

When proxy agreement is not accuracy: Closing-price construction and range-based volatility

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Anam Giri: Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft.

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Abstract

Closing-price construction can reverse volatility comparisons. In 40,000 simulated sessions with 195 trades and a 20% closing window, Parkinson's standard-deviation-scale ratio is 1.013 against a squared-return proxy but 0.920 against the latent target. Holding paths and trades fixed isolates the closing window's effect. For Brownian log prices, independent finite-grid sampling and Gamma-weighted log-price averaging, we derive the proxy's exact expectation and a model-specific correction. Stochastic-variance and arithmetic-price controls retain the separation. The findings concern unconditional measurement levels; Monte Carlo intervals quantify simulation error. Proxy agreement alone does not establish the correct latent scale.

Keywords: volatility measurement; volatility proxies; closing prices; range estimators; Parkinson estimator; Monte Carlo; market microstructure

JEL: C15, C58, G14

1 Introduction

A volatility estimator can appear high relative to a return-based benchmark while understating the latent target. In our known-target experiment, Parkinson's ratio is 1.013 against a squared open-to-close log-return proxy and 0.920 against latent volatility. Both comparisons use the same 40,000 simulated sessions and the standard-deviation scale. This reversal can arise even when prices are recorded correctly.

Daily high-low ranges provide volatility information beyond endpoint returns. Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991) give familiar estimators;

later work studies their properties and the effects of sampling and microstructure (Hansen and Lunde, 2006; Martens and van Dijk, 2007; Molnár, 2012).

Averaging-induced distortions in random walks are established (Working, 1960), as is the effect of discrete observation on range estimation (Christensen and Podolskij, 2007). Our contribution is an exact joint characterization of finite-grid sampling, closing-window length and random closing weights, linked to the comparison between range estimators and a squared-return benchmark.

Patton (2011b) establishes conditions under which loss functions preserve forecast rankings with a conditionally unbiased volatility proxy. Patton (2011a) addresses data-based ranking of realized-volatility estimators with correlated measurement errors. We examine whether agreement between unconditional mean levels establishes the correct latent scale when the benchmark itself is biased. Our comparison uses squared open-to-close log returns; it does not address close-to-close returns containing overnight movement.

The matched experiment identifies the role of price construction. With 195 trades and no rounding, changing the closing window from the last trade to the final 40% of trades moves Parkinson's proxy comparison from 0.944 to 1.102 while its range estimate stays fixed. Our exact finite-grid calculation explains the benchmark side of this reversal: how sparse sampling and closing-price averaging lower the proxy's expected level. Under Brownian prices, independent sampling and Gamma-weighted closing log prices, it gives this expected attenuation and a model-specific adjustment restoring the proxy's expected latent variance level. The range side of the comparison is measured by simulation.

2 Known-target design

Let the latent log price satisfy $dX_t = \sigma dW_t$ on $[0, 1]$, with integrated variance $IV = \sigma^2$.

Potential trade times are i/D , $i = 1, \dots, D$. Removing r points at each boundary leaves

$M = D - 2r$ available points. We sample n times uniformly without replacement, independently of prices, and order them as $T_1 < \dots < T_n$.

Let α and σ be positive. The integer grid and sample sizes satisfy

$0 \leq 2r < D$, $2 \leq n \leq M$, $1 \leq m \leq n$. The high H and low L are the maximum and minimum prices across the sampled trades; O and C denote the opening and closing reference prices.

The opening log price is X_{T_1} . The closing log price is a weighted mean of the final m trades, with normalized independent $\text{Gamma}(\alpha, 1)$ weights drawn independently of prices and trade times. Thus the price close is a weighted geometric mean. This is a count-based reference, not a reconstruction of a clock-time price VWAP. We apply the familiar formulas to these observation rules; their usual unbiasedness properties need not carry over. Define

$Q = [\ln(C/O)]^2$ and the standard estimators

$$P = \frac{[\ln(H/L)]^2}{4\ln 2}, \quad G = \frac{[\ln(H/L)]^2}{2} - (2\ln 2 - 1)Q. \quad (1)$$

$$RS = \ln(H/O)\ln(H/C) + \ln(L/O)\ln(L/C). \quad (2)$$

2.1 Exact proxy attenuation

Proposition. Before tick rounding, under the stated Brownian sampling and weighting assumptions,

$$b_{nm} := \frac{E[Q]}{\sigma^2} = \frac{M+1}{D(n+1)} \left[n - m + \frac{(m-1)\{\alpha(2m-1)+3\}}{6(\alpha m+1)} \right]. \quad (3)$$

Proof sketch. Let $I = \{n - m + 1, \dots, n\}$ index the closing trades and $Z_j = X_{T_j} - X_{T_1}$.

