

# Price Discovery in the U.S. Equity Market at Tick-Time Precision

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## Abstract

The accelerated pace of trading in U.S. equity markets, combined with the proliferation of trading venues in recent years, call for price discovery models capable of operating at ultra-high frequencies and across multiple trading venues. We examine price discovery for the 30 stocks in the Dow Jones Industrial Average using an unobserved-components model estimated in tick time. The quote dynamics model directly represents quote updates across trading venues and thereby accommodates irregular sampling and microstructure noise while avoiding the order dependence of conventional information share measures. We find that, on average, the fastest and most liquid venues make the largest contributions to price discovery, with the NYSE holding a modest advantage over competing venues.

**Keywords:** C32, C58, G14

**JEL Classification:** High-frequency, Kalman filter, market microstructure, panel regression, unobserved components

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# 1 Introduction

Financial markets have undergone significant structural changes over the past two decades, driven by technological advances, regulatory reforms, globalization, and changes in market participants' behavior. Whereas traders could previously rely on a single dominant exchange, the emergence of numerous exchanges and alternative trading systems has resulted in increasingly fragmented markets. In the United States, Regulation ATS (alternative trading systems; Reg ATS, 2000) and Regulation National Market System (Reg NMS, 2007), together with the Markets in Financial Instruments Directive (MiFID, 2007) in Europe, established the regulatory framework for multiple trading venues to operate, interact, and compete for liquidity and order flow (Chung and Chuwonganant, 2012; Grammig and Peter, 2018; Dias et al., 2021; Chen and Duffie, 2021; Hagströmer, 2022).

Market fragmentation produces multiple price series for the same asset across trading venues. Despite the law-of-one-price principle stating that prices for the same asset should be consistent and tied to the asset's latent efficient price, deviations between prices on different trading venues are prevalent at high frequencies. These deviations can arise from differences in market microstructure, including the degree of transparency, trading clientele, fee structure, and market design of each venue (O'Hara, 2015; Menkveld, 2016). In the U.S. equity market, quote information from different venues is consolidated in the National Best Bid and Offer (NBBO), allowing quote updates across markets to influence the prevailing reference price of a security. In addition, Reg NMS requires trading venues to protect certain displayed orders against trade-throughs, thereby facilitating access to the best available price.<sup>1</sup> These provisions are intended to promote competition among exchanges. Grammig and Peter (2018) find that traditional exchanges such as the NYSE lost substantial trading volume to newer exchanges and alternative trading systems following the introduction of Reg ATS and Reg NMS.

The extensive empirical literature on price discovery (see, for example, Narayan and Smyth, 2015; Ozturk et al., 2017; Hagströmer and Menkveld, 2019; Hasbrouck, 2021; Dimpfl and Schweikert, 2023) reflects continued interest in understanding how fragmented markets process new information and in identifying which trading venues contribute most to price discovery. This question is relevant for investors seeking to execute trades at favorable prices and for market operators competing to attract order flow. In particular, it is important to determine whether the U.S. equity market under Reg NMS can be viewed as a single virtual market with multiple points of entry (O'Hara and Ye, 2011; Gomber et al., 2017). This interpretation would imply that several exchanges contribute substantially to price discovery rather than a single venue – such as the primary listing exchange – consistently dominating

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<sup>1</sup>A trade-through occurs when an order is executed at a price worse than the best protected price available in another market.

the process.

This question is closely related to the broader market-microstructure literature, which emphasizes that venue-level price formation reflects not only the arrival of fundamental information but also strategic trading, execution design, and venue-specific frictions. For example, [Ellul et al. \(2007\)](#) show that order dynamics on the NYSE contain information about price formation, while [Chakravarty et al. \(2012\)](#) document how informed traders can exploit inter-market routing technologies such as sweep orders. These findings suggest that differences in venue-level price discovery may reflect the interaction of information arrival with order submission, routing, and execution decisions. A better understanding of these price-formation mechanisms is essential for exchanges and trading platforms to offer competitive pricing and attract trading volume.

While several measures of a market’s contribution to price discovery have been proposed (see, for example, [Hasbrouck, 1995](#); [Lien and Shrestha, 2009](#); [de Jong and Schotman, 2010](#); [Grammig and Peter, 2013](#)), the Hasbrouck information share remains the most widely used. The information share measures the contribution of each venue to the variance of innovations in the common efficient price and is typically estimated using a reduced-form vector error correction model (VECM). Although the VECM is intuitive and well understood, it has several important limitations in the context of high-frequency, fragmented markets.

First, VECMs were originally developed to describe systems of relatively modest dimension and lag order. Consequently, they can be difficult to estimate for high-frequency financial data, which may require many lags ([Hasbrouck, 2021](#)), or for systems comprising a large number of markets. Second, estimation over short, fixed calendar intervals requires prices from different markets to be observed at a common sampling frequency. However, trading venues operate at different speeds, price changes and quote updates occur asynchronously. The literature therefore typically recommends either synchronizing prices on a refresh-time grid ([Harris et al., 1995](#); [Barndorff-Nielsen et al., 2011](#)) or sampling at the frequency of the slowest market to avoid long sequences of zero price changes ([Harris et al., 2002](#); [de Jong and Schotman, 2010](#)). In turn, longer sampling intervals generally increase the contemporaneous correlation between markets and, consequently, widen the bounds of the information shares. Moreover, Hasbrouck information shares computed from a VECM are not invariant to the ordering of the variables. The standard approach to reducing contemporaneous residual correlation and narrowing the information-share bounds is to use a higher sampling frequency. This, however, complicates the specification of the dynamic model and the construction of intervals containing observations from all markets.<sup>2</sup> Third, applying reduced-form models to ultra-high-frequency data may lead to estimates of the relevant (co)variances being dominated by market microstructure noise ([Nguenang, 2016](#); [Dias et al., 2022](#)). This is particularly

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<sup>2</sup>Several order-independent price-discovery measures based on VECMs have been proposed ([Lien and Shrestha, 2009](#); [Fernandes and Scherrer, 2018](#)), but the Hasbrouck approach remains the most widely used.

important because the innovation covariance matrix is a central component of the information-share calculation. At subsecond frequencies, covariance estimates may be affected by the Epps effect (Epps, 1979), whereby estimated return correlations tend to zero as the sampling frequency increases. As recognized by Buccheri et al. (2021), the resulting narrowing of the information-share bounds may therefore be driven partly by the Epps effect and may mask the underlying correlation between innovation terms.

Although reduced-form models dominate the price-discovery literature, several studies employ structural models. de Jong and Schotman (2010) develop a unobserved-components model in calendar time for fragmented markets and show that its information shares coincide with the Hasbrouck information shares when the error terms are uncorrelated. Their approach, however, relies on regular sampling intervals and therefore requires a careful choice of sampling frequency. De Jong (2021) show that information shares from unobserved-components models are invariant to time aggregation when noninformational pricing errors are uncorrelated. Ozturk et al. (2017) apply a calendar-time unobserved-components model to 50 S&P 500 stocks and incorporate intraday seasonal components, allowing information shares to vary over the trading day.

In this paper, we study price discovery in U.S. equity markets using a structural model that accommodates irregular sampling and market microstructure noise at ultra-high frequencies. Analyzing data in tick time provides a detailed representation of quote dynamics and avoids imposing a common calendar-time sampling frequency across venues. Following Frijns and Schotman (2009), we employ an unobserved-components model, which we refer to as the *quote dynamics model*, and estimate its parameters by quasi-maximum likelihood using the Kalman filter. The model directly represents quote updates at each trading venue. Consequently, stale quotes at a prespecified calendar-time frequency do not enter the model as repeated observations; instead, the timing of quote updates is captured through the measured duration between successive updates. This reduces the relevance of staleness and limits the computational burden associated with high-frequency sampling. In addition, the model’s parameters are time-invariant up to a scale factor, allowing it to be estimated separately for individual trading days.

The resulting information shares may nevertheless reflect mechanical features of the quoting process, including price discreteness, quote clustering, and venue-specific liquidity conditions. As emphasized by Hagströmer (2021) in the context of liquidity measures, such effects can vary across stocks, venues, and trader types. We therefore interpret the estimated information shares as venue-level contributions to the observed price-discovery process rather than as pure measures of informational advantage. We further examine the determinants of daily variation in these contributions using observed covariates, including measures of trading activity and liquidity.

We apply the quote dynamics model to tick data for all 30 stocks in the Dow Jones

Industrial Average over the period from March 1 to May 30, 2024. We first estimate daily information shares for the five major exchanges. We find that the primary listing exchange remains the informationally dominant venue, with an average information share of 27% for stocks primarily listed on the NYSE and 19% for stocks primarily listed on Nasdaq. This result is robust to alternative choices regarding timestamp precision. We then conduct a panel analysis of the determinants of daily variation in information shares. The results indicate that faster and more liquid venues make larger contributions to price discovery. After controlling for the covariates, the NYSE retains a modest advantage over competing venues, which can be attributed in part to its status as the primary listing exchange for 23 of the 30 stocks in our sample.

The remainder of the paper is organized as follows: Section 2 presents the quote dynamics model and the relevant price-discovery measures. Section 3 reports a simulation study evaluating the performance of the commonly used reduced-form model when prices are generated by the quote dynamics model. Section 4 presents the empirical analysis of price discovery for DJIA stocks and examines its daily variation using observable covariates. Section 5 concludes.

