present study is to demonstrate that these three elements can be unified within a single analytical framework based on capital accumulation under a common mean-reverting profitability process.

The theoretical developments presented in Sections 3–5 build directly upon this perspective. Separate policy-specific derivations are developed for complete cash-dividend distribution, complete earnings retention, and partial earnings retention under a common return structure. Under the additional mean-reverting specification, the present value of the common terminal shareholder wealth converges to the same closed-form expression. The resulting dividend policy invariance theorem therefore follows naturally from the common dynamics governing productive capital rather than from assumptions regarding market equilibrium or investor preferences.

2.5 Position of the Present Study

The preceding sections review four major streams of research related to equity valuation. Traditional valuation models establish the theoretical foundations of intrinsic value. Dividend policy theories examine whether payout decisions influence firm value. Profitability research demonstrates that return on equity evolves dynamically through persistence and mean reversion. Finally, the capital accumulation perspective proposed in this study provides a unified interpretation linking these research streams.

Despite their substantial contributions, existing studies leave an important theoretical question unresolved. Dividend discount models generally assume a given dividend policy, while accounting-based valuation models treat payout decisions as implicit accounting assumptions. Dividend policy theories investigate whether payout affects firm value but do not derive valuation equations under alternative payout mechanisms. Mean-reverting valuation models successfully incorporate profitability dynamics into valuation but likewise assume a specific earnings-allocation mechanism. Although prior valuation research has established important dividend-irrelevance and dividend-displacement results, comparatively less attention has been devoted to deriving and directly comparing complete payout, complete retention, and partial retention as alternative capital-accumulation mechanisms under a common dynamic profitability process.

The present study addresses this theoretical gap by integrating the four research streams into a unified analytical framework. Rather than treating dividend policy as an external assumption, the study develops separate policy-specific derivations for three alternative earnings-allocation mechanisms under a common return structure: complete cash-dividend distribution, complete earnings retention, and partial earnings retention. All three derivations are constructed under the same mean-reverting profitability process, allowing their mathematical structures to be compared directly.

The principal contribution of this study is the demonstration that these three capital accumulation mechanisms converge to an identical closed-form valuation equation. The resulting Dividend Policy Invariance Theorem shows that dividend policy affects the organizational path through which capital accumulates but, under the stated assumptions, does not alter terminal shareholder wealth. Consequently, equity valuation is reformulated as a theory of capital accumulation governed by common profitability dynamics rather than as a theory determined by a particular dividend policy.

Figure 1 summarizes the theoretical positioning of the present study within the existing valuation literature, while Figure 2 presents the overall research framework. Together, they illustrate how equity valuation, dividend policy, profitability dynamics, and capital accumulation are integrated into a single theoretical framework that culminates in the Dividend Policy Invariance Theorem.

To summarize the relationship between previous research and the present study, Table 3 compares the major research streams, their primary research questions, principal limitations, and the contribution of the present study. Building upon these theoretical foundations, Sections 3–5 develop separate mathematical derivations for complete cash-dividend distribution, complete earnings retention, and partial earnings retention, respectively, under a common period-by-period return sequence. The resulting equivalence of terminal shareholder wealth across the three earnings-

allocation mechanisms establishes the Dividend Policy Invariance Theorem presented later in this study.

Table 3 provides an integrated comparison of the principal research streams reviewed in this chapter. It highlights their primary research questions, remaining theoretical limitations, and the specific contribution of the present study. The comparison shows that existing studies generally assume a particular earnings-allocation mechanism, whereas the present study explicitly derives valuation equations under three alternative dividend policies and demonstrates their analytical equivalence.

Table 3. Comparison of Major Research Streams and the Position of the Present Study

Research Stream	Representative Literature	Main Question	Principal Limitation	Present Study
Dividend Discount Models	Williams (1938); Gordon (1959, 1962)	How do expected dividends determine intrinsic value?	Assumes a given dividend policy and perpetual dividend process.	Extends valuation to alternative dividend policies within a unified framework.
Residual Income Models	Ohlson (1995); Feltham & Ohlson (1995)	How do book value and residual income determine equity value?	Does not directly compare complete payout, complete retention, and partial retention as alternative capital-accumulation mechanisms under a common return sequence.	Derives valuation equations under alternative earnings-allocation mechanisms.
Dividend Policy Theory	Lintner (1956); Miller & Modigliani (1961); Jensen (1986)	Does dividend policy affect firm value?	Explains economic effects but does not derive a common valuation equation.	Demonstrates mathematical invariance of valuation across dividend policies.
Profitability Dynamics	Fama & French (2000); Nissim &	How does profitability	Focuses on profitability	Uses mean-reverting

Research Stream	Representative Literature	Main Question	Principal Limitation	Present Study
Mean-Reverting Valuation	Penman (2001); Penman (2013)	evolve over time?	forecasting rather than valuation equivalence.	profitability as the common analytical foundation.
	Yeh, 2024, 2025; Yeh & Hsu, 2011; Yeh & Chi 2025	How does mean reversion determine intrinsic value?	Assumes a particular earnings-allocation mechanism.	Generalizes the framework to complete retention, complete distribution, and partial retention.

2.6 Common Assumptions and Profitability Dynamics

Before deriving the policy-specific valuation models, we first establish the common assumptions and notation used throughout the theoretical analysis. These assumptions apply to the complete cash-dividend distribution, complete earnings-retention, and partial earnings-retention policies analyzed in the subsequent sections. Presenting these assumptions only once avoids repeated definitions and provides a unified mathematical foundation for all three dividend policies.

Let R_t denote the return on equity (ROE) earned during period t , and let r denote the constant required rate of return. To simplify subsequent derivations, we define the relative profitability factor

$$X_t = \frac{1 + R_t}{1 + r} \quad (1)$$

The relative profitability factor X_t measures the profitability of the firm relative to the required rate of return. When $X_t > 1$, the firm's profitability exceeds investors' required return and contributes positively to value creation.

For mathematical convenience, we further define

$$x_t = \ln X_t \quad (2)$$

The analysis assumes that $r > -1$ and $R_t > -1$ for all t , ensuring that $X_t > 0$ and that $x_t = \ln X_t$ is well defined. This logarithmic transformation converts the multiplicative accumulation process into an additive representation, thereby simplifying the derivation of the infinite-horizon valuation formula.

Following Yeh and Chi (2025), profitability is assumed to follow a mean-reverting process. Specifically, the relative profitability factor evolves according to

$$X_{t+1} = X_t^a, \quad 0 < a < 1. \quad (3)$$

For analytical tractability, the present study adopts a deterministic mean-reversion path rather than a stochastic profitability process.

Taking natural logarithms on both sides yields

$$x_{t+1} = ax_t \quad (4)$$

Applying Equation (4) recursively gives

$$x_t = a^{t-1}x_1, \quad (5)$$

which shows that profitability gradually converges toward its long-run equilibrium. Because $0 < a < 1$, the magnitude of x_t declines geometrically over time, implying that abnormal profitability diminishes progressively rather than remaining permanently constant.

The long-run equilibrium is normalized to the required rate of return r . Economically, this specification reflects the assumption that competitive forces gradually eliminate abnormal profitability, so that ROE converges toward the return required by investors in the long run. This is a theoretical steady-state condition of the present model rather than a claim that the observed long-run ROE of every firm must equal its cost of equity.

Equations (1)–(5) constitute the common mathematical assumptions used throughout the remainder of this study. In the following sections, these assumptions are applied successively to complete cash-dividend distribution, complete earnings retention, and partial earnings retention.

Terminal realization convention.

For each finite investment horizon T , the terminal book equity accumulated within the firm is assumed to be fully realizable by shareholders at its book amount at the terminal date. This realization may be interpreted as a terminal distribution, redemption, or equivalent conversion of the accumulated equity claim into shareholder wealth. Accordingly, B_T represents the model-implied terminal value of the shareholders' claim under this convention, rather than merely an unrecoverable accounting balance. The infinite-horizon value derived below is defined as the limit of the discounted terminal values associated with these finite investment horizons; it does not require a literal liquidation at an infinite date.

2.7 Valuation Objects and Terminology

To avoid ambiguity, three related but distinct valuation objects should be distinguished in the present framework. First, firm equity value refers to the value at

time zero of the equity claim on the original firm. Second, cum-dividend shareholder wealth refers to the combined value of the equity position and cash dividends received by shareholders over the investment horizon. Third, reinvested shareholder wealth refers to shareholder wealth after distributed dividends are reinvested externally and allowed to accumulate through the terminal date.

The equivalence results derived in Sections 3–5 directly concern terminal shareholder wealth under the terminal realization convention specified in Section 2.6. Under the model's common period-by-period return sequence, alternative earnings-retention ratios generate identical terminal shareholder wealth after accounting for both internally accumulated equity and the external reinvestment of distributed dividends. Discounting this terminal wealth to time zero yields the model-implied intrinsic value under the capital-accumulation framework.

This model-implied intrinsic value is conditional on two features of the framework. First, terminal book equity is assumed to be fully realizable by shareholders at its book amount at each finite valuation horizon. Second, distributed dividends are assumed to be reinvested externally under the same period-by-period return sequence as internally retained capital. The resulting measure should therefore be distinguished from an unconditional market value or from a conventional ex-dividend valuation that does not include subsequent returns earned on externally reinvested dividends.

3. Valuation under Complete Cash-Dividend Distribution

3.1 Wealth Accumulation under Complete Cash-Dividend Distribution

We begin the policy-specific valuation analysis with complete cash-dividend distribution, under which all accounting earnings are distributed immediately to shareholders. This policy provides a natural starting point because shareholder benefits are represented directly by cash distributions and their subsequent external reinvestment. Under this policy, the earnings-retention ratio is

$$v=0. \tag{6}$$

Unlike the complete-retention case, book equity no longer grows through retained earnings because every period's accounting income is paid out immediately to shareholders. For analytical purposes, accounting earnings are defined after the expenses and depreciation required to maintain the firm's initial book-capital base. Accordingly, complete payout refers to the distribution of accounting earnings rather

than the distribution of the initial book equity B_0 , which is treated as a maintained capital base throughout the valuation horizon. Consequently, capital accumulation no longer occurs inside the firm. Instead, shareholders receive each dividend at the end of its originating period and reinvest it externally according to the profitability process established in Section 2.6.

