

Dynamic Governance and Trading with ESG*

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Abstract

We study a dynamic model of shareholder trading and voting. The model features a population of small investors and a large fund with heterogeneous preferences over ESG policies. We characterize both the term structure and the limiting distributions of shareholdings and corporate policy. Our results show that (i) it might be difficult to learn about a fund's preference for pro-social policy from static snapshots of its trading and voting; the dynamic term structure needs to be examined. (ii) moderately pro-social investors tend to hold shares in more pro-social firms and can prevent a highly demanding fund from moving a firm's policy as far as it would prefer. Overall, the paper provides a dynamic framework for understanding how trading, voting, and heterogeneous ESG preferences jointly shape corporate policy.

Keywords: Dynamic Responsible Investing, Dynamic Shareholder Voting, Tiebout Sorting, Governance.

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1 Introduction

How do investors who care about financial returns and social impact make trading and governance decisions? Do investors with pro-social preferences trade or vote differently than investors primarily concerned with making money? When are the differences most stark? Whose preferences are most strongly represented by the policies that emerge from shareholder voting? How do the answers to these questions vary over time?

In order to gain traction on questions of this form, we develop and analyze a model in which a pool of small investors and a large fund exhibit heterogeneity in how they balance firm profit and pro-social (say, green) concerns. In each period, investors can buy or sell shares in a collection of firms and then vote on governance decisions for the firms in their portfolio. We study the stable, or so-called absorbing, limiting distributions of shareholdings and policy, as well as the equilibrium dynamics (so-called term structure) of this model. The model is rich enough to paint a nuanced picture of the forces at work when both a population of small investors and a fund with access to substantial capital are motivated by pro-social and financial concerns. Results about both the dynamics and the mapping from underlying investor preferences to stable portfolios and profiles of firm policy are studied.

Several key insights emerge. Initially, the fund's trading and voting behavior will not fully reflect the strength of its pro-social preferences. To casual observers, the fund might appear to have modest pro-social preferences. This stems from an income effect. Even a fund with access to a large capital stock might appreciate that it faces scarcity. When firms are not particularly green, the marginal impact on pro-social outcomes of very green investing and voting is lower, and the financial return from brown investments is higher than the fund anticipates in the future when it moves some firms to be greener. As a result, the fund might choose to accumulate capital early in order to maximize the impact of its green behavior later. In particular, the fund finds it optimal to build cash reserves/wealth by investing in more profitable firms and directing the firms it owns to place greater emphasis on maximizing financial returns. There is some greening, but less than one would expect from myopic fund activities. But as the fund's wealth accumulates and some firms become a bit greener, the fund will shift towards greener firms and vote in ways that make the firms it owns select even greener policies. The term structure shows a form of investing in profit when the fund has relatively low assets in order to maximize its impact later, and then increased consumption of greener policies at the expense of financial return as the fund's wealth rises.

This pattern of behavior suggests that inferences about an investor’s commitment to pro-social behavior based on a short time series of trading and voting behavior may be biased. Pronouncements about the resolve of a new “green fund” that doesn’t appear very green might be premature.

Second, even in the long run, there is a potentially strong equilibrium force that dampens the influence of large, high-demand funds. If a fund wants to dramatically influence the policy of a firm, it needs to accumulate enough shares so that its vote is decisive, or it needs to induce enough sorting so that a large fraction of the owners are high demanders. But as a firm’s policy moves away from a moderate starting point, the differentiation between it and other firms induces a form of sorting whereby investors who favor this new moderately green policy accumulate shares. Because these investors value this firm more than others, it is costly for the fund to buy their shares, and at the same time, because these investors only have moderate preferences for green policy, these investors become a force of friction that works against continued efforts by the highly demanding large fund; acquiring the shares needed for more extreme policy becomes too costly.

Third, characterizing how the fund optimally balances these cross pressures allows us to overturn the simple view that responsible investors should always buy green firms and divest from brown firms. A brown firm can be an attractive target when marginal votes are especially productive there. For example, a firm may be far below the fund’s desired policy, and a small increase in ownership may substantially move the weighted median. Conversely, a green firm may have low marginal governance value if the fund already has enough votes there or if additional ownership is expensive. The relevant object is not current greenness alone, but marginal governance value per dollar.

Related Literature When investors trade and vote, we cannot help but think about feedback loops between firm governance and ownership decisions. In other words, investors in firms are likely subject to a form of Tiebout sorting that is analogous to how businesses and individuals move between towns/states based on tax policy. Since [Tiebout \(1956\)](#) this kind of problem has received interest in Political Economy. Recent contributions are [Gieczewski \(2021\)](#) and [Gieczewski and Kosterina \(2024\)](#). In the more recent models, the focus is on a single organization, and in nearly all work in the so-called Tiebout tradition, the decision to enter or exit the organization is not governed by explicitly modeled market forces. In our context, this feature is important because investors, by definition, are participants in well-defined and often liquid markets for

ownership. As such, reliance on reduced form outside options to model decisions to vote with one's feet misses something central to our conception of what should be governed by equilibrium forces.

While these dynamics might be a natural consideration as one considers any dimensions upon which there is potentially firm heterogeneity, we see the role of ESG or pro-social behavior as a particularly relevant area. Many commentators highlight investor heterogeneity in terms of a willingness to trade off financial return for pro-social behavior. In addition, this is thought to be one area where large blocks are sometimes systematically different than the typical retail investor. In this paper, we focus on the feedback between governance decisions and ownership decisions to study the dynamics of pro-social (so-called ESG) voting and firm ownership by a population of investors that exhibits preference and scale heterogeneity.

In the finance literature, our paper contributes to the literature on blockholder governance through trading and intervention. Classic models study how market liquidity and block formation shape incentives to monitor or intervene, and how the threat of informed exit can discipline management ([Maug, 1998](#); [Kahn and Winton, 1998](#); [Admati and Pfleiderer, 2009](#); [Edmans, 2009](#); [Edmans and Manso, 2011](#)). The dynamic ownership problem in [DeMarzo and Urošević \(2006\)](#) is especially relevant: current prices capitalize a large shareholder's future trades, and optimal block adjustment can be gradual. More recently, [Marinovic and Varas \(2026\)](#) study the dynamic strategic trading of an informed blockholder. Our model shifts the source of dynamics from private information and monitoring to the accumulation and allocation of governance rights. The fund trades in several firms, saves cash, and votes directly over policy; its current portfolio affects both its future financial capacity and the future distribution of voters. Blockholder activity, therefore, has a term structure: early trading and voting can be chosen partly to build the resources needed for later influence.

Our closest antecedents are [Levit et al. \(2024\)](#) and [Levit et al. \(2026\)](#). [Levit et al. \(2024\)](#) show that trading changes the composition of the shareholder base and is complementary with voting, generating self-fulfilling expectations and potentially multiple equilibria. [Levit et al. \(2026\)](#) study how a minority blockholder's purchases from dissenting shareholders generate a voting premium. We retain their basic sequence of competitive trading followed by shareholder voting, but embed it in an infinite-horizon, multi-firm environment. This makes the value of a vote endogenous to the fund's future cash, its governance opportunities in other firms, and the evolving identity of the marginal seller. The multi-firm dimension also connects the paper to [Edmans et al.](#)

(2019). In their analysis, common ownership affects governance through the information conveyed by selective exit. Here, common ownership arises endogenously even without private information: as concentrated ownership raises the price demanded by increasingly selected sellers and can lower the marginal governance benefit in one firm, the fund reallocates scarce governance capital across firms.

The paper also contributes to the theory of sustainable and socially responsible investing. One branch of this literature studies how investor tastes, portfolio restrictions, and exclusion affect asset prices, expected returns, and firms' incentives to become greener (Heinkel et al., 2001; Pástor et al., 2021; Pedersen et al., 2021). Another branch studies when responsible investors create impact through voice, active ownership, financing, or investment mandates, and how these mechanisms compare with exit (Broccardo et al., 2022; Gollier and Pouget, 2022; Oehmke and Opp, 2025; Landier and Lovo, 2025; Gupta et al., 2026). Our distinctive mechanism is the dynamic market acquisition of voting rights. A highly pro-social fund may initially hold a high-dividend, relatively brown firm and vote less aggressively in order to build internal resources for later governance. At the same time, greener policies attract moderately pro-social shareholders, who become increasingly important marginal sellers and can make further block accumulation costly. Responsible investing is therefore not simply a static choice between green and brown assets, or between exit and voice; it is an intertemporal allocation of cash and governance rights across firms.

2 The Model

We consider a dynamic setting with discrete time, $t = 0, 1, 2, \dots$. There are $K \geq 2$ firms, indexed by $j \in \{1, \dots, K\}$. Each firm has one unit of shares outstanding. In each period, a population of investors trade shares and vote over a firm-level ESG policy

$$x_{j,t} \in [0, 1].$$

2.1 Firms

Firm j 's financial dividend and social output are

$$\pi_j(x) = \bar{\pi}_j - \frac{1}{2}x^2, \quad \bar{\pi}_j > \frac{1}{2},$$

and

$$s_j(x) = x - \frac{d_j}{2}x^2, \quad d_j > 0.$$

A larger d_j means that ESG production is more costly. When useful, firms are ordered so that

$$d_1 \leq \dots \leq d_K.$$

2.2 Investors, preferences, and flows

There is one long-lived active fund and a continuum of competitive short-horizon small shareholders. The active fund trades and votes dynamically she can also save money in a risk free security. A unit of money that is not invested in one of the firms yields the risk-free return of $R = 1 + r > 1$. The fund has discount factor $\delta \in (0, 1)$, with

$$\delta R < 1.$$

The fund has a social type

$$\beta \in \left(\frac{1}{2}, 1\right],$$

meaning that it values one unit of social output attached to a share at β .

At the beginning of period t , the fund enters with inherited holdings

$$\alpha_{t-1} = (\alpha_{1,t-1}, \dots, \alpha_{K,t-1}) \in [0, 1]^K$$

and cash

$$c_t \geq 0.$$

It then receives a net dollar flow

$$f_t \in \mathcal{F} \subset \mathbb{R}.$$

A positive value $f_t > 0$ is an inflow, while a negative value $f_t < 0$ is an outflow. The flow process follows a finite-state Markov chain,

$$f_{t+1} \sim P(\cdot \mid f_t).$$

The self-financing case is nested by setting $f_t \equiv 0$.

Small shareholders are indexed by $b \in [0, 1]$. A small shareholder of type b values one unit of social output attached to a share at b . Types are distributed according to

G , with density g . The baseline assumes a uniform distribution:

$$G(b) = b, \quad g(b) = 1, \quad \mathbb{E}[b] = \frac{1}{2}.$$

Small shareholders are short-horizon investors. They do not solve a dynamic portfolio problem and do not internalize how their individual trades affect future prices or policies. Instead, each evaluates her position one period at a time, taking as given the expected current policy and the expected next-period market value of a share.

Let $h_{j,t}^-(b)$ denote the pre-trade holdings of type- b small shareholders in firm j , per unit mass of type b . Thus, total pre-trade dispersed ownership in firm j is

$$\int_0^1 h_{j,t}^-(b) dG(b).$$

Since the fund enters period t with ownership $\alpha_{j,t-1}$, we have

$$\int_0^1 h_{j,t}^-(b) dG(b) = 1 - \alpha_{j,t-1}.$$

We allow dispersed ownership to be partially persistent across periods. By $\lambda \in [0, 1)$ we denote the persistence of dispersed ownership. After trading in period t , the holdings of firm j of a small-shareholder with type b are denoted $h_{j,t}^+(b)$. The next period's pre-trade dispersed holdings are

$$h_{j,t+1}^-(b) = \lambda h_{j,t}^+(b) + (1 - \lambda)(1 - \alpha_{j,t}).$$

Here $\lambda = 0$ is the full-turnover case: the residual float is reassigned each period according to the type distribution. The case $\lambda = 1$ is the full-persistence case. Intermediate values, $\lambda \in (0, 1)$, capture partial turnover of dispersed ownership.

2.3 Trading

Trading precedes voting. The fund chooses post-trade ownership

$$\alpha_t = (\alpha_{1,t}, \dots, \alpha_{K,t}) \in [0, 1]^K.$$

Its net purchase in firm j is

$$q_{j,t} \equiv \alpha_{j,t} - \alpha_{j,t-1}.$$

A positive $q_{j,t}$ is a purchase by the fund, while a negative $q_{j,t}$ is a sale.

Small shareholders are competitive and submit price-contingent net demands. Let

$$z_{j,t}(p, b)$$

denote the net demand of a type- b small shareholder in firm j at price p , again measured per unit mass of type b . Let

$$p_{j,t+1}^e \equiv \mathbb{E}_t[p_{j,t+1}]$$

denote the expected next-period market value of one share of firm j .

Given expected policy $x_{j,t}^e$, execution price $p_{j,t}$, and expected next-period market value $p_{j,t+1}^e$, a type- b small shareholder's end-of-period payoff from firm j is

$$u_{j,t}(b) = [h_{j,t}^-(b) + z_{j,t}(p_{j,t}, b)] [\pi_j(x_{j,t}^e) + b s_j(x_{j,t}^e) + p_{j,t+1}^e] - R p_{j,t} z_{j,t}(p_{j,t}, b) - \frac{\rho}{2} z_{j,t}(p_{j,t}, b)^2, \quad (1)$$

where

$$h_{j,t}^+(b) = h_{j,t}^-(b) + z_{j,t}(p_{j,t}, b)$$

is the post-trade position. We typically suppress the price in this end of period summary. The term $R p_{j,t} z_{j,t}(p_{j,t}, b)$ is the end-of-period opportunity cost of the net trade. If $z_{j,t}(p_{j,t}, b) > 0$, the type buys shares and pays $R p_{j,t} z_{j,t}(p_{j,t}, b)$ in end-of-period dollars. If $z_{j,t}(p_{j,t}, b) < 0$, the type sells shares and receives $-R p_{j,t} z_{j,t}(p_{j,t}, b)$ in end-of-period dollars.

