

RENTAL LEASE ARRANGEMENT AS AN INVESTMENT FINANCING VEHICLE: COMPOUND REAL OPTIONS THEORY AND EVIDENCE IN KOREA

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SUMMARY

Chonseï is a popular rental lease arrangement in Korea, similar forms of which are found in other countries. Previous studies acknowledge that chonseï is an informal financing mechanism, whereby a landlord borrows a deposit from the tenant and returns it in full at maturity. However, few studies have examined the role of chonseï from the perspective of landlords (or investors). By building a theoretical bridge between investment option and rental lease option, I show that chonseï promotes investment. Through the landlord default option attached to the lease, the landlord has an incentive to take less marginal benefit from waiting to maturity and brings forward the decision on optimal investment timing. This theory is empirically supported by cointegration models for housing starts in Korea over the period 2005 to 2023, which reveal that housing starts depend on rental lease market conditions.

Key Words: investment option, defaultable rental lease, housing starts, chonseï

I. INTRODUCTION

Chonsei is a popular rental lease arrangement in Korea under which a landlord receives from the tenant an upfront lump-sum deposit for return in full at maturity.¹ The deposit ranges from 40 to 80 per cent of property value and, during the contract period, the landlord receives no regular rental payments. Data from the Ministry of Data and Statistics (MoDS) indicate that, in 2000, chonsei comprised 61.6 per cent of the rental market.²

The rental lease deposit allows chonsei landlords to finance an investment in real property. In Korea, this “gap financing” in housing has received attention not only in the real estate market but also in the political arena, as the financing only fuels housing speculation; an official report from the Ministry of Land, Infrastructure and Transport (MoLIT) released in 2021 indicated that more than 50 per cent of chonsei contracts were for landlords who could achieve homeownership only with the gap (the difference between house value and deposit) financed with the tenant’s deposit.³ From the tenant perspective, previous studies highlight the role of chonsei as an informal savings vehicle (Kim, 2013), a stepping stone to homeownership (Kim and Park, 2016), or an important component of household portfolio (Cho, 2010). The role of chonsei from the landlord perspective, meanwhile, has received little attention in the literature. This paper, therefore, examines the mechanism of chonsei from the perspective of landlords as “gap investors”.

The paper treats a rental lease contract or arrangement as a financing vehicle for investments. Following Moon (2018) and Ambrose and Kim (2003), I consider the value of such lease to include the landlord’s default option. Consequently, I conjecture that the landlord’s option to implement a default strategy interacts with his real property

¹ Landlords around the world use rental lease arrangements similar to chonsei, including bogey and girvi contracts in India (Gilbert, 2003) and antichresis contracts in Bolivia and several civil law countries (Navarro and Turnbull, 2010), including Louisiana in the United States (Slovenko, 1958). In Australia, aged care home residents can pay a lump-sum Refundable Accommodation Deposit (RAD) or a Daily Accommodation Payment (DAP). See the Australian Government’s Department of Health and Aged Care (<https://www.health.gov.au/initiatives-and-programs/residential-aged-care/managing-residential-aged-care-services/managing-accommodation-payments-and-contributions-for-residential-aged-care>, as of April 2025). Such rental lease arrangements, however, are unlike chonsei in that they represent only a minor share of their respective rental markets.

² According to the Korea Housing Survey, in 2019, 58.0 per cent of households lived in their own homes, 15.1 per cent lived in chonsei-financed housing, and 23.0 per cent lived in monthly rental housing, either with deposits (19.7 per cent) or without deposits (3.3 per cent).

³ <https://www.asiae.co.kr/article/2021081517581795118> (as of Sep 1, 2026; in Korean).

investment strategy. By building a real options model, this paper shows that chonsei as a defaultable lease expedites investment timing for landlords as “gap investors”.

The logic behind my theory is that at the maturity of a rental lease, a landlord can avoid bad “states of nature” by making an *ex-post* strategic decision to default on the lease.⁴ The model presented in this paper delivers the insight that making an *ex-ante* investment decision with a defaultable lease attached can depend on such an *ex-post* decision. It is worth noting that this theory can be applied to other investment decisions involving defaultable lease, which can be found in commercial real estate, aviation, and shipbuilding markets.

My model derives the value of a compound option, which differs from a compound option in Geske (1979).⁵ In order to develop a gap financing model, we rely on backward induction strategy. In signing a rental lease contract, in the first stage, a landlord earns the right to default at contract maturity; in the second stage, the landlord determines the optimal timing for investing in a housing unit, which is to be financed through the defaultable lease. As consequence, our landlord has an American investment option on a European rental lease option; the American option model utilizes the models of Capozza and Helsley (1990) and You (2012), and the European option model utilizes the model of Ambrose and Kim (2003).

To support this theory, I provide empirical evidence by assuming the presence of a representative developer, as in Topel and Rosen (1988), Mayer and Somerville (2000), and Miles (2009). The empirical model of housing starts from Q1 2005 through to Q3 2023 and is constructed with housing prices and costs related to housing starts. Taking into account long-run relationships in housing markets, as in Grimes and Aitken (2010) and Gunnewig-Monert and Lyons (2024), I measure gap financing through chonsei deposit by the ratio of chonsei deposit value to house value. Due to potential endogeneity, I also adopt a dynamic ordinary least squares (DOLS) approach as in Stock and Watson (1993) and a fully modified ordinary least squares (FMOLS) approach for robustness check; the results

⁴This paper distinguishes between *ex-ante* and *ex-post* decisions according to the moment at which a chonsei rental lease contract is signed.

⁵ Our theory requires the assumption that all claims, including an investment option and a default option, are tradable in the absence of any market frictions such as taxes, bankruptcy costs, information asymmetry, and investment lags. The assumption is shared by several studies on real option development, such as Williams (1991), Riddiough (1997), and You (2012).

produced by an instrumental variables approach are also provided. The empirical evidence reveals that national housing starts is a function of rental lease market condition.

The remainder of the paper is structured as follows. In Section 2, we review the literature on real options approach to housing development and defaultable rental lease arrangements and in Section 3 we build an investment model that includes a defaultable rental lease. The theoretical model has a closed-form solution for both an investment option value with a defaultable option contract and its trigger value. A numerical example is also provided for comparative statics. In Section 4 we provide empirical evidence for the theory, and Section 5 concludes the paper.

II. RELATED LITERATURE

This section briefly examines two fields of literature: real options in housing investment and chonse rental lease, as the theoretical model of this paper examines the effects of a defaultable rental lease on investment decision for housing investors.

1. Real Options Approach to Housing Investment

The value of an option to invest or develop in land under uncertainty is extensively investigated in the literature. With exogenously determined rent or value, for example, Capozza and Helsley (1990) and Bar-Ilan and Strange (1996) examine optimal policies for exercising an option to invest under uncertainty. This paper argues that the value of such an investment option can include the value of a defaultable rental lease.

You (2012) derives the value of an investment option consisting of a defaultable loan. Upon the completion of a development project, You (2012)'s investor, as a borrower, can get access to debt capital through a loan. In contrast, our housing investor, as a landlord, borrows from the tenant through a defaultable rental lease.

There is extensive empirical research on real options. Proposing a representative developer in the United States housing market as in Topel and Rosen (1988), Mayer and Somerville (2000) suggest that housing starts at the national level can be a function of economic variables such as housing prices and costs associated with construction

activities. Relying on real options theory, Somerville (2001) and Miles (2009) analyze housing starts with a time-series data set. With housing starts, we prove our real options theory by adopting a representative investor's approach, as in Mayer and Somerville (2000), Somerville (2001), Miles (2009), and recently Gunnewig-Monert and Lyons (2024).

2. Chonsei Rental Lease

In a real options framework, we demonstrate Kim (1990)'s argument that a landlord's investment strategy can treat a chonsei deposit as an investment opportunity. Renaud (1989) similarly argued that the chonsei system serves as a vehicle for informal financing between tenants and landlords in Korea's undeveloped housing finance market. Gyourko and Han (1989) and more recently Kim (2013) predicted that in well-functioning housing finance markets such a rental system would fade away; indeed, it is entirely possible that chonsei would have fallen out of use if they no longer benefitted both landlords and tenants.⁶

As mentioned above, this paper highlights the landlord's perspective.⁷ To value chonsei rental lease arrangements, Ambrose and Kim (2003) adopt the Grenadier (1995, 1996) approach to rental lease valuation. Nevertheless, the option to default (Moon, 2018; Park *et al.*, 2023) in chonsei rental lease arrangements presents landlords with a strategic option that needs to be examined further.