Accordingly, shareholder wealth is accumulated through two distinct components. The first is the firm's initial book value, B_0 , which is treated as a maintained capital base throughout the investment horizon. Under the terminal realization convention, this maintained capital base is assumed to be fully realizable by shareholders at the terminal date. The second is the accumulated value of all cash dividends after external reinvestment. The total terminal shareholder wealth therefore consists of

1. the maintained initial book-capital base at the terminal date; and
2. the accumulated value of all reinvested dividends received during the investment horizon.

Although this accumulation mechanism differs fundamentally from the internal compounding process examined in Section 4, both mechanisms are governed by the common profitability process established in Section 2.6. The analytical objective of this section is therefore to determine whether these different organizational paths of capital accumulation ultimately generate different terminal shareholder wealth.



Figure 3. External Capital Accumulation under Complete Cash-Dividend Distribution

The mathematical derivation follows the timing convention specified in Appendix A.1. Each dividend is distributed at the end of its originating period and begins to earn investment returns only from the following period. This convention prevents a dividend

from earning the return of a period that has already ended and ensures consistency between the accounting timing and the investment process. The detailed derivation is presented in Appendix A.1, while the intermediate algebraic transformations are also provided in the Appendix.

Figure 3 illustrates the wealth-accumulation mechanism under complete cash-dividend distribution. Unlike Figure 4, where shareholder wealth grows entirely through internally retained earnings, shareholder wealth in the present case is accumulated through the successive reinvestment of distributed cash dividends while the firm's initial book-capital base remains maintained throughout the investment horizon.

3.2 Terminal Shareholder Wealth under Complete Cash-Dividend Distribution

Under the complete cash-dividend distribution policy, all accounting earnings generated during each period are distributed immediately to shareholders. Since no earnings are retained within the firm, shareholders' equity remains constant throughout the investment horizon.

During period t , the firm earns accounting income equal to

$$R_t B_0,$$

because the firm's book value remains fixed at its initial level B_0 . This entire amount is distributed as a cash dividend at the end of period t .

Following the timing convention established in Appendix A.1, each dividend begins to earn investment returns only from the beginning of the following period. Consequently, the dividend received at the end of period t is reinvested externally for the remaining $T-t$ periods. Its accumulated value at the terminal date is therefore

$$D_t \prod_{s=t+1}^T (1 + R_s), \quad (7)$$

where

$$D_t = R_t B_0. \quad (8)$$

Within the model's terminal-wealth representation, total shareholder wealth at time T consists of the maintained initial book-capital base plus the accumulated reinvestment account:

$$W_T^{\text{pay}} = B_0 + B_0 \sum_{t=1}^T R_t \prod_{s=t+1}^T (1 + R_s). \quad (9)$$

To simplify this expression, we utilize the following algebraic identity:

$$1 + \sum_{t=1}^T R_t \prod_{s=t+1}^T (1 + R_s) = \prod_{s=1}^T (1 + R_s). \quad (10)$$

The proof of Equation (10) can be found in Appendix A.1.

Substituting Equation (10) into Equation (9) yields

$$W_T^{\text{pay}} = B_0 \prod_{s=1}^T (1 + R_s). \quad (11)$$

Equation (11) is the terminal-wealth expression under complete cash-dividend distribution. Unlike the complete-retention case, where shareholder wealth is represented entirely by accumulated book equity, the present case separates terminal wealth into two components: the unchanged initial investment and the accumulated value of reinvested dividends.

Although Equation (11) appears structurally different from the terminal book-value expression subsequently derived in Section 4, the two expressions are shown to be mathematically equivalent. The next subsection demonstrates that the two apparently different wealth-accumulation mechanisms are connected by a fundamental algebraic identity. Consequently, both dividend policies generate identical terminal shareholder wealth.

3.3 Equivalence of Terminal Shareholder Wealth

Although the complete cash-dividend distribution policy produces a terminal wealth expression that differs substantially from that obtained under complete earnings retention, this apparent difference arises solely from the organizational path through which capital accumulates. Under complete earnings retention, shareholder wealth is accumulated entirely through the continuous growth of book equity within the firm. Under complete cash-dividend distribution, by contrast, shareholder wealth is accumulated through the external reinvestment of distributed dividends while the firm's initial book value remains unchanged.

At first sight, these two accumulation mechanisms appear to generate different terminal wealth expressions. However, Appendix A.1 demonstrates that they are connected by a fundamental algebraic identity. After rearranging the accumulated dividend terms, the terminal shareholder wealth under complete cash-dividend distribution can be written in exactly the same product form as that obtained under complete earnings retention.

Consequently, according to Equations (11) and (20), we obtain

$$W_T = B_T. \quad (12)$$

Equation (12) establishes that the two earnings-allocation mechanisms generate identical terminal shareholder wealth despite their fundamentally different capital accumulation processes. The difference between the two policies therefore lies only in the organizational location of capital accumulation rather than in the amount of wealth ultimately created.

The detailed algebraic transformation leading from the accumulated-dividend expression in Equation (11) to Equation (12) is presented in Appendix A.1. Additional intermediate derivations and supporting identities are also provided in the Appendix, which establish the equivalence rigorously, whereas the present section summarizes only the principal analytical result.

The wealth-equivalence result obtained here provides the key theoretical bridge between the complete-retention and complete-payout policies. The wealth-equivalence result established here concerns terminal shareholder wealth under the common reinvestment opportunity specified in the model. It does not by itself imply equivalence with the conventional dividend-discount valuation of the original firm, because distributed dividends are assumed to continue accumulating outside the firm after distribution. The next subsection therefore derives the present value of this common terminal shareholder wealth within the capital-accumulation framework adopted in this study.

3.4 Infinite-Horizon Present Value of Terminal Shareholder Wealth

Section 3.3 establishes the central analytical result of the complete cash-dividend distribution policy: although shareholder wealth is accumulated through a fundamentally different mechanism, the terminal shareholder wealth is identical to that obtained under complete earnings retention.

Since

$$W_T = B_T, \quad (13)$$

the present value of the common terminal shareholder wealth is obtained by discounting W_T at the required rate of return,

$$V_T^{\text{div}} = \frac{W_T}{(1+r)^T}. \quad (14)$$

Substituting Equation (13) into Equation (14) gives

$$V_T^{\text{div}} = \frac{B_T}{(1+r)^T}, \quad (15)$$

which will be shown in Section 4 to be identical to the valuation expression obtained under complete earnings retention.

Consequently, the remaining analytical development follows the logarithmic transformation and infinite-product procedure presented in Section 4.4. Applying the common profitability dynamics established in Section 2.6 yields

$$V_{\infty}^{\text{div}} = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}} \quad (16)$$

This valuation equation is identical to the closed-form solution obtained under complete earnings retention. Therefore, changing the dividend policy from complete earnings retention to complete cash-dividend distribution does not alter terminal shareholder wealth under the common reinvestment opportunity specified in the model. The only difference lies in the organizational path through which capital is accumulated before the terminal date.

To avoid duplicating the logarithmic transformation and infinite-product derivation provided in Section 4.4, the corresponding closed-form result is stated directly here. Once the equality of terminal shareholder wealth is established, the closed-form valuation equation follows from the common profitability dynamics specified in Section 2.6.

3.5 Theorem 1. Closed-Form Present Value of Terminal Shareholder Wealth under Complete Cash-Dividend Distribution

The analytical development presented in Sections 3.2–3.4 establishes the intrinsic valuation equation under the complete cash-dividend distribution policy. The principal result is summarized below.

Theorem 1

Under the common assumptions and the terminal realization convention established in Section 2.6, suppose that the firm distributes all accounting earnings as cash dividends throughout the investment horizon ($v=0$), maintains the initial book-capital base B_0 , and that each distributed dividend is fully reinvested externally according to the common period-by-period return sequence. Then the model-implied intrinsic value, defined as the present value of terminal shareholder wealth under the capital-accumulation framework, is

$$V_{\infty}^{\text{pay}} = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}}. \quad (17)$$

where

- B_0 denotes the initial book value of shareholders' equity;
- R_1 denotes the initial return on equity;
- r is the constant required rate of return; and
- a is the mean-reversion parameter defined in Section 2.6.

This model-implied intrinsic value is conditional on the maintained initial book-capital base being fully realizable by shareholders at the terminal date and on all distributed dividends being fully reinvested under the common period-by-period return sequence. It should therefore be distinguished from the conventional dividend-discount value of the original firm, which discounts dividends when they are distributed and does not include either the terminal realization of the maintained book-capital base or the subsequent returns earned on externally reinvested dividends.

Theorem 1 demonstrates that complete cash-dividend distribution produces the closed-form valuation equation stated above. Section 4 subsequently demonstrates that complete earnings retention produces exactly the same closed-form present-wealth expression. Although shareholder wealth is accumulated through the external reinvestment of distributed dividends rather than through internal growth of book equity, the two accumulation mechanisms generate identical terminal shareholder wealth, as established in Section 3.3. Consequently, discounting this common terminal wealth yields the same closed-form valuation equation derived previously in Section 3.4.

The significance of Theorem 1 is that dividend distribution changes the organizational location of capital accumulation but does not alter the mathematical structure of the present value of terminal shareholder wealth under the model's assumptions. The remaining question is whether this result continues to hold when earnings are allocated simultaneously between retained earnings and cash dividends. Section 5 addresses this more general case.

4. Closed-Form Valuation under Complete Earnings Retention

4.1 Complete Earnings Retention and Its Relation to the Full-Payout Benchmark

Section 2.6 establishes the common assumptions and mean-reverting profitability process used throughout the analysis. Section 3 derives the valuation equation under complete cash-dividend distribution. This section considers the opposite boundary policy of complete earnings retention and demonstrates that internal capital accumulation produces the same closed-form valuation equation.

4.2 Book-Equity Accumulation under Complete Earnings Retention

Under the complete earnings-retention policy, the firm distributes no cash dividends during the investment horizon. All accounting earnings are retained within the firm and immediately added to shareholders' equity. Consequently, book equity grows solely through internally generated earnings.

Let B_t denote shareholders' book equity at the end of period t . During period t , the firm earns accounting income equal to

$$R_t B_{t-1}, \quad (18)$$

where R_t is the return on equity defined in Section 2.6.

Since all earnings are retained, the entire amount is added to book equity at the end of the period. The one-period evolution of book equity is therefore

$$B_t = B_{t-1} + R_t B_{t-1} = B_{t-1}(1 + R_t). \quad (19)$$

Equation (19) is the fundamental accounting identity governing capital accumulation under complete earnings retention. Unlike the dividend-distribution policies examined in Sections 3 and 5, no cash leaves the firm during the investment horizon. As a result, shareholders' wealth remains fully invested within the firm, allowing book equity to compound continuously over time.