Since the term

$$h_{j,t}^-(b) [\pi_j(x_{j,t}^e) + b s_j(x_{j,t}^e) + p_{j,t+1}^e]$$

is fixed at the trading stage, type b 's net demand $z_{j,t}(p_{j,t}, b)$ solves

$$\max_{z \geq -h_{j,t}^-(b)} \left\{ z [\pi_j(x_{j,t}^e) + b s_j(x_{j,t}^e) + p_{j,t+1}^e - R p_{j,t}] - \frac{\rho}{2} z^2 \right\}. \quad (2)$$

The lower bound ensures that final holdings are nonnegative. The quadratic term is the transaction cost for trading z shares, where ρ captures trading frictions, such as illiquidity or transaction costs.

Market clearing in firm j requires

$$q_{j,t} + \int_0^1 z_{j,t}(p_{j,t}, b) dG(b) = 0. \quad (3)$$

Post-trade small-shareholder holdings are

$$h_{j,t}^+(b) = h_{j,t}^-(b) + z_{j,t}(p_{j,t}, b).$$

Equivalently, market clearing implies

$$\alpha_{j,t} + \int_0^1 h_{j,t}^+(b) dG(b) = 1.$$

The fund's post-trade cash is

$$c_t^+ = c_t + f_t - \sum_{j=1}^K p_{j,t} q_{j,t}.$$

We impose fairly standard feasibility conditions that ensure that choice sets are compact.

We assume that

$$c_t^+ \geq 0, \quad \sum_{j=1}^K p_{j,t} \alpha_{j,t} \leq \bar{W}.$$

The first constraint rules out borrowing after trading. The second is a reduced-form portfolio-size constraint: at clearing prices, the dollar value of the fund's risky shareholdings cannot exceed $\bar{W}$. We assume throughout that the feasible set is nonempty in every state.

2.4 Voting

After trading, shareholders vote over $x_{j,t} \in [0, 1]$. Voting is one-share-one-vote. In firm j , the fund has voting weight $\alpha_{j,t}$, and small shareholders have voting-weight density $h_{j,t}^+(b)$.

Small shareholders are atomistic and vote behaviorally as if pivotal. When voting, they take the next-period market value as given and choose the policy that maximizes their current per-share flow payoff. Thus type b votes for

$$x_j^*(b) = \arg \max_{x \in [0,1]} \{ \pi_j(x) + b s_j(x) \}.$$

Under the primitives above,

$$x_j^*(b) = \frac{b}{1 + b d_j},$$

with projection onto $[0, 1]$ if needed.

The fund is long-lived. Its vote

$$x_{j,t}^B \in [0, 1]$$

is part of its dynamic strategy and need not equal its static favorite policy. Current policy affects dividends, future cash, and therefore future governance capacity.

The implemented policy is the weighted median which depends on the platform vote of the fund and small-investors as well as the ownership of these agents:

$$x_{j,t} = M(x_{j,t}^B, x_j^*(\cdot); \alpha_{j,t}, h_{j,t}^+(\cdot)).$$

Equivalently, if $C_{j,t}(x)$ denotes cumulative voting weight weakly below policy x ,

$$C_{j,t}(x) = \alpha_{j,t} \mathbf{1}\{x_{j,t}^B \leq x\} + \int_{\{b: x_j^*(b) \leq x\}} h_{j,t}^+(b) dG(b),$$

then $x_{j,t}$ is a policy at which $C_{j,t}$ crosses one half. Given the form of $x_j^*(\cdot)$ the integral is well-defined.

2.5 The fund's objective

After trading and voting, the fund's end-of-period financial resources are

$$\mathcal{R}_t = Rc_t^+ + \sum_{j=1}^K \alpha_{j,t} \pi_j(x_{j,t}).$$

Equivalently,

$$\mathcal{R}_t = R \left(c_t + f_t - \sum_{j=1}^K p_{j,t} q_{j,t} \right) + \sum_{j=1}^K \alpha_{j,t} \pi_j(x_{j,t}).$$

The fund chooses next-period cash

$$c_{t+1} \in [0, \mathcal{R}_t].$$

This choice is the fund's optimal cash buffer for the next period. The residual

$$D_t = \mathcal{R}_t - c_{t+1} \geq 0$$

is the gross distribution to its investors. Since f_t is a capital contribution when positive and a redemption when negative, the end-of-period net financial payout to investors is

$$D_t - Rf_t.$$

This term may be negative in inflow periods.

The fund's nonpecuniary social payoff is

$$\beta \sum_{j=1}^K \alpha_{j,t} s_j(x_{j,t}).$$

The fund chooses trades, votes, and cash saving to maximize

$$\mathbb{E} \sum_{t=0}^{\infty} \delta^t \left[D_t - Rf_t + \beta \sum_{j=1}^K \alpha_{j,t} s_j(x_{j,t}) \right], \quad (4)$$

subject to trading, voting, market clearing, cash feasibility, the portfolio-size constraint, and the law of motion for dispersed ownership.

2.6 Timing

Each period has five stages.

- (i) The fund enters with (α_{t-1}, c_t) , the pre-trade dispersed ownership distribution h_t^- , and the dollar flow f_t .
- (ii) The fund and small shareholders trade shares. Small shareholders evaluate one-period positions using expected policies and expected next-period market values. Prices clear the share markets. Post-trade fund holdings α_t , small-shareholder holdings h_t^+ , and fund cash c_t^+ are determined.
- (iii) Shareholders vote. Small shareholders vote for their current-flow ideal policies, and weighted medians are implemented.
- (iv) Dividends and social outputs are realized. The fund chooses the optimal next-period cash buffer c_{t+1} ; the residual $D_t = \mathcal{R}_t - c_{t+1}$ is distributed.
- (v) Dispersed ownership partially persists. For each firm j ,

$$h_{j,t+1}^-(b) = \lambda h_{j,t}^+(b) + (1 - \lambda)(1 - \alpha_{j,t}).$$

The fund carries (α_t, c_{t+1}) into the next period, and f_{t+1} is drawn according to $P(\cdot \mid f_t)$.

2.7 Equilibrium Concept

We focus on equilibria that are Markovian in a simple state. The beginning-of-period state is

$$\omega_t = (\alpha_{t-1}, h_t^-, c_t, f_t).$$

We study Equilibria that are Markov-perfect with respect to this state and which involve stage-weakly undominated voting reports.

Definition 1 (Markov-perfect equilibrium). *A Markov-perfect equilibrium with stage-undominate voting consists of a fund value function, fund policies for post-trade holdings, voting reports, and cash saving, small-shareholder demand and voting rules, price functions, and implemented policy functions that depend only on the state such that:*

- (i) *small-shareholder demands solve (2);*
- (ii) *markets clear according to (3);*
- (iii) *small shareholders vote for their favorite current-flow policies, and the implemented policies are weighted medians;*
- (iv) *the fund's post-trade holdings, voting, and cash-saving choices maximize (4) subject to ownership, cash, portfolio-size, trading, voting, and dispersed-ownership feasibility;*
- (v) *expectations about implemented policies and next-period market values are correct;*
- (vi) *expectations are Markov and anonymous. At the trading stage, small shareholders take as given the equilibrium policy and next-period price functions evaluated at the public state induced by the fund's post-trade ownership choice. They do not condition on their own infinitesimal order. Thus, for every candidate fund order, $x_{j,t}^e$ and $p_{j,t+1}^e$ are the equilibrium expectations associated with the resulting public state.*
- (vii) *the state evolves according to*

$$\omega_{t+1} = (\alpha_t, h_{t+1}^-, c_{t+1}, f_{t+1}), \quad f_{t+1} \sim P(\cdot \mid f_t),$$

with

$$h_{j,t+1}^-(b) = \lambda h_{j,t}^+(b) + (1 - \lambda)(1 - \alpha_{j,t}).$$

3 Equilibrium Analysis

3.1 Equilibrium Building Blocks

This section characterizes the competitive small-shareholder behavior. We take as given the fund's net order $q_{j,t}$, the expected policy $x_{j,t}^e$, the expected next-period market value $p_{j,t+1}^e$, and the pre-trade dispersed ownership distribution $h_{j,t}^-$. We first solve for small-shareholder demand and the market-clearing price. We then characterize post-trade ownership and the sorting in ownership from equilibrium trading of the small-shareholders. We term this Tiebout tilt in dispersed holdings.

3.1.1 Small-shareholder demand, prices, and ownership

Fix a firm j and period t . A type- b small shareholder enters the trading stage with pre-trade holdings $h_{j,t}^-(b)$. Given the expected policy $x_{j,t}^e$, current execution price $p_{j,t}$, and expected next-period market value $p_{j,t+1}^e \equiv \mathbb{E}_t[p_{j,t+1}]$, her net demand solves

$$\max_{z \geq -h_{j,t}^-(b)} \left\{ z \left[\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t} \right] - \frac{\rho}{2} z^2 \right\}.$$

The lower bound ensures that final holdings are nonnegative. The term $p_{j,t+1}^e$ is the next-period market value of a share.

Lemma 1 (Post-trading ownership and price). *Fix $x_{j,t}^e$, $q_{j,t}$, $h_{j,t}^-$, and $p_{j,t+1}^e$. The type- b small-shareholder net demand is*

$$z_{j,t}(p_{j,t}, b) = \max \left\{ -h_{j,t}^-(b), \frac{\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho} \right\}.$$

Thus post-trade holdings are

$$h_{j,t}^+(p_{j,t}, b) = \left[h_{j,t}^-(b) + \frac{\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho} \right]_+.$$

The clearing price $p_{j,t}$ is determined by

$$\int_0^1 h_{j,t}^+(p_{j,t}, b) db = 1 - \alpha_{j,t}.$$

For each $p_{j,t}$ there exists a cutoff

$$b_{j,t}^* \in [0, 1]$$

such that

$$h_{j,t}^+(p_{j,t}, b) = 0 \quad \text{for } b < b_{j,t}^*,$$

and

$$h_{j,t}^+(p_{j,t}, b) > 0 \quad \text{for } b > b_{j,t}^*.$$

The case $b_{j,t}^* = 0$ corresponds to the case in which no positive mass of small shareholders is exhausted.

If no type is exhausted ($b^* = 0$), then

$$Rp_{j,t} - p_{j,t+1}^e = \pi_j(x_{j,t}^e) + \frac{1}{2}s_j(x_{j,t}^e) + \rho q_{j,t},$$

and

$$h_{j,t}^+(b) = h_{j,t}^-(b) + \frac{s_j(x_{j,t}^e)}{\rho} \left(b - \frac{1}{2} \right) - q_{j,t}.$$

If $b_{j,t}^* > 0$, the cutoff type is indifferent at zero final holdings:

$$h_{j,t}^-(b_{j,t}^*) + \frac{\pi_j(x_{j,t}^e) + b_{j,t}^* s_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho} = 0.$$

Equivalently,

$$Rp_{j,t} - p_{j,t+1}^e = \pi_j(x_{j,t}^e) + b_{j,t}^* s_j(x_{j,t}^e) + \rho h_{j,t}^-(b_{j,t}^*).$$

For types $b > b_{j,t}^*$,

$$h_{j,t}^+(b) = h_{j,t}^-(b) - h_{j,t}^-(b_{j,t}^*) + \frac{s_j(x_{j,t}^e)}{\rho} (b - b_{j,t}^*).$$

Proof. The objective is strictly concave in z . Ignoring the lower bound, the first-order condition is

$$\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t} - \rho z = 0.$$

Thus the unconstrained net demand is

$$\frac{\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho}.$$

Imposing the lower bound $z \geq -h_{j,t}^-(b)$ gives the stated demand. Since

$$h_{j,t}^+(b) = h_{j,t}^-(b) + z_{j,t}(p_{j,t}, b),$$

we obtain the post-trade holdings formula. Market clearing requires

$$\int_0^1 h_{j,t}^+(b) db = 1 - \alpha_{j,t}.$$

On the upper-tail active branch, define

$$b_{j,t}^* = \inf\{b : h_{j,t}^+(b) > 0\}.$$

If $b_{j,t}^* > 0$, the cutoff type is exactly indifferent at zero final holdings:

$$h_{j,t}^-(b_{j,t}^*) + \frac{\pi_j(x_{j,t}^e) + b_{j,t}^* s_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho} = 0.$$

Rearranging gives

$$Rp_{j,t} - p_{j,t+1}^e = \pi_j(x_{j,t}^e) + b_{j,t}^* s_j(x_{j,t}^e) + \rho h_{j,t}^-(b_{j,t}^*).$$

For active types $b > b_{j,t}^*$, the lower bound does not bind, so

$$h_{j,t}^+(b) = h_{j,t}^-(b) + \frac{\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho}.$$

Substituting the cutoff expression gives

$$h_{j,t}^+(b) = h_{j,t}^-(b) - h_{j,t}^-(b_{j,t}^*) + \frac{s_j(x_{j,t}^e)}{\rho}(b - b_{j,t}^*).$$

If no type is exhausted, the lower bound does not bind for any type. Market clearing gives

$$q_{j,t} + \int_0^1 \frac{\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho} db = 0.$$

Since $\int_0^1 b \, db = 1/2$, this implies

$$Rp_{j,t} - p_{j,t+1}^e = \pi_j(x_{j,t}^e) + \frac{1}{2}s_j(x_{j,t}^e) + \rho q_{j,t}.$$

Substituting this expression into individual demand gives

$$z_{j,t}(b) = \frac{s_j(x_{j,t}^e)}{\rho} \left(b - \frac{1}{2} \right) - q_{j,t}.$$

Since $h_{j,t}^+(b) = h_{j,t}^-(b) + z_{j,t}(b)$, the no-exhaustion ownership formula follows. $\square$

Lemma 1 gives the market-clearing price and post-trading ownership. Small-shareholder demand depends on the current execution price net of the expected next-period market value,

$$Rp_{j,t} - p_{j,t+1}^e.$$

The expected next-period market value is common across types, so it affects the price level but not the cross-sectional Tiebout tilt. When $b_{j,t}^* = 0$ and no type is exhausted, this market-value-adjusted price term equals the average small-shareholder flow valuation plus the fund's price impact. When some types are exhausted, the average type $1/2$ is no longer the relevant marginal holder. The cutoff type $b_{j,t}^*$ is the marginal holder: she is indifferent between being exhausted and holding a positive position.