Brownian covariance gives $E[Z_j Z_k | T] = \sigma^2 [\min(T_j, T_k) - T_1]$. The normalized weights

satisfy $E[w_j w_k] = (\alpha + 1_{\{j=k\}}) / [m(m\alpha + 1)]$, and uniform grid sampling gives

$E[\min(T_j, T_k) - T_1] = (M + 1)[\min(j, k) - 1]/[D(n + 1)]$. Since $\ln(C/O) = \sum_{j \in I} w_j Z_j$,

independence yields

$$\frac{E[Q]}{\sigma^2} = \frac{M+1}{D(n+1)m(m\alpha+1)} \sum_{j,k \in I} (\alpha + 1_{\{j=k\}}) [\min(j, k) - 1]. \quad (4)$$

Evaluating the double sum and its diagonal in Eq. (4) gives Eq. (3).

Summation details are in Supplementary Material S1, Section 1, and exhaustive finite-grid checks are in Section 2. For $m = 1$, it reduces to the expected first-to-last trade span. As D and n tend to infinity with fixed positive α , $r/D \rightarrow s$ and $m/n \rightarrow f$, it tends to $(1 - 2s)(1 - 2f/3)$.

A population SD ratio is the square root of the ratio of the mean variance estimate to the mean benchmark. Its subscripts identify the estimator and benchmark.

Writing $a_n = E[P]/\sigma^2$, the population comparisons satisfy

$$R_{P,Q}^2 = \frac{a_n}{b_{nm}}, \quad R_{P,IV}^2 = a_n. \quad (5)$$

The directional reversal is characterized by $b_{nm} < a_n < 1$: Parkinson exceeds the proxy while remaining below latent variance on the mean-variance scale.

Agreement occurs when $a_n = b_{nm}$. The bracket in Eq. (3) never exceeds $n - 1$, and $(M + 1)(n - 1) < D(n + 1)$, so $b_{nm} < 1$ for every admissible design. Before rounding, agreement therefore implies $a_n < 1$: a range estimator that matches this proxy understates latent variance. Changing only the closing window leaves a_n unchanged. Under these assumptions, Q/b_{nm} has mean σ^2 and provides an adjusted benchmark. This correction

requires the observation model; Eq. (3) is not asserted for stochastic variance, price averaging or tick rounding.

The attenuation factor in Eq. (3) is independent of any positive volatility scale when the sampling and weighting parameters are fixed. This Brownian scale invariance holds before rounding. The observation parameters determine whether equality or reversal occurs through Eq. (5).

Garman–Klass shares the same proxy-equality crossing:

$$G = 2\ln 2 P - (2\ln 2 - 1)Q, \quad R_{G,Q}^2 = 1 + 2\ln 2 (R_{P,Q}^2 - 1). \quad (6)$$

Eq. (6) implies that the Garman–Klass proxy comparison crosses one exactly when Parkinson’s does. We therefore use Parkinson as the primary demonstration and Rogers–Satchell as an additional estimator check on the same simulated paths. Because G also contains Q , its latent comparison, $R_{G,IV}^2 = 2\ln 2 a_n - (2\ln 2 - 1)b_{nm}$, moves with the closing window and equals a_n at the proxy-equality crossing. Only Parkinson’s latent comparison is unchanged across windows.

2.2 Simulation and uncertainty

We use $D = 4000$, $r = 80$, $\alpha = 2$, a reference price of 1,000 Nepalese rupees (NPR) and NPR 0.10 rounding. The window contains $m = \max(1, \lceil fn \rceil)$ trades. The three empirical calibration constants are $n_{0.10} = 52$, $n_{0.50} = 195$ and $\sigma = 0.02810448817655783$, calculated from 72,102 ordinary-equity stock-days in the Nepal Stock Exchange (NEPSE) calibration slice, March 4, 2024–June 16, 2025. NEPSE supplies baseline trade counts and return scale for the experiment. All other observation settings are illustrative. These disclosed constants suffice for replication without market records.

The exploratory simulation uses seed 20260910 and 40 independent batches of 1,000 sessions. Fourteen trade counts from 20 to 500 cross six windows: 0%, 5%, 10%, 20%, 30% and 40%. Latent paths are shared across trade counts. Within each count, every window also shares the same sampled trades and underlying Gamma weights. We also run an unrounded control for Eq. (3) and an arithmetic price-average control at 195 trades and 20%.