Applying Equation (19) recursively gives

$$\begin{aligned} B_1 &= B_0(1 + R_1), \\ B_2 &= B_1(1 + R_2) = B_0(1 + R_1)(1 + R_2), \\ &\vdots \\ B_T &= B_0 \prod_{t=1}^T (1 + R_t). \end{aligned} \quad (20)$$

Equation (20) shows that terminal book equity equals the initial book value multiplied by the cumulative growth generated by the sequence of returns on equity. This expression provides the accounting foundation for the valuation model. The next subsection converts the accumulated terminal book value into present value by discounting future shareholder wealth and expressing the valuation equation in terms of the relative profitability factor introduced in Section 2.6.

Figure 4 illustrates the capital accumulation process under complete earnings retention. All accounting earnings remain within the firm, increasing book value and generating future profitability through internal investment until terminal shareholder wealth is realized.

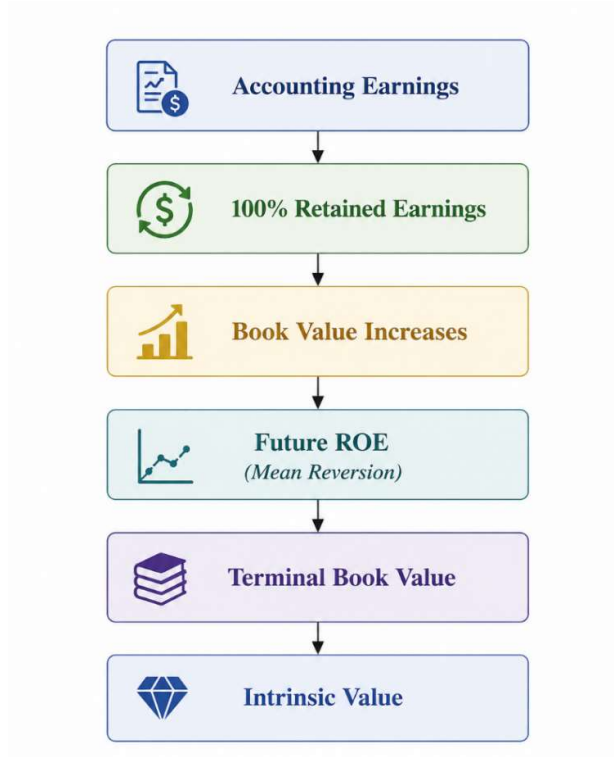


Figure 4. Internal Capital Accumulation

4.3 Present Value of Terminal Shareholder Wealth

The terminal book equity derived in Section 4.2 represents the wealth accumulated within the firm at the end of the investment horizon. Because shareholders receive no intermediate cash dividends under the complete-retention policy, all shareholder wealth remains represented by the accumulated equity claim. Under the terminal realization convention specified in Section 2.6, this accumulated book equity is assumed to be fully realizable by shareholders at its book amount at the terminal date. Accordingly, B_T represents the model-implied terminal shareholder wealth under the complete-retention policy.

The model-implied intrinsic value at time zero is obtained by discounting this terminal shareholder wealth at the required rate of return.

Accordingly, the present value of terminal shareholder wealth is

$$V_T^{\text{ret}} = \frac{B_T}{(1+r)^T}. \quad (21)$$

Substituting Equation (20) into Equation (21) gives

$$V_T^{\text{ret}} = \frac{B_0 \prod_{t=1}^T (1+R_t)}{(1+r)^T}. \quad (22)$$

Using the relative profitability factor introduced in Equation (1),

$$X_t = \frac{1 + R_t}{1 + r},$$

Equation (22) can be rewritten as

$$V_T^{\text{ret}} = B_0 \prod_{t=1}^T X_t. \quad (23)$$

Equation (23) expresses the present value of shareholder wealth as the product of the initial book value and the cumulative relative profitability factors. This transformation is important because it separates the accounting accumulation process from the discounting process and converts the valuation problem into the analysis of the dynamic behavior of X_t .

At this stage, no specific assumption has yet been imposed on the time path of profitability beyond the general mean-reverting framework established in Section 2.6. The next subsection incorporates the mean-reverting profitability process into Equation (23) and derives a closed-form valuation formula for the infinite-horizon case.

4.4 Infinite-Horizon Closed-Form Valuation

Equation (23) expresses the present value of terminal shareholder wealth under complete earnings retention as the product of the initial book value and the sequence of relative profitability factors. To obtain a closed-form valuation formula, the mean-reverting profitability process established in Section 2.6 is now incorporated into the valuation equation.

Taking natural logarithms of Equation (23) yields

$$\ln \left(\frac{V_T^{\text{ret}}}{B_0} \right) = \sum_{t=1}^T \ln X_t. \quad (24)$$

Using the logarithmic variable introduced in Equation (2),

$$x_t = \ln X_t,$$

Equation (24) becomes

$$\ln \left(\frac{V_T^{\text{ret}}}{B_0} \right) = \sum_{t=1}^T x_t. \quad (25)$$

According to the mean-reverting profitability process derived in Equation (5),

$$x_t = a^{t-1} x_1,$$

where $0 < a < 1$. Substituting Equation (5) into Equation (25) gives

$$\ln \left(\frac{V_T^{\text{ret}}}{B_0} \right) = x_1 \sum_{t=1}^T a^{t-1}. \quad (26)$$

As the investment horizon approaches infinity, $T \rightarrow \infty$, the geometric series converges to

$$\sum_{t=1}^{\infty} a^{t-1} = \frac{1}{1-a}. \quad (27)$$

Consequently,

$$\ln \left(\frac{V_{\infty}^{\text{ret}}}{B_0} \right) = \frac{x_1}{1-a}. \quad (28)$$

Substituting

$$x_1 = \ln X_1$$

into Equation (28) gives

$$V_{\infty}^{\text{ret}} = B_0 X_1^{\frac{1}{1-a}}. \quad (29)$$

Finally, using the definition of the relative profitability factor in Equation (1),

$$X_1 = \frac{1 + R_1}{1 + r},$$

the infinite-horizon model-implied intrinsic value under complete earnings retention is obtained as

$$V_{\infty}^{\text{ret}} = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}} \quad (30)$$

Equation (30) is the closed-form expression for the model-implied intrinsic value under the complete earnings-retention policy. Under the assumptions specified in Section 2.6, this value depends on three fundamental determinants: the initial shareholders' equity, the firm's initial profitability relative to the required rate of return, and the speed of profitability mean reversion. Because the profitability process converges geometrically, the infinite sequence of future profitability can be summarized by a single closed-form expression.

4.5 Theorem 2. Closed-Form Valuation under Complete Earnings Retention

The derivation presented in Sections 4.2–4.4 establishes the intrinsic valuation formula under the complete earnings-retention policy. The principal result is summarized below.

Theorem 2

Under the common assumptions and the terminal realization convention established in Section 2.6, suppose that the firm retains all accounting earnings throughout the investment horizon ($v=1$). Then the model-implied intrinsic value, defined as the present value of terminal shareholder wealth under the capital-accumulation framework, is

$$V = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}} \quad (31)$$

Equation (31) shows that the intrinsic value under complete earnings retention depends exclusively on three economic determinants: the initial shareholders' equity, the firm's initial profitability relative to the required rate of return, and the speed at which profitability converges toward its long-run equilibrium. Because all earnings remain within the firm, shareholder wealth accumulates entirely through internal capital formation, and no intermediate dividend payments occur during the investment horizon.

This proposition serves as the benchmark result for the remainder of the paper. Section 3 demonstrates that the same closed-form valuation equation is obtained under complete cash-dividend distribution, while Section 5 extends the result to partial earnings retention despite the fundamentally different mechanisms through which shareholder wealth accumulates.

5. Valuation under Partial Earnings Retention

5.1 Economic Setting under a General Dividend Policy

Sections 3 and 4 examine the two extreme earnings-allocation policies. Under complete earnings retention, all accounting earnings remain within the firm and shareholder wealth accumulates entirely through the growth of book equity. Under complete cash-dividend distribution, all earnings are distributed to shareholders and wealth accumulates externally through dividend reinvestment.

This section considers the general case in which the firm retains only a fraction of its accounting earnings and distributes the remainder as cash dividends. Let v denote the earnings-retention ratio, where

$$0 \leq v \leq 1. \quad (32)$$

In each period, the firm retains the fraction v of accounting earnings and distributes the remaining fraction $1-v$. The two extreme policies analyzed previously are therefore nested within this general framework:

- $v=1$ corresponds to complete earnings retention;
- $v=0$ corresponds to complete cash-dividend distribution; and
- $0 < v < 1$ represents partial earnings retention.

Under this general policy, shareholder wealth accumulates through two simultaneous channels. The retained portion of earnings remains inside the firm, increases shareholders' book equity, and generates future returns through internal investment. The distributed portion is received by shareholders as a cash dividend and is subsequently reinvested outside the firm.

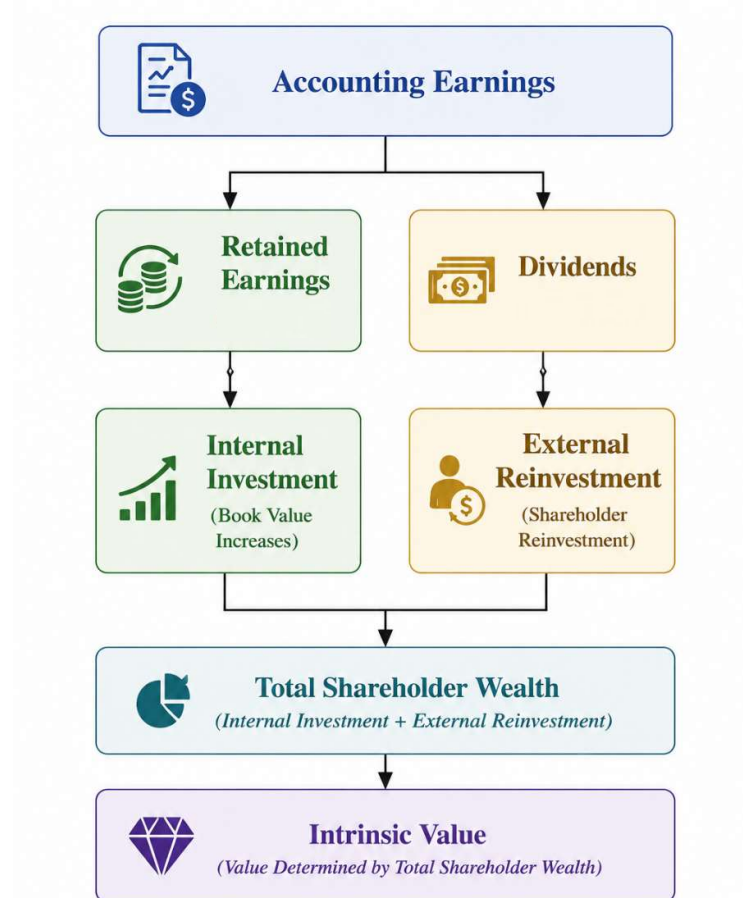


Figure 5. Hybrid Capital Accumulation under Partial Earnings Retention

Both channels are assumed to follow the same sequence of returns R_t and the common mean-reverting profitability process established in Section 2.6. Thus, the distinction between retained earnings and distributed dividends concerns the organizational location of capital rather than its underlying productivity.