3.1.2 Tiebout sorting

The trading block also determines the composition of the shareholder base. This is the Tiebout-sorting channel.

Lemma 2 (Tiebout sorting). *Suppose $s_j(x_{j,t}^e) > 0$. Among active small-shareholder types, trading tilts ownership toward higher- b shareholders. In particular, if two types $b_2 > b_1$ both hold positive post-trade amounts, then*

$$h_{j,t}^+(b_2) - h_{j,t}^+(b_1) = h_{j,t}^-(b_2) - h_{j,t}^-(b_1) + \frac{s_j(x_{j,t}^e)}{\rho}(b_2 - b_1).$$

Thus, relative to the pre-trade distribution, higher- b active shareholders increase their holdings by more.

Proof. If both types are active, their lower bounds do not bind. Therefore

$$h_{j,t}^+(b) = h_{j,t}^-(b) + \frac{\pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e) + p_{j,t+1}^e - Rp_{j,t}}{\rho}.$$

Taking the difference between b_2 and b_1 eliminates both the current execution price and the expected next-period market value:

$$h_{j,t}^+(b_2) - h_{j,t}^+(b_1) = h_{j,t}^-(b_2) - h_{j,t}^-(b_1) + \frac{s_j(x_{j,t}^e)}{\rho}(b_2 - b_1).$$

Since $s_j(x_{j,t}^e) > 0$ and $b_2 > b_1$, the last term is positive. $\square$

The lemma gives the basic Tiebout force. If a firm produces positive social output, high- b shareholders value its shares more and therefore hold relatively more of the firm. If the lower bound binds, low- b shareholders are the first to be exhausted. The persistence law then determines how much of this sorting survives into future periods:

$$h_{j,t+1}^-(b) = \lambda h_{j,t}^+(b) + (1 - \lambda)(1 - \alpha_{j,t}).$$

When $\lambda > 0$, past Tiebout sorting affects the future shareholder base. This is the channel that later generates endogenous marginal holders in the stationary analysis.

3.2 Voting and the Term Structure of Responsible Ownership

This section characterizes the voting stage. Trading has already occurred, so the fund's post-trade ownership $\alpha_{j,t}$, small-shareholder holdings $h_{j,t}^+$, and post-trade cash c_t^+ are fixed. Small shareholders vote behaviorally for their static favorite policies. The fund is long-lived and may vote differently from its static favorite policy because current policy affects current dividends, future cash, and future governance capacity.

3.2.1 Small-shareholder ideal points

Lemma 3 (Small-shareholder ideal points). *A type- b small shareholder in firm j votes for*

$$x_j^*(b) = \arg \max_{x \in [0,1]} \{\pi_j(x) + bs_j(x)\}.$$

Thus the ideal point is

$$x_j^*(b) = \frac{b}{1 + bd_j},$$

which is increasing in b and decreasing in d_j .

Proof. The type- b objective is

$$\bar{\pi}_j - \frac{1}{2}x^2 + b \left(x - \frac{d_j}{2}x^2 \right).$$

This is strictly concave in x . The first-order condition for an interior optimum is

$$-x + b(1 - d_j x) = 0.$$

Solving gives

$$x_j^*(b) = \frac{b}{1 + bd_j}.$$

The comparative statics follow immediately:

$$\frac{\partial x_j^*(b)}{\partial b} = \frac{1}{(1 + bd_j)^2} > 0,$$

and

$$\frac{\partial x_j^*(b)}{\partial d_j} = -\frac{b^2}{(1 + bd_j)^2} < 0$$

for $b > 0$. □

3.2.2 The fund's implementable policy set

For any candidate policy x , define the dispersed voting weight strictly below x by

$$L_{j,t}(x) \equiv \int_{\{b: x_j^*(b) < x\}} h_{j,t}^+(b) dG(b).$$

Lemma 4 (Fund implementable policy set). *Fix post-trade ownership $\alpha_{j,t}$ and post-trade dispersed holdings $h_{j,t}^+$. The fund can implement policy $x \in [0, 1]$ in firm j by placing its vote at x if and only if*

$$L_{j,t}(x) \leq \frac{1}{2} \leq L_{j,t}(x) + \alpha_{j,t}.$$

Hence the fund's implementable policy set is

$$\mathcal{X}_{j,t} = \left\{ x \in [0, 1] : L_{j,t}(x) \leq \frac{1}{2} \leq L_{j,t}(x) + \alpha_{j,t} \right\}.$$

Proof. Suppose the fund reports policy x . Small shareholders with ideal policies strictly

below x cast total voting weight $L_{j,t}(x)$. Since the fund is an atom of voting weight $\alpha_{j,t}$ at x , cumulative voting weight at x is

$$L_{j,t}(x) + \alpha_{j,t},$$

up to zero-measure small-shareholder ties. The policy x is a weighted median if the voting weight strictly below x is no more than one half and the voting weight weakly below x , including the fund's atom, is at least one half. Therefore x is implementable if and only if

$$L_{j,t}(x) \leq \frac{1}{2} \leq L_{j,t}(x) + \alpha_{j,t}.$$

□

When $\alpha_{j,t} \geq 1/2$, the fund can implement any policy in $[0, 1]$.

3.2.3 Cash saving and the shadow value of resources

Before characterizing the fund's voting strategy, we characterize its cash-saving decision. After trading and voting, the fund has end-of-period financial resources

$$\mathcal{R}_t = Rc_t^+ + \sum_{j=1}^K \alpha_{j,t} \pi_j(x_{j,t}).$$

It chooses next-period cash

$$c_{t+1} \in [0, \mathcal{R}_t].$$

Thus the fund first chooses an optimal cash buffer for the next period; any resources above that buffer are paid out to investors.

Let

$$\Phi_t(c) \equiv \delta \sum_{f' \in \mathcal{F}} P(f' \mid f_t) V(\alpha_t, h_{t+1}^-, c, f')$$

denote the continuation value from carrying cash c into the next period, holding current ownership and the next-period dispersed ownership distribution fixed.

Lemma 5 (Optimal cash buffer). *Given $\mathcal{R}_t$, the fund chooses next-period cash c_{t+1} to solve*

$$\max_{0 \leq c \leq \mathcal{R}_t} \{ \mathcal{R}_t - c - Rf_t + \Phi_t(c) \}.$$

Equivalently,

$$c_{t+1}^* \in \arg \max_{0 \leq c \leq \mathcal{R}_t} \{\Phi_t(c) - c\}.$$

Suppose Φ_t is differentiable and concave. Then the optimal cash buffer is characterized by

$$\begin{aligned} c_{t+1}^* &= 0 & \text{if } \Phi_t'(0) \leq 1, \\ \Phi_t'(c_{t+1}^*) &= 1 & \text{if } 0 < c_{t+1}^* < \mathcal{R}_t, \end{aligned}$$

and

$$c_{t+1}^* = \mathcal{R}_t \quad \text{if } \Phi_t'(\mathcal{R}_t) \geq 1.$$

Proof. At the cash-saving stage, the fund solves

$$\max_{0 \leq c \leq \mathcal{R}_t} \{\mathcal{R}_t - c - Rf_t + \Phi_t(c)\}.$$

The terms $\mathcal{R}_t$ and Rf_t are fixed with respect to c , so the problem is equivalent to

$$\max_{0 \leq c \leq \mathcal{R}_t} \{\Phi_t(c) - c\}.$$

The derivative with respect to c is $\Phi_t'(c) - 1$. The interior and boundary conditions follow from this derivative and the concavity of Φ_t . $\square$

Corollary 1 (Shadow value of end-of-period resources). *Let Λ_t denote the shadow value of one additional dollar of end-of-period resources $\mathcal{R}_t$. Then*

$$\Lambda_t \geq 1.$$

If the optimal cash buffer is below available resources,

$$c_{t+1}^* < \mathcal{R}_t,$$

then marginal resources are paid out and

$$\Lambda_t = 1.$$

If the fund retains all resources,

$$c_{t+1}^* = \mathcal{R}_t,$$

then

$$\Lambda_t = \Phi'_t(\mathcal{R}_t) \geq 1.$$

Thus $\Lambda_t > 1$ precisely when the fund retains all marginal resources and the marginal continuation value of cash exceeds the payout value of cash.

Proof. Let $W_t(\mathcal{R}_t)$ denote the optimized value of the cash-saving problem:

$$W_t(\mathcal{R}_t) = \max_{0 \leq c \leq \mathcal{R}_t} \{\mathcal{R}_t - c - Rf_t + \Phi_t(c)\}.$$

If $c_{t+1}^* < \mathcal{R}_t$, an additional dollar of resources can be paid out immediately, so $\Lambda_t = 1$. If $c_{t+1}^* = \mathcal{R}_t$, an additional dollar of resources becomes an additional dollar of next-period cash, so by the envelope theorem,

$$\Lambda_t = \Phi'_t(\mathcal{R}_t) \geq 1.$$

□

3.2.4 The fund's continuation-adjusted ideal

We now characterize the policy the fund wants to implement when it is pivotal. At the voting stage, the fund's post-trade ownership and post-trade cash are fixed. Current policy affects the fund through two channels. First, it affects current social utility. Second, it affects financial dividends, which enter the fund's cash resources and may be used to build future governance capacity.

Proposition 1 (The fund's continuation-adjusted ideal policy). *Fix firm j and period t . Let Λ_t be the equilibrium shadow value of one dollar of end-of-period resources, evaluated at the fund's optimal cash-buffer choice. By the envelope theorem for the cash-saving problem, when differentiating with respect to the current policy, the induced adjustment of the optimal cash buffer is not differentiated through; the effect of policy on end-of-period resources is summarized by Λ_t .*

Because a dollar of dividend can be retained and used to acquire future governance rights, the fund's voting objective over firm j 's policy is proportional to

$$\Lambda_t \pi_j(x) + \beta s_j(x).$$

It follows that the fund's continuation-adjusted ideal policy for firm j is

$$\hat{x}_{j,t}^B = \frac{\beta}{\Lambda_t + \beta d_j}.$$

Moreover,

$$\frac{\partial \hat{x}_{j,t}^B}{\partial \Lambda_t} = -\frac{\beta}{(\Lambda_t + \beta d_j)^2} < 0.$$

Proof. At the voting stage, post-trade ownership and post-trade cash are fixed. If policy x is implemented in firm j , the fund receives nonpecuniary payoff $\alpha_{j,t}\beta s_j(x)$ and financial dividends $\alpha_{j,t}\pi_j(x)$. A marginal dollar of end-of-period resources is worth Λ_t . Therefore, conditional on other firms' policies, the fund's relevant voting objective in firm j is

$$\alpha_{j,t} [\Lambda_t \pi_j(x) + \beta s_j(x)].$$

Since $\alpha_{j,t} > 0$, this is equivalent to maximizing

$$\Lambda_t \pi_j(x) + \beta s_j(x).$$

Substituting the primitives gives

$$\Lambda_t \left(\bar{\pi}_j - \frac{1}{2}x^2 \right) + \beta \left(x - \frac{d_j}{2}x^2 \right).$$

The first-order condition is

$$-\Lambda_t x + \beta(1 - d_j x) = 0.$$

Solving gives

$$\hat{x}_{j,t}^B = \frac{\beta}{\Lambda_t + \beta d_j}.$$

The derivative with respect to Λ_t is immediate. □

When $\Lambda_t > 1$, the fund places more weight on current dividends and its continuation-adjusted ideal is below its static ideal. This term-structure force is a transition or liquidity-scarcity force. In a mature self-financing stationary allocation with positive distributions, Lemma 8 below implies $\Lambda = 1$.

Remark 1 (Joint voting fixed point). *Outside a distribution-paying state with $\Lambda_t = 1$, the fund's desired votes across firms are linked through the common shadow value Λ_t .*

Indeed, Λ_t depends on end-of-period resources,

$$\mathcal{R}_t = Rc_t^+ + \sum_{k=1}^K \alpha_{k,t} \pi_k(x_{k,t}),$$

which depends on the vector of implemented policies. Therefore the continuation-adjusted ideals should be understood as the solution to a joint system:

$$x_{j,t} = \text{proj}_{\mathcal{X}_{j,t}} \left(\frac{\beta}{\Lambda_t(x_{1,t}, \dots, x_{K,t}) + \beta d_j} \right), \quad j = 1, \dots, K.$$

In stationary self-financing equilibria with positive distributions, Lemma 8 gives $\Lambda = 1$, so this cross-firm link through Λ disappears. Along transition paths, the common Λ_t creates an additional cross-firm linkage in voting incentives.