III. THEORETICAL MODEL

1. Basic Assumptions

⁶ Previous studies assert that it was financial repression that engendered the chonsei system, and that the development of a well-functioning mortgage market should bring it to its end. Unlike previous studies, this paper does not address why this system emerged or how the market might be reshaped. For the former, please see Shin and Kim (2013) and for the latter, please see Ryu and Kim (2018).

⁷ From the tenant perspective, on the other hand, it is well documented in the housing literature that a chonsei deposit represents a rung on the homeownership ladder (Cho, 2005; and Kim and Park, 2016). Tenants prefer chonsei leases over monthly rental leases (Kim, 2013) as they can save more with an asset-based contract than with a Western-style income-based rental contract. The rental lease arrangement can affect household financial strategies (Cho, 2010). With an exogenous value, again, it is worth noting that previous studies such as Ambrose and Kim (2003) derive the value of a defaultable rental lease recursively.

My model begins with an agent who owns a parcel of land on which he builds a residential building in compliance with local zoning regulations. Such an agent can choose when to start the project, the construction of which is completed immediately with no investment lag. We assume that after completing the project, the agent will operate the residential building as a landlord.⁸ For convenience, the building is a single housing unit. Its value follows a geometric Brownian motion

$$dH(t) = (\mu - \delta)H(t)dt + \sigma H(t)dz(t), \quad (1)$$

where d is difference, $t(>0)$ is the time, μ is the total expected return on the house, δ is rental flows, σ is the volatility of housing returns, and $dz(t)$ is a standard Wiener process. As in Kau and Kim (1994) and Kau *et al.* (1993), the future rental flows in equation (1) are a constant proportion of $H(t)$.

The model derives the value of an investment option with a rental lease. By investing cost k , an agent owns a unit to be leased according to a chonseil arrangement. The agent receives upfront deposit D , which is to be determined, where $D \leq k$.⁹ The deposit is to be fully returned to the tenant at the termination of the arrangement. The maturity of such an arrangement is legally determined by $T > t$;¹⁰ such an agent may default on the rental lease if $H(T) \leq D$, as in Ambrose and Kim (2003) and Moon (2018).¹¹

The optimal timing of an investment decision can be analyzed by backward induction, as in Wang and Zhou (2006) and Yao and Pretorius (2014). In the first stage, we derive the value of the deposit by assuming that we know the optimal timing of the investment τ , where $0 < \tau < T$ and where $T - \tau$ is 2 years based on the Housing Lease Protection Act. Given the value of the deposit, in the second stage, we derive the value of an investment option and the optimal trigger $H(\tau)$.

⁸ The model produces the same implications as long as a housing unit is traded at fair market value in the market with no friction, even though the developer, the landlord, and the investor are distinct agents.

⁹ In a development project, the value of a housing unit is known after the completion of the project. Taking into account the value of such a unit, market participants negotiate its rental value. This sequential process seems to be reasonable for new developments.

¹⁰ Before July 2020, the legal maturity of a chonseil rental lease contract was 2 years. Starting in July 2020, however, such a contract can be extended once for another 2 years if the tenant wants to do so. Nonetheless, we treat T as two years, as our dates were collected when legal maturity was 2 years.

¹¹ Such a decision is considered rational in the mortgage literature, as in Kau *et al.* (1993) and Kau and Kim (1994).

We work in a risk-neutral world by replacing equation (1) with

$$dH(t) = (r - \delta)H(t)dt + \sigma H(t)dz(t), \quad (2)$$

where $r (\geq \delta)$ is the fixed risk-free interest rate, by following Buttimer *et al.* (2008) and Grenadier (1995) on the lease market and Ambrose *et al.* (1997), Kau and Keenan (1996), Hilliard *et al.* (1998), and Bardhan *et al.* (2006) on the mortgage market.¹²

1. The Value of a Chonsei Rental Lease Arrangement

The value of a rental lease arrangement is the sum of the expected discounted service flows until maturity. At origination, the value with known $H(\tau)$ is

$$\begin{aligned} Y(H(\tau), \tau, T - \tau) &= \int_{\tau}^T e^{-r(t-\tau)} E_{\tau} (\delta H(t)) dt \\ &= \int_{\tau}^T (e^{-r(t-\tau)} \delta H(\tau) e^{(r-\delta)(t-\tau)}) dt = \int_{\tau}^T (\delta H(\tau) e^{-\delta(t-\tau)}) dt \\ &= (1 - e^{-\delta(T-\tau)}) H(\tau), \end{aligned} \quad (3)$$

as in Majd and Pindyck (1987: 11) and Grenadier (1995, 1996). In equation (3), as $T \rightarrow \infty$, $Y(H(\tau), \tau, T - \tau) = H(\tau)$; if a tenant stays for $T = \infty$, the value of the lease is the same as the value of the house. The value of an asset at τ can be divided into the use of the asset for the contract period: $T - \tau$ and the discounted value of the expected house value:

$$H(\tau) = Y(H(\tau), \tau, T - \tau) + e^{-r(T-\tau)} E_{\tau}(H(T - \tau)) = \int_{\tau}^T e^{-r(t-\tau)} E_{\tau}(\delta H(\tau)) dt + e^{-r(T-\tau)} E_{\tau}(H(T - \tau)), \quad (4)$$

as in Dixit and Pindyck (1994: 124). The value of a chonsei rental lease arrangement D is included in

¹² See Kau *et al.* (1994: 540-542) and Hilliard *et al.* (1998: 434) for details on deriving equation (2) from equation (1).

$$Y(H(\tau), \tau, T - \tau) = D (1 - e^{-r(T-\tau)}) + e^{-r(T-\tau)} E_{\tau}[\max\{D - H(T - \tau), 0\}], \quad (5)$$

as the agent has an implicit put option to transfer the property to the tenant, as represented by the second term of the right-hand side of equation (5) (Ambrose and Kim, 2003). Therefore, chonse tenants are considered to be in an “economically weak position” (Ryu and Kim, 2018: 414).¹³ Moon (2018) proposes that the foreclosure process requires a court decision; after the court auction, according to the Civil Execution Act amended in 2003, landlords are not allowed to redeem a property.¹⁴

By equating equation (3) to equation (5), we can derive the value of a chonse rental lease arrangement

$$D = H(\tau) - [e^{-\delta(T-\tau)} H(\tau) \Phi(\omega) - e^{-r(T-\tau)} D * \Phi(\omega - \sigma\sqrt{T-\tau})] \quad (6)$$

where

$$\omega = [\ln(H(\tau)/D) + (r - \delta + \sigma^2/2)(T - \tau)] / \sigma\sqrt{T - \tau}, \quad (7)$$

where Φ is the standard cumulative normal distribution function. The proof is shown in Appendix 1 A).

The second term on the right-hand side of equation (6) is the value of a call option on a dividend-paying asset. The value of the deposit in equation (6) is the sum of the long position in a housing asset and the short position in a call option, such that the agent has a right to take the asset back if the value of the asset exceeds the value of the deposit at maturity.¹⁵

¹³ Our theory assumes no costs related to default, which is a common assumption, as in Ambrose *et al.* (1997), Riddiough (1997), and Adkins and Paxson (2017). Considering default costs would make our model more complicated, even though its implications would remain invariant.

¹⁴ The purpose of the Civil Execution Act is to “prescribe the procedures for compulsory execution, an auction to exercise a security right, an auction.” As a court decision is final and conclusive, the old landlord is not allowed to revoke the decision; no call option in Ambrose and Kim (2003) is embedded in the default option by the amendment of the Civil Execution Act. A call option embedded in the default option can be incorporated into our model. Nonetheless, this would require a numerical analysis, as we fail to provide a closed-form solution for both a default strategy and an investment strategy with a defaultable rental lease. The solution helps us understand the implications of the theory more clearly.