## 2 Methodology

### 2.1 The quote dynamics model

In the following, we outline the quote dynamics model that is used to measure the contribution to price discovery in fragmented markets. We define  $q_j$  as the  $(2M \times 1)$  vector of  $M$  bid and ask quote pairs for each market at time  $t_j$ , where the bid (ask) of market  $i$  corresponds to element  $2i - 1(2i)$  of  $q_j$ . The model structure is based on the simple but flexible [Glosten and Harris \(1988\)](#) asymmetric information model which decomposes the observed quotes into permanent and transitory components. Particularly, it allows us to summarize the market microstructure by estimating market-specific bid/ask asymmetry parameters. Analyzing the information transmission between fragmented markets, we do not distinguish between different dealers on each market or order-book updates. Instead, we only consider the best bid or best ask at a given tick. This is in contrast to [Frijns and Schotman \(2009\)](#) who specifically model the quote setting behaviour of dealers in the Nasdaq system. However, we are primarily interested in the different points of entry to the U.S. equity market which comprise dealer markets and limit order markets. To make the analysis feasible, we simply register the latest quote update posted in a given market. The duration between two consecutive quote arrivals is measured by  $\tau_j = t_j - t_{j-1}$ .

The model for the quote dynamics in tick time is given by

$$S_j q_j = S_j(c + \iota m_j + u_j), \quad j = 1, \dots, J$$

$$\begin{aligned}
m_j &= m_{j-1} + \sigma \tau_j^{\delta_1} \nu_j \\
u_j &= \alpha \tau_j^{\delta_2} \sigma \nu_j + e_j,
\end{aligned} \tag{1}$$

where  $S_j$  is the  $(k_j \times 2M)$  selection matrix which contains the rows of the identity matrix that correspond to the  $k_j$  elements of  $q_j$  that are updated at  $t_j$ .  $k_j$  is a multiple of 2 which means that we define a quote update, if either the best bid or best ask or both change at a given tick.<sup>3</sup>  $m$  is the asset's latent efficient price modeled as a random walk.  $\iota$ ,  $c$ , and  $u_j$  are  $2M$ -vectors of ones, constants and temporary deviations from  $m_j$ , respectively. The efficient price innovation's magnitudes are determined by the duration-dependent volatility  $\sigma_j = \sigma \tau_j^{\delta_1}$ . The parameter  $\delta_1$  measures the impact of durations on volatility. [Frijns and Schotman \(2009\)](#) assume that  $\delta_1 > 0$ . In case of a negative value, the information flow would be slower than the arrival rate of quote updates. However, this might be relevant in relatively noisy markets where quotes change often but do not necessarily reflect changes in the fundamental price of the asset. Generally, for positive  $\delta_1$ , we consider two important boundary values. If  $\delta = 1/2$ , the variance of the efficient price is proportional to the duration between quote updates and the efficient price essentially evolves in calendar time. If  $\delta_1 = 0$ , the variance of the efficient price is not affected by quote durations and the efficient price is said to evolve in tick time. In this case, the calendar-time variance is proportional to the number of quote updates. The transitory component  $u_j$  captures market microstructure effects. Following [Glosten and Harris \(1988\)](#) and [Frijns and Schotman \(2009\)](#), we allow for endogenous errors that are correlated with changes in the efficient price. The idiosyncratic quote noise,  $e_j$ , is assumed to be uncorrelated with the efficient price innovations  $\nu_j$  and the durations  $\tau_j$ , and has covariance matrix  $\Omega$  with block-diagonal structure

$$\Omega = \begin{pmatrix} \Omega_1 & & \\ & \ddots & \\ & & \Omega_M \end{pmatrix}, \tag{2}$$

where each  $\Omega_i$ ,  $i = 1, \dots, M$  is of dimension  $(2 \times 2)$ . The diagonal elements of  $\Omega_i$  measure the variance of quote innovations that is not attributable to innovations in the efficient price. A lower idiosyncratic noise variance therefore indicates that a venue's quotes track the efficient price more closely. The off-diagonal elements of  $\Omega_i$  allow bid and ask quote noise within the same venue to be correlated. This correlation is economically plausible because inventory imbalances or other venue-specific liquidity shocks may induce liquidity providers to adjust both sides of the market simultaneously. By contrast, the block-diagonal structure rules out residual correlation between quote innovations at different venues. Common price-relevant

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<sup>3</sup>[Frijns and Schotman \(2009\)](#) define an update as an event when both the best bid and the best ask change. Naturally, the sample sizes are smaller following their definition but the results do not change qualitatively.

information that affects multiple venues is instead captured through the component associated with the efficient price.<sup>4</sup> The endogenous microstructure noise  $u_j$  is determined by two components: the efficient price innovations through  $\alpha_j \nu_j$  and the idiosyncratic component  $e_j$ . The  $2M$  vector  $\alpha$  measures the informational asymmetry. Two parameters for each market (potentially different values for bid and ask quotes) determine how each market reacts to newly arriving price-relevant information but their interpretation is not straightforward as their scaling depends on  $\delta_2$  and  $\sigma$ . The exponent  $\delta_2$  measures the impact of quote durations on the asymmetric information. We assume  $\delta_2 < 0$ , implying that quotes are less informative at long durations (Easley and Hara, 1992; Dufour and Engle, 2000; Engle and Patton, 2004).

As in Frijns and Schotman (2009), we consider multiple measures to characterize the price discovery process in tick time. The first measure is called “dealer/market liquidity” defined by

$$\beta_j = \iota + \frac{\alpha_j}{\sigma_j}. \quad (3)$$

It indicates how much of the change in the efficient price is immediately reflected in a respective market’s quote innovation.<sup>5</sup> Plugging-in for the duration-dependent informational asymmetry and volatility parameters, this yields a function of  $\tau$ :

$$\beta(\tau) = \iota + \alpha \tau^{\delta_2 - \delta_1}. \quad (4)$$

If  $\delta_1 > \delta_2$ , then the dealer/market liquidity measure converges to  $\iota$  for long durations. When  $\alpha$  is positive, then  $\beta(\tau)$  is larger than one and this means that this quote incorporates more information of the random walk and vice versa.

Before we can define the information shares of each market, we need to define  $\gamma_j$  measuring the impact of quote innovations on the efficient price change<sup>6</sup>

$$\gamma_j = \frac{\sigma_j^2}{1 + \sigma_j^2 \beta_j' \Omega^{-1} \beta_j} \Omega^{-1} \beta_j. \quad (5)$$

This measure depends inversely on the covariance matrix of the idiosyncratic noise component, i.e., the measure increases if the market has low idiosyncratic noise. Thus, we can control for the different levels of microstructure noise in each market. Again, we express this as a

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<sup>4</sup>The block-diagonal structure is a plausible modeling approximation because we sample at the highest available frequency. It may nevertheless be violated by intermarket sweep orders or other routing activity that generates contemporaneous quote revisions across venues, inducing cross-venue dependence in the idiosyncratic quote innovations.

<sup>5</sup>Frijns and Schotman (2009) derive the measure as a regression coefficient, regressing the quote innovations on the efficient price innovations.

<sup>6</sup>Again, the measure is derived as a regression coefficient, in this case regressing the efficient price innovations on the quote innovations.

function of the durations  $\tau$ :

$$\gamma(\tau) = \frac{\sigma^2 \tau^{2\delta_1} \Omega^{-1} \beta(\tau)}{1 + \sigma^2 \tau^{2\delta_1} \beta(\tau)' \Omega^{-1} \beta(\tau)}. \quad (6)$$

Next, we determine the fraction of the variance of the efficient price change that can be attributed to quote innovations. This can be computed as the  $R^2$  of the corresponding regression and amounts to

$$R_j^2 = \frac{\gamma_j' \Sigma_j \gamma_j}{\sigma_j^2} = \beta_j' \gamma_j, \quad (7)$$

where  $\Sigma_j$  denotes the variance of the quote innovations. Finally, the information shares are given by the decomposition of  $R_j^2$  into the contributions from each market (or dealer as in the original paper):

$$IS_j = \beta_j \circ \gamma_j, \quad (8)$$

which is an  $M \times 1$  vector. Note that the information shares do not necessarily add up to one. In contrast to the reduced-form model in [Hasbrouck \(1995\)](#), the change in the efficient price is not a deterministic function of the quote innovations. This is a convenient model choice that allows us to select a limited number of venues in a fragmented market and accounts automatically for unavailable or smaller ones, thereby reducing the dimensionality of the estimation problem.

After obtaining the tick-time measures, it is possible to aggregate them to fixed calendar-time intervals. A summary on how this can be accomplished is included in the Supplementary Material and a detailed exposition is given in [Frijns and Schotman \(2009\)](#).

## 2.2 Inference

The duration-dependent measures are given as functions of  $\tau$  which makes it difficult to provide confidence bands with the correct coverage for the full support. However, it is possible to derive the asymptotic distribution for a pre-specified value  $\tau$ , for example the average or median duration, using the delta method. The parameters  $\{\alpha, \Omega, \sigma, \delta_1, \delta_2\}$  are estimated with quasi maximum likelihood incorporating the Kalman filter. The estimator  $\hat{\theta}$  is asymptotically normally distributed,

$$\sqrt{J}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, \Sigma_{\theta_0}^{-1}), \quad (9)$$

where  $\theta_0$  denotes the true value and

$$\Sigma_{\theta_0} = - \lim_{J \rightarrow \infty} E \frac{1}{J} \frac{\partial^2 \log L}{\partial \theta \partial \theta'} \bigg|_{\theta_0}.$$

Numerically optimizing the likelihood, directly yields the Hessian or the OPG as estimates of  $\Sigma_{\theta_0}$ . A proof can be provided analogously to the proof of Theorem 3.3 in [Newey and McFadden \(1994\)](#).