3.2.5 Static ideal as a special case

Corollary 2 (Static ideal if and only if $\Lambda_t = 1$). *The fund's continuation-adjusted ideal equals its static favorite policy if and only if*

$$\Lambda_t = 1.$$

In that case,

$$\hat{x}_{j,t}^B = \frac{\beta}{1 + \beta d_j}.$$

If $\Lambda_t > 1$, then

$$\hat{x}_{j,t}^B < \frac{\beta}{1 + \beta d_j}.$$

The difference between the static ideal and the continuation-adjusted ideal is

$$\frac{\beta}{1 + \beta d_j} - \frac{\beta}{\Lambda_t + \beta d_j} = \frac{\beta(\Lambda_t - 1)}{(1 + \beta d_j)(\Lambda_t + \beta d_j)}.$$

Proof. The fund's static favorite policy solves

$$\max_{x \in [0,1]} \{ \pi_j(x) + \beta s_j(x) \},$$

so it is

$$\frac{\beta}{1 + \beta d_j}.$$

The continuation-adjusted ideal is

$$\frac{\beta}{\Lambda_t + \beta d_j}.$$

These two expressions are equal if and only if $\Lambda_t = 1$. The remaining claims follow by inspection and subtraction. $\square$

3.2.6 Governance value and binding voting constraints

Proposition 2 (Governance value only when voting constraints bind). *Fix firm j and period t . Suppose that, locally, the implemented policy can be written as a differentiable function of the fund's ownership,*

$$x_{j,t} = x_j(\alpha_{j,t}).$$

Consider the fund's current voting-stage value from firm j , measured in resource units:

$$\alpha_{j,t} [\Lambda_t \pi_j(x_j(\alpha_{j,t})) + \beta s_j(x_j(\alpha_{j,t}))].$$

The derivative with respect to $\alpha_{j,t}$ is

$$\underbrace{\Lambda_t \pi_j(x_{j,t}) + \beta s_j(x_{j,t})}_{\text{direct value of one more share}} + \underbrace{\alpha_{j,t} [\Lambda_t \pi'_j(x_{j,t}) + \beta s'_j(x_{j,t})]}_{\text{current governance value}} x'_j(\alpha_{j,t}).$$

If the fund's continuation-adjusted ideal lies in the interior of the implementable set and is implemented, then

$$\Lambda_t \pi'_j(x_{j,t}) + \beta s'_j(x_{j,t}) = 0,$$

and the current governance value is zero. If instead the implemented policy is constrained by the boundary of the implementable set, then the bracketed term is generally nonzero, and an additional share has current governance value to the extent that it relaxes the voting constraint and changes $x_{j,t}$.

Proof. Differentiate the displayed value with respect to $\alpha_{j,t}$. The product rule gives exactly the stated derivative. If the implemented policy equals the unconstrained continuation-adjusted ideal and lies in the interior of the implementable set, then the first-order condition from Proposition 1 holds. Therefore the policy-effect term vanishes. At a boundary of the implementable set, this first-order condition need not hold, and a marginal share can have first-order governance value. $\square$

3.3 Fund Trading and Cross-Firm Governance Spillovers

This section characterizes the fund's trading problem. The previous sections characterized prices, dispersed ownership, and voting outcomes conditional on post-trade ownership. We now study how the fund chooses ownership across firms.

3.3.1 Fund Bellman problem and KKT conditions

Lemma 6 (Fund recursive problem and shadow values). *At the beginning of period t , the state is*

$$\omega_t = (\alpha_{t-1}, h_t^-, c_t, f_t).$$

In a Markov equilibrium, the fund's value satisfies

$$\begin{aligned} V(\alpha_{t-1}, h_t^-, c_t, f_t) = \max_{\alpha_t, x_t^B, c_{t+1}} \left\{ \mathcal{R}_t - c_{t+1} - Rf_t + \beta \sum_{j=1}^K \alpha_{j,t} s_j(x_{j,t}) \right. \\ \left. + \delta \sum_{f' \in \mathcal{F}} P(f' | f_t) V(\alpha_t, h_{t+1}^-, c_{t+1}, f') \right\}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} q_{j,t} &= \alpha_{j,t} - \alpha_{j,t-1}, \\ c_t^+ &= c_t + f_t - \sum_{j=1}^K p_{j,t} q_{j,t}, \\ \mathcal{R}_t &= Rc_t^+ + \sum_{j=1}^K \alpha_{j,t} \pi_j(x_{j,t}), \end{aligned}$$

and

$$h_{j,t+1}^-(b) = \lambda h_{j,t}^+(b) + (1 - \lambda)(1 - \alpha_{j,t}).$$

The maximization is subject to market clearing, weighted-median voting, the law of motion for dispersed ownership, and the feasibility constraints fghfghghghgf

$$c_t^+ \geq 0, \quad \sum_{j=1}^K p_{j,t} \alpha_{j,t} \leq \bar{W}, \quad 0 \leq \alpha_{j,t} \leq 1, \quad 0 \leq c_{t+1} \leq \mathcal{R}_t.$$

Proof. The Bellman equation follows directly from the fund's objective and the timing of the model. $\square$

We now state the first-order condition for ownership on a smooth interior branch. The derivative below is evaluated at the optimal cash buffer c_{t+1}^* . By the envelope theorem, we do not differentiate through the response of c_{t+1}^* to a marginal ownership change; the effect of ownership on end-of-period resources is captured by Λ_t .

Let $\eta_t \geq 0$ be the multiplier on the post-trade cash constraint $c_t^+ \geq 0$, and let $\nu_t \geq 0$ be the multiplier on the portfolio-size constraint

$$\sum_{j=1}^K p_{j,t} \alpha_{j,t} \leq \bar{W}.$$

Proposition 3 (Fund KKT conditions). *Consider a smooth region in which the fund's choice of $\alpha_{j,t}$ is interior, $0 < \alpha_{j,t} < 1$. Then the fund's optimality condition for firm j is*

$$\begin{aligned} & \underbrace{\Lambda_t \pi_j(x_{j,t}) + \beta s_j(x_{j,t})}_{\text{direct value of one more share}} + \underbrace{\alpha_{j,t} [\Lambda_t \pi'_j(x_{j,t}) + \beta s'_j(x_{j,t})]}_{\text{current governance value}} \frac{\partial x_{j,t}}{\partial \alpha_{j,t}} \\ & + \underbrace{\delta \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{f' \in \mathcal{F}} P(f' | f_t) V(\alpha_t, h_{t+1}^-, c_{t+1}^*, f') \right]}_{\text{continuation value}} \\ & = \underbrace{(R\Lambda_t + \eta_t) \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{k=1}^K p_{k,t} q_{k,t} \right]}_{\text{shadow cost of acquiring shares}} + \underbrace{\nu_t \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{k=1}^K p_{k,t} \alpha_{k,t} \right]}_{\text{shadow cost of portfolio capacity}}. \end{aligned} \tag{6}$$

Proof. Differentiate the Bellman objective with respect to $\alpha_{j,t}$ on a smooth interior branch. A marginal share gives current direct value

$$\Lambda_t \pi_j(x_{j,t}) + \beta s_j(x_{j,t}).$$

It also affects the implemented policy, generating the current governance term. It affects the next state and therefore continuation value. On the cost side, acquiring shares uses trading-stage cash. A dollar spent at the trading stage reduces end-of-period resources by R , and a marginal dollar of end-of-period resources is worth Λ_t . The post-trade cash multiplier adds η_t . The portfolio-size constraint contributes the term with multiplier ν_t . This gives (6). $\square$

Remark 2 (Smooth-branch interpretation). *Proposition 3 is a local characterization on smooth interior branches. At ownership bounds, voting thresholds, and small-shareholder*

exhaustion cutoffs, the same comparison can be written using one-sided directional derivatives.

3.3.2 Cross-firm spillovers and selective engagement

The KKT condition shows that the value of a share in firm j depends on the fund's opportunities in other firms. A dollar used to buy governance capital in firm j cannot be used to buy governance capital in firm k . Thus, even when the portfolio-size constraint is slack, the acquisition cost of one firm is an opportunity cost in all other firms.

Define the marginal benefit of ownership in firm j by the left-hand side of (6):

$$MB_{j,t} \equiv \Lambda_t \pi_j(x_{j,t}) + \beta s_j(x_{j,t}) + \alpha_{j,t} [\Lambda_t \pi'_j(x_{j,t}) + \beta s'_j(x_{j,t})] \frac{\partial x_{j,t}}{\partial \alpha_{j,t}} + \delta \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{f'} P(f' | f_t) V(\alpha_t, h_{t+1}^-, c_{t+1}^*, f') \right]. \quad (7)$$

Define the marginal trading cost by

$$TC_{j,t} \equiv \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{k=1}^K p_{k,t} q_{k,t} \right].$$

Proposition 4 (Cross-firm governance spillovers). *On a smooth interior branch with slack portfolio-size constraint and slack post-trade cash constraint, the fund equalizes*

$$\frac{MB_{j,t}}{TC_{j,t}}$$

across all firms held at the interior ownership levels. Hence, the value of buying one firm depends on the marginal governance opportunities available in the other firms.

Proof. If the post-trade cash and portfolio-size constraints are slack, then $\eta_t = \nu_t = 0$. The KKT condition becomes

$$MB_{j,t} = R \Lambda_t TC_{j,t}.$$

For any two interior firms j and k ,

$$\frac{MB_{j,t}}{TC_{j,t}} = R \Lambda_t = \frac{MB_{k,t}}{TC_{k,t}}.$$

□

Corollary 3 (Selective engagement). *The fund may buy a brown firm when its marginal governance value per dollar is high. In particular, a firm with high d_j can be an attractive target if the implemented policy is far from the fund's continuation-adjusted ideal and additional ownership relaxes the voting constraint.*

Proof. The current governance term in $MB_{j,t}$ is

$$\alpha_{j,t} [\Lambda_t \pi'_j(x_{j,t}) + \beta s'_j(x_{j,t})] \frac{\partial x_{j,t}}{\partial \alpha_{j,t}}.$$

Using

$$\hat{x}_{j,t}^B = \frac{\beta}{\Lambda_t + \beta d_j},$$

we have

$$\Lambda_t \pi'_j(x_{j,t}) + \beta s'_j(x_{j,t}) = (\Lambda_t + \beta d_j) (\hat{x}_{j,t}^B - x_{j,t}).$$

Thus, governance value is positive when the fund is constrained below its desired policy, and marginal ownership raises the implemented policy. $\square$

3.4 Limiting Ownership and Policies

We now study the limiting allocation of ownership and policy along a deterministic equilibrium path. We ask what dispersed ownership, prices, and policies must look like when the fund's ownership and firm policies settle down.

Time-invariant equilibrium strategies need not generate a constant state, and a dynamic equilibrium path need not converge for every parameter vector. Our results therefore have three layers. First, conditional on convergence of fund ownership and policy, partial persistence implies convergence of dispersed ownership to a unique limiting distribution. Second, we characterize that distribution and the associated limiting price in closed form. Third, we derive the fund's limiting no-purchase condition and state sufficient conditions under which it pins down an interior limiting block.

Definition 2 (Convergent equilibrium path). *A deterministic self-financing equilibrium path is ownership-policy convergent if, for some $\alpha^\infty \in [0, 1]^K$, $x^\infty \in [0, 1]^K$, $c^\infty \geq 0$, we have*

$$\alpha_t \rightarrow \alpha^\infty, \quad x_t \rightarrow x^\infty, \quad c_t \rightarrow c^\infty,$$

and equilibrium prices are bounded. Since $q_{j,t} = \alpha_{j,t} - \alpha_{j,t-1}$, ownership convergence

implies

$$q_{j,t} \rightarrow 0 \quad \text{for every } j.$$

3.4.1 Convergence of dispersed ownership

Partial persistence makes the dispersed-ownership transition contractive once fund ownership and policy settle down. The key observation is that, for fixed fund ownership and policy, market clearing chooses a common shift in all small shareholders' demands so that their post-trade holdings are nonnegative and integrate to the residual float. Equivalently, the market-clearing holdings profile is the metric projection of

$$h_{j,t}^-(b) + \frac{bs_j(x_{j,t})}{\rho}$$

onto the set of nonnegative profiles that integrate to $1 - \alpha_{j,t}$. This projection is nonexpansive.

Lemma 7 (Limiting dispersed ownership). *Suppose $h_{j,0}^-$ is continuous and Lipschitz. Consider an ownership–policy convergent equilibrium path and fix a firm j with positive limiting residual float, $1 - \alpha_j^\infty > 0$. Then there exist unique profiles $h_j^{-,\infty}$ and $h_j^{+,\infty}$ such that*

$$h_{j,t}^- \rightarrow h_j^{-,\infty}, \quad h_{j,t}^+ \rightarrow h_j^{+,\infty}$$

uniformly in b . Moreover,

$$p_{j,t} \rightarrow p_j^\infty,$$

and the limiting objects satisfy

$$h_j^{+,\infty}(b) = \left[1 - \alpha_j^\infty + \frac{\pi_j(x_j^\infty) + bs_j(x_j^\infty) - (R-1)p_j^\infty}{\rho(1-\lambda)} \right]_+, \quad (8)$$

$$h_j^{-,\infty}(b) = \lambda h_j^{+,\infty}(b) + (1-\lambda)(1 - \alpha_j^\infty), \quad (9)$$

and

$$\int_0^1 h_j^{+,\infty}(b) db = 1 - \alpha_j^\infty. \quad (10)$$

Proof. Small-shareholder optimality and market clearing give

$$h_{j,t}^+(b) = \left[h_{j,t}^-(b) + \frac{\pi_j(x_{j,t}) + bs_j(x_{j,t}) + p_{j,t+1} - Rp_{j,t}}{\rho} \right]_+, \quad (11)$$

with

$$\int_0^1 h_{j,t}^+(b) db = 1 - \alpha_{j,t}.$$

For fixed $\alpha_{j,t}$ and $x_{j,t}$, the market-clearing profile in (11) is the metric projection described above. Hence, if two economies have the same fund ownership and policy but inherited profiles h and $\tilde{h}$, their post-trade profiles satisfy

$$\|h^+ - \tilde{h}^+\|_2 \leq \|h - \tilde{h}\|_2.$$

The persistence law therefore contracts differences in inherited profiles by the factor λ :

$$\|h_{t+1}^- - \tilde{h}_{t+1}^-\|_2 \leq \lambda \|h_t^- - \tilde{h}_t^-\|_2.$$

Because $\alpha_{j,t} \rightarrow \alpha_j^\infty$ and $x_{j,t} \rightarrow x_j^\infty$, the corresponding market-clearing projections converge to the projection associated with α_j^∞ and x_j^∞ . The standard argument for a sequence of contractions with common modulus $\lambda < 1$ gives

$$h_{j,t}^- \rightarrow h_j^{-,\infty} \quad \text{in } L^2,$$

where the limit is the unique fixed point of the limiting ownership transition. Continuity of the market-clearing projection then gives

$$h_{j,t}^+ \rightarrow h_j^{+,\infty} \quad \text{in } L^2.$$

The convergence is uniform. Since $s_j(x)$ is bounded on $[0, 1]$, the positive-part formula implies a uniform bound on the Lipschitz constants of the post-trade profiles. The persistence law gives the same property for the pre-trade profiles. Uniform equicontinuity together with L^2 convergence therefore implies uniform convergence.