The analytical objective of this section is to determine whether the earnings-retention ratio affects the total wealth ultimately accumulated by shareholders. At first sight, a change in v appears likely to alter terminal wealth because it changes both the growth rate of corporate book equity and the amount of cash distributed for external reinvestment. The following subsections demonstrate, however, that these two effects exactly offset one another under the assumed profitability dynamics.

Figure 5 illustrates the hybrid capital-accumulation mechanism. Accounting earnings are divided between internal retention and cash-dividend distribution, while both components continue to contribute to terminal shareholder wealth.

5.2 Terminal Shareholder Wealth under Partial Earnings Retention

Under the general dividend policy introduced in Section 5.1, shareholder wealth accumulates through internal book-equity growth and external dividend reinvestment. This subsection derives the terminal value of each component and combines them into total shareholder wealth at time T .

5.2.1 Internal accumulation through retained earnings

Let B_t denote the book value of shareholders' equity at the end of period t . During period t , the firm earns accounting income equal to

$$R_t B_{t-1}.$$

Because the firm retains the fraction v , retained earnings during the period equal $v R_t B_{t-1}$.

The one-period evolution of book equity is therefore

$$B_t = B_{t-1} + v R_t B_{t-1} = B_{t-1}(1 + v R_t). \quad (33)$$

Iterating Equation (33) from period 1 through period T gives

$$B_T = B_0 \prod_{s=1}^T (1 + v R_s). \quad (34)$$

Equation (34) represents the shareholder wealth accumulated internally through retained earnings at time T .

5.2.2 External accumulation through reinvested dividends

The remaining fraction $1-v$ of accounting earnings is distributed as a cash dividend at the end of each period. Because the book value at the beginning of period t is B_{t-1} , the dividend paid at the end of that period is

$$D_t = (1 - v) R_t B_{t-1}. \quad (35)$$

From Equation (34), the beginning-of-period book value can be expressed as

$$B_{t-1} = B_0 \prod_{s=1}^{t-1} (1 + vR_s). \quad (36)$$

Substituting Equation (36) into Equation (35) gives

$$D_t = B_0(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s). \quad (37)$$

Following the timing convention established in Section 3, the dividend paid at the end of period t begins earning returns only in period $t+1$. Its accumulated value at time T is therefore

$$D_t \prod_{s=t+1}^T (1 + R_s). \quad (38)$$

Accordingly, the accumulated terminal value of all reinvested dividends is

$$A_T = \sum_{t=1}^T D_t \prod_{s=t+1}^T (1 + R_s). \quad (39)$$

Substituting Equation (37) into Equation (39) yields

$$A_T = B_0 \sum_{t=1}^T \left[(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s) \right]. \quad (40)$$

5.2.3 Total terminal shareholder wealth

At the terminal date, total shareholder wealth consists of the terminal book equity accumulated within the firm and the accumulated value of all externally reinvested dividends. Total terminal shareholder wealth is therefore

$$W_T^{\text{gen}} = B_T + A_T. \quad (41)$$

Substituting Equations (34) and (40) into Equation (41) gives

$$W_T^{\text{gen}} = B_0 \prod_{s=1}^T (1 + vR_s) + B_0 \sum_{t=1}^T \left[(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s) \right]. \quad (42)$$

The first term in Equation (42) represents internal capital accumulation through retained earnings. The second term represents external capital accumulation through reinvested dividends.

At this stage, the retention ratio v appears explicitly throughout the terminal-wealth expression. Equation (42) therefore seems to imply that different dividend policies may generate different amounts of shareholder wealth. Section 5.3 examines this apparent dependence and applies the core algebraic identity established in Appendix A.3 to simplify the expression.

5.3 General Wealth Identity and Simplification

The terminal-wealth expression derived in Equation (42) contains two components: the book value accumulated through retained earnings and the terminal value of externally reinvested dividends. Although both components depend explicitly on the earnings-retention ratio v , Appendix A.3 establishes a generalized algebraic identity that combines them into a single product independent of v .

The core identity is

$$\prod_{s=1}^T (1 + R_s) = \prod_{s=1}^T (1 + vR_s) + \sum_{t=1}^T \left[(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s) \right]. \quad (43)$$

Equation (43) is the fundamental algebraic identity established in Appendix A.3. The complete proof is presented in the Appendix and is omitted here for brevity.

Substituting Equation (43) into Equation (42) immediately gives

$$W_T^{\text{gen}} = B_0 \prod_{s=1}^T (1 + R_s). \quad (44)$$

Equation (44) is the central result of the general dividend-policy case. Although shareholder wealth is divided between internally accumulated book equity and externally reinvested dividends, their combined terminal value is exactly equal to the terminal wealth generated under complete earnings retention.

Most importantly, the earnings-retention ratio v has completely disappeared from Equation (44). Thus,

$$W_T^{\text{gen}} = W_T^{\text{ret}} = W_T^{\text{pay}} = B_0 \prod_{s=1}^T (1 + R_s) \quad (45)$$

The equality in Equation (45) does not yet represent the infinite-horizon valuation result. It establishes only that, for any finite horizon T , all three earnings-allocation mechanisms generate identical terminal shareholder wealth. The next subsection discounts this common terminal wealth and applies the closed-form valuation procedure developed in Section 4.4.

5.4 Infinite-Horizon Closed-Form Valuation

Section 5.3 establishes that, under a general dividend policy, the total shareholder wealth accumulated through the combined effects of retained earnings and externally reinvested dividends is identical to the terminal wealth obtained under both complete earnings retention and complete cash-dividend distribution. Specifically,

$$W_T^{\text{gen}} = B_0 \prod_{s=1}^T (1 + R_s). \quad (46)$$

Within the capital-accumulation framework, the present value of the common terminal shareholder wealth is obtained by discounting W_T to time zero,

$$V_T^{\text{gen}} = \frac{W_T^{\text{gen}}}{(1 + r)^T}. \quad (47)$$

Substituting Equation (46) into Equation (47) yields

$$V_T^{\text{gen}} = B_0 \prod_{s=1}^T \frac{1 + R_s}{1 + r} = B_0 \prod_{s=1}^T X_s, \quad (48)$$

where X_s is the relative profitability factor defined in Equation (1).

Equation (48) is identical to the finite-horizon valuation equation derived previously under complete earnings retention in Equation (23). Consequently, no further mathematical derivation is required. Applying the common mean-reverting profitability process established in Section 2.6 and the infinite-product evaluation developed in Section 4.4 immediately gives

$$V^{\text{gen}} = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}} \quad (49)$$

Thus, the present value of terminal shareholder wealth under a general dividend policy is identical to the corresponding present-wealth expressions obtained under complete earnings retention and complete cash-dividend distribution.

Unlike the derivation presented in Section 4.4, the present section does not repeat the logarithmic transformation, geometric-series evaluation, or infinite-product derivation. Those mathematical developments are common to all three dividend policies and have already been established in Section 4.4. The only new result of the present chapter is the proof, provided in Section 5.3, that the terminal shareholder wealth under partial earnings retention is identical to that under the two extreme payout policies. Once this equivalence is established, the closed-form valuation equation follows directly from the common valuation framework.

5.5 Theorem 3. Closed-Form Valuation under a General Dividend Policy

The analytical development presented in Sections 5.2–5.4 establishes the intrinsic valuation equation under a general dividend policy in which the firm retains a fraction v of its earnings and distributes the remaining fraction $1-v$ as cash dividends. The principal result is summarized below.

Theorem 3

Under a common period-by-period return sequence for internal investment and external shareholder reinvestment, alternative earnings-retention ratios generate identical terminal shareholder wealth for any finite investment horizon. Accordingly, dividend policy affects the organizational path through which shareholder wealth accumulates but not the amount of terminal shareholder wealth generated under the common return dynamics assumed in the model.

Under the additional mean-reverting profitability specification and the terminal realization convention, the model-implied intrinsic value, defined as the limit of the discounted terminal shareholder wealth over finite investment horizons, converges to the following closed-form expression as $T \rightarrow \infty$.

$$V = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}} \quad (50)$$

Theorem 3 demonstrates that terminal shareholder wealth is independent of the earnings-retention ratio under the common period-by-period return sequence assumed in the model. Although shareholder wealth is accumulated jointly through internally retained earnings and externally reinvested dividends, the generalized wealth identity established in Section 5.3 shows that these two accumulation channels always combine to produce the same terminal shareholder wealth. Consequently, discounting this common terminal wealth yields exactly the same closed-form valuation equation obtained under both complete earnings retention and complete cash-dividend distribution.

Furthermore, Theorem 3 naturally includes the two extreme dividend policies as special cases. When $v=1$, the valuation equation reduces to the complete earnings-retention model developed in Section 4. When $v=0$, it reduces to the complete cash-dividend distribution model developed in Section 3. Therefore, the general dividend policy unifies the two extreme cases within a single theoretical framework. This relationship is established analytically in Appendix A.3.

Theorem 3 establishes the Dividend Policy Invariance Theorem, which constitutes the principal theoretical result of this study. The next section discusses its economic interpretation, scope, and relation to existing valuation theories.