¹⁵ Shin and Kim (2013) propose that a chonse rental lease contract can be interpreted as a financial arrangement in which by transferring a house as collateral, the landlord holding a right to repurchase the house borrows from the tenant. This concept of a “housing repurchase agreement or housing repo” is reflected in equation (6).

Proposition 1. The D has a unique solution.

The proof of Proposition 1 is shown in Appendix 1. Equations (6) and (7) provide the closed-form solution of the value of the deposit.¹⁶ Robichek and Van Horne (1967) propose that capital budgeting needs to consider possible abandonment; their argument is reflected in the second term of the right-hand side of equation (5). With known $H(\tau)$, we can derive the value of the deposit; immediately after completing an investment project, D is determined at fair market value with no sources of market friction.

Corollary 1.

1) With an increase in δ , the equilibrium D increases; 2) with an increase in σ , the equilibrium D decreases; and 3) with an increase in r , the equilibrium D decreases.

See Appendix 1 C) for the proof.

It is straightforward to show the effects of δ on D , as the size of the deposit increases with an increase in δ . With an increase in σ , the value of the option increases and, with given $H(\tau)$, D decreases as the value of the call option increases. An increase in r leads to an increase in the value of the call option, which reduces the value of such an arrangement.

2. The Value of an Investment Option under a Chonse Rental Lease Arrangement

Now suppose an agent determines when to invest in a housing unit that is to be rented through a defaultable rental lease in equation (6).

¹⁶ In addition to the proofs of equations (6) and (7) that appear in the appendix, the put-call parity relationship in Guo and Su (2006) can be used for the derivation of those equations.

1) Analytic Results

The value of an investment option $F(t)$ that an agent holds satisfies the partial differential equation

$$\frac{1}{2}\sigma_H^2 H^2 F''(H) + (r - \delta)HF'(H) - rF(H) = 0. \quad (8)$$

The general solution to equation (8) is $F(H) = \Omega_1 H^\beta + \Omega_2 H^\gamma$, where both Ω_1 and Ω_2 are to be determined. As in Dixit and Pindyck (1994:142-144), $\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + 2\frac{r}{\sigma^2}} > 1$ and $\gamma = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + 2\frac{r}{\sigma^2}} < 0$, where $\frac{\partial\beta}{\partial r} < 0$, $\frac{\partial\beta}{\partial\delta} > 0$, and $\frac{\partial\beta}{\partial\sigma} < 0$.

Three boundary conditions are required: initial, value-matching, and smooth-pasting. The initial condition $\lim_{H \rightarrow 0} F(H) = 0$ leads to $F(H) = \Omega_1 H^\beta$. The value-matching condition is

$$F(H(\tau)) = \int_{\tau}^{\infty} e^{-\mu(t-\tau)} E_{\tau} (\delta H(t)) dt - k, \quad (9)$$

where k is the investment cost, which is assumed not to be lower than the amount of chonse deposit, documented in Ryu and Kim (2018, 411) to range from 40 to 80% of $H(\tau)$.

The smooth-pasting condition is

$$\partial F(H(\tau))/\partial H = \partial \left[\int_{\tau}^{\infty} e^{-\mu(t-\tau)} E_{\tau} (\delta H(t)) dt - k \right] / \partial H. \quad (10)$$

At the optimal trigger level $H(\tau)$, when the option in equation (8) is to be exercised, the agent receives D , which is to be returned to the tenant at $T - \tau$. Consequently, the agent has an incentive to take the less marginal benefits of waiting further, as he takes no account of uncertainty transferred through the defaultable rental lease. The option protects the housing investor from bad states of nature at the maturity of the lease.

When assuming the abovementioned conditions, the optimal trigger is

$$H(\tau) = [\beta / (\beta - e^{-\delta(T-\tau)} \Phi(\omega))] k. \quad (11)$$

See Appendix 1 D) for the details.

Proposition 2. There exists a unique solution that generates the optimal trigger $H(\tau)$.

The proof of Proposition 2 is shown in Appendix 1.

The value of a development option is

$$F(H) = (H(\tau) - k) * [H / H(\tau)]^\beta, \quad (12)$$

where we take $H(\tau)$ as in equation (11). As a result, we can prove Proposition 3.

Proposition 3. An agent exercises an investment option earlier under a chonse rental lease arrangement than he would with no chonse rental lease arrangement.

The proof of Proposition 3 is shown in Appendix 1.

In an uncertain business environment, the optimal trigger with no defaultable rental leases is $H(\tau_n) = [\beta / (\beta - 1)]k$. This paper shows that with a defaultable rental lease, $H(\tau)$ is lower than $H(\tau_n)$.¹⁷ This theory is similar to those proposed in Williams (1991), You (2012), and Adkins and Paxson (2017). Note that You (2012) proposes an investor with a default option on a loan, while the others consider an investor with an abandonment option. Those options are clearly distinguished from a default option on a rental lease that our agent owns.

Corollary 2.

1) With an increase in δ , the equilibrium $H(\tau)$ decreases; 2) with an increase in r , the equilibrium $H(\tau)$ increases; and 3) with an increase in k , the equilibrium $H(\tau)$ increases.

The proof of Corollary 2 is in Appendix 1.

¹⁷ Please note that the difference between the two investment timings may be significantly large, even though the difference between the trigger values is very small.

Our agent is more likely to start a project at the higher level of δ , as future rental flows increase. In addition, with an increase in δ the equilibrium D increases, as shown in Corollary 1, and the amount of net investment ($k - D$) that the agent requires increases. With an increase in r , the present value of the rental flows decreases and our agent has an incentive to wait longer for a project, also with the larger amount of the net investment. With an increase in k , in addition, the agent is more likely to hesitate to start an investment project.

2) Numerical Simulations

An advantage of the closed-form solution in equation (11) is that the solution may require no numerical analyses. Nevertheless, Appendix 2 includes a numerical analysis of Propositions 2 and 3. The average quarterly 1-year bond rate between Q1 2001 and Q1 2020 in Korea ranges from 6.03 per cent to 1.18 per cent with an average of 3.31 per cent. We assume that δ and σ are 3.0 per cent and 1.0 per cent, respectively; it is unfortunate that δ and σ cannot be measured with market data. In addition, k is assumed to be 100 and the investment cost is financed through a 2-year chonseil arrangement; according to the real options literature, an investment trigger value that we expect is higher than the cost.¹⁸ The first graph shows that the optimal trigger under the rental lease $H(\tau)$ is determined at the intersection of $f(H)$ and $g(H)$, both of which are defined in Appendix 1 B). This trigger is also lower than the trigger with no chonseil, as indicated by the red circle in the first graph in Appendix 2 A).

The literature on real options has widely held that as uncertainty increases an agent delays an investment decision, which can be proved by $\frac{\partial H(\tau_n)}{\partial \sigma} = \frac{-1}{(\beta - 1)^2} \frac{\partial \beta}{\partial \sigma} > 0$. However, we cannot prove this relationship analytically for the trigger with a defaultable rental lease arrangement in equation (11).¹⁹ Therefore, the numerical relationship between $H(\tau)$ and σ is shown in the second graph in Appendix 2 B), where $H(\tau)$ increases with an increase in σ . As is the case with other models, my model shows that an agent (who depends on a defaultable rental lease) delays investment with greater uncertainty.

¹⁸ Using a backward induction methodology, we derive $H(\tau)$ with known D . For our numerical analysis, nevertheless, D must be assumed.

¹⁹ The sign of $\frac{\partial g(H)}{\partial \sigma} = \frac{e^{-\delta(\tau-\tau)} \Phi(\omega)}{\beta^2} \frac{\partial \beta}{\partial \sigma} - \frac{e^{-\delta(\tau-\tau)} \varphi(\omega)}{\beta} \frac{\partial \omega}{\partial \sigma}$ is undetermined, as we have no analytic information indicating the actual values of both $\frac{\Phi(\omega)}{\beta} \frac{\partial \beta}{\partial \sigma}$ and $\varphi(\omega) \frac{\partial \omega}{\partial \sigma}$.