From the previous subsection, we know that  $\beta(\tau)$  and  $\gamma(\tau)$  are functions of the parameters  $\theta$ . Defining the function

$$h_i(\theta) = \beta_i(\tau, \theta)' \gamma_i(\tau, \theta), \quad (10)$$

to obtain the information share of market  $i$ , we can compute the gradient

$$g = \frac{\partial h_i(\theta)}{\partial \theta} \Big|_{\theta_0}. \quad (11)$$

Now, applying the delta method, the asymptotic distribution of the information shares is given by

$$\hat{IS}_i \overset{a}{\sim} \mathcal{N}(IS_i, g' \Sigma_{\theta_0}^{-1} / Jg), \quad i = 1, \dots, M. \quad (12)$$

In finite samples, we approximate the gradient numerically and, in combination with the Hessian or OPG, compute the pointwise standard errors of the information shares for each  $\tau$ .

### 3 Simulation: structural models versus reduced-form models

Before presenting the empirical results, we conduct a simulation study to assess how well a reduced-form model recovers the informational roles of markets when prices are generated by a structural model with irregular sampling. We use the quote dynamics model to simulate quote data under varying levels of trading intensity and quote informativeness. We then construct midquotes, aggregate them to a regular sampling frequency, and estimate Hasbrouck information shares using the reduced-form approach. Our primary objective is to identify the conditions under which the reduced-form model correctly identifies the informationally dominant market. Empirically, this requires both correctly ranking markets by their information shares and obtaining narrow spreads between the upper and lower information-share bounds.

[Putniņš \(2013\)](#) conduct a related analysis using a uniform one-second sampling grid and simplified model structure. Whereas the author examines how the relative speed and noisiness of markets affect information and component shares, we investigate how the pace of contemporary markets influences the ability of reduced-form models to identify the informationally dominant venue. Using a partial-adjustment model, [Yan and Zivot \(2010\)](#) show that information shares can be biased and residual correlation can increase when the sampling frequency is lower than the underlying trading frequency. Rather than imposing a fixed calendar-time frequency, our simulation specifies market activity through quote durations and the associated parameters governing volatility. This setup more closely reflects the irregular timing of quote updates in high-frequency financial data.

We generate data according to Equation (2) for a two market system with parameters chosen in line with those obtained for IBM quotes recorded on NYSE and Nasdaq. The average quote duration is set to  $\lambda = 0.1$  for the main results which corresponds to quote updates on average every 100 milliseconds. We set the scaling factor for the efficient price variance to  $\sigma = \{0.0025, 0.025\}$ . The informational asymmetry is given by  $\alpha = (-0.35, 0.35, -0.25, 0.25)'$  and the block-diagonal covariance matrix of the idiosyncratic quote noise is given by

$$\Omega = \begin{pmatrix} 0.00024 & -0.00008 & & \\ -0.00008 & 0.00025 & & \\ & & 0.00018 & -0.00008 \\ & & -0.00008 & 0.00019 \end{pmatrix}. \quad (13)$$

The parameter measuring the impact of durations on volatility is our primary focus and takes values  $\delta_1 \in \{-1, 0.5, 0, 0.25, 0.5, 1\}$ , while the second exponent parameter is fixed at  $\delta_2 = -1$ . The constants determining the average spread are  $c = (-0.5, 0.5, -0.5, 0.5)$ . With this setup, we are able to scale the variance of the efficient price with  $\delta_1$  and thereby change the signal-to-noise ratio. If  $\delta_1$  is negative (positive), the efficient price variance is scaled up (down), while the noise variance stays fixed. In principle, a change in  $\sigma$  affects both components. However, the impact of durations on the idiosyncratic errors are upscaled if  $\delta_2 < 0$ . This implies that a larger  $\sigma$  leads to a better signal-to-noise ratio if  $\delta_1 > 0$  or more generally  $\delta_1 > \delta_2$ .

After the quote data is generated, we follow the usual procedure for calendar-time models that require regular sampling intervals and compute the midquotes for the chosen calendar-time frequency. The high-resolution VECM proposed by [Hasbrouck \(2021\)](#) is then estimated for different frequencies with a maximum lag length fixed at 10 seconds. The model is given by

$$\begin{aligned} \Delta y_t &= \Pi_{t-1} + \Gamma^{(1)} \Delta y_{t-1|t-k_1} + \Gamma^{(2)} \Delta y_{t-k_1-1|t-k_2} + \dots \\ &+ \Gamma^{(d)} \Delta y_{t-k_{d-1}-1|t-k_d} + \epsilon_t, \quad t = 1, \dots, T, \end{aligned} \quad (14)$$

where  $\Delta y_{p|q} = \sum_{i=p}^q \Delta y_i$ ,  $d$  denotes the time-scales considered, and  $k_i$ ,  $i = 1, \dots, d$  denotes the range of a time-scale. Specifying the VECM in this form constrains the  $k$  coefficients  $\Gamma^{(i)}$  to be constant for each time-scale  $s \in [t - k_{i-1} - 1, t - k_i]$ . For example, at a frequency of 100-ms, we set  $d = 3$ ,  $k_1 = 1$ ,  $k_2 = 10$ , and  $k_3 = 100$ . Note that we estimate the model anew for each trading day so that  $T$  denotes the number of available intra-day observations.

As our main results, we report the average HIS midpoints over all trading days and the average residual correlation as the main determinant of the spread between upper and lower bound of the HIS (see [Table 1](#)). The main takeaways are as follows. We observe a generally higher residual correlation if the information flow is slower than the quote arrival, i.e. setting  $\delta_1 < 0$ . In these cases, the quotes are mainly driven by the fundamental price variation (high

signal-to-noise ratio) without substantial microstructure noise to differentiate the quote series through the informational asymmetry parameters  $\alpha$ . Hence, the information in each quote change is highly correlated across markets. After computing the midpoints, this feature of the data also translates to a high residual correlation in the reduced-form model obscuring the source of price innovations.

If the quote arrival rate is slower than the chosen sampling frequency for the midquotes ( $\lambda = 1$ , i.e., quote updates arrive on average every second and the reduced-form model is estimated on a 100-ms grid), then a higher resolution helps to reduce the residual correlation. This is also one of the few cases where the leading market, indicated by the true structural information share at the average duration, is correctly identified (right panel in [Table 1](#)). If the quote arrival rate is faster than the midquote sampling frequency, the residual correlation seems to be slightly higher.

Higher values of  $\sigma$  tend to increase the residual correlation. As outlined above,  $\sigma$  determines the efficient price variance but also enters the microstructure noise equation. Since a larger  $\sigma$  increases the signal-to-noise ratio, the reduced-form model has difficulties determining the contribution from each market which leads to higher residual correlation and wider spreads.

Overall, the Hasbrouck approach does not reliably identify the leading market when applied to data generated by the structural model. When residual correlation is high, the midpoint of the information-share bounds approaches  $(0.5, 0.5)$ , obscuring differences in the markets' informational contributions. When residual correlation is low, the midpoint may instead identify the wrong market as the leader. Because the structural information share depends on the duration  $\tau$ , we use its value at the average duration as the benchmark. Although constructing midquotes may affect the sampling frequency that is optimal for the reduced-form model, the resulting bias in the Hasbrouck information shares is apparent when the model is estimated at sampling frequencies both above and below the quote-arrival rate.

Table 1: Simulation results for  $\delta_2 = -1$  and  $\lambda = \{0.01, 0.1, 1\}$ . High-resolution VECM estimated at 100-ms intervals.