6. Discussion

6.1 Position of the Proposed Valuation Framework

The unified valuation framework developed in this study represents a different theoretical perspective from existing approaches to equity valuation. Rather than beginning with dividend payments, accounting identities, or market equilibrium conditions, the proposed framework starts from the dynamic accumulation of productive capital governed by a common mean-reverting profitability process. Within this framework, alternative dividend policies are interpreted as different capital allocation mechanisms rather than distinct determinants of intrinsic value.

Traditional Dividend Discount Models (DDMs), pioneered by Williams (1938) and later simplified by Gordon (1959, 1962), value equity as the present value of expected future dividends. Dividends therefore constitute the primary valuation variable, and assumptions regarding payout policy and perpetual dividend growth are embedded directly in the valuation equation. Although these models are elegant and analytically tractable, they implicitly associate valuation with a particular dividend distribution mechanism.

Accounting-based valuation models, particularly the Residual Income Model (RIM) developed by Ohlson (1995) and Feltham and Ohlson (1995), shift the analytical emphasis from dividends to accounting information. Intrinsic value is expressed as current book value plus the discounted value of future residual income, thereby reducing the dependence on explicit dividend assumptions. Nevertheless, dividend policy remains embedded within the accounting framework rather than being analyzed explicitly as an independent theoretical variable.

More recent valuation research has incorporated profitability dynamics into the valuation process. Instead of assuming constant perpetual growth, these models recognize that return on equity exhibits persistence and gradual mean reversion (Fama & French, 2000; Nissim & Penman, 2001; Penman, 2013). Mean-reverting valuation models therefore derive firm value from the dynamic evolution of profitability rather than from fixed growth assumptions. However, existing formulations generally assume a given earnings-allocation mechanism throughout the derivation and do not investigate whether alternative dividend policies lead to different valuation equations.

The present study extends this line of research by introducing capital accumulation as the central organizing principle of equity valuation. Under complete earnings

retention, productive capital remains within the firm and accumulates through corporate investment. Under complete cash-dividend distribution, productive capital is transferred to shareholders and accumulates through external reinvestment. Under partial earnings retention, both accumulation channels coexist. Although these mechanisms differ operationally, they are shown to generate identical terminal shareholder wealth when governed by the same mean-reverting profitability process.

Accordingly, the proposed framework reformulates equity valuation as a theory of productive capital accumulation. Dividend policy determines where capital accumulates, whereas profitability determines how efficiently capital accumulates. The intrinsic value of equity is therefore determined by the dynamic productivity of invested capital rather than by the specific organizational channel through which capital is invested.

This perspective leads directly to the central theoretical result of this study. Because all three earnings-allocation mechanisms converge to the same closed-form valuation equation, dividend policy becomes an allocation mechanism rather than a valuation determinant. The resulting Dividend Policy Invariance Theorem provides a conceptual bridge among dividend discount models, accounting-based valuation models, and mean-reverting valuation theory by interpreting these approaches from a common capital-accumulation perspective.

Table 4 compares the analytical characteristics of the principal valuation frameworks. Unlike previous approaches, the present framework simultaneously incorporates mean-reverting profitability, alternative dividend policies, and a unified closed-form valuation equation. This comparison highlights the distinctive theoretical position of the proposed model within the equity valuation literature.

Table 5 provides a conceptual comparison of the dividend discount model, the residual income model, and the proposed framework. The three approaches differ in their primary valuation basis, their treatment of dividend policy, and the analytical mechanism through which intrinsic value is represented. The dividend discount model expresses value through expected future dividends, whereas the residual income model relies on current book value and expected residual income. The proposed framework instead emphasizes capital accumulation under mean-reverting profitability and explicitly analyzes dividend policy as an allocation mechanism. The comparison also clarifies that each framework is conditional on a different set of analytical assumptions rather than implying that one model is universally superior to the others.

As shown in Table 5, the principal distinction of the proposed framework lies in its explicit treatment of internal and external capital accumulation under alternative dividend policies. This distinction does not eliminate the assumptions required by valuation analysis. In particular, the dividend-policy invariance result depends on retained earnings and shareholder reinvestment being governed by the same mean-

reverting profitability process. The proposed framework should therefore be regarded as a complementary theoretical perspective rather than a universal replacement for the DDM or RIM.

Figure 6 compares the proposed framework with the principal valuation models discussed in the literature. Unlike previous approaches, the present study provides a common capital-accumulation perspective for examining alternative dividend policies and mean-reverting profitability. Under the additional mean-reverting specification, the common terminal shareholder wealth admits a closed-form present-value expression in the infinite-horizon limit.

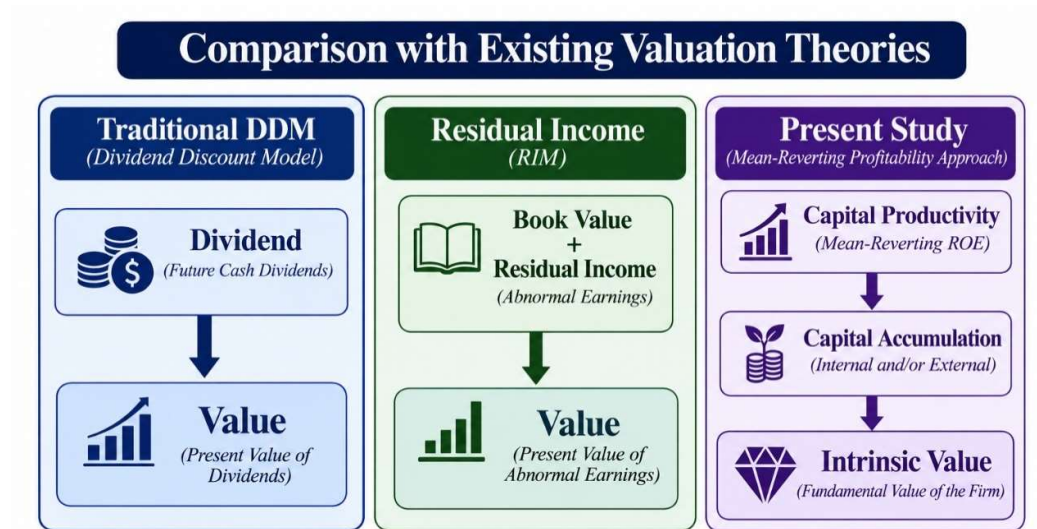


Figure 6. Position of the Proposed Framework among Major Equity Valuation Models

Table 4. Comparison of Major Equity Valuation Frameworks

Theory	Valuation Driver	Dividend Policy	Mean Reversion	Closed-form
DDM	Dividend	Fixed	X	✓
RIM	Residual Income	Implicit	Partial	✓
M&M	Market Equilibrium	Main Topic	X	X
Mean-Reverting Valuation	ROE	Fixed	✓	✓
Present Study	Mean-Reverting ROE	Alternative Policies	✓	✓

Table 5. Conceptual Comparison of Major Equity Valuation Frameworks

Concept	DDM	RIM	Proposed Framework
Primary valuation basis	Expected future dividends	Current book value and expected residual income	Capital accumulation under mean-reverting profitability
Treatment of dividend policy	Explicitly determines projected cash distributions	Embedded in the accounting relations	Explicitly analyzed as a capital-allocation mechanism
Core valuation perspective	Distribution of shareholder cash flows	Accounting value creation through residual income	Accumulation of productive capital
Central analytical focus	Expected dividend stream	Book value and abnormal earnings	Profitability dynamics and internal/external capital accumulation
Principal analytical condition	Requires assumptions about future dividend timing and growth	Requires clean-surplus accounting and forecasts of future residual income	Requires common profitability dynamics for retained earnings and reinvested dividends

Note: The proposed framework's dividend-policy invariance is conditional on retained earnings and shareholder reinvestment following the same mean-reverting profitability process.

6.2 Relation to Existing Dividend Policy Theories

Dividend policy has been one of the most extensively studied topics in corporate finance. Over the past several decades, numerous theories have been proposed to explain why firms distribute dividends and whether payout decisions influence firm value. Although these theories provide important economic insights into corporate payout behavior, they generally focus on the economic consequences of dividend policy rather than on the mathematical structure of equity valuation.

The classical Dividend Irrelevance Proposition developed by Miller and Modigliani (1961) argues that, under perfect capital markets, dividend policy has no effect on firm value because investors can replicate any desired cash-flow pattern through homemade dividends. Their conclusion is derived from an equilibrium argument based on arbitrage and market completeness. Consequently, the irrelevance

of dividend policy follows from assumptions regarding investor behavior and capital-market efficiency.

Subsequent theories explain why dividend policy may appear to influence firm value when these idealized assumptions are relaxed. Signaling theory argues that dividend changes convey managers' private information regarding future profitability (Bhattacharya, 1979; Miller & Rock, 1985). Agency theory suggests that dividend payments reduce managerial discretion over free cash flow and thereby mitigate agency conflicts between managers and shareholders (Jensen, 1986; Easterbrook, 1984). Life-cycle theory further proposes that payout policies evolve systematically as firms progress through different stages of corporate development (DeAngelo et al., 2006). These theories therefore interpret dividend policy primarily as an economic response to market frictions, information asymmetry, or organizational incentives.

The present study investigates whether alternative earnings-allocation mechanisms generate different terminal shareholder wealth under a common return structure. The analysis is therefore conducted within a valuation framework rather than an equilibrium framework.

A fundamental distinction between the present study and existing dividend policy theories lies in the source of dividend-policy neutrality. In the Miller–Modigliani proposition, neutrality results from investors' ability to replicate alternative payout policies through market transactions. In contrast, the neutrality established in this study arises from the mathematical equivalence of capital accumulation. When retained earnings and shareholder reinvestment are governed by the same mean-reverting profitability process, alternative dividend policies generate identical terminal shareholder wealth and therefore converge to the same closed-form valuation equation.

Accordingly, the proposed Dividend Policy Invariance Theorem provides a different analytical formulation of dividend-policy invariance within a capital-accumulation framework. Whereas Miller and Modigliani explain why dividend policy does not affect firm value from the perspective of market equilibrium, the present study explains why alternative dividend policies generate identical terminal shareholder wealth under a common period-by-period return sequence, with the same closed-form present-wealth expression obtained under the additional mean-reverting specification.

The two approaches should therefore be regarded as complementary rather than competing. The Miller–Modigliani proposition provides an equilibrium interpretation of dividend irrelevance in capital markets, whereas the present study establishes a valuation interpretation based on common profitability dynamics and capital accumulation. Together, they suggest that dividend policy neutrality can emerge from both market equilibrium and valuation theory, even though the two approaches employ different analytical mechanisms.