IV. EMPIRICAL EVIDENCE

In the previous chapter, my theory is that a defaultable rental lease hastens an agent's housing investment decision. Following Mayer and Somerville (2000), Somerville (2001) and Miles (2009), we build an empirical model of housing investment using time-series data on nationwide housing starts. In Korea, an advantage of these statistics is that the Housing Act (Article 88) requires all the housing starts to be officially reported to the MoLIT.

1. Empirical model

Our empirical analysis uses quarterly time-series data of all housing unit starts from Q1 2005 through to Q3 2023.²⁰ Housing starts is a function of housing prices and costs related to housing supply such as cost of land, construction costs, and interest rates.²¹ In an equilibrium urban growth model, as in Capozza and Helsley (1989) and Rosenthal and Helsley (1994), housing prices have a long-run relationship with supply costs. Grimes and Aitken (2010) propose, among those variables, the existence a cointegration relationship, which is also found in several countries by Winkler (2016) and Gunnewig-Monert and Lyons (2024).

To analyze housing supplies, following the latter two studies, I adopted a DOLS approach, as in Stock and Watson (1993). The logarithm of the number of national housing starts is

$$\text{LH}_t = \alpha_0 + \beta X_t + \sum_{j=-k_1}^{k_2} \theta_j \Delta X_{t+j} + e_t, \quad (13)$$

²⁰ Although new housing supply may include both housing starts and housing permits, with the former a coincidental indicator according to the MoLIT, and the latter a leading indicator, housing starts seem more appropriate for our analysis. Even though both housing permits and housing starts can proxy for new housing supply (Somerville, 2001; Miles, 2009) some housing permits can be cancelled; for official statistics in Korea, some cancelled permits appear to be included in the number of housing permits. Note that housing units include condominiums, multi-unit buildings, single-family detached houses, and other types of housing.

²¹ Our use of housing starts is distinguished from other housing supply proxies such as capital investments (Caldera and Johansson, 2013; Winkler, 2016), the growth of existing stock (Ball et al., 2010), and supply constraints (Glaeser *et al.*, 2006).

where X includes a vector of regressors that include housing prices, the cost of land, construction costs, and interest rates, as in <Table 1>. β and θ_j are the vectors of coefficients and α_0 is a constant term, while k_1 and k_2 are the lag and the lead, respectively, Δ is the differential, and e_t is the residual.²²

My theory indicates that we can expect more housing starts, as landlords are more likely to finance through chonseil deposit as shown in Proposition 3. The likelihood of an investor's default on a rental lease increases with an increase in the amount of rental lease deposit. At the same time, the risks that the investor takes by starting a housing project decrease because when facing bad states of nature, the investor can transfer the leased house to the tenant.

It is worth noting that the value of a house and the value of its chonseil deposit are correlated; simply implementing both values, therefore, our empirical model is vulnerable to an issue of the multicollinearity. In equation (13), therefore, I measure gap financing by the ratio of chonseil deposit value to condominium value, widely known as Chonseil Deposit Rate (CDR). This ratio has been released by KB Kookmin Bank (KB Bank) since December 1998.

Again, we build a representative developer model of housing starts that includes the CDR. Owing to investment lag (Bar-Ilan and Strange, 1996), we may set aside the issue of endogeneity.²³ There are far fewer housing unit starts than the existing units stock;²⁴ real estate agents affiliated with the KB Bank provide chonseil information on existing units but do not report predicted values for units under construction. Moreover, we use the CDR for condominiums, even though housing starts includes all types of residential housing units.²⁵ As in Gunnewig-Monert and Lyons (2024), we also perform an instrumental variables regression analysis using a two-stage least square (2SLS) method.

²² The optimal k_1 and k_2 are determined automatically based on Schwarz information criterion (SIC) with maximum lags of 3 (i.e. within a year).

²³ A CDR for a new development is determined when the development is completed. For general condominium buildings, it is known in Korea that the development lag between start and completion is at least two and a half years.

²⁴ The average number of housing unit starts is 107 K from Q1 2005 to Q3 2023 in <Table 2>, while the number of existing condominium units is 9,806 K according to the 2015 Population and Housing Census of MoDS.

²⁵ Its affiliated real estate agents provide the bank with assessed chonseil deposits of the condominium and the assessed value of the same condominiums. With those figures the bank announces the ratio, considering the distribution of condominium dwellings at the national level according to the National Census. Too many observations are missed if the CDR "for all types of housing units," which is available from Q2 2011, is used.

2. Data and Their Pretests

1) Variables and definitions

LH_t in equation (13) is the logarithm of the number of quarterly housing unit starts (HS), which is the sum of monthly housing unit starts released by the MoLIT and MoDS.²⁶ The regressors are housing prices (HP), land costs (LP), construction costs (CC), and the rate of negotiable certificate of deposit (CD).²⁷ HP, LP, CC, and CD are converted to real terms with the consumer price index (CPI). The variables are drawn from KB Bank, the Korea Appraisal Board, the Korea Institute of Civil Engineering and Building Technology, the Bank of Korea (BoK), and MoDS, as in <Table 1>. Except for the real CD rate (RCD), the real-term variables are converted to logarithmic variables: LHP for real housing prices, LLP for real land costs, and LCC for real construction costs.

<Table 2> shows that the average HS is 107.8 thousand during the sample period, varying from a minimum of 30.0 thousand in Q1 2009 to a maximum of 254.5 thousand in Q4 2015. Over the 75 quarters running from Q1 2005 to Q3 2023, the average CDR is 64.1 per cent, varying from a minimum of 52.4 per cent in Q1 2009 to a maximum of 75.7 per cent in Q3 2017.²⁸ HP, LP, CC, and CPI are an index; the CD rate averages 2.72 per cent.

From the demand side, collected are instrumental variables such as monthly household disposable income (HINC), ratio of the population to households (HHS), and the mortgage outstanding balance (MORT) as credit market condition; their summary statistics are in <Table 2>. Those instrumental variables are suggested by Gunnewig-Monert and Lyons (2024), who propose that house supply seems not to be a direct function of those instrumental variables (Glaeser *et al.*, 2005). Due to a structural change of the Household Income and Expenditure Survey, HINC (in real terms) is available only from the year 2006. Quarterly HHS captures only an annual trend of

²⁶ The official HS statistics are publicly available from 2011 and our analysis includes unofficial MoLIT statistics from Q1 2005. The supply statistics released by the MoLIT according to the Housing Act are more reliable than other statistics produced by private institutions, such as R114 (private real estate market information provider), which takes little account of housing units supplied by public agencies.

²⁷ Similar to the now-discontinued London Interbank Offered Rate (LIBOR) in international real estate markets, the CD rate plays the role of a benchmark rate for development financing in Korean real estate markets.

²⁸ The CDR of 63.7 per cent implies that, for example, if the value of a condominium is KRW 100 million, the chonseil deposit the owner receives by renting the same condominium through a 2-year chonseil contract is KRW 637 million.

demographics, as the two estimates are available on an annual basis. In addition, the BOK Economic Statistics System has released the outstanding mortgage loan balance since Q4 2007.²⁹

2) Pretests of the data

To determine the stationarity of time-series data, I run the augmented Dickey–Fuller (ADF) test, based on

$$\Delta y_t = \alpha_u + \rho y_t + \sum_{j=1}^k \theta_j \Delta y_{t-j} + e_t, \quad (14)$$

where y_t is each variable in the model, α_u is the constant term, ρ is the autoregressive coefficient and k is an optimal lag to be determined by the Schwarz information criterion (SIC). The null hypothesis of a unit root is $H_0: \rho = 1$; <Table 3> shows the results. LS is I (1), which is similar to Miles' (2009) results with US monthly housing starts and to Gunnewig-Monert and Lyons' (2024) results with Ireland quarterly permissions and completions. Except for RCD, the four regressors (CDR, LHP, LLP, and LCC) are integrated of order one: I(1). Using EViews statistical package, I performed a cointegration test of Johansen (1991), which produced the same results as expected; <Table 4> reveals the existence of a cointegration relationship.³⁰

3. Main Empirical Results

1) Basic DOLS regression

With a cointegration relationship found in the previous section, I use the DOLS regression model with equation (13). <Table 5> delivers our empirical regression results. Model 1 is the base model with four covariates, which are considered in the existing literature. Models 2–6 include the key covariate, CDR, which is salient for my theory.