| $\delta_1$        | $\lambda = 0.01$ |               |                   |                   |              | $\lambda = 0.1$ |               |                   |                   |              | $\lambda = 1$ |               |                   |                   |              |
|-------------------|------------------|---------------|-------------------|-------------------|--------------|-----------------|---------------|-------------------|-------------------|--------------|---------------|---------------|-------------------|-------------------|--------------|
|                   | $IS_1^{tick}$    | $IS_2^{tick}$ | $\widehat{HIS}_1$ | $\widehat{HIS}_2$ | $\hat{\rho}$ | $IS_1^{tick}$   | $IS_2^{tick}$ | $\widehat{HIS}_1$ | $\widehat{HIS}_2$ | $\hat{\rho}$ | $IS_1^{tick}$ | $IS_2^{tick}$ | $\widehat{HIS}_1$ | $\widehat{HIS}_2$ | $\hat{\rho}$ |
| $\sigma = 0.0025$ |                  |               |                   |                   |              |                 |               |                   |                   |              |               |               |                   |                   |              |
| -1                | 0.391            | 0.609         | 0.500             | 0.500             | 0.999        | 0.372           | 0.580         | 0.500             | 0.500             | 0.999        | 0.065         | 0.100         | 0.500             | 0.500             | 0.999        |
| -0.5              | 0.563            | 0.426         | 0.499             | 0.501             | 0.994        | 0.325           | 0.400         | 0.473             | 0.527             | 0.878        | 0.065         | 0.100         | 0.400             | 0.600             | 0.353        |
| 0                 | 0.606            | 0.382         | 0.389             | 0.612             | 0.199        | 0.278           | 0.210         | 0.384             | 0.616             | 0.063        | 0.065         | 0.100         | 0.384             | 0.616             | 0.046        |
| 0.25              | 0.606            | 0.381         | 0.384             | 0.616             | 0.090        | 0.270           | 0.182         | 0.383             | 0.617             | 0.026        | 0.065         | 0.100         | 0.384             | 0.616             | 0.041        |
| 0.5               | 0.606            | 0.381         | 0.384             | 0.616             | 0.062        | 0.268           | 0.172         | 0.383             | 0.617             | 0.015        | 0.065         | 0.100         | 0.384             | 0.616             | 0.046        |
| 1                 | 0.606            | 0.381         | 0.384             | 0.616             | 0.049        | 0.268           | 0.169         | 0.383             | 0.617             | 0.009        | 0.065         | 0.100         | 0.385             | 0.615             | 0.087        |
| $\sigma = 0.025$  |                  |               |                   |                   |              |                 |               |                   |                   |              |               |               |                   |                   |              |
| -1                | 0.391            | 0.609         | 0.500             | 0.500             | 0.999        | 0.391           | 0.609         | 0.500             | 0.500             | 0.999        | 0.372         | 0.580         | 0.500             | 0.500             | 0.999        |
| -0.5              | 0.569            | 0.431         | 0.500             | 0.500             | 0.999        | 0.447           | 0.550         | 0.500             | 0.500             | 0.999        | 0.372         | 0.580         | 0.496             | 0.504             | 0.980        |
| 0                 | 0.613            | 0.386         | 0.481             | 0.519             | 0.915        | 0.563           | 0.426         | 0.465             | 0.535             | 0.835        | 0.372         | 0.580         | 0.462             | 0.538             | 0.822        |
| 0.25              | 0.614            | 0.386         | 0.417             | 0.583             | 0.534        | 0.591           | 0.397         | 0.424             | 0.576             | 0.592        | 0.372         | 0.580         | 0.458             | 0.542             | 0.803        |
| 0.5               | 0.614            | 0.386         | 0.388             | 0.612             | 0.199        | 0.601           | 0.386         | 0.397             | 0.603             | 0.348        | 0.372         | 0.580         | 0.462             | 0.538             | 0.821        |
| 1                 | 0.614            | 0.386         | 0.384             | 0.616             | 0.069        | 0.606           | 0.382         | 0.384             | 0.616             | 0.108        | 0.372         | 0.580         | 0.478             | 0.522             | 0.902        |

Note:  $IS_1^{tick}$  and  $IS_2^{tick}$  denote the structural information share evaluated at the mean duration for market one and two, respectively.  $\widehat{HIS}_1$  and  $\widehat{HIS}_2$  are the corresponding midpoints between the upper and lower bound of the Hasbrouck information shares estimated from the high-resolution VECM.  $\hat{\rho}$  denotes the estimated residual correlation, the main driver for the spread between upper and lower bound.

## 4 Price discovery in the U.S. equity market

### 4.1 Data

We study venue-level contributions to price discovery for all 30 DJIA stock prices. The sample comprises 64 trading days from March 1 through May 31, 2024, and covers the five major U.S. exchanges: NYSE (N), Nasdaq (T), Arca (P), Cboe EDGX (K), and Cboe BZX (Z). We retrieve quote data from Tick History and apply the same cleaning filters as in [Barndorff-Nielsen et al. \(2009\)](#). The cleaning process entails discarding observations with zero quotes, negative bid-ask spreads, or those occurring outside the primary trading hours. The trading day usually starts with a single-price double-sided auction. Because the opening auction differs substantially from continuous trading, we exclude it and restrict the sample to 9:45 a.m. to 4:00 p.m. EST. We also remove observations for which the bid-ask spread exceeds 50 times the daily median spread. Finally, we merge quotes at the same time stamp. The resulting aggregation depends on timestamp precision, which is available at nanosecond resolution in Tick History. Reducing timestamp precision is analogous to aggregating observations over calendar-time intervals (the number of ticks at different decimal places is reported in [Table S.5](#) of the Supplementary Material). Naturally, if quotes are downsampled, the probability of multiple markets updating their quotes at the same tick increases. To balance the need for precision and the computational burden, we first compute the average duration between quote updates and then estimate the tick-time model for the required precision. Across the DJIA stocks, average durations range from 0.032 to 3.20 seconds, indicating substantial heterogeneity in quote-arrival rates. The quote dynamics model accommodates these differences through the volatility-scaling parameters  $\delta_1$  and  $\delta_2$ . If timestamp precision is substantially finer than the average quote-arrival duration, the estimated  $\delta_1$  may be negative, indicating that information arrives more slowly than the selected tick-time resolution.

The observed quote data are the best bid and best ask quotes in a given markets. We are not concerned with non-marketable orders in the exchanges because they have no immediate effect on the price formation process. [Figure 1](#) shows the observed quotes for a representative stock (IBM) during a brief interval on March 1, 2024. All quote series appear to share a common stochastic trend, while bid-ask spreads vary both over time and across venues. In addition, some bid and ask quotes remain unchanged for short intervals at particular venues.

The ideal quote updating frequency that balances precision and computational complexity for our DJIA sample is 100-ms or 10-ms if we are limited by rounding to a decimal place. In [Table 2](#) (see [Table S.7](#) of the Supplementary Material for 10-ms results), we report the average parameter estimates over all trading days. We find exclusively positive  $\delta_1$  parameters. Still, we estimate  $\delta_1$  parameters close to zero for some stocks at 10-ms precision (particularly for those stocks that have longer average durations). A similar pattern appears for the 20 stocks selected by [Frijns and Schotman \(2009\)](#) whose stocks with the longest average durations

Figure 1: Price quotes in a fragmented market

This figure depicts bid and ask quotes for IBM stock in five main trading venues in the U.S.: NYSE (green), Nasdaq (gray), Arca (orange), Cboe EDGX (lightblue), and Cboe BZX (yellow) on March 1, 2024 at 100-milliseconds precision.

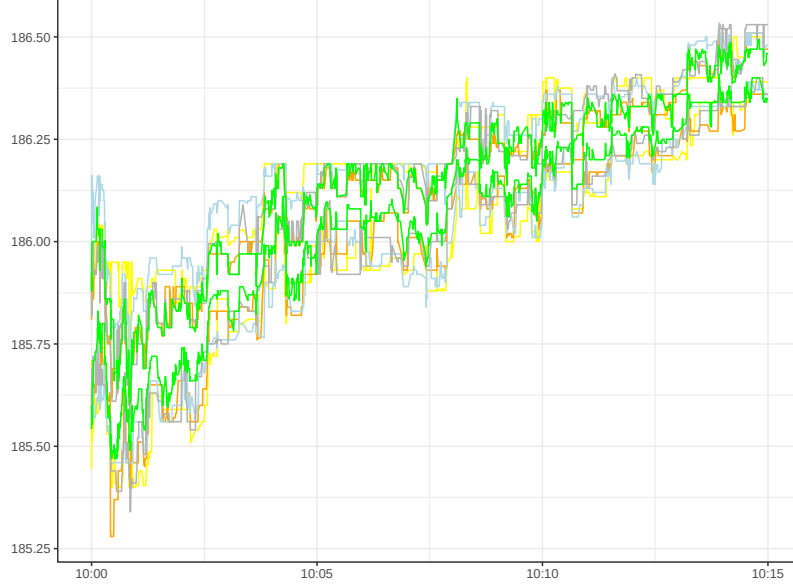


exhibit negative estimates of  $\delta_1$ . The choice between 100- and 10-millisecond precision is motivated not only by computational considerations but also by the fact that quote updates do not necessarily correspond to trades and that information may be incorporated into prices at a lower frequency. We therefore expect these resolutions to capture the main features of price discovery even for highly liquid stocks such as AAPL, MSFT, and NVDA, for which some venues have average quote durations below 10 milliseconds.<sup>7</sup>

At highest precision (nanosecond precision, i.e. ninth decimal place), almost 100% of all ticks include only a quote change for a single market (at least bid or ask changes). This number decreases to 70% at 10-ms precision (17% have two markets report a quote change and 13% at least three markets changing their quotes at the same time) and further to 58% at 100-ms precision (20% two markets and 22% three or more). In [Table S.6](#) of the Supplementary Material, we report the average number of markets simultaneously updating quotes at a tick at 100-ms and 10-ms precision.

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<sup>7</sup>Daily tick-time information shares estimated at nanosecond precision are reported in [Table S.9](#) of the Supplementary Material. They exhibit the same qualitative patterns as the estimates based on 100- and 10-millisecond precision.

## 4.2 Tick-time estimates for IBM stock

The quote dynamics model can be estimated separately for each trading day or jointly over the entire sample by concatenating trading days. Daily estimation allows us to study time variation in price discovery and to relate it to covariates capturing other aspects of the trading process (see, for a related panel analysis of the relative accuracy of realized-variance estimators, Liu et al., 2015). The computational burden of daily estimation is relatively modest. In the joint estimation, we reset the Kalman filter with a diffuse prior at the beginning of each trading day while estimating the parameters using the full sample. This approach yields more precise estimates but requires substantially more computation.<sup>8</sup>

First, to illustrate the method and provide an overview of the data, we present full-sample results for IBM using timestamps at three levels of precision: 10 milliseconds, 100 milliseconds, and 1 second. IBM is a stock with medium liquidity within the DJIA. By contrast, highly liquid stocks such as AAPL have faster quote updates and consequently shorter average quote durations. For IBM, the average duration between quote updates ranges from 0.33 seconds on its primary listing exchange, the NYSE, to 1.07 seconds on NYSE Arca. These durations suggest that 100-millisecond precision provides an appropriate balance between temporal resolution and computational cost. We also report results based on 10-millisecond precision to assess whether finer timestamps improve estimation accuracy or merely increase the computational burden of estimating the quote-dynamics model.