6.3 Economic Interpretation of the Dividend Policy Invariance

Theorem

The Dividend Policy Invariance Theorem established in this study concerns the equivalence of shareholder wealth accumulation under alternative dividend policies. Under the assumptions of the present model, including a common period-by-period return sequence for internal investment and external shareholder reinvestment, terminal shareholder wealth is invariant to the earnings-retention ratio for any finite investment horizon. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression as the horizon approaches infinity. Dividend policy affects only the path through which capital continues to accumulate, not the economic capacity of that capital to generate future wealth.

This interpretation shifts the focus of equity valuation from cash distribution to capital productivity. Traditional valuation models often regard dividends as the primary source of shareholder value because future dividends constitute the discounted cash flows received by investors. By contrast, the present framework shows that dividends merely transfer capital from the corporation to its shareholders. Whether capital remains inside the firm or is distributed and subsequently reinvested, terminal shareholder wealth depends on the common return sequence governing that capital rather than on its organizational location.

Accordingly, retained earnings and cash dividends should not be viewed as competing sources of value creation. Instead, they represent alternative capital accumulation mechanisms through which the same economic capital continues to earn future returns. Under the assumptions of the present model, both internal corporate investment and external shareholder reinvestment are governed by an identical mean-reverting profitability process. As a consequence, the long-run evolution of shareholder wealth remains unchanged despite differences in dividend policy.

Under the common return dynamics assumed in the model, dividend policy influences the allocation path of capital but not the resulting terminal shareholder wealth. This distinction separates the operational decision of where capital is invested from the economic question of how much value that capital is capable of generating. Consequently, the dividend policy invariance theorem does not imply that dividend policy is economically unimportant. Rather, it indicates that, under common profitability dynamics, alternative earnings-allocation mechanisms generate identical terminal shareholder wealth because internal investment and external shareholder reinvestment are assumed to follow the same period-by-period return sequence.

Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression as the investment horizon approaches infinity.

More broadly, the theorem suggests that the fundamental object of equity valuation is productive capital rather than dividends, retained earnings, or accounting book value individually. These variables are different accounting or financial representations of the same underlying economic process. Once the dynamic evolution of productive capital is specified, the valuation equation becomes independent of the particular accounting channel through which earnings are allocated. This perspective provides a unified interpretation linking equity valuation, dividend policy, and profitability dynamics within a single analytical framework.

Box 1. Economic Interpretation of the Dividend Policy Invariance Theorem

Why is terminal shareholder wealth invariant to dividend policy under the model's assumptions?

- Terminal shareholder wealth is determined by the common return dynamics governing invested capital rather than by the allocation of earnings between internal retention and external reinvestment.
- Retained earnings and cash dividends represent alternative capital accumulation mechanisms rather than alternative sources of value.
- Internal corporate investment and external shareholder reinvestment generate identical shareholder wealth when governed by the same mean-reverting profitability process.
- Dividend policy determines where capital is invested, whereas profitability determines how much value capital creates.
- Consequently, alternative dividend policies converge to the same closed-form valuation equation under the assumptions of the proposed model.

6.4 Scope and Assumptions of the Dividend Policy Invariance Theorem

Like any analytical theorem, the dividend policy invariance theorem established in this study is valid within the scope of its underlying assumptions. These assumptions define the conditions under which the mathematical equivalence among alternative dividend policies can be rigorously established. They should therefore be interpreted as the theoretical domain of the theorem rather than as empirical restrictions.

The most fundamental assumption is that productive capital generates returns according to a common mean-reverting profitability process regardless of whether capital remains within the firm or is transferred to shareholders through cash dividends. This common profitability dynamics provides the mathematical foundation linking the

three capital accumulation mechanisms. If internal corporate investment and external shareholder reinvestment are governed by substantially different profitability processes, the analytical equivalence established in this study may no longer hold.

More precisely, a sufficient condition for the finite-horizon wealth-invariance result established in Sections 3–5 is that internal investment and external shareholder reinvestment be governed by the same period-by-period return sequence. This wealth-equivalence result does not itself require the specific mean-reverting functional form. Mean reversion is an additional structural assumption that permits the common return sequence to be evaluated analytically in the infinite-horizon limit and thereby yields the particular closed-form valuation equation derived in this study. Thus, common period-by-period returns provide the basis for dividend-policy invariance, whereas the mean-reverting specification determines the particular closed-form solution. The common return sequence is treated as exogenous to the earnings-retention ratio; thus, the present theorem does not address settings in which payout policy itself alters investment productivity.

A second assumption concerns the required rate of return. For analytical tractability, the model assumes a constant required return throughout the investment horizon. Although this assumption facilitates the derivation of a closed-form valuation equation, future studies may extend the framework by incorporating stochastic discount rates, time-varying risk premia, or changing macroeconomic conditions.

The present framework also abstracts from institutional frictions such as taxation, transaction costs, financing constraints, agency conflicts, and asymmetric information. These factors undoubtedly influence firms' actual dividend policies and investors' financing decisions. However, they are intentionally excluded because the objective of the present study is to establish the mathematical properties of the valuation equation rather than to explain observed payout behavior.

The valuation object considered in the present framework should be distinguished from the conventional ex-dividend value of the original firm. Under the complete- and partial-payout cases, distributed dividends are assumed to be reinvested externally and their subsequent accumulation forms part of terminal shareholder wealth. The resulting present-value measure therefore represents the discounted value of terminal shareholder wealth under the model's common reinvestment opportunity, rather than a conventional dividend-discount valuation that attributes only the firm's own future distributions and terminal value to the original equity claim. The Dividend Policy Invariance Theorem should accordingly be interpreted as an invariance result for shareholder wealth accumulation under alternative payout policies, conditional on the common return dynamics specified in the model.

Accordingly, the theorem should be interpreted as a fundamental analytical result under a clearly specified theoretical framework. Similar to many classical results in

finance and economics, its significance lies in identifying the essential mechanism governing valuation before additional institutional complexities are introduced. Extensions incorporating market frictions and empirical settings therefore represent natural directions for future theoretical development rather than limitations of the theorem itself.

Box 2. Scope of the Dividend Policy Invariance Theorem**The theorem holds under the following assumptions:**

- Internal investment and external shareholder reinvestment are governed by the same period-by-period return sequence.
- The common return sequence is treated as exogenous to the earnings-retention ratio.
- Distributed dividends are fully reinvested during the modeled accumulation horizon.
- Under complete payout, the initial book-capital base is treated as maintained rather than distributed. Terminal book equity, including the maintained initial book-capital base under complete payout, is assumed to be fully realizable by shareholders at its book amount at each finite valuation horizon.
- The required rate of return remains constant over the valuation horizon.
- The framework abstracts from taxes, transaction costs, financing constraints, agency conflicts, and asymmetric information.

The theorem does not claim that:

- All firms should adopt identical dividend policies.
- Market prices are identical under all institutional environments.
- Dividend policy has no practical implications for corporate finance.

6.5 Future Research

The dividend policy invariance theorem established in this study opens several promising directions for future theoretical research.

A natural extension is to relax some of the simplifying assumptions adopted in the present framework. Future studies may investigate valuation models with stochastic discount rates, time-varying risk premia, or alternative profitability dynamics. Such extensions would improve the realism of the model while preserving its analytical foundation.

A second direction is to examine whether the proposed theorem remains valid under more complex institutional environments. Incorporating taxation, transaction costs, financing constraints, agency conflicts, or asymmetric information may help identify the conditions under which dividend policy invariance continues to hold or requires modification.

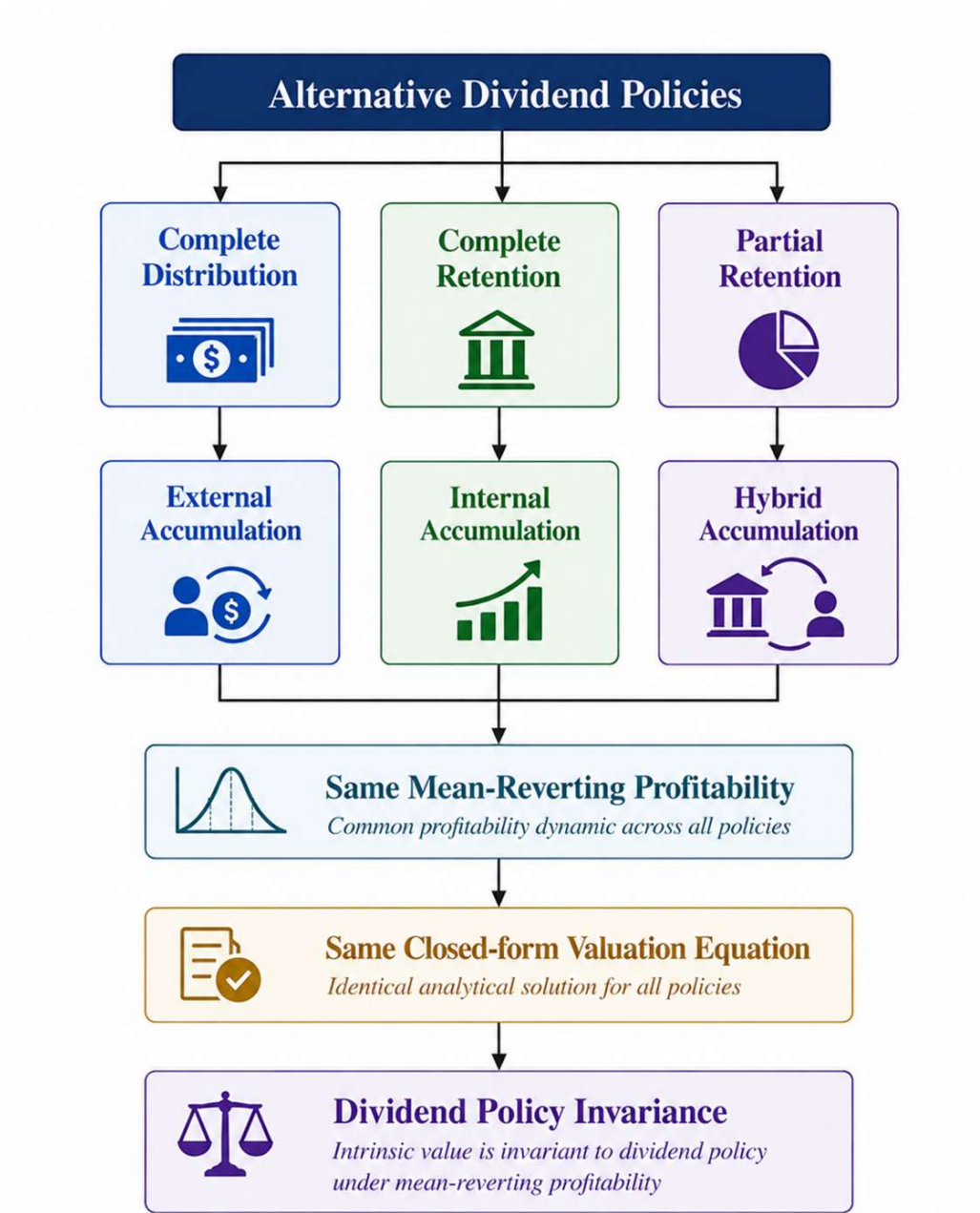


Figure 7. Dividend Policy Invariance Framework

Beyond dividend policy itself, the capital accumulation framework developed in this study may be generalized to other corporate financial decisions. Share repurchases, seasoned equity offerings, debt financing, and alternative capital allocation policies may all be analyzed within the same dynamic profitability framework. Such extensions could lead to a broader analytical theory of capital allocation and firm valuation.