A Heston-style variance process (Heston, 1993), with zero log-price drift, uses initial and long-run variance σ^2 , mean reversion 4, variance diffusion coefficient 0.04 and shock correlation -0.5 . Full-truncation Euler uses the same positive left-endpoint variances to generate log-price increments and calculate each path's numerical integrated variance. Supplementary Material S1, Section 3, specifies the recursion. This is one discretized stochastic-variance check.

Reported standard-deviation (SD) ratios are square roots of ratios of pooled mean variances. Approximate 95% Monte Carlo intervals use a paired log-delta calculation across the 40 batches with a t_{39} critical value. Crossing intervals use 5,000 paired resamples of complete batch vectors, preserving dependence across the grid. We interpolate the mean difference $P - Q$ linearly in log trade count, without extrapolation. These intervals describe simulation error, not empirical NEPSE uncertainty or specification uncertainty.

Supplementary Material S2 provides executable code, simulation settings and verification records. Python scripts generate all results and figures; Supplementary Material S1 describes computational verification and AI assistance.

3 Results

At 195 trades and a 20% window, Parkinson's SD ratio is 1.013 against the proxy (95% MC interval 1.009–1.018), but 0.920 against latent volatility (0.917–0.923). Its variance bias is -15.43% . Table 1 reports all three estimators. Under the Heston-style process, the

corresponding Parkinson ratios are 1.013 and 0.919. The Brownian baseline using all 4,000 grid points, without boundary omission or rounding is 0.988 against latent volatility, documenting finite-grid range bias separately.

[Insert Table 1 here]

Figure 1 isolates the closing mechanism. At 195 trades, the same Parkinson estimate remains fixed while the window changes its proxy comparison from 0.944 with a last-trade close to 1.102 with a 40% window. In the unrounded 20% case, Eq. (3) gives $b_{nm}=0.825945$; adjustment changes the proxy comparison to 0.921, close to the latent comparison. Across all 84 unrounded cells, simulated proxy means differ from Eq. (3) by at most 1.14 Monte Carlo standard errors. These cells share latent paths, so their discrepancies are correlated; the exact checks in Supplementary Material S1, Section 2, are the direct verification. With a tick of 0.01% of the reference price, rounding changes mean P and Q by at most 0.003% across the grid.

[Insert Figure 1 here]

Figure 2 shows the joint sensitivity. At a 20% window, the estimated proxy-equality crossing is 132 trades (95% MC interval 115–148). At higher sampled trade counts in this grid, Parkinson exceeds the proxy; this does not establish greater accuracy. Its latent SD ratio remains below 0.95 across this grid. The crossing moves to about 52 and 28 trades for 30% and 40% windows; the 0% and 5% curves do not cross within 20–500 trades. The 10% curve reaches 0.998 (0.993–1.003) at 500 trades, and only 1,141 of 5,000 bootstrap draws cross within the grid (Supplementary Material S1, Table S1). The threshold depends on the closing rule.

[Insert Figure 2 here]

The arithmetic price-average control changes Parkinson's proxy ratio by 0.000016 (95% MC interval 0.000002–0.000030) under Brownian prices and 0.000098 (0.000082–0.000115) under stochastic variance (Supplementary Material S1, Table S2). These particular controls preserve the level separation; they do not extend the exact formula to arithmetic averaging.

4 Discussion and conclusion

The closing reference can reverse the apparent direction of measurement error: Parkinson lies above the observed proxy while remaining below the latent target. The matched experiment shows why. Altering the closing window moves the benchmark's expected level while the sampled range stays fixed. The exact attenuation formula explains this reversal and supplies an adjusted benchmark. Correlated errors enter the uncertainty calculation, but are not required for the mean identity in Eq. (5).

The known-target design isolates unconditional level distortion. Applying the adjustment to market data requires a suitable model for trading times and closing weights, which are independent of prices here. The numerical controls assess specific observation rules and one stochastic-variance parameterization. Daily estimation accuracy, forecast performance and the empirical size of NEPSE bias require separate evaluation.

Validation should begin with the target period and the construction of the benchmark. Where the Brownian observation assumptions are appropriate, dividing the squared-return proxy by the factor in Eq. (3) restores its expected variance level and exposes the range estimator's remaining shortfall. For other observation rules, a separately justified benchmark or sensitivity analysis provides the appropriate comparison.