For Model 1, the estimated coefficient of LHP is positive and the other estimated coefficients are negative; these base model results are as expected. The estimated price elasticity of the new housing supply is larger than one unit;

²⁹ Instead of the balance of newly originated loans, the outstanding balance of all mortgage loans in Korea is used. During a certain period of some presidential terms in Korea, moreover, both loan to value ratio and new origination of mortgage loan are directly and tightly controlled by financial supervisory bodies. On the other hand, market information on LTV ratio, used as an instrument variable in Gunnewig-Monert and Lyons (2024), is limited in Korea.

³⁰ The cointegration test of Engle and Granger (1987) also supports the existence of cointegration in the Korean housing market. The results can be provided upon request.

a 1 per cent increase in real house prices is associated with a 7 per cent increase in the number of housing unit starts.³¹ Phang *et al.* (2010) find that the price elasticities of housing starts tend to be higher than the price elasticities of changes in the housing stock. As predicted in Corollary 2, the estimated coefficient of LLP and the estimated coefficient of LCC are negative; the former is statistically significant at the 1 per cent level of significance, while the latter is not statistically significant.

2) The effects of chonse market condition on housing starts

To see the effects of rental market condition on housing unit starts, Model 2 includes CDR. A 1 per cent increase in the CDR leads to a 0.03(-0.02) per cent increase in the number of housing starts. These results support our Proposition 3.

In a real options model, uncertainty matters. Using a GARCH (generalized autoregressive conditional heteroskedasticity) model, as in Bulan *et al.* (2009), we propose a proxy for the volatility of LHP. In Appendix 3 we explain the process of estimating $\hat{\sigma}_{\text{lh}}^2$. As predicted by real options theory, the estimated coefficient of $\hat{\sigma}_{\text{lh}}^2$ is negative; its sign supports the numerical analysis we present in Appendix 2.

During the sample period, moreover, the housing market suffered through the Global Financial Crisis (GFC). To stimulate dampened housing markets during this time, the Korean government proposed several policy measures. For example, in June through November 2008 it announced seven policy packages (Kim and You, 2020). In Model 4, nevertheless, the estimated coefficient of GFC is negative. However, it is worth noting that the estimated coefficient of $\hat{\sigma}_{\text{lh}}^2$ is statistically significant at the 1 per cent level. From 2020 to 2021, in addition, the global economy went through the COVID-19 pandemic and thus our housing supply model may need to control for the pandemic effects with a time dummy variable. In its wake, we observed more housing starts and other advanced housing markets soared (Igan *et al.*, 2022).

³¹ Oh (2018) estimates that the price elasticity of housing starts in Korea is around 5, even though the price elasticity of residential investments is lower than 1. By focusing on the value of national investments in the residential sector, her model can be distinguished from our model. Malpezzi and Maclennan (2001) propose that the price elasticity of housing starts in the United States ranges from 4 to 16 and that of housing completions in the United Kingdom ranges from 0 to 4.

In Model 6, lastly, we follow an instrumental variable approach of Gunnewig-Monert and Lyons (2024). In the first stage, we estimate housing demand equation, where LHP is a function of instrumental variables: LHINC, HHS, LMORT. In the second stage, estimated LHP is used for Model 6, which produces results similar to previous models.

3) Robustness test: Fully modified ordinary least squares (FMOLS)³²

With the cointegration relationship in the previous subsection, I implemented a parametric DOLS model, which can manage endogeneity efficiently. In the presence of a cointegrating relationship, this subsection adopts a semiparametric approach: FMOLS, which adjusts for the effects of serial correlations and endogeneity (Phillips, 1995; Phillips and Hansen, 1990). The key variable is CDR, whose estimated coefficients under Models 2–6 are positive and statistically significant at the 1 per cent level.

V. CONCLUSION

The benefits of chonse-i-type rental lease from the tenant perspective are well documented in the literature. Nevertheless, a chonse-i rental lease includes an option to default, the effects of which have rarely been examined from the landlord's perspective. By building a theoretical bridge between American investment option and European rental lease option, this paper examines the optimal timing of investment in a housing asset, which will be rented through a defaultable rental lease. In particular, we propose a compound real options theory and find its evidence in the Korean housing market by arguing that housing supply can depend on market conditions for defaultable chonse-i rental lease. The theory of this paper can be applied in other markets to a general decision on investment consisting of a defaultable rental lease arrangement. Adopting cointegration models used in international housing markets in the literature, moreover, our empirical models strongly support that gap financing through chonse-i deposit, measured by the ratio of chonse-i deposit value to house value, promotes housing starts in Korea.

³² In addition to the housing unit starts used in our empirical model, residential building starts have also been publicly released by the MoLIT since Q1 2000. A previous version of this paper analyzed the official statistics of residential building starts as LH_t in equation (13). The empirical results, which are available upon request, are invariant with empirical results of this paper.

This paper provides policy implications for emerging housing markets. Chonsei-type rental lease arrangements, which are accepted in several civil law countries, can produce direct effects on housing investment. From the housing policy perspective in Korea, these effects are often overlooked. In international housing markets, moreover, this paper offers the implication that housing supply can be determined by rental lease market condition, which needs to be examined further in future research.

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<Table 1> Quarterly Variables and Their Sources

	Description	Source
HS	Residential housing unit starts	MoLIT
CDR	Ratio of chonse deposit to value of condominium	KB Bank
HP	House price index	KB Bank
LP	Land price index	Korea Appraisal Board
CC	Construction cost index (residential building)	Korea Institute of Civil Engineering and Building Technology
CD	Negotiable certificate of deposit rate	BoK
CPI	Consumer price index	MoDS
HINC	Monthly household disposal Income (real value as of 2020)	MoDS
EP	(Estimated) Population	MoDS
EH	(Estimated) Number of households	MoDS
HHS	EP/EH	
MORT	Outstanding balance of mortgage loan	BoK
GFC	GFC dummy (1 if $2018 \leq \text{year} \leq 2010$, 0 otherwise)	
COVID	Covid dummy (1 if $2020 \leq \text{year} \leq 2021$, 0 otherwise)	
LH	Log(HS)	
LCR	Log(CDR)	
LHP	Log(HP/CPI)	
LLP	Log(LP/CPI)	
LCC	Log(CC/CPI)	
RCD	CD rate - annual % change in CPI	
LHINC	Log(HINC)	
LMORT	Log(MORT/CPI)	

<Table 2> Summary Statistics: from Q1 2005 through Q3 2023

	Obs.	Mean	Max.	Min.	Std. Dev.	Unit
HS	75	107,834.9	254,515	30,067	47,972.2	No. of housing unit starts
CDR	75	64.1	75.7	52.4	7.5	%
HP	75	73.6	100.9	51.9	12.3	100 as of Jan 2022
LP	75	77.6	100.3	60.4	11.4	100 as of Oct 2022
CC	75	101.6	152.8	69.9	22.4	100 as of 2015
CD	75	2.72	5.69	0.65	1.3	%
CPI	75	92.5	112.1	73.9	9.9	100 as of 2020
HINC	71	3.438.2	3,672.9	3,123.7	123.5	KRW thousand (real value as of 2020)
HHS	75	2.69	3.00	2.34	1.90	Population/households ¹⁾
MORT	64	148.9	52.7	283.3	74.1	KRW trillion

Note 1) Annual numbers are used.