The market-clearing common shift in (11) depends continuously on inherited ownership, fund ownership, and policy. Hence, $Rp_{j,t} - p_{j,t+1}$ converges. Since equilibrium prices are bounded, iterating the asset-pricing identity gives

$$p_{j,t} = \sum_{n=0}^{\infty} \frac{Rp_{j,t+n} - p_{j,t+n+1}}{R^{n+1}}.$$

Dominated convergence therefore implies $p_{j,t} \rightarrow p_j^\infty$. Taking limits in small-shareholder optimality, persistence, and market clearing gives (8)–(10). $\square$

Remark 3 (Full ownership). *If $\alpha_j^\infty = 1$, market clearing implies that limiting dispersed ownership is zero. In that corner, the dispersed market alone does not uniquely pin down the share price. The pricing results below therefore concern firms with a positive limiting residual float.*

3.4.2 Limiting prices and endogenous marginal holders

The limiting formula nests two regions. When the residual float is sufficiently large, every type remains active and price reflects the average small shareholder. When the residual float is small, low- b types are exhausted and price is pinned down by an endogenous marginal seller.

Proposition 5 (Limiting sorting and endogenous marginal holders). *Fix a firm j with $1 - \alpha_j^\infty > 0$ and $s_j(x_j^\infty) > 0$. If*

$$1 - \alpha_j^\infty \geq \frac{s_j(x_j^\infty)}{2\rho(1 - \lambda)}, \quad (12)$$

then no positive mass of types is exhausted. The limiting price satisfies

$$(R - 1)p_j^\infty = \pi_j(x_j^\infty) + \frac{1}{2}s_j(x_j^\infty), \quad (13)$$

and

$$h_j^{+, \infty}(b) = 1 - \alpha_j^\infty + \frac{s_j(x_j^\infty)}{\rho(1 - \lambda)} \left(b - \frac{1}{2} \right). \quad (14)$$

If

$$1 - \alpha_j^\infty < \frac{s_j(x_j^\infty)}{2\rho(1 - \lambda)}, \quad (15)$$

then low- b shareholders are exhausted. The lowest active type is

$$b_j^* = 1 - \sqrt{\frac{2\rho(1 - \lambda)(1 - \alpha_j^\infty)}{s_j(x_j^\infty)}}, \quad (16)$$

and

$$h_j^{+, \infty}(b) = \frac{s_j(x_j^\infty)}{\rho(1 - \lambda)} (b - b_j^*)_+. \quad (17)$$

The limiting price satisfies

$$(R - 1)p_j^\infty = \pi_j(x_j^\infty) + b_j^*s_j(x_j^\infty) + \rho(1 - \lambda)(1 - \alpha_j^\infty). \quad (18)$$

Proof. If no type is exhausted, the positive-part operator in (8) is slack. Integrating over b and imposing market clearing gives (13); substitution gives (14). The lowest type has nonnegative holdings exactly when (12) holds.

If that condition fails, let $b_j^* > 0$ be the lowest active type. Its limiting holding is zero, so

$$1 - \alpha_j^\infty + \frac{\pi_j(x_j^\infty) + b_j^* s_j(x_j^\infty) - (R-1)p_j^\infty}{\rho(1-\lambda)} = 0.$$

This gives (18). Substitution into (8) gives (17). Finally, market clearing implies

$$1 - \alpha_j^\infty = \frac{s_j(x_j^\infty)}{2\rho(1-\lambda)}(1 - b_j^*)^2,$$

which gives (16). □

Corollary 4 (Ownership, persistence, and acquisition prices). *On the low-type-exhaustion branch, holding the limiting policy fixed,*

$$\frac{\partial b_j^*}{\partial \alpha_j^\infty} = \frac{1 - b_j^*}{2(1 - \alpha_j^\infty)} > 0, \tag{19}$$

$$\frac{\partial b_j^*}{\partial \lambda} = \frac{1 - b_j^*}{2(1 - \lambda)} > 0, \tag{20}$$

$$\frac{\partial((R-1)p_j^\infty)}{\partial \alpha_j^\infty} = \rho(1-\lambda) \frac{b_j^*}{1 - b_j^*} > 0, \tag{21}$$

and

$$\frac{\partial((R-1)p_j^\infty)}{\partial \lambda} = \rho(1 - \alpha_j^\infty) \frac{b_j^*}{1 - b_j^*} > 0. \tag{22}$$

Thus both greater fund ownership and greater persistence move the marginal seller upward in b and raise the price at which the fund can acquire additional shares.

Proof. Differentiate (16) to obtain (19) and (20). Then differentiate (18), holding x_j^∞ fixed, and use the cutoff equation to obtain (21) and (22). □

The corollary isolates the endogenous supply force. Low- b shareholders sell out first. Once they are exhausted, an additional purchase must be absorbed by increasingly pro-social holders. Persistence strengthens this force because more of the sorted post-trade distribution survives into the next period.

3.4.3 Limiting cash and policy

A limiting allocation becomes a *limiting rest point* when its limiting no-trade action reproduces the state and solves the fund's Bellman problem at that state. The next results require this global no-deviation property; convergence of the state alone is not sufficient.

Lemma 8 (Limiting cash and the shadow value of resources). *At any limiting rest point,*

$$c^\infty = 0.$$

If, in addition, $\sum_{j=1}^K \alpha_j^\infty > 0$, then

$$D^\infty = \sum_{j=1}^K \alpha_j^\infty \pi_j(x_j^\infty) > 0$$

and

$$\Lambda^\infty = 1.$$

Proof. At a limiting rest point,

$$D^\infty = (R-1)c^\infty + \sum_{j=1}^K \alpha_j^\infty \pi_j(x_j^\infty).$$

Suppose $c^\infty > 0$. The fund can distribute an additional $\varepsilon \in (0, c^\infty)$ at the limiting state and thereafter follow the same no-trade policy while carrying $c^\infty - \varepsilon$. The current distribution rises by ε , while every future distribution falls by $(R-1)\varepsilon$. The present-value gain is

$$\varepsilon - \sum_{\tau=1}^{\infty} \delta^\tau (R-1)\varepsilon = \varepsilon \frac{1 - \delta R}{1 - \delta} > 0,$$

contradicting Bellman optimality. Hence $c^\infty = 0$.

If total limiting fund ownership is positive, then

$$\pi_j(x_j^\infty) = \bar{\pi}_j - \frac{1}{2}(x_j^\infty)^2 > 0.$$

Thus $D^\infty > 0$, marginal resources are distributed, and $\Lambda^\infty = 1$. □

At an engaged limiting rest point, the fund's continuation-adjusted ideal coincides

with its static ideal. Thus, if $\alpha_j^\infty > 0$,

$$x_j^\infty \in \arg \min_{x \in \mathcal{X}_j(\alpha_j^\infty, h_j^{+, \infty})} \left| x - \frac{\beta}{1 + \beta d_j} \right|, \quad (23)$$

where $\mathcal{X}_j(\alpha_j^\infty, h_j^{+, \infty})$ is the implementable set defined earlier. If $\alpha_j^\infty = 0$, the fund's report is irrelevant and the limiting policy is the dispersed weighted median.

3.4.4 Voluntary stopping of block accumulation

Let MB_j^∞ denote the limiting value of the marginal dynamic benefit $MB_{j,t}$ defined above. It includes the direct financial and social value of another share, all current policy effects, and all continuation effects. Let $\eta^\infty \geq 0$ and $\nu^\infty \geq 0$ denote the limiting multipliers on the post-trade cash constraint and the portfolio-size constraint.

Proposition 6 (Limiting voluntary-stopping condition). *At a smooth limiting rest point,*

$$\frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{k=1}^K p_{k,t}(\alpha_t)(\alpha_{k,t} - \alpha_{k,t-1}) \right] \Big|_{\alpha_t = \alpha_{t-1} = \alpha^\infty} = p_j^\infty. \quad (24)$$

Therefore the one-sided no-purchase condition is

$$MB_j^\infty \leq (R\Lambda^\infty + \eta^\infty)p_j^\infty + \nu^\infty \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{k=1}^K p_{k,t}(\alpha_t)\alpha_{k,t} \right] \Big|_{\alpha_t = \alpha^\infty}. \quad (25)$$

If $0 < \alpha_j^\infty < 1$ and both local ownership directions are feasible, this condition holds with equality.

At an engaged limiting rest point, $\Lambda^\infty = 1$, so

$$MB_j^\infty \leq (R + \eta^\infty)p_j^\infty + \nu^\infty \frac{\partial}{\partial \alpha_{j,t}} \left[\sum_{k=1}^K p_{k,t}(\alpha_t)\alpha_{k,t} \right] \Big|_{\alpha_t = \alpha^\infty}. \quad (26)$$

If the portfolio-size constraint is slack and two firms j and k are held at interior levels, the fund equalizes marginal dynamic value per dollar:

$$\frac{MB_j^\infty}{p_j^\infty} = R + \eta^\infty = \frac{MB_k^\infty}{p_k^\infty}. \quad (27)$$

Proof. At a limiting rest point every net trade is zero. Differentiating trading expendi-

ture with respect to α_j , all terms involving derivatives of prices are multiplied by zero net trades. Only the direct expenditure on firm j remains, proving (24). The limiting KKT condition from Proposition 3 then gives (25). The remaining claims follow from Lemma 8 and complementary slackness. $\square$

The stopping condition makes the role of persistence precise. On the low-type-exhaustion branch, greater α_j^∞ or greater λ raises p_j^∞ , holding policy fixed. This raises the price component of the shadow acquisition cost. Persistence therefore creates an endogenous force against further accumulation. It does not, by itself, prove that an interior stopping level exists, because governance and continuation benefits can also vary with ownership.

Proposition 7 (Interior limiting block under single crossing). *Fix a smooth limiting branch for firm j , parameterized by ownership*

$$a \in [\underline{a}_j, \bar{a}_j] \subset (0, 1),$$

with policies, prices, multipliers, and continuation values chosen optimally on the branch. Suppose the difference between the marginal dynamic benefit and the shadow acquisition cost on the two sides of (25) is continuous and strictly decreasing in a . Suppose also that this difference is positive at $\underline{a}_j$ and negative at $\bar{a}_j$. Then there is a unique

$$a \in (\underline{a}_j, \bar{a}_j)$$

at which (25) holds with equality. Any interior limiting rest point on this branch must have $\alpha_j^\infty = a$.

If an equilibrium path eventually remains on this branch, ownership is monotone from that date onward, and its limit is a Bellman-verified interior rest point, then its limit is the unique ownership level described above and

$$q_{j,t} \rightarrow 0.$$

Proof. Existence and uniqueness follow from continuity, strict monotonicity, and the intermediate value theorem. At an interior limiting rest point, the KKT condition requires equality in (25); uniqueness of the equality point therefore identifies α_j^∞ . If ownership is eventually monotone and remains in the compact interval, it converges. The assumed Bellman verification makes its interior limit the unique equality point, and

ownership convergence implies $q_{j,t} \rightarrow 0$. □

3.4.5 Characterization of a limiting rest point

Definition 3 (Limiting rest point). *A limiting rest point consists of*

$$\left\{ \alpha_j^\infty, p_j^\infty, x_j^\infty, x_j^{B,\infty}, h_j^{-,\infty}, h_j^{+,\infty} \right\}_{j=1}^K$$

and D^∞ , satisfying:

(i) Share feasibility:

$$0 \leq \alpha_j^\infty \leq 1.$$

(ii) Limiting trading, persistence, and market clearing: *for every firm with positive residual float,*

$$h_j^{+,\infty}(b) = \left[1 - \alpha_j^\infty + \frac{\pi_j(x_j^\infty) + bs_j(x_j^\infty) - (R-1)p_j^\infty}{\rho(1-\lambda)} \right]_+,$$

$$\int_0^1 h_j^{+,\infty}(b) db = 1 - \alpha_j^\infty,$$

and

$$h_j^{-,\infty}(b) = \lambda h_j^{+,\infty}(b) + (1-\lambda)(1 - \alpha_j^\infty).$$

(iii) Voting: x_j^∞ is a weighted median generated by the fund's report, fund ownership, and limiting dispersed voting weights. If $\alpha_j^\infty > 0$, it satisfies (23); if $\alpha_j^\infty = 0$, it is the dispersed weighted median.