Data availability

Code, disclosed calibration constants, batch means, numerical tables, and figure scripts are supplied as Supplementary Material S2, with derivations and implementation details in

Supplementary Material S1. All reported simulation results can be regenerated without the underlying NEPSE records. Calibration provenance is documented in the replication package; the original market records are not redistributed due to commercial licensing and proprietary restrictions on trade-level data. Access to the raw NEPSE and NSE datasets requires direct acquisition from authorized providers such as ShareSansar. The code, simulated data, and supplementary materials supporting this study are openly available in Zenodo at <https://doi.org/10.5281/zenodo.22804154>.

Supplementary materials

Supplementary Material S1: finite-grid derivation, mathematical checks, simulation recursion, uncertainty calculations, controls, and calibration provenance.

Supplementary Material S2 (06_Replication_S2.zip): executable simulation and analysis code, dependency versions, disclosed calibration constants, batch means, result tables, figure scripts and verification records.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the authors used OpenAI Codex (GPT-6, OpenAI) and Claude (Claude Opus 5, Anthropic) to assist with mathematical and code reviews and

reproducibility checks. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the published article. Numerical results and figures are produced by the disclosed Python scripts.

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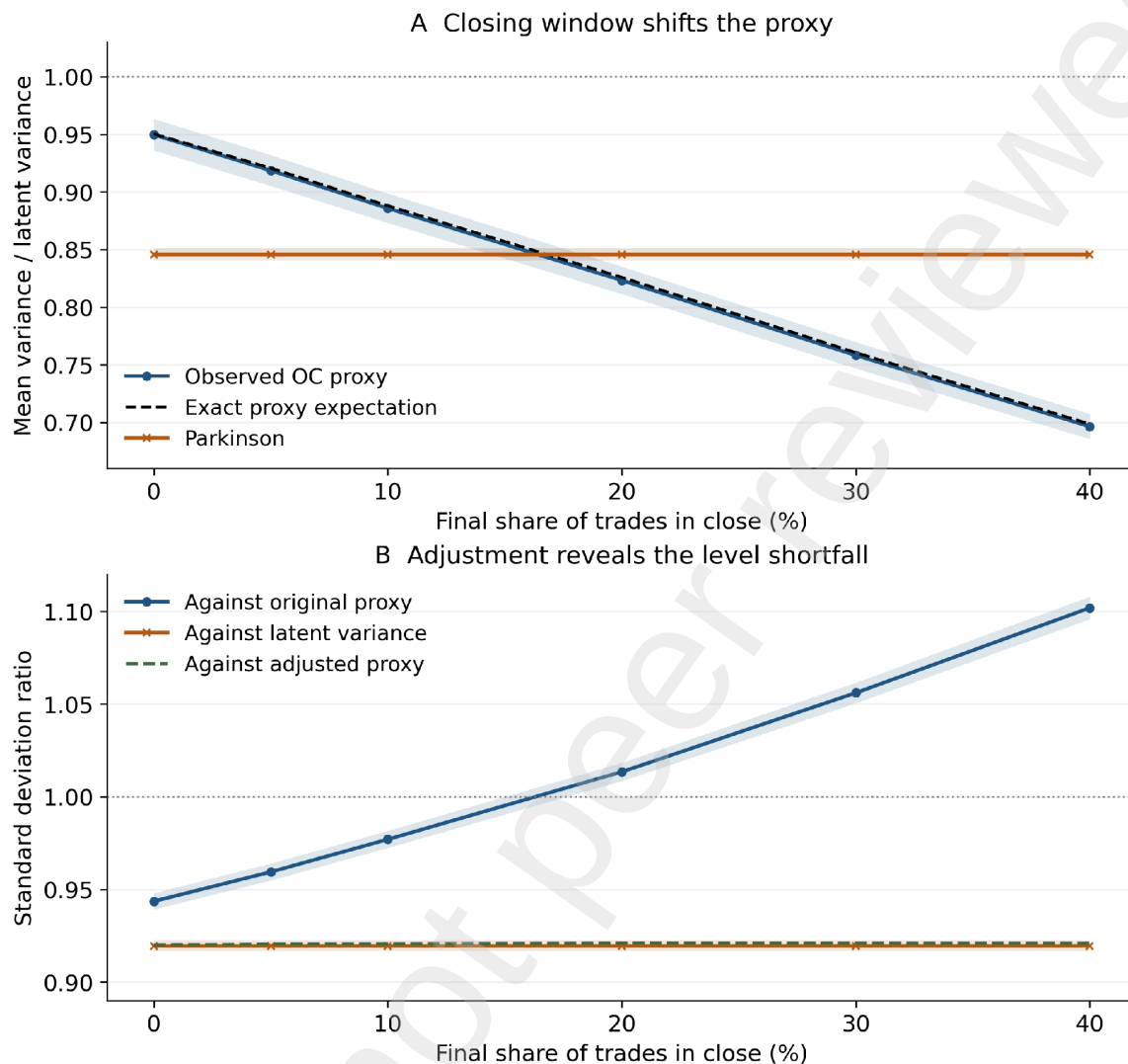
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Table 1 Proxy agreement and latent scale under a 20% closing window