<Table 3> Augmented Dickey–Fuller Test Results

		Lag ¹⁾	t-statistics	Prob. ²⁾
LH	Level	8	-1.64	0.46
	1st Difference	7	-1.80	0.06*
CDR	Level	2	-1.47	0.55
	1st Difference	1	-3.58	0.00***
LHP	Level	1	-2.19	0.20
	1st Difference	0	-4.71	0.00***
LLP	Level	2	-0.76	0.38
	1st Difference	1	-3.24	0.00***
LCC	Level	1	1.53	0.96
	1st Difference	0	-5.88	0.00***
RCD	Level	0	-1.99	0.04**
	1st Difference	2	-8.47	0.00***

Note: 1) The number of optimal lags is determined based on Schwarz information criterion (SIC)

2) *** p<0.01, ** p<0.05, * p<0.1

<Table 4> Cointegration Test: Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.
None *	0.411179	116.2924	95.75366	0.0009
At most 1 *	0.352369	78.15880	69.81889	0.0093
At most 2	0.243013	46.87959	47.85613	0.0616
At most 3	0.213013	26.83415	29.79707	0.1058
At most 4	0.122256	9.587023	15.49471	0.3139
At most 5	0.002749	0.198169	3.841466	0.6562

Note: 1) Trace test indicates 2 cointegrating eqn(s) at the 0.05 level

2) * Denotes rejection of the hypothesis at the 0.05 level

<Table 5> Cointegration Regression Results: Dynamic Ordinary Least Squares (DOLS)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
CDR		0.03***	0.03***	0.02***	0.02***	0.02**
		(0.01)	(0.00)	(0.00)	(0.00)	(0.01)
LHP	7.08***	6.04***	7.26***	5.79***	7.47***	14.01***
	(2.07)	(1.84)	(1.55)	(1.26)	(0.97)	(4.63)
LLP	-5.40***	-1.21	-2.90**	-0.87	-2.92***	-0.20*
	(0.98)	(1.71)	(1.35)	(1.21)	(0.97)	(2.16)
LCC	-0.87	-3.24**	-1.77*	-2.72***	-3.54***	-8.83**
	(1.12)	(1.25)	(1.03)	(0.84)	(0.64)	(3.33)
RCD	-0.01	-0.00	0.09**	-0.02	-0.00	-0.13**
	(0.05)	(0.04)	(2.31)	(0.04)	(0.03)	(0.05)
$\hat{\sigma}_{lhp}^2$			-2.17	-4.97***	-2.81*	
			(1.98)	(1.82)	(1.44)	
GFC				-0.37***	-0.22**	
				(0.13)	(0.10)	
COVID					0.41***	
					(0.14)	
C	12.19***	10.59***	10.67***	11.4***	11.0***	13.8***
	(0.49)	(0.45)	(0.61)	(0.59)	(0.43)	(1.79)
Obs	75	75	75	75	75	61
R ²	0.749	0.812	0.864	0.887	0.903	0.849

Note 1) To measure $\hat{\sigma}_{lhp}^2$, for Models 3-5, we adopt a GARCH (generalized autoregressive conditional heteroskedasticity) model.

2) For Model 6, LHP is estimated with instrumental variables of LHINC, HHS, and LMORT.

3) *** p<0.01, ** p<0.05, and * p<0.1.

Standard errors are reported in parentheses.

<Table 6> Robustness Test: Fully Modified Least Squares (FMOLS)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
CDR		0.05***	0.05***	0.03***	0.03***	0.05***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LHP	6.61***	6.03***	6.33***	4.92***	4.66***	5.70**
	(1.68)	(1.32)	(1.43)	(1.12)	(0.99)	(2.58)
LLP	-5.06***	-0.70	-0.93	0.62	-0.42	0.72
	(0.10)	(0.96)	(1.04)	(0.83)	(0.77)	(4.13)
LCC	-0.35	-4.04***	-4.00***	-4.30***	-4.26***	-4.85**
	(0.85)	(0.82)	(0.83)	(0.65)	(0.57)	(1.81)
RCD	-0.07	0.00	0.01	-0.08**	-0.07**	-0.08
	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.05)
$\hat{\sigma}_{lhp}^2$			0.05	-4.72***	-3.44**	
			(1.19)	(1.74)	(1.54)	
GFC				-0.61***	-0.55***	
				(0.12)	(0.10)	
COVID					0.34***	
					(0.09)	
C	12.11***	9.71***	9.75***	11.4***	10.9***	9.92***
	(0.37)	(0.66)	(0.56)	(0.56)	(0.50)	(1.02)
Obs	75	75	75	75	75	63
R ²	0.183	0.595	0.594	0.662	0.678	0.507

Note 1) To measure $\hat{\sigma}_{lhp}^2$, for Models 3-5, we adopt a GARCH model.

2) For Model 6, LHP is estimated with instrumental variables of LHINC, HHS, and LMORT.

3) *** p<0.01, ** p<0.05, and * p<0.1.

Standard errors are reported in parentheses.

[Appendix 1] The Proofs

A) Proof of equation (6)

Following Buchanan (2012), who derives the value of a European put option on an asset that pays a continuous dividend, we can define the value of the put option in equation (5). As equation (3) is equivalent to equation (5),

$$\begin{aligned} H(\tau) - H(\tau)e^{-\delta(T-\tau)} \\ = D(1 - e^{-r(T-\tau)}) + De^{-r(T-\tau)} * \Phi(\sigma\sqrt{T-\tau} - \omega) - H(\tau)e^{-\delta(T-\tau)} * \Phi(-\omega) \\ = D - De^{-r(T-\tau)} * \Phi(\omega - \sigma\sqrt{T-\tau}) + H(\tau)e^{-\delta(T-\tau)}(\Phi(\omega) - 1) \end{aligned}$$

$$\text{where } \omega = \frac{\ln\left(\frac{H(\tau)}{D}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-\tau)}{\sigma\sqrt{T-\tau}}.$$

B) Proof of a unique D

From equation (6), we can define both $f(D) = H(\tau) - D$, where $0 < D < H(\tau)$ and $g(D) = H(\tau)e^{-\delta(T-\tau)}\Phi(\omega) - De^{-r(T-\tau)}\Phi(\omega - \sigma\sqrt{T-\tau})$, which is a call option with the underlying asset-paying dividend.

A unique solution is satisfied by the following four conditions:

- 1) $\frac{\partial g(D)}{\partial D} = H(\tau)e^{-\delta(T-\tau)}\frac{\partial \Phi(\omega)}{\partial D} - De^{-r(T-\tau)}\frac{\partial \Phi(\omega - \sigma\sqrt{T-\tau})}{\partial D} - e^{-r(T-\tau)}\Phi(\omega - \sigma\sqrt{T-\tau})$
 $= H(\tau)e^{-\delta(T-\tau)}\frac{1}{2\sqrt{\pi}}e^{-\frac{\omega^2}{2}}\frac{1}{\sigma\sqrt{T-\tau}}\left(-\frac{1}{D}\right) - De^{-r(T-\tau)}\frac{1}{2\sqrt{\pi}}e^{-\frac{\omega^2}{2}}\frac{H(\tau)}{D}e^{(r-\delta)(T-\tau)}\frac{1}{\sigma\sqrt{T-\tau}}\left(-\frac{1}{D}\right)$
 $- e^{-r(T-\tau)}\Phi(\omega - \sigma\sqrt{T-\tau})$
 $= -e^{-r(T-\tau)}\Phi(\omega - \sigma\sqrt{T-\tau}) > -1.$
- 2) $\frac{\partial^2 g(D)}{\partial D^2} = -e^{-r(T-\tau)}\left\{-\frac{\phi(\omega - \sigma\sqrt{T-\tau})}{D\sigma\sqrt{T-\tau}}\right\} > 0,$
- 3) As $\lim_{D \rightarrow 0^+} H(\tau)e^{-\delta T}\Phi(\omega_\tau) = H(\tau)e^{-\delta T}$, $g(D) = H(\tau)e^{-\delta T}\Phi(\omega_\tau) < H(\tau)$, and
- 4) $g(H(\tau)) > 0.$