(iv) Cash and distributions:

$$c^\infty = 0, \quad D^\infty = \sum_{j=1}^K \alpha_j^\infty \pi_j(x_j^\infty).$$

If total limiting fund ownership is positive, then

$$D^\infty > 0 \quad \text{and} \quad \Lambda^\infty = 1.$$

(v) Portfolio feasibility:

$$\sum_{j=1}^K p_j^\infty \alpha_j^\infty \leq \bar{W}.$$

(vi) Fund optimality: *at the state*

$$(\alpha^\infty, h^{-,\infty}, 0, 0),$$

the no-trade action

$$(\alpha^\infty, x^{B,\infty}, 0)$$

solves the fund's Bellman problem. No feasible ownership, voting-report, or cash deviation raises value after accounting for the prices, policies, and next-period dispersed ownership induced by the entire deviation.

Proposition 8 (Characterization of a limiting rest point). *A candidate limiting allocation is a limiting rest point if and only if it satisfies Definition 3.*

Proof. Necessity follows by taking limits in small-shareholder optimality, market clearing, persistence, weighted-median voting, feasibility, and Bellman optimality. Lemma 8 gives the cash and shadow-value conditions. Conversely, the conditions in Definition 3 imply that the limiting no-trade allocation reproduces ownership, dispersed holdings, prices, policies, and cash, while the fund's action is globally Bellman optimal. Hence the candidate is a limiting rest point. $\square$

Remark 4 (Convergence, multiplicity, and finite-time stopping). *The section does not claim that every equilibrium path converges. The contraction result establishes a unique limiting dispersed-ownership distribution conditional on limiting fund ownership and policy. The joint ownership–policy system can still have multiple limiting rest points or, without additional monotonicity, fail to converge.*

Likewise,

$$q_{j,t} \rightarrow 0$$

is an asymptotic stopping result. With continuous ownership and quadratic trading costs, the fund may make successively smaller purchases forever. Exact finite-time cessation would require an additional nonconvexity, such as a fixed trading cost or a discrete ownership grid. Persistence nevertheless provides a clear endogenous limiting force: it moves the marginal seller upward and raises the acquisition price as the fund accumulates.

4 A Welfare Benchmark: Central Planner

This section introduces a welfare benchmark: a utilitarian central planner directly chooses firm policies and assigns the unit supply of each firm's shares to the fund and to small shareholders.

4.1 The planner's problem

The planner chooses policies

$$x = (x_1, \dots, x_K) \in [0, 1]^K,$$

fund ownership

$$\alpha^P = (\alpha_1^P, \dots, \alpha_K^P) \in [0, 1]^K,$$

and an assignment of residual small-shareholder shares

$$z^P(b) = (z_1^P(b), \dots, z_K^P(b)),$$

across firms.

Let $a^P \equiv \sum_{j=1}^K \alpha_j^P$ denote the aggregate number of shares assigned to the fund. The planner faces a cap $a^P \leq \bar{A} \in [0, K]$. This cap is the analog of the portfolio-size constraint in the trading-voting economy. Since the planner directly assigns shares and does not clear markets, the cap is stated in shares rather than dollars.

For an aggregate fund size a^P , the residual small-shareholder ownership has total mass $K - a^P$. Under the uniform type distribution $b \sim U[0, 1]$, an interval of types of length db contains $(K - a^P) db$ small-shareholder share units. The planner chooses an assignment rule $z^P(b) = (z_1^P(b), \dots, z_K^P(b))$, where $z_j^P(b) \geq 0$, and $\sum_{j=1}^K z_j^P(b) = 1$ for every b . Thus, $z_j^P(b)$ is the fraction of type- b residual small-shareholder share units assigned to firm j . The holdings density of type b in firm j is $h_j^P(b) = (K - a^P) z_j^P(b)$.

Each firm has one unit of shares. Therefore, firm-by-firm share feasibility is

$$\alpha_j^P + (K - a^P) \int_0^1 z_j^P(b) db = 1, \quad j = 1, \dots, K.$$

The planner's welfare function is

$$\begin{aligned} \mathcal{W}^P(x, \alpha^P, z^P) &= \sum_{j=1}^K \alpha_j^P [\pi_j(x_j) + \beta s_j(x_j)] \\ &\quad + (K - a^P) \int_0^1 \sum_{j=1}^K z_j^P(b) [\pi_j(x_j) + b s_j(x_j)] db. \end{aligned} \quad (28)$$

Using the firm share feasibility, this can be written as

$$\mathcal{W}^P(x, \alpha^P, z^P) = \sum_{j=1}^K \left\{ \pi_j(x_j) + \bar{\beta}_j^P s_j(x_j) \right\}, \quad (29)$$

where

$$\bar{\beta}_j^P \equiv \beta \alpha_j^P + (K - a^P) \int_0^1 b z_j^P(b) db$$

is the ownership-weighted mean social valuation assigned to firm j .

We summarize the central planner's problem below.

Definition 4 (Planner's problem). *The planner's arrangement of policies and allocation of shares solves*

$$\max_{x, \alpha^P, z^P} \mathcal{W}^P(x, \alpha^P, z^P)$$

subject to

$$x_j \in [0, 1], \quad 0 \leq \alpha_j^P \leq 1, \quad z_j^P(b) \geq 0,$$

$$\sum_{j=1}^K z_j^P(b) = 1 \quad \text{for every } b,$$

firm share feasibility,

$$\alpha_j^P + (K - a^P) \int_0^1 z_j^P(b) db = 1 \quad \text{for every } j,$$

and the fund's cap,

$$a^P = \sum_{j=1}^K \alpha_j^P \leq \bar{A}.$$

4.2 Policy choice and the mean-valuation reduction

Conditional on an ownership assignment, the planner's policy problem depends on the assignment only through the ownership-weighted mean valuation

$$\bar{\beta}_j^P = \beta \alpha_j^P + (K - a^P) \int_0^1 b z_j^P(b) db.$$

Lemma 9 (Mean-valuation reduction). *Fix firm j . Given $\bar{\beta}_j^P$, the planner's indirect welfare from firm j is*

$$\mathcal{V}_j(\bar{\beta}_j^P) \equiv \max_{x \in [0,1]} \{ \pi_j(x) + \bar{\beta}_j^P s_j(x) \}.$$

Under the primitives $\pi_j(x) = \bar{\pi}_j - \frac{1}{2}x^2$ and $s_j(x) = x - \frac{d_j}{2}x^2$, the planner's conditional first-best policy is

$$x_j^P(\bar{\beta}_j^P) = \frac{\bar{\beta}_j^P}{1 + d_j \bar{\beta}_j^P},$$

and indirect welfare is

$$\mathcal{V}_j(\bar{\beta}_j^P) = \bar{\pi}_j + \frac{(\bar{\beta}_j^P)^2}{2(1 + d_j \bar{\beta}_j^P)}.$$

Moreover,

$$\mathcal{V}'_j(\bar{\beta}) = s_j(x_j^P(\bar{\beta})) = \frac{\bar{\beta} + \frac{1}{2}d_j\bar{\beta}^2}{(1 + d_j\bar{\beta})^2} > 0,$$

and

$$\mathcal{V}''_j(\bar{\beta}) = \frac{1}{(1 + d_j\bar{\beta})^3} > 0.$$

Finally, for fixed $\bar{\beta} > 0$,

$$\frac{\partial}{\partial d_j} \mathcal{V}'_j(\bar{\beta}) = -\frac{\bar{\beta}^2(3 + d_j\bar{\beta})}{2(1 + d_j\bar{\beta})^3} < 0.$$

Thus the marginal value of assigning a higher social type is larger in lower- d_j firms.

Proof. For fixed $\bar{\beta}$, the planner solves

$$\max_x \left\{ \bar{\pi}_j - \frac{1}{2}x^2 + \bar{\beta} \left(x - \frac{d_j}{2}x^2 \right) \right\}.$$

The first-order condition is

$$-x + \bar{\beta}(1 - d_j x) = 0.$$

Solving gives

$$x_j^P(\bar{\beta}) = \frac{\bar{\beta}}{1 + d_j \bar{\beta}}.$$

Substituting this expression into the objective gives

$$\mathcal{V}_j(\bar{\beta}) = \bar{\pi}_j + \frac{\bar{\beta}^2}{2(1 + d_j \bar{\beta})}.$$

The expression for $\mathcal{V}'_j$ follows by the envelope theorem:

$$\mathcal{V}'_j(\bar{\beta}) = s_j(x_j^P(\bar{\beta})).$$

Substituting the policy gives

$$s_j(x_j^P(\bar{\beta})) = \frac{\bar{\beta} + \frac{1}{2}d_j \bar{\beta}^2}{(1 + d_j \bar{\beta})^2}.$$

Differentiating yields

$$\mathcal{V}''_j(\bar{\beta}) = \frac{1}{(1 + d_j \bar{\beta})^3} > 0$$

and

$$\frac{\partial}{\partial d_j} \mathcal{V}'_j(\bar{\beta}) = -\frac{\bar{\beta}^2(3 + d_j \bar{\beta})}{2(1 + d_j \bar{\beta})^3} < 0.$$

□

This lemma delivers the key policy comparison. The planner chooses the ideal policy of the ownership-weighted mean social valuation. In the trading-voting equilibrium, policy is instead determined by a weighted median, with the fund's effective voting type reduced to β/Λ_t when cash is scarce.

4.3 Efficient ownership assignment by the central planner

We now solve the planner's ownership assignment. Since firms are ordered by ESG cost,

$$d_1 \leq d_2 \leq \dots \leq d_K,$$

Lemma 9 implies that the marginal value of assigning higher social types is highest in lower- d_j firms. Hence the planner assigns higher-valuation investors to lower- d_j firms.

Let σ denote the ordering from lowest marginal social value to highest marginal social value:

$$d_{\sigma(1)} \geq d_{\sigma(2)} \geq \cdots \geq d_{\sigma(K)}.$$

Thus $\sigma(1)$ is the highest- d , brownest firm, and $\sigma(K)$ is the lowest- d , greenest firm. This ordering is pinned down by the technology and is not a fixed point over realized policies.

For a given fund size a , the small shareholders form a continuous mass $K - a$ with types uniformly distributed on $[0, 1]$, while the fund is an atom of mass a located at type β . The cumulative mass of investor-share units up to type b is

$$M_a(b) = (K - a)b + a\mathbf{1}\{b \geq \beta\}.$$

The fund atom occupies the rank interval

$$[r_-(a), r_+(a)], \quad r_-(a) = (K - a)\beta, \quad r_+(a) = (K - a)\beta + a.$$

Firm $\sigma(m)$ has capacity interval

$$I_m = (m - 1, m], \quad m = 1, \dots, K.$$

Proposition 9 (Efficient assignment of ownership). *The planner assigns the lowest unit of investor rank to firm $\sigma(1)$, the next unit to firm $\sigma(2)$, and so on. Equivalently, firm $\sigma(m)$ receives the investor mass with cumulative ranks in*

$$I_m = (m - 1, m].$$

The fund ownership assigned to firm $\sigma(m)$ is

$$\alpha_{\sigma(m)}^P(a) = [\min\{m, r_+(a)\} - \max\{m - 1, r_-(a)\}]_+, \quad (30)$$

where $[y]_+ \equiv \max\{y, 0\}$.

The small-shareholder assignment is

$$z_{\sigma(m)}^P(b; a) = \mathbf{1}\{b < \beta, (K - a)b \in I_m\} + \mathbf{1}\{b > \beta, (K - a)b + a \in I_m\}, \quad (31)$$

up to cutoff values of b , which are irrelevant for welfare.

Proof. For fixed policies and a fixed vector of mean valuations, assigning a marginal increase in social type to a lower- d_j firm is weakly more valuable than assigning it to a higher- d_j firm by Lemma 9. Equivalently, for $d_i < d_j$ and $\omega_H \geq \omega_L$,

$$[\mathcal{V}_i(\omega_H) - \mathcal{V}_i(\omega_L)] - [\mathcal{V}_j(\omega_H) - \mathcal{V}_j(\omega_L)] = \int_{\omega_L}^{\omega_H} [\mathcal{V}'_i(u) - \mathcal{V}'_j(u)] du \geq 0.$$

Thus higher social types should be matched to lower- d_j firms.

The combined investor distribution consists of a continuous mass $K - a$ of small-shareholder types uniformly distributed on $[0, 1]$, plus a fund atom of mass a at type β . Hence cumulative investor mass up to type b is

$$M_a(b) = (K - a)b + a\mathbf{1}\{b \geq \beta\}.$$

Since each firm has capacity one, firm $\sigma(m)$ receives cumulative rank interval $I_m = (m - 1, m]$. The fund atom occupies the interval

$$[r_-(a), r_+(a)] = [(K - a)\beta, (K - a)\beta + a].$$

The overlap of this interval with I_m is

$$[\min\{m, r_+(a)\} - \max\{m - 1, r_-(a)\}]_+,$$

which gives (30). Small shareholders below β have rank $(K - a)b$, and small shareholders above β have rank $(K - a)b + a$. Therefore type b is assigned to firm $\sigma(m)$ if this rank lies in I_m , giving (31). $\square$

Given a , the assignment in Proposition 9 determines

$$\bar{\beta}_j^P(a) = \beta\alpha_j^P(a) + (K - a) \int_0^1 b z_j^P(b; a) db.$$

The corresponding planner policy is

$$x_j^P(a) = \frac{\bar{\beta}_j^P(a)}{1 + d_j \bar{\beta}_j^P(a)}.$$

The planner's indirect welfare is

$$\mathcal{W}^P(a) = \sum_{j=1}^K \left[\bar{\pi}_j + \frac{(\bar{\beta}_j^P(a))^2}{2(1 + d_j \bar{\beta}_j^P(a))} \right]. \quad (32)$$

The planner chooses

$$a^P \in \arg \max_{a \in [0, \bar{A}]} \mathcal{W}^P(a).$$

4.3.1 Summary of the central planner's choices

The planner's allocation has two steps. First, the planner assigns shares directly. The fund is an atom of mass a^P at type β , and the residual small-shareholder ownership is sorted by type. Higher- b ownership units are assigned to firms where social valuation is most productive, which under the baseline ordering means toward lower- d_j firms. Second, given this ownership assignment, the planner chooses each firm's policy as the ideal policy of the ownership-weighted mean social valuation:

$$x_j^P = \frac{\bar{\beta}_j^P}{1 + d_j \bar{\beta}_j^P}, \quad \bar{\beta}_j^P = \beta \alpha_j^P + (K - a^P) \int_0^1 b z_j^P(b) db.$$

The planner, therefore, produces sharp ownership sorting and mean-preference policy choice, in contrast to the trading-voting equilibrium, where ownership is price-mediated, and policy is determined by a weighted median. We further analyze the differences between the central planner's choices and the trading-voting equilibrium below.