Latent process	Estimator	Variance bias (%)	SD vs latent 95% MC interval	SD vs proxy 95% MC interval
Brownian	Parkinson	-15.43	0.920 (0.917, 0.923)	1.013 (1.009, 1.018)
Brownian	Garman-Klass	-14.57	0.924 (0.922, 0.927)	1.019 (1.012, 1.025)
Brownian	Rogers-Satchell	-16.53	0.914 (0.911, 0.916)	1.007 (0.999, 1.014)
Stochastic variance	Parkinson	-15.55	0.919 (0.916, 0.922)	1.013 (1.008, 1.018)
Stochastic variance	Garman-Klass	-14.71	0.924 (0.921, 0.926)	1.018 (1.011, 1.025)
Stochastic variance	Rogers-Satchell	-16.46	0.914 (0.912, 0.916)	1.008 (1.000, 1.015)

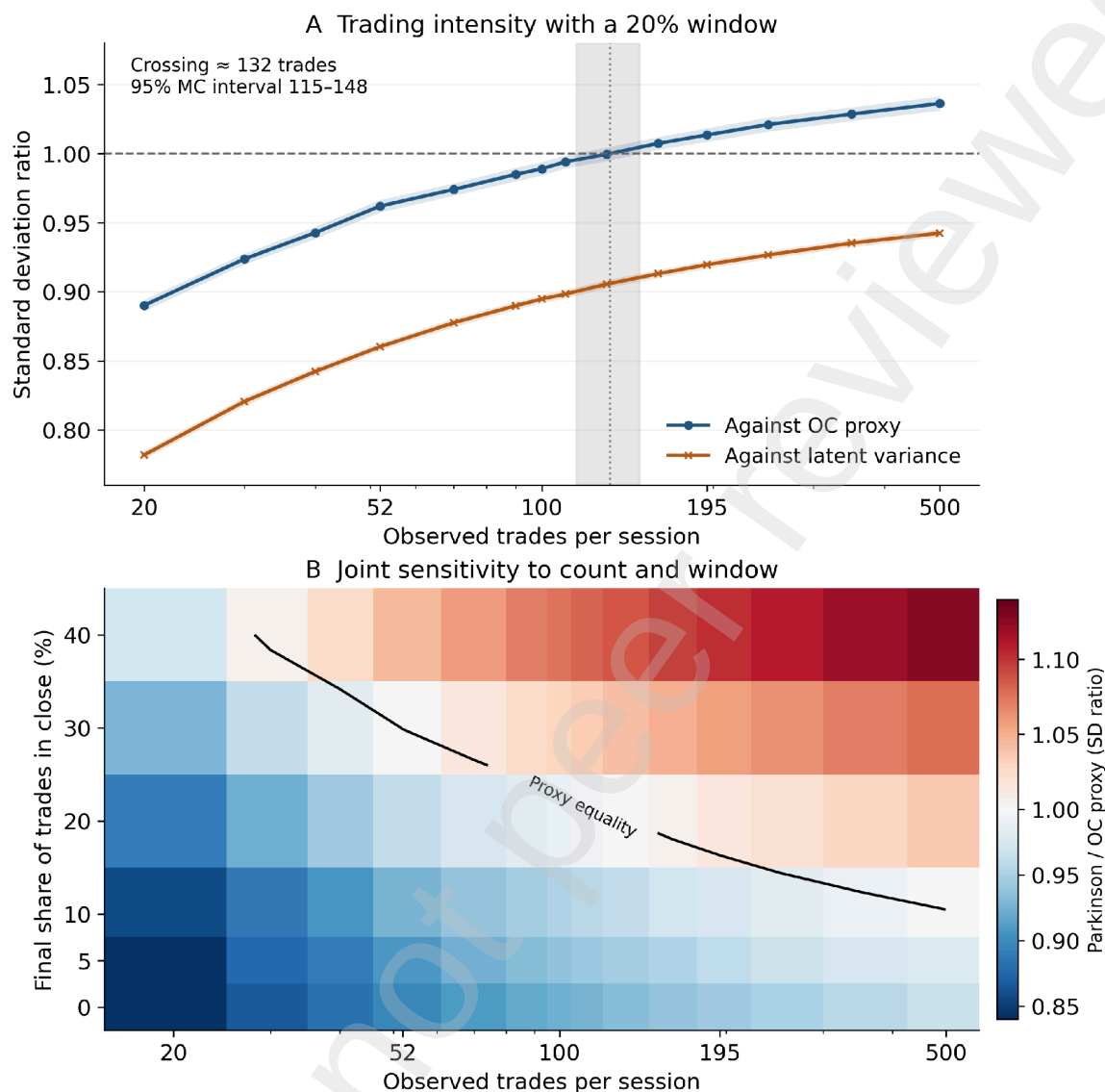
Settings: 195 trades, a 20% closing window, 80 omitted grid points per boundary and NPR 0.10 rounding. Each row uses 40,000 sessions. SD = standard deviation; MC = Monte Carlo. SD ratios are square roots of pooled mean-variance ratios. The proxy is the squared open-to-close log return; latent variance is integrated variance of the simulated path. Parentheses show approximate pointwise 95% Monte Carlo intervals.