Although the present study is purely theoretical, the proposed theorem also provides a foundation for future empirical research. The analytical predictions developed here may be evaluated using accounting and market data from different

countries, industries, and institutional settings to assess the empirical applicability of the dividend policy invariance framework.

More fundamentally, the present theorem suggests that dividend policy invariance may represent only one special case of a broader capital accumulation invariance principle, under which alternative capital allocation mechanisms produce identical intrinsic valuation whenever productive capital follows the same underlying economic dynamics. Exploring this broader theoretical possibility may provide a promising direction for future research in equity valuation and corporate finance.

The theoretical result established in this chapter is summarized in Figure 7. Although the three dividend policies involve different capital accumulation mechanisms, they converge to the same closed-form valuation equation under a common mean-reverting profitability process, thereby establishing the Dividend Policy Invariance Theorem.

7. Conclusion

7.1 Summary of the Study

This study develops a common capital-accumulation framework for examining shareholder wealth under alternative dividend policies. Unlike traditional valuation models, which typically assume a particular earnings-allocation mechanism, the proposed framework explicitly considers three alternative policy cases: complete cash-dividend distribution, complete earnings retention, and partial earnings retention.

For each case, a separate mathematical derivation is presented under a common mean-reverting profitability process. Rather than assuming the equivalence of different dividend policies, the analysis establishes their equivalence analytically. The derivations demonstrate that all three capital accumulation mechanisms converge to the same closed-form valuation equation, despite their different operational structures.

Accordingly, the study establishes a Dividend Policy Invariance Theorem, showing that terminal shareholder wealth is independent of the earnings-retention ratio under the stated assumptions. The theorem provides a unified analytical framework linking equity valuation, dividend policy, and profitability dynamics.

7.2 Principal Findings

The present study produces three principal findings.

First, three fundamentally different earnings-allocation mechanisms—complete cash-dividend distribution, complete earnings retention, and partial earnings retention—can each be analyzed within a common mean-reverting profitability

framework. Separate policy-specific derivations under a common return structure demonstrate that these mechanisms generate identical terminal shareholder wealth.

Second, terminal shareholder wealth is invariant to the earnings-retention ratio under the common period-by-period return sequence assumed in the model. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression as the investment horizon approaches infinity.

Third, the analysis demonstrates that, within the capital-accumulation framework, terminal shareholder wealth is governed by the return dynamics of invested capital rather than by the accounting allocation of earnings between retained earnings and cash dividends. Consequently, dividend policy determines the channel of capital accumulation rather than the source of value creation.

7.3 Theoretical Contributions

The present study contributes to the literature in several important respects.

First, it extends equity valuation theory by introducing a unified analytical framework that explicitly incorporates alternative dividend policies into a single valuation model. Existing valuation models generally assume a particular payout mechanism, whereas the present study derives the valuation equation independently under three different earnings-allocation mechanisms.

Second, the study establishes a new Dividend Policy Invariance Theorem. Unlike existing dividend policy theories that primarily examine whether payout decisions influence firm value, the proposed theorem demonstrates mathematically that alternative dividend policies converge to an identical closed-form valuation equation under a common mean-reverting profitability process.

Third, the study provides a new interpretation of equity valuation by emphasizing the role of productive capital. Within the proposed framework, dividends, retained earnings, and shareholder reinvestment are viewed as alternative channels through which productive capital continues to generate future returns. This perspective unifies equity valuation, dividend policy, and profitability dynamics within a single analytical system.

7.4 Concluding Remarks

The analytical results presented in this study suggest that the fundamental object of equity valuation is not the accounting allocation of earnings but the dynamic productivity of invested capital. Whether earnings remain within the firm or are distributed and externally reinvested by shareholders, terminal shareholder wealth

depends on the common return sequence governing invested capital rather than on the organizational channel through which capital is held.

The dividend policy invariance theorem established in this paper therefore provides a new theoretical perspective on equity valuation. Rather than treating dividend policy as an independent determinant of value, the theorem identifies it as an alternative capital accumulation mechanism operating within a common profitability dynamics. Under the assumptions of the proposed framework, alternative dividend policies generate identical terminal shareholder wealth because internal investment and external shareholder reinvestment are assumed to follow the same period-by-period return sequence. Under the additional mean-reverting profitability specification, the present value of this common terminal wealth converges to the same closed-form expression as the investment horizon approaches infinity.

More broadly, the present study suggests that dividend policy invariance may represent one example of a more general principle governing capital accumulation and firm valuation. Exploring whether similar invariance properties exist for other corporate financial decisions may provide a fruitful direction for future theoretical research. The unified framework developed in this study therefore offers not only a new analytical theorem for dividend policy but also a foundation for further advances in dynamic equity valuation theory.

Finally, it is important to distinguish the proposed Dividend Policy Invariance Theorem from the classical dividend irrelevance proposition of Miller and Modigliani (1961). The latter is an equilibrium theory explaining why payout policy does not affect market value under perfect capital markets. The theorem developed in this study addresses a different question. It demonstrates that alternative dividend policies generate identical terminal shareholder wealth under a common period-by-period return sequence and, under the additional mean-reverting profitability specification, the same closed-form present-wealth expression. The resulting invariance therefore originates from the mathematics of capital accumulation rather than from arbitrage, market equilibrium, or investors' homemade dividends.

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Notation

Symbol	Definition
B_t	Book value of shareholders' equity at the end of period t .
B_0	Initial book value of shareholders' equity.
R_t	Return on equity (ROE) during period t .
R_1	Initial return on equity at the valuation date.
r	Constant required rate of return (discount rate).
a	Mean-reversion parameter governing the evolution of ROE.
T	Investment horizon (number of periods).
V	Infinite-horizon present value of terminal shareholder wealth under the capital-accumulation framework.
V_T	Present value obtained by discounting terminal shareholder wealth at time T .
W_T	Total shareholder wealth accumulated to time T .
D_t	Cash dividend distributed at the end of period t .
v	Earnings-retention ratio, $0 \leq v \leq 1$.
$1 - v$	Dividend payout ratio.
X_t	Relative profitability factor, $X_t = \frac{1 + R_t}{1 + r}$.

Appendix A: Proof of Valuation Equivalence under Alternative Dividend Policies

Preliminaries and Assumptions

This appendix provides a rigorous proof that terminal shareholder wealth is identical under three alternative dividend policies: full payout, full retention, and partial payout.

Let $R_t = ROE_t$ denote the return on equity in period t , and let r be the constant discount rate.

Define the gross growth factor of the equity value as

$$X_t \equiv \frac{1 + R_t}{1 + r}, \quad (A1)$$

and let $x_t = \ln X_t$. The assumed law of motion governing the evolution of the ROE is given by

$$X_{t+1} = X_t^a, \quad 0 < a < 1, \quad (A2)$$

which implies

$$x_{t+1} = ax_t, \quad \text{and thus} \quad x_t = a^{t-1}x_1. \quad (A3)$$

A.1. Policy I: Full Payout with Shareholder Reinvestment

Under this alternative policy, the firm distributes all earnings as cash dividends each period.

Consequently, the firm's book value remains constant at B_0 . The dividend paid at the end of period t is

$$D_t = R_t B_0.$$

Shareholders receive this dividend and immediately reinvest it. Crucially, the dividend received at time t starts earning the *next* period's ROE, i.e., R_{t+1} . Thus, the terminal value at time T of the dividend received in period t is

$$R_t B_0 \prod_{s=t+1}^T (1 + R_s).$$

The total wealth of the shareholders at time T consists of the maintained initial book-capital base plus the accumulated reinvestment account.

$$W_T^{pay} = B_0 + B_0 \sum_{t=1}^T R_t \prod_{s=t+1}^T (1 + R_s). \quad (A4)$$

To simplify this expression, we utilize the following algebraic identity:

$$1 + \sum_{t=1}^T R_t \prod_{s=t+1}^T (1 + R_s) = \prod_{s=1}^T (1 + R_s). \quad (A5)$$

Substituting (A5) into (A4) yields

$$W_T^{pay} = B_0 \prod_{s=1}^T (1 + R_s).$$

Discounting this terminal wealth back to the present gives

$$V_T^{pay} = \frac{W_T^{pay}}{(1+r)^T} = B_0 \prod_{s=1}^T \frac{1+R_s}{1+r} = B_0 \prod_{s=1}^T X_s.$$

Taking the limit as $T \rightarrow \infty$, we obtain

$$V_T^{pay} = B_0 \lim_{T \rightarrow \infty} \prod_{s=1}^T X_s = B_0 \exp \left(\sum_{s=1}^{\infty} x_s \right).$$

Using (A3), the infinite sum is a geometric series:

$$\sum_{s=1}^{\infty} x_s = \sum_{s=1}^{\infty} a^{s-1} x_1 = \frac{x_1}{1-a}.$$

Therefore,

$$V^{pay} = B_0 \left(\frac{1+R_1}{1+r} \right)^{\frac{1}{1-a}} \quad (\text{A6})$$

Proof of the Key Identity via Mathematical Induction

We now prove the identity stated in (A5):

$$1 + \sum_{t=1}^T R_t \prod_{s=t+1}^T (1 + R_s) = \prod_{s=1}^T (1 + R_s).$$

We proceed by induction on T .