The average bid-ask spreads for IBM at 100-millisecond precision are 0.065 on the NYSE, 0.087 on Nasdaq, 0.145 on NYSE Arca, 0.127 on Cboe EDGX, and 0.267 on Cboe BZX. The NYSE, IBM's primary listing exchange, has the narrowest spread, followed by Nasdaq.<sup>9</sup> Figure 3 presents the duration-dependent information shares. At 100-millisecond precision, the NYSE has the largest information share, primarily because its quotes have a larger impact on the efficient price, as shown in Figure 2. By contrast, the venues differ relatively little in the estimated liquidity measure (see Figure S.2 of the Supplementary Material). The results are similar at 10-millisecond precision, suggesting that increasing the precision beyond the average quote-arrival rate does not drastically improve the estimates. At 1-second precision, however, the estimated information shares are less sensitive to quote durations, indicating that this sampling frequency is too coarse to capture their effect. The smaller exchanges also receive larger information shares at this frequency, although the NYSE remains the informationally dominant venue.

Closer examination of the parameter estimates reveals that the informational asymme-

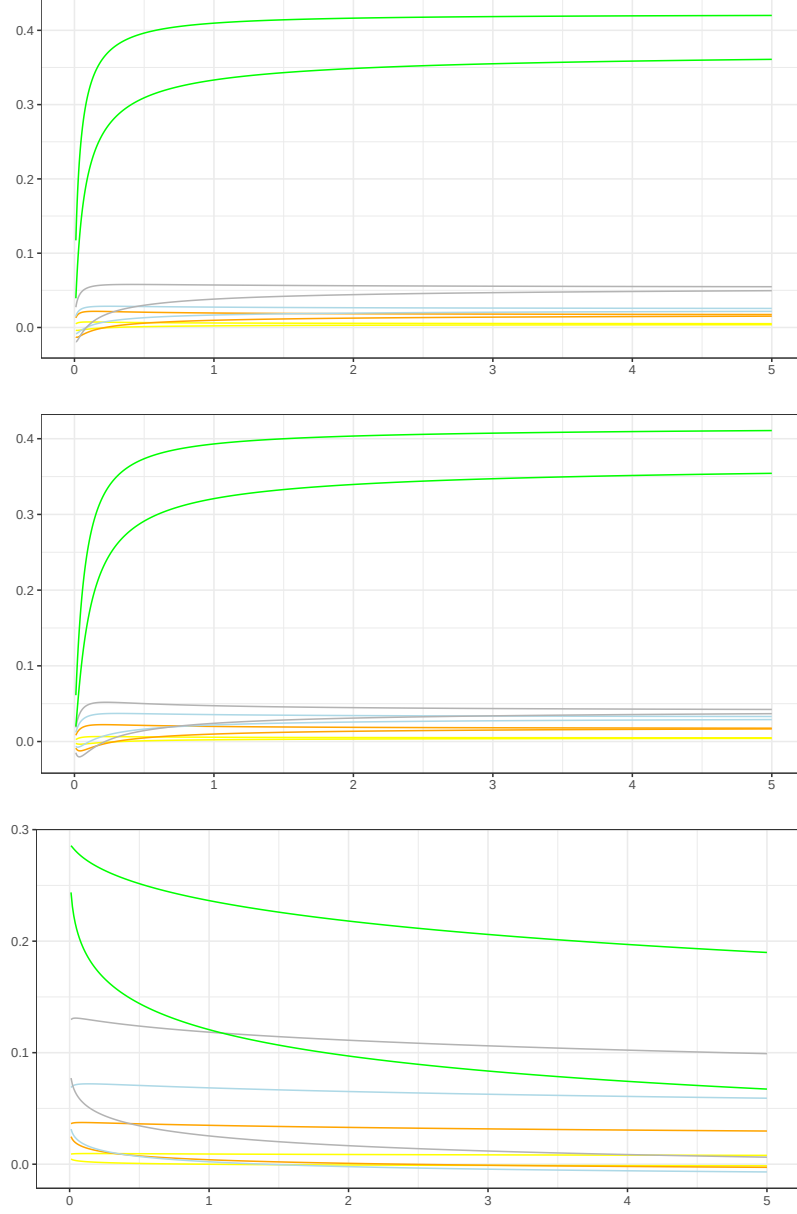
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<sup>8</sup>The model is implemented in R using precompiled C++ code via the Rcpp package and can be downloaded here: <https://github.com/SiSanchopancho/QuoteDynamics>.

<sup>9</sup>For comparison, the corresponding average spreads for AAPL are 0.030 on the NYSE, 0.013 on Nasdaq, 0.020 on NYSE Arca, 0.019 on Cboe EDGX, and 0.024 on Cboe BZX. Nasdaq, AAPL's primary listing exchange, has the narrowest spread.

Figure 2: Quote impact on the efficient price change for durations ranging from 10 milliseconds to 5 seconds

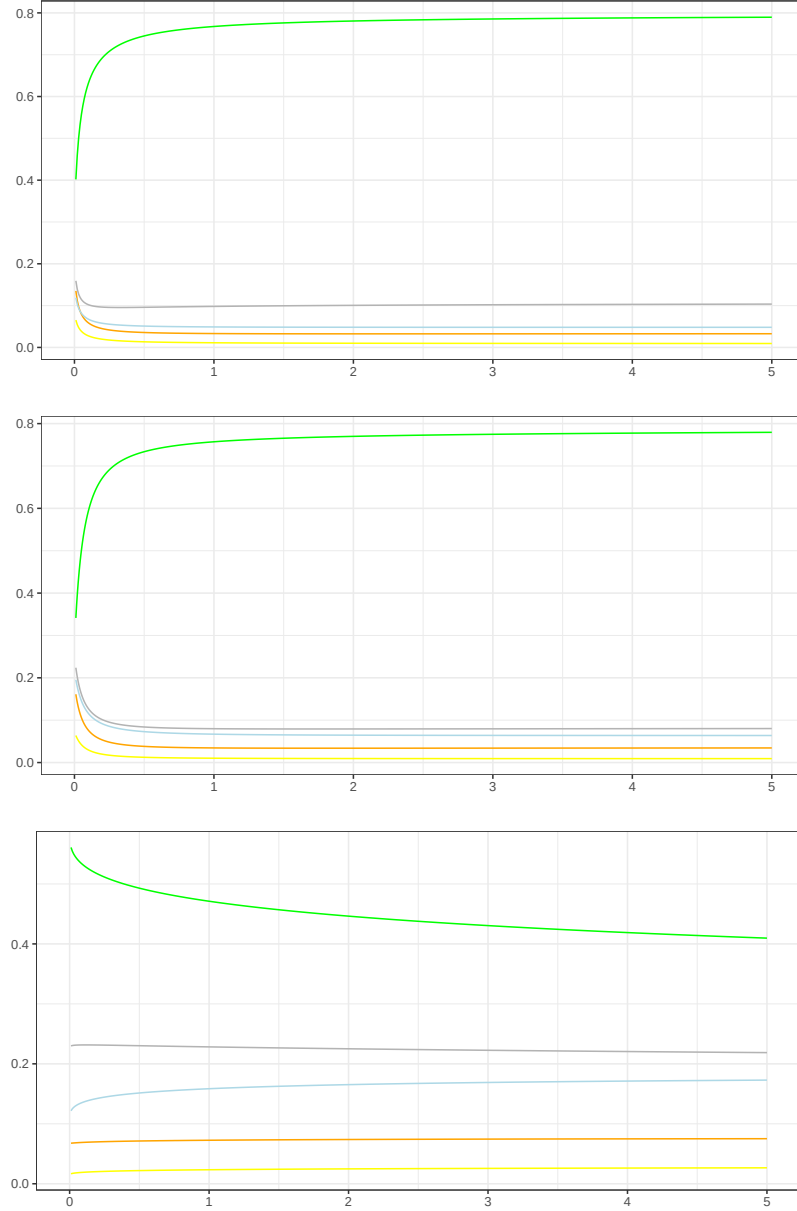
This figure depicts  $\gamma(\tau)$  for IBM: NYSE (green), Nasdaq (gray), Arca (orange), Cboe EDGX (lightblue), and Cboe BZX (yellow) estimated for the full sampling period (March 1, 2024, to May 31, 2024) using data at 10-milliseconds, 100-milliseconds, and 1-second precision.



try parameters  $\alpha$  are of opposite sign for bid and ask quotes series (results are reported in [Table S.12](#) and [Table S.13](#) of the Supplementary Material). This pattern motivates a more parsimonious specification, setting the coefficients to  $\alpha_i = -\alpha_{i+1}$  for  $i = 1, 3, 5, \dots, 2M - 1$ . This hypothesis cannot be rejected based on an appropriate Wald test but throughout this

Figure 3: Information shares for durations ranging from 10 milliseconds to 5 seconds

This figure depicts the information shares for IBM: NYSE (green), Nasdaq (gray), Arca (orange), Cboe EDGX (lightblue), and Cboe BZX (yellow) estimated for the full sampling period (March 1, 2024, to May 31, 2024) using data at 10-milliseconds, 100-milliseconds, and 1-second precision.



paper, we still report the results for the more general specification. At subsecond precision, we estimate positive  $\delta_1$  and negative  $\delta_2$  parameters. [Frijns and Schotman \(2009\)](#) note that a negative value for  $\delta_1$  can arise when the information flow is slower than the arrival rate of quote updates as quote updates are more frequent than transactions and possibly not all quote updates are equally informative. More generally,  $\delta_1$  is not straightforward to interpret

because its value depends on the observed quote durations and, consequently, on the chosen timestamp precision. In case of IBM, we estimate a value of  $\delta_1$  that is close to  $1/2$  at 100-ms. This can be interpreted as the variance of the efficient price being roughly proportional to the duration between quote updates which, in turn, can be considered as evidence that the efficient price evolves in calendar time. Instead, the value of  $\delta_1$  drops to just below  $1/4$  at 10-ms indicating that the price discovery process is less responsive to quote durations and hence evolves in tick time. This pattern can be observed for most stocks in the DJIA sample. A negative  $\delta_2$  indicates that long durations lead to less asymmetric (also less informative) quotes as the longer durations are down-weighted by a large negative exponent.