Figure 1 Closing windows alter the benchmark with sampled trades held fixed



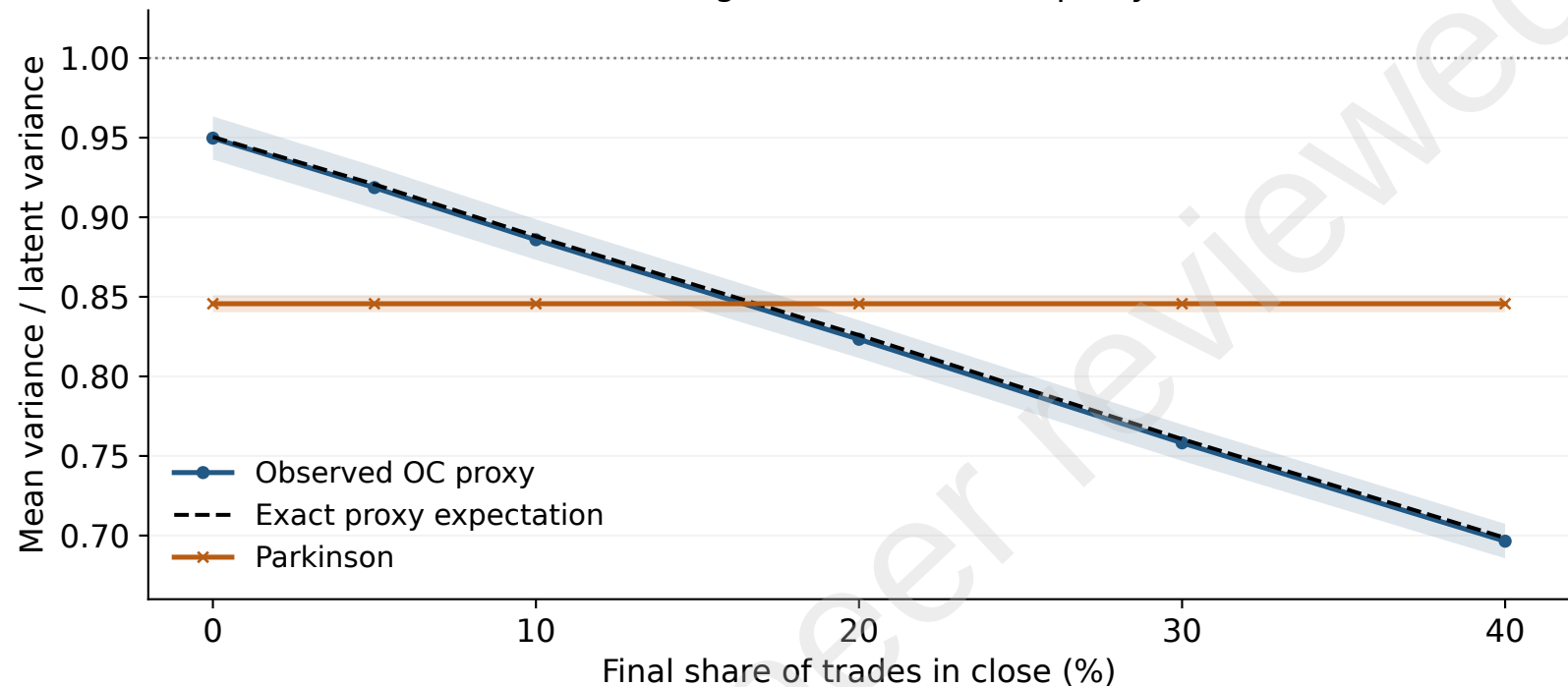
Unrounded Brownian prices with 195 trades. OC = open-to-close; Q is the squared OC log-return proxy; b is its exact expected value divided by latent variance under the Brownian observation model. Panel A compares the observed proxy, its exact expectation and Parkinson. Panel B gives Parkinson's standard-deviation-scale ratios against Q , latent variance and the adjusted proxy Q/b . Shading shows pointwise 95% Monte Carlo intervals for the original-proxy and latent comparisons. The adjusted-proxy line gives point estimates; its intervals are in results/summary.csv in Supplementary Material S2. A 0% window denotes the last trade. The figure is generated programmatically from the simulation outputs.