4.4 Comparison with the trading-voting equilibrium

The planner and the trading-voting equilibrium differ in two dimensions: policies and ownership. The planner assigns shares directly and chooses policies using ownership-weighted mean valuations. The trading-voting equilibrium generates ownership through markets, prices, transaction costs, cash constraints, and governance value; policies are then determined by weighted-median voting.

Let (α^P, h^P, x^P) denote the capped planner allocation, and let $(\alpha^E, h^{E,+}, x^E, \Lambda^E)$ denote a stationary trading-voting equilibrium.

4.4.1 Policy wedge

For the planner, define the ownership-weighted mean social valuation in firm j :

$$\bar{\beta}_j^P = \beta \alpha_j^P + \int_0^1 b h_j^P(b) db.$$

The planner's policy is

$$x_j^P = x_j^*(\bar{\beta}_j^P) = \frac{\bar{\beta}_j^P}{1 + d_j \bar{\beta}_j^P}.$$

For the trading-voting equilibrium, let m_j^E denote the weighted median type in firm j . The dispersed shareholders are located at their true types b , with weights $h_j^{E,+}(b)$. The fund is an atom of mass α_j^E , but because of the shadow value of cash, it votes as if it were located at the effective type $\frac{\beta}{\Lambda^E}$. Thus, the equilibrium policy is

$$x_j^E = x_j^*(m_j^E) = \frac{m_j^E}{1 + d_j m_j^E}.$$

Proposition 10 (Policy wedge: mean assignment versus median voting). *For each firm j , the policy difference between the capped planner and the stationary trading-voting equilibrium is*

$$x_j^P - x_j^E = \frac{\bar{\beta}_j^P}{1 + d_j \bar{\beta}_j^P} - \frac{m_j^E}{1 + d_j m_j^E}.$$

Equivalently,

$$x_j^P - x_j^E = \frac{\bar{\beta}_j^P - m_j^E}{(1 + d_j \bar{\beta}_j^P)(1 + d_j m_j^E)}.$$

Therefore,

$$x_j^P > x_j^E \iff \bar{\beta}_j^P > m_j^E.$$

The planner is greener than the trading-voting equilibrium in firm j if and only if the mean valuation assigned to firm j by the planner exceeds the equilibrium weighted median type in that firm.

Proof. The planner policy follows from the mean-valuation reduction:

$$x_j^P = x_j^*(\bar{\beta}_j^P) = \frac{\bar{\beta}_j^P}{1 + d_j \bar{\beta}_j^P}.$$

The equilibrium policy is the weighted median of shareholder ideal points. Since $x_j^*(b) =$

$b/(1 + d_j b)$ is strictly increasing in b , the weighted median of ideal policies is the ideal policy of the weighted median type:

$$x_j^E = x_j^*(m_j^E) = \frac{m_j^E}{1 + d_j m_j^E}.$$

Subtracting gives

$$x_j^P - x_j^E = \frac{\bar{\beta}_j^P}{1 + d_j \bar{\beta}_j^P} - \frac{m_j^E}{1 + d_j m_j^E} = \frac{\bar{\beta}_j^P - m_j^E}{(1 + d_j \bar{\beta}_j^P)(1 + d_j m_j^E)}.$$

The denominator is strictly positive, so the sign of the policy wedge equals the sign of $\bar{\beta}_j^P - m_j^E$. $\square$

Proposition 10 shows that the planner does not necessarily make every firm greener. The sign of the policy wedge is firm-specific. A firm is too brown in the trading-voting equilibrium if its equilibrium median type is below the planner's assigned mean valuation. But a firm can be too green in the trading-voting equilibrium if the fund is pivotal, if the dispersed shareholder base is strongly sorted toward high- b types, or if the equilibrium weighted median exceeds the planner's mean valuation.

4.4.2 Ownership wedge

The planner assigns ownership by efficient type sorting, whereas the trading equilibrium assigns ownership where the marginal value exceeds the acquisition price. As a result, the planner tends to produce sharper ownership sorting. The trading-voting equilibrium tends to produce smoother, price-mediated sorting, and common ownership can arise because the fund equalizes marginal governance value per dollar across firms.

The planner ranks investor ownership units by social valuation and assigns them directly to firms. Higher social types are assigned to firms where their valuation has the highest welfare value. In the baseline ordering $d_1 \leq \dots \leq d_K$, this means that higher- b ownership units, including the fund's atom at type β , are assigned toward lower- d_j firms. There are no acquisition prices, no trading costs, and no need to buy votes.

The trading-voting equilibrium is different. The fund cannot assign itself shares. It must buy them. As the fund accumulates ownership in a firm, Tiebout sorting changes the identity of the remaining sellers. Low- b shareholders sell out first. Once they hold zero shares, they cannot sell more. Additional purchases must come from higher- b

shareholders, who value the stock more. Thus, with the market, Tiebout sorting is turned into an endogenous supply curve.

In the low-type-exhaustion region, the stationary marginal seller is

$$b_j^* = 1 - \sqrt{\frac{2\rho(1-\lambda)(1-\alpha_j)}{s_j(x_j)}},$$

and the stationary price satisfies

$$Rp_j = \pi_j(x_j) + b_j^* s_j(x_j) + \rho(1-\lambda)(1-\alpha_j).$$

Holding x_j fixed, b_j^* is increasing in α_j . Moreover, in the exhaustion region,

$$\frac{\partial(Rp_j)}{\partial\alpha_j} = \rho(1-\lambda) \left(\frac{b_j^*}{1-b_j^*} \right) > 0.$$

Thus, as the fund concentrates ownership in firm j , the marginal seller moves upward in b , and the acquisition price rises.

This rising acquisition price is absent from the planner problem. It is a market force created by the fact that the fund must buy shares from shareholders who are increasingly selected toward high valuations. It prevents the trading allocation from generally reproducing the planner's sharp assignment. Even when the planner would assign the fund's block primarily to one firm, the market fund may optimally stop short of full concentration because additional shares in that firm become expensive.

This force also helps explain common ownership. In the trading-voting equilibrium, a share has direct payoff value, governance value, continuation value, and an acquisition cost. As the fund buys more of one firm, the marginal governance value of additional shares in that firm may fall, especially once the fund approaches its implementable ideal, while the marginal acquisition cost rises because the marginal seller becomes more pro-social. At the same time, another firm may still have a high marginal governance value and a lower acquisition cost. The fund then spreads ownership across firms.

Proposition 11 (Ownership wedge: sharp assignment versus endogenous marginal sellers). *Consider a stationary trading-voting equilibrium on the low-type-exhaustion branch*

for firm j . Holding policy x_j fixed, the marginal-seller cutoff is

$$b_j^* = 1 - \sqrt{\frac{2\rho(1-\lambda)(1-\alpha_j)}{s_j(x_j)}},$$

and the stationary price satisfies

$$Rp_j = \pi_j(x_j) + b_j^* s_j(x_j) + \rho(1-\lambda)(1-\alpha_j).$$

Then

$$\frac{\partial b_j^*}{\partial \alpha_j} > 0$$

and

$$\frac{\partial(Rp_j)}{\partial \alpha_j} = \rho(1-\lambda) \left(\frac{b_j^*}{1-b_j^*} \right) > 0.$$

Thus, as the fund concentrates ownership in firm j , the marginal seller moves upward in b , and the acquisition price rises.

By contrast, in the capped planner benchmark, ownership is assigned directly and there is no acquisition-price schedule. Hence the planner can implement sharp type sorting, while the trading-voting equilibrium generally features price-mediated, less sharp sorting. If marginal governance value is positive in more than one firm and marginal acquisition costs rise with concentration, an interior trading equilibrium naturally involves common ownership across those firms.

Proof. On the low-type-exhaustion branch,

$$b_j^* = 1 - \sqrt{\frac{2\rho(1-\lambda)(1-\alpha_j)}{s_j(x_j)}}.$$

Differentiating with respect to α_j , holding x_j fixed, gives

$$\frac{\partial b_j^*}{\partial \alpha_j} = \frac{\rho(1-\lambda)}{s_j(x_j)} \left[\frac{2\rho(1-\lambda)(1-\alpha_j)}{s_j(x_j)} \right]^{-1/2} > 0.$$

The stationary price is

$$Rp_j = \pi_j(x_j) + b_j^* s_j(x_j) + \rho(1-\lambda)(1-\alpha_j).$$

Therefore

$$\frac{\partial(Rp_j)}{\partial\alpha_j} = s_j(x_j)\frac{\partial b_j^*}{\partial\alpha_j} - \rho(1 - \lambda).$$

Using

$$1 - b_j^* = \sqrt{\frac{2\rho(1 - \lambda)(1 - \alpha_j)}{s_j(x_j)}},$$

we can write

$$s_j(x_j)\frac{\partial b_j^*}{\partial\alpha_j} = \frac{\rho(1 - \lambda)}{1 - b_j^*}.$$

Thus

$$\frac{\partial(Rp_j)}{\partial\alpha_j} = \rho(1 - \lambda) \left(\frac{1}{1 - b_j^*} - 1 \right) = \rho(1 - \lambda) \left(\frac{b_j^*}{1 - b_j^*} \right) > 0.$$

The planner does not face this acquisition price because it directly assigns shares. The final statement follows from the fund's trading optimality condition. At an interior trading optimum, the fund cannot profitably reallocate a dollar from one firm to another, so marginal value per dollar is equalized across interior holdings. Rising marginal acquisition costs in a concentrated firm, together with positive marginal governance value in other firms, make common ownership the natural interior outcome. $\square$

Proposition 11 identifies the ownership wedge. The planner can assign ownership sharply because it does not have to buy shares. The trading fund must buy from an endogenous supply curve. As it concentrates in one firm, low- b sellers are exhausted, the marginal seller becomes more pro-social, and the acquisition price rises. This prevents the trading allocation from generally reproducing the planner's sharp assignment.

5 A Two-Firm Numerical Illustration

This section illustrates the model in a deterministic two-firm economy with no external fund flows. The numerical example highlights five points. First, the fund may initially increase its ownership in the high-dividend brown firm and only later rotate toward the lower-ESG-cost firm. Second, the fund's continuation-adjusted voting ideal are different from her static ideal. Third, policies change endogenously along the transition. Fourth, the market allocation generates common ownership: the fund holds both firms in the trading-voting equilibrium. By contrast, under the central planner the same aggregate fund footprint is concentrated in a single firm. Fifth, the policy allocation under the planner differs from the policy allocation under the market.

We set $K = 2$ and impose $f_t \equiv 0$. Firm 1 has the lower ESG cost, $d_1 < d_2$, while firm 2 has the higher baseline dividend, $\bar{\pi}_2 > \bar{\pi}_1$. The parameters is summarily reported in Table 1. The fund begins with positive ownership in both firms and a finite cash buffer.

Table 1: Parameters for the two-firm illustration

Parameter	Value
Number of firms K	2
Baseline dividends $(\bar{\pi}_1, \bar{\pi}_2)$	(1.00, 1.50)
ESG costs (d_1, d_2)	(0.50, 2.20)
Fund type β	0.90
Gross risk-free return R	1.020
Fund discount factor δ	0.978
Trading cost ρ	8.0
Ownership persistence λ	0.70
Initial fund ownership $(\alpha_{1,-1}, \alpha_{2,-1})$	(0.04, 0.14)
Initial cash c_0	14.00
External flows	$f_t \equiv 0$

Figure 1 shows the ownership path. The fund first increases its ownership in firm 2, the high-dividend brown firm. Later it rotates toward firm 1, the lower-ESG-cost firm. This is the harvest-to-build-voice mechanism: early dividends from firm 2 help finance later purchases of voting rights in firm 1.

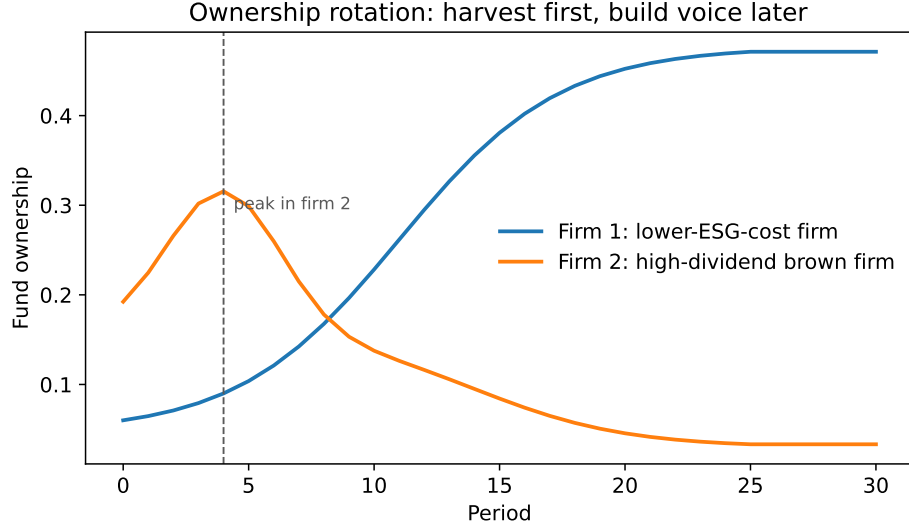


Figure 1: Ownership rotation. The fund first increases its ownership in the high-dividend brown firm and later shifts toward the lower-ESG-cost firm.