C) Proof of Corollary 1: Comparative statics of D

- 1) $\frac{\partial g(D)}{\partial \delta} = -H(\tau)(T-\tau)e^{-\delta(T-\tau)}\phi(\omega) + H(\tau)e^{-\delta(T-\tau)}\frac{\partial \Phi(\omega)}{\partial \delta} - De^{-r(T-\tau)}\frac{\partial \Phi(\omega - \sigma\sqrt{T-\tau})}{\partial \delta}$
 $= -H(\tau)(T-\tau)e^{-\delta(T-\tau)}\phi(\omega) + H(\tau)e^{-\delta(T-\tau)}\frac{1}{2\sqrt{\pi}}e^{-\frac{\omega^2}{2}}\frac{\sqrt{T-\tau}}{\sigma}$
 $- De^{-r(T-\tau)}\frac{1}{2\sqrt{\pi}}e^{-\frac{\omega^2}{2}}\frac{H(\tau)}{D}e^{(r-\delta)(T-\tau)}\frac{\sqrt{T-\tau}}{\sigma}$
 $= -H(\tau)(T-\tau)e^{-\delta(T-\tau)}\phi(\omega) < 0,$
- 2) $\frac{\partial g(D)}{\partial \sigma} = H(\tau)e^{-\delta(T-\tau)}\phi(\omega)\frac{\partial \omega}{\partial \sigma} - De^{-r(T-\tau)}\phi(\omega - \sigma\sqrt{T-\tau})\left(\frac{\partial \omega}{\partial \sigma} - \sqrt{T-\tau}\right)$
 $= H(\tau)\sqrt{T-\tau}e^{-\delta(T-\tau)}\phi(\omega), \text{ and}$
- 3) $\frac{\partial g(D)}{\partial r} = H(\tau)e^{-\delta(T-\tau)}\phi(\omega)\frac{\partial \omega}{\partial r} - De^{-r(T-\tau)}\phi(\omega - \sigma\sqrt{T-\tau})\frac{\partial \omega}{\partial r} + D(T-\tau)e^{-r(T-\tau)}\Phi(\sigma\sqrt{T-\tau} - \omega)$
 $= D(T-\tau)e^{-r(T-\tau)}\Phi(\sigma\sqrt{T-\tau} - \omega).$

D) The derivation of the value of a development option and its trigger value

Upon investment at τ , our agent receives D from the tenant and $H(\tau) = \int_{\tau}^{\infty} e^{-\mu(t-\tau)} E_{\tau} (\delta H(t)) dt = Y(H(\tau), \tau, T - \tau) + \int_{\tau}^{\infty} e^{-\mu(t-\tau)} E_{\tau} (\delta H(t)) dt = D + e^{-\delta(T-\tau)} H(\tau) \Phi(\omega) - e^{-r(T-\tau)} D * \Phi(\omega - \sigma\sqrt{T-\tau})$ from the investment. Signing a chonseil rental lease arrangement, the agent has an option to default at the maturity of the arrangement.

From the value-matching condition, $\Omega_1 = (H(\tau) - k)H(\tau)^{-\beta}$ and from the smooth-pasting condition, $\frac{\partial F(H(\tau))}{\partial H} = \frac{\partial [H - k]}{\partial H} = e^{-\delta(T-\tau)} \Phi(\omega) + e^{-\delta(T-\tau)} H(\tau) \varphi(\omega) \frac{1}{\sigma\sqrt{T-\tau}} \frac{1}{H} - e^{-r(T-\tau)} D * \varphi(\omega - \sigma\sqrt{T-\tau}) \frac{1}{\sigma\sqrt{T-\tau}} \frac{1}{H}$.

Therefore, we know that $\beta(H(\tau) - k) = e^{-\delta(T-\tau)} \Phi(\omega) H(\tau) + \left[e^{-\delta(T-\tau)} \varphi(\omega) \frac{1}{\sigma\sqrt{T-\tau}} H(\tau) - e^{-r(T-\tau)} D * \varphi(\omega - \sigma\sqrt{T-\tau}) \frac{1}{\sigma\sqrt{T-\tau}} \right]$, where $e^{-\delta(T-\tau)} H(\tau) \varphi(\omega) \frac{1}{\sigma\sqrt{T-\tau}} - e^{-r(T-\tau)} D * \varphi(\omega - \sigma\sqrt{T-\tau}) \frac{1}{\sigma\sqrt{T-\tau}} = 0$, which can be proven by $e^{-\delta(T-\tau)} H(\tau) \varphi(\omega) - e^{-r(T-\tau)} D \varphi(\omega - \sigma\sqrt{T-\tau})$

$$\begin{aligned}
 &= e^{-\delta(T-\tau)} H(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - e^{-r(T-\tau)} D e^{-\frac{(\omega - \sigma\sqrt{T-\tau})^2}{2}} \\
 &= e^{-\delta(T-\tau)} H(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - e^{-r(T-\tau)} D e^{-\frac{\omega^2 - 2\omega\sigma\sqrt{T-\tau} + \sigma^2(T-\tau)}{2}} \\
 &= e^{-\delta(T-\tau)} H(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - e^{-r(T-\tau)} D e^{-\frac{\omega^2}{2}} e^{\ln\left(\frac{H(\tau)}{D}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-\tau)} e^{-\frac{\sigma^2(T-\tau)}{2}} \\
 &= e^{-\delta(T-\tau)} H(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - e^{-r(T-\tau)} D e^{-\frac{\omega^2}{2}} \frac{H(\tau)}{D} e^{(r-\delta)(T-\tau)} = 0.
 \end{aligned}$$

We have $\beta(H(\tau) - k) = e^{-\delta(T-\tau)} \Phi(\omega) H(\tau)$, which can be converted to $(\beta - e^{-\delta T} \Phi(\omega)) * H(\tau) = \beta k$. As a result, we derive $H(\tau) = \frac{\beta k}{\beta - e^{-\delta(T-\tau)} \Phi(\omega)}$.

E) Proof of Proposition 2 and Proposition 3

From $H(\tau) = \frac{\beta k}{\beta - e^{-\delta(T-\tau)} \Phi(\omega)}$, we have $\frac{H}{k} = \frac{\beta}{\beta - e^{-\delta(T-\tau)} \Phi(\omega)}$. In order to show a unique solution of equation (11), we can define $f(H) = \frac{k}{H}$, then $\frac{\partial f(H)}{\partial H} < 0$ and $\frac{\partial^2 f(H)}{\partial H^2} > 0$ and also define $g(H) = \frac{\beta - e^{-\delta(T-\tau)} \Phi(\omega)}{\beta}$, then $\frac{\partial g(H)}{\partial H} = \frac{-e^{-\delta(T-\tau)} \varphi(\omega)}{\beta} \frac{1}{\sigma\sqrt{T-\tau}} \frac{1}{H} < 0$ and $\frac{\partial^2 g(H)}{\partial H^2} = \frac{e^{-\delta(T-\tau)} \varphi(\omega)}{\beta} \frac{1}{\sigma\sqrt{T-\tau}}$

$$\frac{1}{H^2} - \frac{e^{-\delta(T-\tau)}}{\beta} \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} (-\omega) \left(\frac{1}{\sigma\sqrt{T-\tau}} \frac{1}{H} \right)^2 > 0.$$

We know $f(k) = 1$ and $g(k) = \frac{\beta - e^{-\delta(T-\tau)} \Phi(\omega_*)}{\beta} < f(k)$ where $\omega_* = \left[\ln(k/D) + (r - \delta + \sigma^2/2)(T - \tau) \right] / \sigma\sqrt{T - \tau}$ and $\beta > 1$, where $k \geq D$.

Our model assumes that the amount of chonseil deposit does not exceed initial investment cost. In the real options literature such as Capozza and Helsey (1990) and Capozza and Li (1994), the value of a house tends to be higher than its initial investment or construction costs, as the value includes the value of irreversible premium, which is equal to equation (12) in our model.

In addition, we know both $f\left(\frac{\beta}{\beta-1}k\right) = \frac{\beta-1}{\beta}$ and $g\left(\frac{\beta}{\beta-1}k\right) = \frac{\beta - e^{-\delta T}\Phi(\omega_{**})}{\beta} > f\left(\frac{\beta}{\beta-1}k\right)$ where $\omega_{**} = \left[\ln\left(\frac{\beta}{\beta-1} * k/D\right) + (r - \delta + \sigma^2/2)(T - \tau)\right]/\sigma\sqrt{(T - \tau)}$. Moreover, it is simple to show that $\lim_{H \rightarrow \infty} f(H) = 0$ but $\lim_{H \rightarrow \infty} g(H) = [\beta - e^{-\delta(T-\tau)}]/\beta$.