Figure 2 Trading intensity and closing windows jointly determine proxy agreement

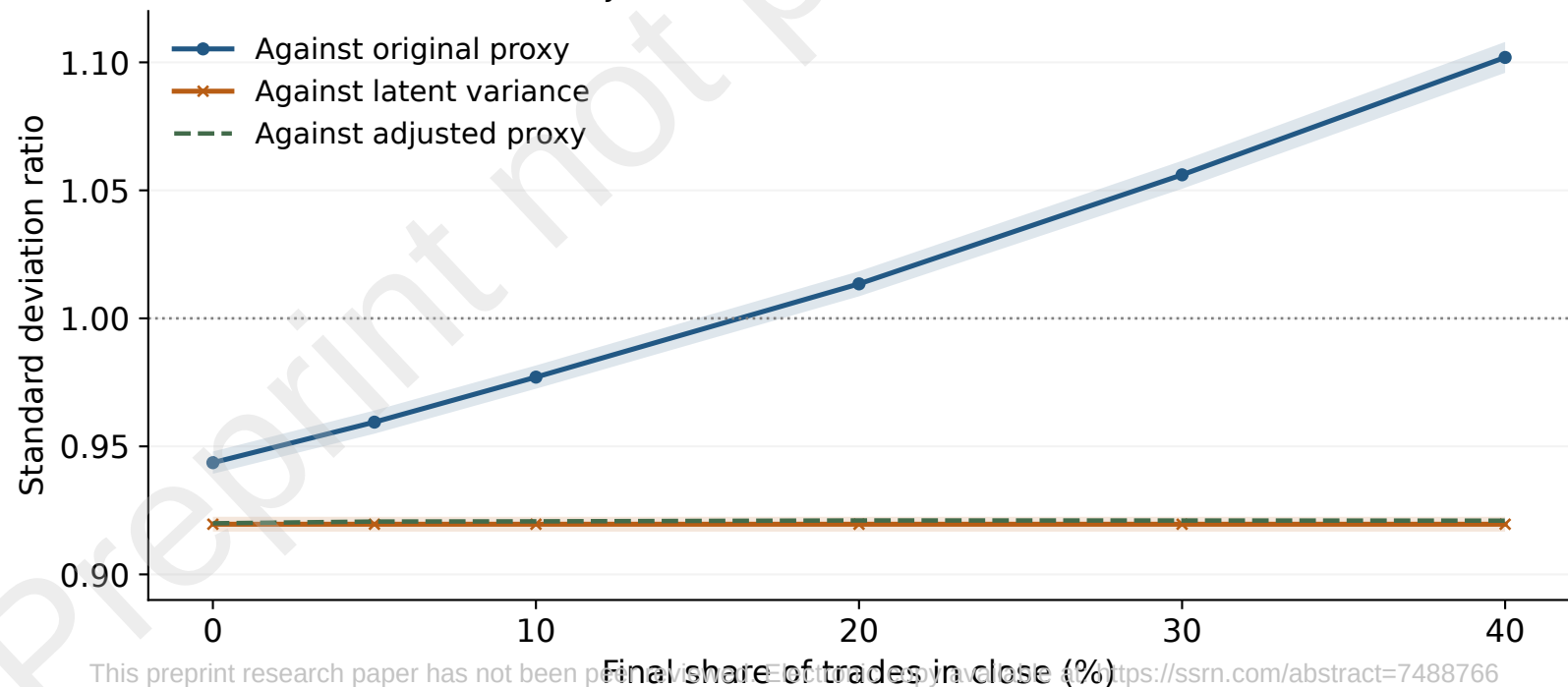


Rounded Brownian observations. OC = open-to-close; SD = standard deviation; MC = Monte Carlo. The proxy is the squared OC log return. SD ratios are square roots of ratios of pooled mean variances. Panel A uses a 20% closing window; colored bands show pointwise 95% MC intervals. The vertical band is the paired-bootstrap interval for the interpolated proxy-equality