### 4.3 DJIA panel analysis

Market share in terms of trading volume is usually considered to be a good predictor for informational importance of a market. However, as shown in [Figure 4](#), market shares differ substantially between Tape A stocks, which are primarily listed on the NYSE, and Tape C stocks, which are primarily listed on Nasdaq.<sup>10</sup> The NYSE’s role varies accordingly: it has the largest market share for Tape A stocks but the smallest for Tape C stocks. Market shares do not, however, indicate how informative the underlying trades are or distinguish informational trading from noise trading. They therefore provide only a preliminary, albeit potentially strong, predictor of daily information shares.

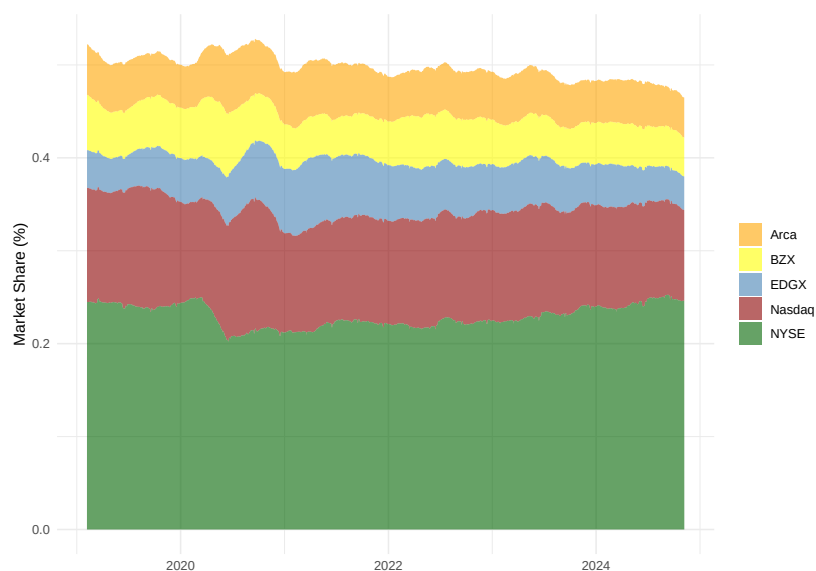
Several empirical studies examine the determinants of daily variation in price-discovery measures using variables that characterize market microstructure and trading activity. An early example is [Chakravarty et al. \(2004\)](#), who study the contribution of options markets to price discovery. After estimating daily Hasbrouck information shares for 60 stocks over 1988-1992, they regress the options market’s information share on variables including the ratio of option volume to stock volume, effective spread ratio, and volatility measures. [Mizrach and Neely \(2008\)](#) conduct a cross-sectional analysis of the contribution of futures markets to price discovery in the U.S. Treasury market, considering relative bid-ask spreads, number of trades, and the daily realized volatility as potential determinants. More recent studies use panel regressions to account for unobserved heterogeneity across firms. For example, [Frijns et al. \(2015a,b\)](#) examine 38 U.S.-Canadian cross-listed stocks and regress reduced-form price-discovery measures on variables including trading volume, bid-ask spreads, and, in the latter study, macroeconomic announcements. Overall, the literature suggests that conventional measures of trading activity, such as volume, are important predictors of a market’s daily information share but do not fully explain its variation ([Chen and Tsai, 2017](#)). Their predictive power may also depend on the financial instrument and the price-discovery measure used. Moreover, previous studies show that daily information shares can be biased

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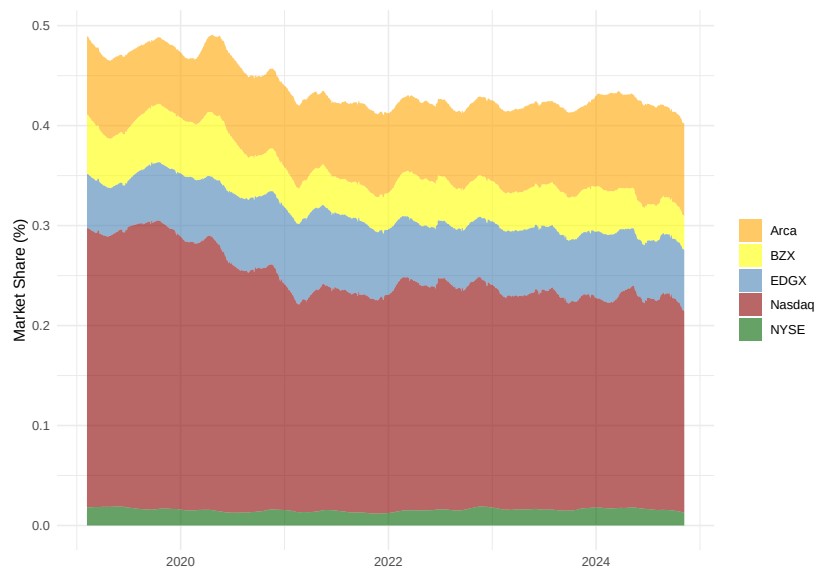
<sup>10</sup>Market share data originate from <https://www.cboe.com/markets/us/equities/market-statistics/>.

Figure 4: U.S. equity market shares

This figure depicts the overall market share in terms of trading volume of the five main trading venues in the U.S.: NYSE (darkgreen), Nasdaq (darkred), Arca (orange), Cboe EDGX (steelblue), and Cboe BZX (yellow) from November 2018 to November 2024.



(a) Tape A stocks



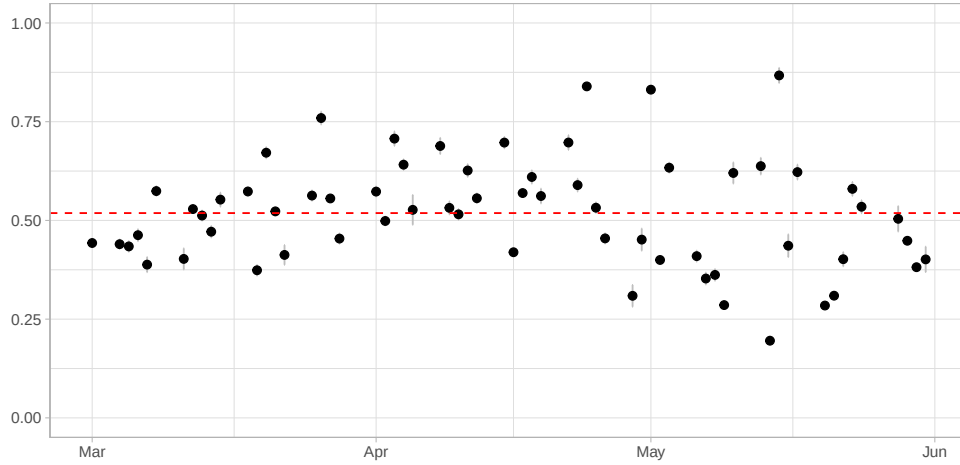
(b) Tape C stocks

estimates of a market’s contribution to price discovery (see, for example, [Dias and Schweikert, 2026](#); [Schweikert, 2026](#)). We therefore compare the panel regression results based on tick-time information shares with those based on a high-resolution VECM estimated after calendar-time aggregation.

The average daily information shares, evaluated at the median duration, and the total fraction of the variance of efficient-price changes attributable to quote innovations ( $R^2$ ) are reported for all DJIA constituents in [Table 3](#). [Figure 5](#) shows the daily variation in the NYSE’s information share for IBM at 100-millisecond precision. The  $R^2$  decreases from 100-ms to 10-ms as the additional quote updates contain more noise and therefore explain a smaller fraction of efficient-price variation. The NYSE has the largest average information share across venues for both Tape A and Tape C stocks. Nevertheless, as in the market-share analysis, the information-share patterns differ substantially between the two groups of stocks.

Figure 5: Daily information shares of NYSE for IBM stock at 100-ms precision

This figure depicts the daily information shares of the primary listing exchange for IBM (NYSE) at the average duration. The information shares are estimated for trading days spanning from March 1, 2024, to May 31, 2024. The vertical bars indicate the 95% confidence interval.