As a result, we can conclude Proposition 3 with $k < H(\tau) < \frac{\beta}{\beta-1}k$, as $\frac{\beta}{\beta-1}k$ is the trigger value with no chonse arrangement.

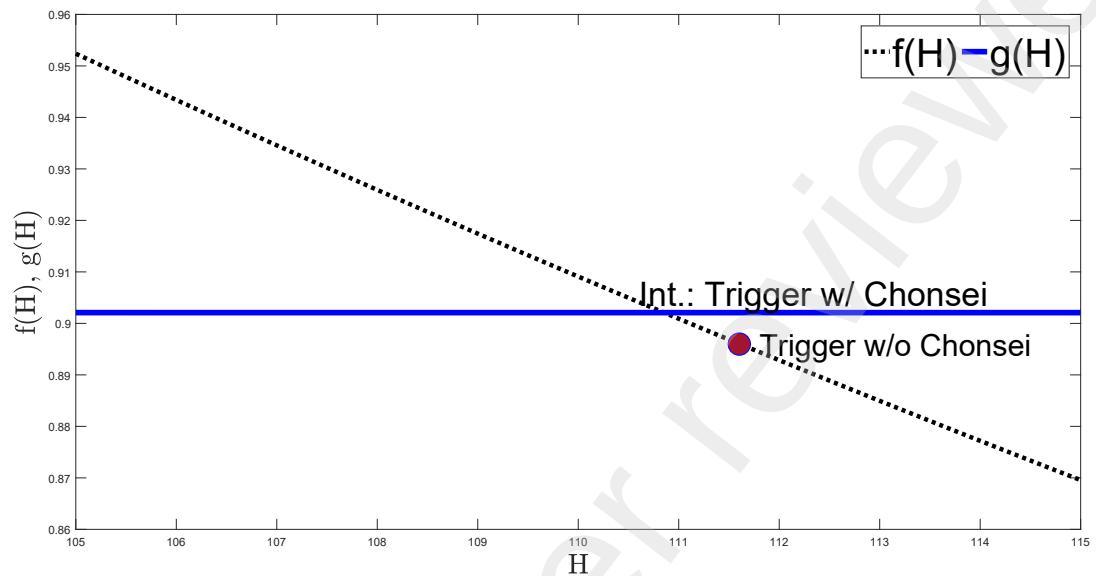
F) Proof of Corollary 2

As $f(H) = \frac{k}{H}$ and $g(H) = \frac{\beta - e^{-\delta(T-\tau)}\Phi(\omega)}{\beta}$, we know $\frac{\partial f(H)}{\partial H} < 0$, $\frac{\partial^2 f(H)}{\partial H^2} > 0$, $\frac{\partial g(H)}{\partial H} < 0$ and $\frac{\partial^2 g(H)}{\partial H^2} > 0$.

- 1) As $\frac{\partial g(H)}{\partial \delta} = \frac{e^{-\delta(T-\tau)}\Phi(\omega)\partial\beta}{\beta^2} + \frac{(T-\tau)e^{-\delta(T-\tau)}\Phi(\omega)}{\beta} + \frac{e^{-\delta(T-\tau)}\varphi(\omega)\sqrt{(T-\tau)}}{\beta\sigma} > 0$, $\frac{\partial H^*}{\partial \delta} < 0$.
- 2) As $\frac{\partial g(H)}{\partial r} = \frac{e^{-\delta(T-\tau)}\Phi(\omega)\partial\beta}{\beta^2} - \frac{e^{-\delta(T-\tau)}\varphi(\omega)\sqrt{(T-\tau)}}{\beta\sigma} < 0$, $\frac{\partial H^*}{\partial r} > 0$.
- 3) As $\frac{\partial f(H)}{\partial k} > 0$ and $\frac{\partial g(H)}{\partial k} = 0$, $\frac{\partial H^*}{\partial k} > 0$.

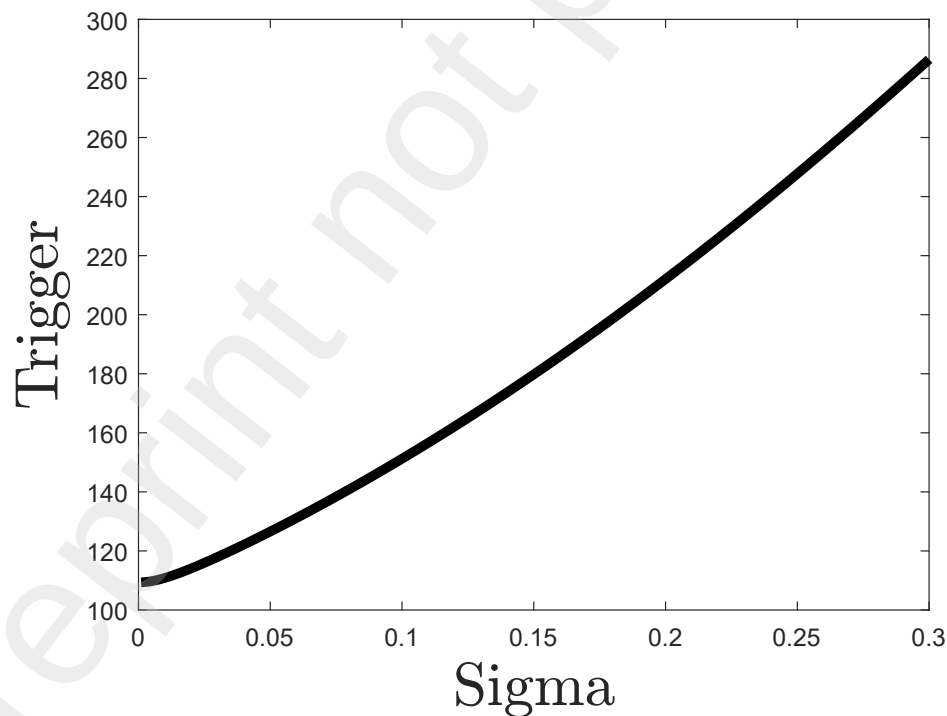
[Appendix 2] Numerical Analysis

A) Optimal Trigger under Chonseil Rental Lease Arrangement



Note: r , δ , and σ are 3.31%, 3% and 1%, respectively. And $k=100$, which is financed by the chonseil deposit for 2 years.

B) Optimal Investment Trigger with respect to Sigma



Note: r and δ are 3.31% and 3%, respectively. $k=100$, which is financed by the chonseil deposit for 2 years.

[Appendix 3] $\hat{\sigma}_{\text{hp}}$ Estimation

To measure the volatility of real house prices, we adopt the simplest GARCH (1,1) model:

$$LHP_t = \alpha_g + \beta_g LHP_{t-1} + e_t,$$

$$\sigma_t^2 = c + \beta_\sigma e_{t-1}^2 + \beta_\theta \sigma_{t-1}^2,$$

where $e_t \sim N(0, \sigma_t)$, c is the constant error term, e_{t-1}^2 is the ARCH term and σ_{t-1}^2 is the GARCH term. The GARCH (1.1) process is weakly stable if and only if the stationary condition ($\beta_\sigma + \beta_\theta < 1$) holds (Teräsvirta, 2009: 20).

For a non-negative variance, as in Nelson and Cao (1992), Palm (1996), and Tsai and Chan (2008), $\sigma^2 = \omega / (1 - \beta_\sigma - \beta_\theta)$.

The estimated results are

$$LHP_t = -0.22^{***} + 0.93^{***} LHP_{t-1},$$

(0.02) (0.03)

$$\sigma_t^2 = 4.48E06 + 0.21^{***} e_{t-1}^2 + 0.76^{***} \sigma_{t-1}^2, \quad \text{where } R^2 = 0.949.$$

(5.09E06) (0.07) (0.06)

The results satisfy the stationarity condition and thus $\hat{\sigma}_{\text{hp}}^2$ can be estimated and its unit is adjusted by multiplying by 100.

In addition, the constant term in the estimated AR(1) model is non-significant. Therefore, we also tried AR(1) model with no constant term. With a new $\hat{\sigma}_{\text{hp}}^2$, DOLS for Models 3, 4 and 5 as reported in <Table 5> produced similar results.