crossing. Panel B displays the 14-by-6 grid of proxy-relative SD ratios and an interpolated equality contour; the heatmap does not display uncertainty. At each trade count, all windows share the same sampled trades. The figure is generated programmatically from the simulation outputs.

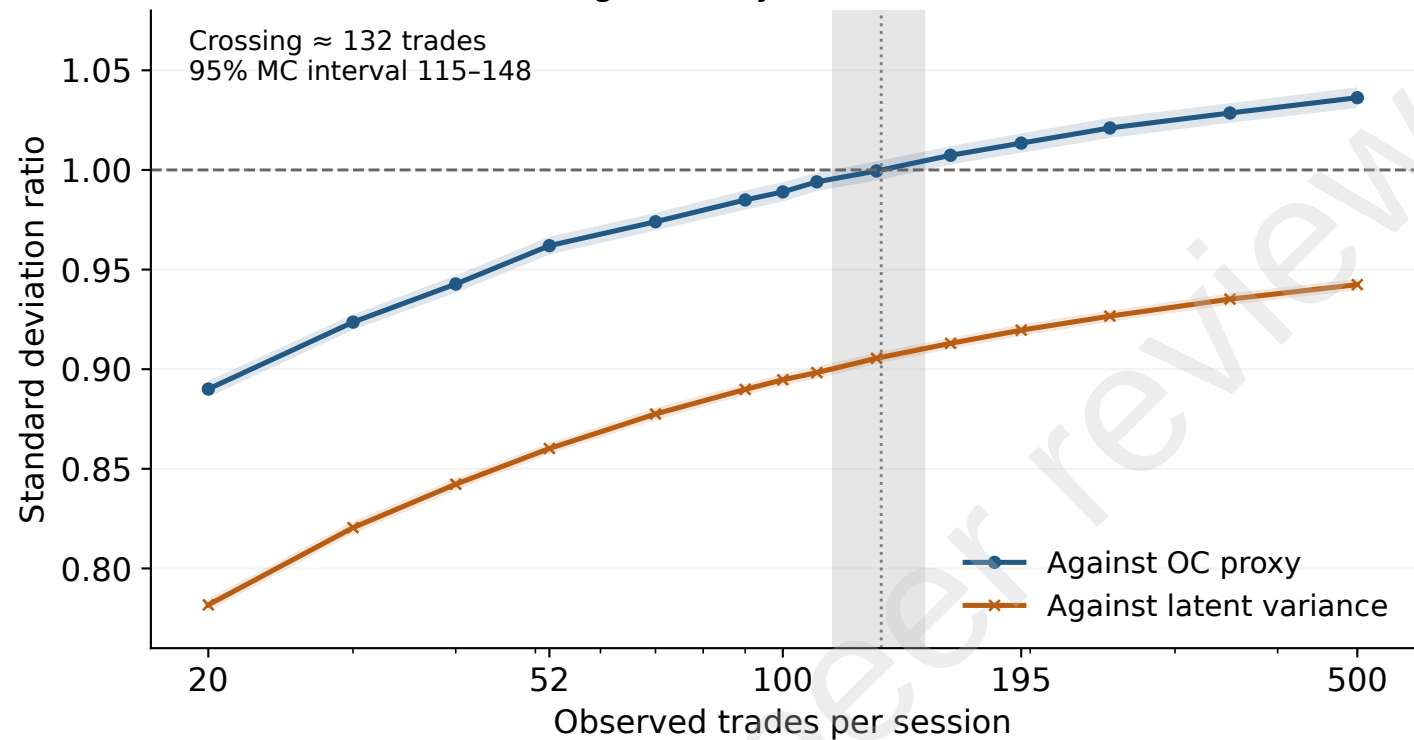
A Closing window shifts the proxy



B Adjustment reveals the level shortfall



A Trading intensity with a 20% window



B Joint sensitivity to count and window