Using a high-resolution VECM is impractical in our setting because the information-share bounds remain wide in a five-venue system at the sampling frequencies considered. Although higher frequencies could reduce residual correlation, they would also amplify the influence of microstructure noise, making the resulting estimates difficult to interpret. Results for 100- and 10-millisecond sampling frequencies, reported in [Table S.10](#) and [Table S.11](#) of the Supplementary Material, show only modest differences across venues. This pattern is consistent with wide information-share bounds compressing their midpoints toward a central value. The Hasbrouck information share midpoints also exhibit substantially less daily variation than the tick-time information shares. This finding is again consistent with residual correlation pushing the Hasbrouck bounds toward the limits of the unit interval. By contrast, the tick-time

information shares are order-invariant and respond to changes in the informativeness of quote updates and market conditions.

We explain daily variation in information shares using covariates that characterize market microstructure and trading activity. Stock fixed effects capture persistent differences in investor interest and overall trading intensity across DJIA constituents. Exchange fixed effects account for unobserved heterogeneity related to trading clientele, fee structures, and market design, while time fixed effects capture changes in the broader economic environment that may affect the relative contributions of venues to price discovery. We primarily focus on high-frequency variables that characterize the market-specific liquidity and overall trading environment. The first variable is *avgDuration* and accounts for the trading speed measuring the average time between ticks in seconds. Then, we also measure the liquidity by the relative daily trading volume, *relTradevol*, allocated to a given market. For the selection of more specific liquidity metrics, we follow [Boudt and Petitjean \(2014\)](#) and [Fong et al. \(2017\)](#) and consider three intraday measures. The effective spread (*effectiveSpread*) is defined as  $ES_t = 2D_t \times [PRICE_t - (BID_t + OFR_t)/2]$ . The variable  $D_t$  is the sign of the trade, indicating whether the buyer (seller) is the initiator of the trade  $D_t = 1$  ( $D_t = -1$ ). The depth imbalance difference (*depthImbalanceRatio*) is defined as  $DI_t = (OFRSIZ_t/BIDSIZ_t)^{D_t}$  and price impact (*priceImpact*) is defined as  $PI_t = (ES_t - RS_t)/2$ , where  $RS_t = 2D_t \times [PRICE_t - (BID_{t+300} + OFR_{t+300})/2]$  indicates the relative spread. Moreover, we estimate the market-specific noise level with the ReMeDI estimator proposed by [Li and Linton \(2022\)](#). The daily *ReMeDI* estimates are computed at the 1-second frequency as choosing a relatively high sampling frequency generally improves the precision of microstructure noise estimators. Alternatively, we capture the daily volatility level with a sparsely sampled RV estimator but this measure exhibits little variation across venues and is largely absorbed by the time fixed effects.

The results for the panel analysis based on the tick-time information shares are reported in the left panel of [Table 4](#) and show only minor differences between 100-ms and 10-ms precision. For comparison, the results based on the HIS can be found in the right panel. The main findings are that longer durations (slower trading speed) are associated with lower tick-time information shares. This relationship does not arise directly for the HIS because all venues are observed at the same calendar-time sampling interval and duration effects can therefore enter only indirectly through the estimated adjustment coefficients.

For both measures, we find a strong impact of daily relative trading volume. Naturally, trading activity in those venues drives the price process and the results do not suggest that noise trading dominates the volume measure. Also, daily market-specific noise levels as measured by the ReMeDI estimator do not seem to matter for either tick-time information shares or HIS. Moreover, we find a (slight) negative effect of the market-specific effective spread and the effect is significant for the HIS specification. The effective spread seems to increase its

predictive power at a higher precision as the 1-ns results show a significantly negative coefficient in the tick-time model (see [Table S.15](#) of the Supplementary Material). Surprisingly, price impact also has a negative coefficient, although it is not statistically significant in the tick-time specification. After controlling for the market-microstructure and trading-activity covariates, Nasdaq has a significantly lower information share than the benchmark venue, Cboe EDGX, under both measures. In the tick-time specification, the NYSE has a significantly higher information share than Cboe EDGX. Thus, even after accounting for observable differences in market conditions, the NYSE retains a relative advantage in the price-discovery process for DJIA stocks.

## 5 Conclusion

The empirical results in this paper show that the U.S. equity market is highly fragmented and multiple markets contribute substantially to the price discovery process. All five venues considered have non-negligible information shares, providing further evidence that the market is strongly interconnected and that trading venues function as alternative points of entry. Trading volume is a useful predictor of a venue’s information share but does not, by itself, identify the informationally dominant venue for a given stock. In our DJIA sample, faster and more liquid venues make larger contributions to price discovery. After controlling for observable market conditions, the NYSE retains a modest advantage over competing venues. This result is partly attributable to the composition of the DJIA, in which 23 of the 30 stocks are primarily listed on the NYSE.

From a methodological perspective, our results indicate that the Hasbrouck information share is poorly suited to estimating daily venue-level contributions in a system consisting of more than two markets. Even at 100- and 10-millisecond sampling frequencies, residual correlation remains substantial and pushes the information-share bounds toward the limits of the unit interval. Together with the measure’s dependence on variable ordering, this makes its daily variation difficult to interpret. Sampling at still higher frequencies entails considerable computational costs and may produce estimates that are distorted by microstructure noise.

The tick-time measures used in this study address several of these limitations. By modeling quote updates directly, they avoid the repeated stale quotes associated with calendar-time sampling and allow volatility to depend on quote durations. The measures are also invariant to the ordering of venues and are based on time-invariant asymmetry parameters. As a result, they provide more informative measures of daily variation, which can be further analyzed using a panel regression framework.

## 6 Acknowledgements

Access to Refinitiv TickHistory, provided by the Hohenheim Datalab (DALAHO), is gratefully acknowledged.

Table 2: DJIA stocks: average duration and estimated quote dynamics parameters at 100-ms precision

|      | Avg. duration |       |       |       |       | Est. parameters |            |            |
|------|---------------|-------|-------|-------|-------|-----------------|------------|------------|
|      | N             | T     | K     | P     | Z     | $\sigma$        | $\delta_1$ | $\delta_2$ |
| AAPL | 0.057         | 0.032 | 0.039 | 0.051 | 0.035 | 0.129           | 1.450      | 2.308      |
| AMGN | 2.519         | 0.867 | 0.791 | 1.429 | 0.630 | 0.010           | 0.139      | -0.913     |
| AMZN | 0.077         | 0.042 | 0.066 | 0.068 | 0.054 | 0.109           | 1.209      | 1.689      |
| AXP  | 0.319         | 0.695 | 0.462 | 0.911 | 0.336 | 0.054           | 0.933      | -0.639     |
| BA   | 0.284         | 0.642 | 0.471 | 0.927 | 0.291 | 0.007           | 0.275      | -0.688     |
| CAT  | 0.489         | 1.122 | 0.813 | 0.756 | 0.470 | 0.001           | 0.172      | -1.022     |
| CRM  | 0.363         | 0.644 | 0.398 | 0.592 | 0.268 | 0.026           | 0.367      | -0.913     |
| CSCO | 0.249         | 0.160 | 0.342 | 0.155 | 0.292 | 0.037           | 1.031      | 0.690      |
| CVX  | 0.125         | 0.297 | 0.195 | 0.327 | 0.161 | 0.033           | 0.782      | -0.501     |
| DIS  | 0.274         | 0.292 | 0.288 | 0.320 | 0.205 | 0.015           | 0.401      | -0.263     |
| GS   | 0.647         | 1.562 | 0.839 | 2.011 | 0.501 | 0.020           | 0.218      | -0.364     |
| HD   | 0.384         | 1.060 | 0.394 | 1.007 | 0.263 | 0.022           | 0.419      | -0.896     |
| HON  | 1.594         | 0.367 | 0.610 | 1.409 | 0.442 | 0.013           | 0.368      | -0.419     |
| IBM  | 0.328         | 0.704 | 0.584 | 1.073 | 0.348 | 0.014           | 0.428      | -0.549     |
| JNJ  | 0.240         | 0.422 | 0.447 | 0.621 | 0.259 | 0.010           | 0.431      | -0.780     |
| JPM  | 0.153         | 0.308 | 0.244 | 0.467 | 0.146 | 0.025           | 0.605      | -0.569     |
| KO   | 0.165         | 0.231 | 0.341 | 0.181 | 0.281 | 0.044           | 1.089      | 0.667      |
| MCD  | 0.452         | 0.981 | 0.667 | 1.267 | 0.462 | 0.006           | 0.222      | -0.579     |
| MMM  | 0.400         | 0.697 | 0.744 | 1.059 | 0.407 | 0.004           | 0.278      | -0.492     |
| MRK  | 0.219         | 0.547 | 0.485 | 0.574 | 0.309 | 0.007           | 0.331      | -0.667     |
| MSFT | 0.186         | 0.071 | 0.117 | 0.202 | 0.103 | 0.136           | 1.264      | -0.241     |
| NKE  | 0.256         | 0.301 | 0.354 | 0.382 | 0.243 | 0.012           | 0.411      | 0.102      |
| NVDA | 0.289         | 0.166 | 0.055 | 0.369 | 0.046 | 0.276           | 0.903      | -0.694     |
| PG   | 0.284         | 0.452 | 0.380 | 0.573 | 0.251 | 0.006           | 0.328      | -0.333     |
| SHW  | 1.172         | 2.835 | 1.903 | 2.543 | 0.973 | 0.009           | 0.142      | -0.314     |
| TRV  | 0.858         | 2.697 | 1.804 | 3.195 | 0.802 | 0.009           | 0.198      | -0.516     |
| UNH  | 0.372         | 1.411 | 0.775 | 1.952 | 0.409 | 0.050           | 0.548      | -0.454     |
| V    | 0.199         | 0.498 | 0.383 | 0.618 | 0.244 | 0.019           | 0.462      | -0.646     |
| VZ   | 0.212         | 0.281 | 0.416 | 0.206 | 0.336 | 0.044           | 1.127      | 0.317      |
| WMT  | 0.222         | 0.262 | 0.385 | 0.259 | 0.299 | 0.019           | 0.813      | 0.536      |