Base Case ($T = 1$):

The left-hand side (LHS) is

$$1 + R_1 \prod_{s=2}^1 (1 + R_s).$$

By the convention of an empty product (which is equal to 1), this becomes

$$1 + R_1 = 1 + R_1.$$

The right-hand side (RHS) is also $1 + R_1$. Thus, the base case holds.

Inductive Step:

Assume the identity holds for $T = k$. That is,

$$1 + \sum_{t=1}^k R_t \prod_{s=t+1}^k (1 + R_s) = \prod_{s=1}^k (1 + R_s). \quad (\text{A7})$$

We must show it holds for $T = k + 1$. Consider the RHS for $T = k + 1$:

$$\prod_{s=1}^{k+1} (1 + R_s) = \left[\prod_{s=1}^k (1 + R_s) \right] \cdot (1 + R_{k+1}).$$

Substituting the induction hypothesis (A7) into the bracket, we get

$$= \left[1 + \sum_{t=1}^k R_t \prod_{s=t+1}^k (1 + R_s) \right] \cdot (1 + R_{k+1}).$$

Expanding the product:

$$= (1 + R_{k+1}) + \sum_{t=1}^k R_t \prod_{s=t+1}^k (1 + R_s) \cdot (1 + R_{k+1}).$$

Notice that $\prod_{s=t+1}^k (1 + R_s) \cdot (1 + R_{k+1}) = \prod_{s=t+1}^{k+1} (1 + R_s)$. Thus, the equation becomes

$$= 1 + R_{k+1} + \sum_{t=1}^k R_t \prod_{s=t+1}^{k+1} (1 + R_s). \quad (\text{A8})$$

Now, observe that the term R_{k+1} can be absorbed into the summation. To do so, we extend the summation index from $t = k$ to $t = k + 1$. Consider the term corresponding to $t = k + 1$:

$$R_{k+1} \prod_{s=(k+1)+1}^{k+1} (1 + R_s) = R_{k+1} \prod_{s=k+2}^{k+1} (1 + R_s). \quad (\text{A9})$$

Since the lower limit $k + 2$ exceeds the upper limit $k + 1$, this product is empty and equals 1 by definition. Hence, this term is simply $R_{k+1} \cdot 1 = R_{k+1}$.

Therefore, (A8) can be rewritten as

$$= 1 + \sum_{t=1}^{k+1} R_t \prod_{s=t+1}^{k+1} (1 + R_s),$$

which is precisely the LHS of (A5) for $T=k+1$. This completes the induction, and the identity is proven.

Clarification on the Empty Product Convention in the Inductive Step

To avoid any potential confusion regarding the absorption of R_{k+1} into the summation in the final step of the induction, we provide an explicit clarification.

In standard mathematical notation, a product of the form $\prod_{s=m}^n f(s)$ is defined to equal 1 when $m > n$. This is analogous to the empty sum being equal to 0. In our specific context,

$$\prod_{s=k+2}^{k+1} (1 + R_s) = 1. \quad (\text{A10})$$

Consequently, when we append the term $t = k + 1$ to the summation, the associated product over an empty set vanishes, leaving only the coefficient R_{k+1} . This justifies the rewriting of

$$1 + R_{k+1} + \sum_{t=1}^k (\dots)$$

as

$$1 + \sum_{t=1}^{k+1} R_t \prod_{s=t+1}^{k+1} (1 + R_s). \quad (\text{A11})$$

Without this convention, the last term of the summation for $t = k + 1$ would be undefined. With this convention, the algebra holds perfectly, ensuring the rigorousness of the proof.

A.2. Policy II: Full Retention of Earnings (No Dividends)

Under this policy, the firm reinvests all earnings internally. The book equity at the end of period T is given by

$$B_T = B_0 \prod_{s=1}^T (1 + R_s). \quad (\text{A12})$$

The present value of terminal shareholder wealth at time zero is

$$V_T^{ret} = \frac{B_T}{(1 + r)^T} = B_0 \prod_{s=1}^T \frac{1 + R_s}{1 + r} = B_0 \prod_{s=1}^T X_s. \quad (\text{A13})$$

Taking the limit as $T \rightarrow \infty$ and applying the same derivation as in (A6), we obtain

$$V^{ret} = B_0 \exp \left(\frac{x_1}{1 - a} \right) = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}} \quad (\text{A14})$$

This is the valuation under the full retention policy. Equations (A14) and (A6) are identical. Hence, the two dividend policies yield the same valuation.

A.3. Policy III: Partial Payout

In this appendix, we extend the analysis to a general dividend policy. Let v be the earnings retention ratio ($0 \leq v \leq 1$), such that the firm retains a fraction v of its earnings and distributes the remaining fraction $1-v$ as cash dividends. We demonstrate that the valuation under this general policy is identical to the two extreme cases (full retention and full payout), thereby confirming the irrelevance of the dividend policy under the assumed ROE dynamics.

Step 1: Dynamics of the Firm's Book Value

Since the firm retains only the fraction v of its earnings each period, the book value of the firm grows at a slower rate. The evolution of the book value is given by

$$B_t = B_{t-1} + vR_t B_{t-1} = B_{t-1}(1 + vR_t).$$

Iterating this forward from period 1 to period T , the terminal book value at time T is

$$B_T = B_0 \prod_{s=1}^T (1 + vR_s). \quad (\text{A15})$$

Step 2: The Shareholder's Reinvestment Account (Dividends)

The dividend paid out at the end of period t is the fraction $1 - v$ of the earnings generated during that period. Since the book value at the beginning of period t is B_{t-1} , the dividend is

$$D_t = (1 - v)R_t B_{t-1}.$$

Substituting the expression for B_{t-1} from (A15) yields

$$D_t = B_0(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s). \quad (\text{A16})$$

Shareholders receive this dividend at time t and immediately reinvest it. This reinvestment earns the subsequent returns $R_{t+1}, R_{t+2}, \dots, R_T$. Therefore, the terminal value at time T of the dividend paid in period t is

$$D_t \prod_{s=t+1}^T (1 + R_s).$$

Step 3: Total Shareholder Wealth at Time T

At the terminal date T , the shareholder's total wealth consists of two components: (i) the liquidated book value of the firm (the retained portion), and (ii) the accumulated value of all reinvested dividends (the paid-out portion). Summing these components gives

$$W_T^{gen} = B_0 \prod_{s=1}^T (1 + vR_s) + B_0 \sum_{t=1}^T \left[(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s) \right]. \quad (\text{A17})$$

Step 4: The Core Algebraic Identity

To simplify (A17), we prove the following generalized identity:

$$\prod_{s=1}^T (1 + R_s) = \prod_{s=1}^T (1 + vR_s) + \sum_{t=1}^T \left[(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s) \right]. \quad (\text{A18})$$

Proof of Identity (A18):

Observe that for every period s , we can decompose the gross return as

$$(1 + R_s) = (1 + vR_s) + (1 - v)R_s. \quad (\text{A19})$$

Thus, the left-hand side of (A18) can be written as

$$\prod_{s=1}^T (1 + R_s) = \prod_{s=1}^T [(1 + vR_s) + (1 - v)R_s].$$

Now, imagine expanding the product on the right-hand side. For each factor (each period s), we have two choices:

1. Choose the first term, $(1 + vR_s)$ (meaning the earnings are retained for this period);
2. Choose the second term, $(1 - v)R_s$ (meaning the earnings are paid out for this period).

We classify the expanded terms according to the **first period** in which we choose the second term.

- **Case 1:** If we choose the first term $(1 + vR_s)$ in **every** period $s = 1, \dots, T$, the product is exactly

$$\prod_{s=1}^T (1 + vR_s),$$

which is the first term on the right-hand side of (A18).

- **Case 2:** Suppose the **first** period in which we choose the second term $(1 - v)R_t$ is period t . Then:
 - For periods 1 to $t - 1$, we must have chosen the first term. Their product is

$$\prod_{s=1}^{t-1} (1 + vR_s).$$

- For period t , we chose the second term, which is $(1 - v)R_t$.
- For periods $t + 1$ to T , we are free to choose either the first or the second term. Summing over all possible choices for these remaining periods yields

$$\prod_{s=t+1}^T [(1 + vR_s) + (1 - v)R_s] = \prod_{s=t+1}^T (1 + R_s).$$

Multiplying these three parts together gives the general term for a fixed t :

$$(1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s).$$

Summing this expression over all possible first-payout periods $t = 1, 2, \dots, T$ covers all cases in which at least one payout occurs. Combining Case 1 and Case 2 proves identity (A18).

Step 5: Simplification of Total Wealth

Substituting the identity (A18) into the bracketed portion of the total wealth expression (A17), we obtain

$$W_T^{gen} = B_0 \left[\prod_{s=1}^T (1 + vR_s) + \sum_{t=1}^T (1 - v)R_t \prod_{s=1}^{t-1} (1 + vR_s) \prod_{s=t+1}^T (1 + R_s) \right].$$

By (A18), the entire bracket simplifies to

$$W_T^{gen} = B_0 \prod_{s=1}^T (1 + R_s). \quad (A20)$$

Notice that the retention ratio v has completely vanished from (A20).

Step 6: Discounting and Taking the Limit

Discounting the terminal wealth (A20) back to the present value yields

$$V_T^{gen} = \frac{W_T^{gen}}{(1 + r)^T} = B_0 \prod_{s=1}^T \frac{1 + R_s}{1 + r} = B_0 \prod_{s=1}^T X_s.$$

Taking the limit as $T \rightarrow \infty$, and following the same geometric series argument used in (A3), we arrive at

$$V^{gen} = B_0 \left(\frac{1 + R_1}{1 + r} \right)^{\frac{1}{1-a}}. \quad (A21)$$

Step 7: Verification of Special Cases

- If $v=1$ (full retention), equation (A15) reduces to

$$B_T = B_0 \prod_{s=1}^T (1 + R_s)$$

and dividends are zero. This matches the result in Appendix A.2.

- If $v=0$ (full payout), Equation (A15) gives $B_T=B_0$, and the proof reduces to the full-payout case in Appendix A.1.

Since (A21) is identical to (A6) and (A14), we conclude that the present value of terminal shareholder wealth is independent of the earnings-retention ratio under the assumed ROE dynamics. The general policy is thus rigorously proven to be equivalent to the two extreme cases.