Figure 2 plots the continuation-adjusted voting ideal together with the static ideal for each firm. At early dates, the continuation-adjusted ideal lies below the static ideal, especially for firm 1. Over time, the two converge. This is the term structure of responsible voting: early in the path, scarce internal resources make the fund effectively less green than a static snapshot of its type would suggest.

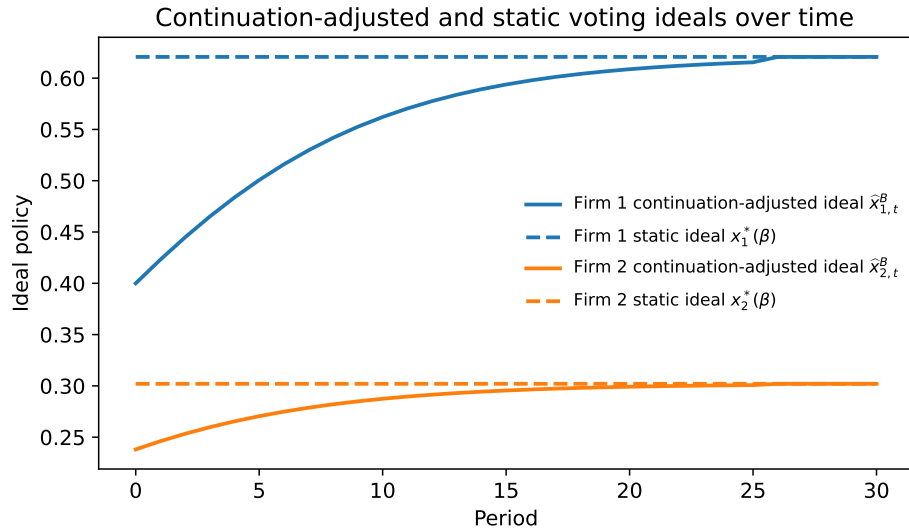


Figure 2: Continuation-adjusted voting ideals and static ideals over time. The continuation-adjusted ideal starts below the static ideal and converges to it.

Figure 3 plots policy dynamics. Firm 1 becomes greener as the fund builds voice in the lower-ESG-cost firm. Firm 2 gradually becomes browner as the fund eventually exits the high-dividend brown firm.

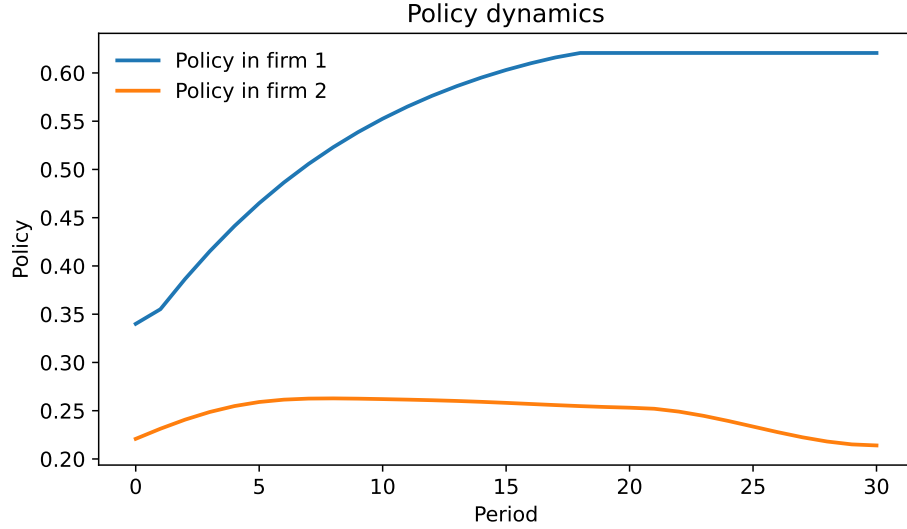


Figure 3: Policy dynamics along the equilibrium path.

We next compare the stationary equilibrium of trading-voting with the planner's allocation at the same aggregate fund footprint. In the market, the fund ends up holding both firms, so common ownership arises endogenously. Under the planner, the fund's footprint is concentrated in firm 1, the lower-ESG-cost firm. Figure 4 makes this contrast explicit.

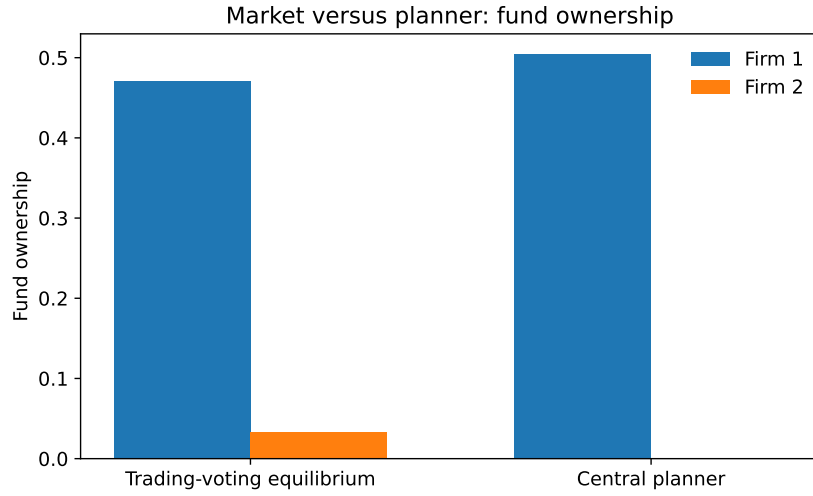


Figure 4: Fund ownership in the stationary equilibrium of trading-voting and in the central planner allocation. The market allocation exhibits common ownership, while the planner concentrates the fund in one firm.

Figure 5 compares the policies under the market and the planner. The planner's policies differ because ownership is assigned directly and policies are chosen using ownership-weighted mean valuations rather than the weighted-median logic of the market equilibrium.

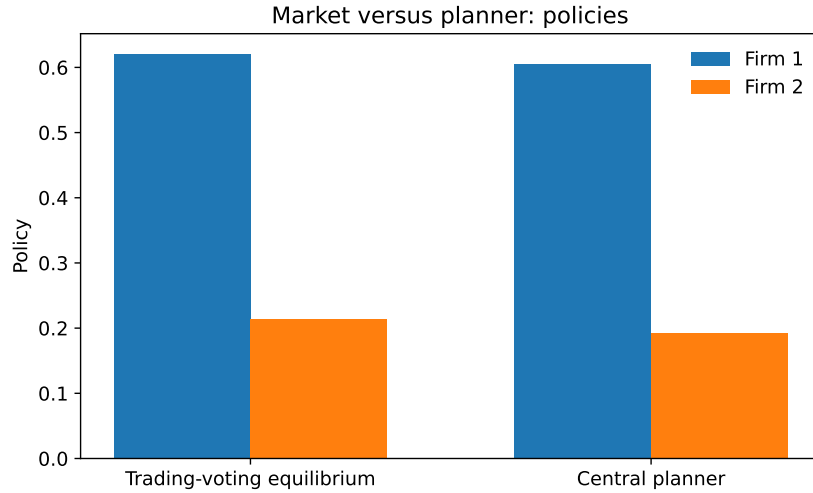


Figure 5: Policies in the stationary equilibrium and in the central planner allocation.

For convenience, Table 2 reports the terminal ownership and policy levels.

Table 2: Market allocation versus planner allocation

	Firm 1	Firm 2
Market ownership α_j^E	0.4713	0.0332
Planner ownership α_j^P	0.5045	0.0000
Market policy x_j^E	0.6207	0.2140
Planner policy x_j^P	0.6050	0.1926

Taken together, the five pictures summarize the economics of the model. The transition path shows the fund using the high-dividend brown firm to build resources, then reallocating toward the lower-ESG-cost firm. The early gap between the continuation-adjusted and static voting ideals makes the term structure of responsible voting visible. The planner comparison shows that the market naturally generates common ownership, whereas direct assignment by a planner can concentrate the fund in a single firm.

5.1 Robustness: Fully Dynamic Small Investors

The main model treats small shareholders as short-horizon investors: each evaluates her position one period at a time and includes its expected next-period market value. This subsection considers infinitely lived small investors who instead solve a dynamic portfolio problem and discount at the market rate $1/R$.

To preserve the partial-turnover interpretation, suppose that at the end of each period a fraction $1 - \lambda$ of each type's position is reset to the per-capita residual float $1 - \alpha_{j,t}$, with the associated position adjustment valued at the next-period market price.

Fix firm j , and define

$$v_{j,t}(b) \equiv \pi_j(x_{j,t}^e) + bs_j(x_{j,t}^e).$$

A type- b dynamic investor with inherited position h^- solves

$$\begin{aligned} U_{j,t}(h^-; b) = \max_{z \geq -h^-} & \left\{ v_{j,t}(b)h^+ - Rp_{j,t}z - \frac{\rho}{2}z^2 \right. \\ & + (1 - \lambda) [h^+ - (1 - \alpha_{j,t})] p_{j,t+1}^e \\ & \left. + \frac{1}{R} \mathbb{E}_t U_{j,t+1} (\lambda h^+ + (1 - \lambda)(1 - \alpha_{j,t}); b) \right\}, \end{aligned} \quad (33)$$

where

$$h^+ = h^- + z.$$

Lemma 10 (Euler equation for dynamic small-shareholder trades). *At an interior optimum,*

$$U'_{j,t}(h^-; b) = Rp_{j,t} + \rho z_{j,t}(b),$$

and trades satisfy

$$Rp_{j,t} + \rho z_{j,t}(b) = v_{j,t}(b) + p_{j,t+1}^e + \frac{\lambda \rho}{R} \mathbb{E}_t z_{j,t+1}(b).$$

Proof. The first-order condition for z is

$$Rp_{j,t} + \rho z = v_{j,t}(b) + (1 - \lambda)p_{j,t+1}^e + \frac{\lambda}{R} \mathbb{E}_t U'_{j,t+1}.$$

The envelope condition is

$$U'_{j,t} = v_{j,t}(b) + (1 - \lambda)p_{j,t+1}^e + \frac{\lambda}{R} \mathbb{E}_t U'_{j,t+1}.$$

Hence

$$U'_{j,t} = Rp_{j,t} + \rho z_{j,t}(b).$$

Advancing this envelope condition one period and substituting it into the first-order condition gives

$$Rp_{j,t} + \rho z_{j,t}(b) = v_{j,t}(b) + p_{j,t+1}^e + \frac{\lambda \rho}{R} \mathbb{E}_t z_{j,t+1}(b).$$

□

Proposition 12 (Stationary equivalence). *Define*

$$\tilde{\rho} \equiv \rho \frac{R - \lambda}{R}.$$

In any stationary equilibrium with fully dynamic small investors,

$$h_j^+(b) = \left[1 - \alpha_j + \frac{\pi_j(x_j) + bs_j(x_j) - (R - 1)p_j}{\tilde{\rho}(1 - \lambda)} \right]_+.$$

On the low-type-exhaustion branch,

$$b_j^* = 1 - \sqrt{\frac{2\tilde{\rho}(1 - \lambda)(1 - \alpha_j)}{s_j(x_j)}},$$

and

$$(R - 1)p_j = \pi_j(x_j) + b_j^* s_j(x_j) + \tilde{\rho}(1 - \lambda)(1 - \alpha_j).$$

In the no-exhaustion region,

$$(R - 1)p_j = \pi_j(x_j) + \frac{1}{2}s_j(x_j).$$

Thus the stationary ownership and pricing formulas are those of the short-horizon model with

$$\rho \text{ replaced by } \tilde{\rho}.$$

When

$$\lambda = 0,$$

we have

$$\tilde{\rho} = \rho,$$

so the two stationary specifications coincide exactly.

Proof. In stationarity,

$$p_{j,t} = p_{j,t+1}^e = p_j \quad \text{and} \quad z_{j,t}(b) = z_{j,t+1}(b) = z_j(b).$$

The Euler equation becomes

$$Rp_j + \rho z_j(b) = \pi_j(x_j) + bs_j(x_j) + p_j + \frac{\lambda\rho}{R}z_j(b).$$

Therefore

$$\tilde{\rho} z_j(b) = \pi_j(x_j) + bs_j(x_j) - (R-1)p_j.$$

Stationarity of positions implies

$$z_j(b) = h_j^+(b) - h_j^-(b) = (1-\lambda) [h_j^+(b) - (1-\alpha_j)].$$

Combining these equations gives

$$h_j^+(b) = 1 - \alpha_j + \frac{\pi_j(x_j) + bs_j(x_j) - (R-1)p_j}{\tilde{\rho}(1-\lambda)}$$

for active types. The positive-part formula follows from nonnegative holdings. The cutoff and price equations then follow exactly as in Proposition 5, with ρ replaced by $\tilde{\rho}$. $\square$

Remark 5 (Stationary and transitional effects). *Because*

$$\tilde{\rho} = \rho \frac{R-\lambda}{R} \leq \rho,$$

fully dynamic small investors respond more strongly to stationary valuation differences. At given fund ownership and policy, this strengthens cross-sectional sorting and can raise the marginal-holder cutoff on the low-type-exhaustion branch.

The equivalence is only stationary. Away from stationarity, the term

$$\frac{\lambda\rho}{R}\mathbb{E}_t z_{j,t+1}(b)$$

links current demand to expected future adjustment, so fully dynamic small investors smooth trades over time.

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