The trading venues are denoted with their ticker symbol: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

Table 3: DJIA stocks: estimated tick-time information shares and  $R^2$ 

|      | 100-ms |       |       |       |       |       | 10-ms |       |       |       |       |       |
|------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | N      | T     | K     | P     | Z     | $R^2$ | N     | T     | K     | P     | Z     | $R^2$ |
| AAPL | 0.064  | 0.208 | 0.184 | 0.249 | 0.166 | 0.872 | 0.052 | 0.191 | 0.138 | 0.177 | 0.131 | 0.690 |
| AMGN | 0.064  | 0.283 | 0.112 | 0.125 | 0.075 | 0.659 | 0.042 | 0.225 | 0.117 | 0.099 | 0.062 | 0.545 |
| AMZN | 0.110  | 0.228 | 0.188 | 0.159 | 0.158 | 0.843 | 0.079 | 0.165 | 0.140 | 0.118 | 0.121 | 0.623 |
| AXP  | 0.202  | 0.154 | 0.129 | 0.148 | 0.115 | 0.749 | 0.171 | 0.111 | 0.106 | 0.138 | 0.085 | 0.611 |
| BA   | 0.484  | 0.098 | 0.095 | 0.048 | 0.069 | 0.795 | 0.435 | 0.074 | 0.056 | 0.032 | 0.046 | 0.644 |
| CAT  | 0.351  | 0.130 | 0.103 | 0.058 | 0.028 | 0.670 | 0.304 | 0.101 | 0.112 | 0.048 | 0.019 | 0.584 |
| CRM  | 0.309  | 0.116 | 0.099 | 0.125 | 0.079 | 0.729 | 0.249 | 0.092 | 0.077 | 0.088 | 0.055 | 0.561 |
| CSCO | 0.142  | 0.185 | 0.239 | 0.157 | 0.126 | 0.849 | 0.127 | 0.152 | 0.185 | 0.167 | 0.122 | 0.754 |
| CVX  | 0.281  | 0.141 | 0.137 | 0.137 | 0.144 | 0.841 | 0.236 | 0.117 | 0.125 | 0.104 | 0.117 | 0.699 |
| DIS  | 0.284  | 0.201 | 0.148 | 0.136 | 0.077 | 0.845 | 0.270 | 0.156 | 0.133 | 0.134 | 0.072 | 0.765 |
| GS   | 0.107  | 0.086 | 0.058 | 0.142 | 0.280 | 0.674 | 0.080 | 0.076 | 0.036 | 0.115 | 0.217 | 0.524 |
| HD   | 0.232  | 0.082 | 0.094 | 0.205 | 0.077 | 0.689 | 0.196 | 0.053 | 0.076 | 0.163 | 0.063 | 0.552 |
| HON  | 0.069  | 0.411 | 0.080 | 0.118 | 0.099 | 0.777 | 0.058 | 0.352 | 0.079 | 0.105 | 0.085 | 0.678 |
| IBM  | 0.422  | 0.137 | 0.118 | 0.075 | 0.038 | 0.789 | 0.362 | 0.116 | 0.100 | 0.055 | 0.032 | 0.665 |
| JNJ  | 0.284  | 0.182 | 0.147 | 0.124 | 0.096 | 0.832 | 0.249 | 0.158 | 0.122 | 0.106 | 0.075 | 0.710 |
| JPM  | 0.256  | 0.126 | 0.130 | 0.174 | 0.119 | 0.805 | 0.226 | 0.098 | 0.107 | 0.134 | 0.093 | 0.658 |
| KO   | 0.280  | 0.154 | 0.218 | 0.123 | 0.122 | 0.897 | 0.173 | 0.170 | 0.167 | 0.136 | 0.104 | 0.751 |
| MCD  | 0.460  | 0.089 | 0.088 | 0.113 | 0.047 | 0.796 | 0.423 | 0.063 | 0.055 | 0.072 | 0.030 | 0.643 |
| MMM  | 0.386  | 0.140 | 0.152 | 0.090 | 0.053 | 0.821 | 0.339 | 0.102 | 0.098 | 0.069 | 0.033 | 0.641 |
| MRK  | 0.367  | 0.103 | 0.123 | 0.128 | 0.087 | 0.808 | 0.296 | 0.088 | 0.111 | 0.126 | 0.067 | 0.688 |
| MSFT | 0.103  | 0.249 | 0.171 | 0.172 | 0.141 | 0.836 | 0.103 | 0.198 | 0.145 | 0.149 | 0.131 | 0.726 |
| NKE  | 0.259  | 0.187 | 0.120 | 0.174 | 0.118 | 0.857 | 0.207 | 0.165 | 0.118 | 0.160 | 0.121 | 0.771 |
| NVDA | 0.057  | 0.091 | 0.060 | 0.115 | 0.505 | 0.828 | 0.058 | 0.067 | 0.042 | 0.120 | 0.433 | 0.721 |
| PG   | 0.309  | 0.135 | 0.121 | 0.126 | 0.093 | 0.784 | 0.283 | 0.103 | 0.101 | 0.106 | 0.069 | 0.662 |
| SHW  | 0.203  | 0.113 | 0.056 | 0.053 | 0.220 | 0.645 | 0.178 | 0.099 | 0.043 | 0.050 | 0.160 | 0.530 |
| TRV  | 0.275  | 0.109 | 0.087 | 0.181 | 0.049 | 0.702 | 0.356 | 0.063 | 0.025 | 0.093 | 0.044 | 0.582 |
| UNH  | 0.304  | 0.076 | 0.049 | 0.073 | 0.224 | 0.726 | 0.389 | 0.057 | 0.028 | 0.050 | 0.069 | 0.592 |
| V    | 0.414  | 0.106 | 0.118 | 0.107 | 0.063 | 0.807 | 0.319 | 0.077 | 0.097 | 0.089 | 0.055 | 0.638 |
| VZ   | 0.266  | 0.173 | 0.231 | 0.156 | 0.115 | 0.941 | 0.203 | 0.191 | 0.195 | 0.149 | 0.127 | 0.865 |
| WMT  | 0.266  | 0.222 | 0.189 | 0.101 | 0.101 | 0.879 | 0.210 | 0.189 | 0.153 | 0.121 | 0.088 | 0.761 |

This table reports the average daily information shares at the median duration along with the total fraction of the variance of the efficient price change that can be attributed to quote innovations (sum over all five trading venues and denoted as  $R^2$ ). The trading venues are denoted with their ticker symbol: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

Table 4: Panel analysis: tick-time and Hasbrouck information shares, liquidity measures and market characteristics

|                          | tick-time IS         |                      | HIS                  |                      |
|--------------------------|----------------------|----------------------|----------------------|----------------------|
|                          | 10-ms                | 100-ms               | 10-ms                | 100-ms               |
| avgDuration              | −0.033***<br>(0.008) | −0.022*<br>(0.012)   | −0.004<br>(0.003)    | −0.006<br>(0.004)    |
| relTradevol              | 0.414***<br>(0.065)  | 0.516***<br>(0.065)  | 0.184***<br>(0.026)  | 0.198***<br>(0.028)  |
| effectiveSpread          | −0.099*<br>(0.053)   | −0.067<br>(0.056)    | −0.065*<br>(0.035)   | −0.072**<br>(0.035)  |
| depthImbalanceRatio      | 0.004*<br>(0.002)    | 0.001<br>(0.002)     | −0.0004<br>(0.001)   | −0.0004<br>(0.001)   |
| priceImpact              | −0.002<br>(0.009)    | 0.014<br>(0.026)     | −0.012***<br>(0.002) | −0.013***<br>(0.002) |
| ReMeDI                   | 0.028<br>(0.054)     | 0.043<br>(0.065)     | 0.007<br>(0.032)     | 0.005<br>(0.033)     |
| N                        | 0.051***<br>(0.018)  | 0.043**<br>(0.017)   | −0.0002<br>(0.006)   | 0.0003<br>(0.007)    |
| P                        | 0.007<br>(0.006)     | −0.001<br>(0.008)    | 0.001<br>(0.002)     | 0.002<br>(0.002)     |
| T                        | −0.069***<br>(0.017) | −0.090***<br>(0.018) | −0.021***<br>(0.004) | −0.024***<br>(0.005) |
| Z                        | 0.005<br>(0.016)     | 0.009<br>(0.020)     | 0.003<br>(0.003)     | 0.008***<br>(0.003)  |
| Observations             | 9,594                | 9,594                | 9,594                | 9,594                |
| $R^2$                    | 0.274                | 0.285                | 0.504                | 0.545                |
| Adjusted $R^2$           | 0.271                | 0.282                | 0.502                | 0.543                |
| F Statistic (df=10;9554) | 361.219***           | 380.400***           | 972.378***           | 1,144.568***         |

Note: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ .

This table reports the results of a panel analysis, regressing the estimated daily information shares on covariates. The panel specification includes entity fixed effects (stock level) and market fixed effects (indicated with dummy variables and K as the reference group). Standard errors are clustered on stock level. The trading venues are denoted with their ticker symbol as the subscript: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

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