

Risk Parity Portfolios with and without Pre-Selection under Different Degrees of Risk Aversion

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Abstract

This paper investigates the impact of asset pre-selection in Risk parity strategies with different degrees of risk aversion. We first prove that Risk Parity portfolios converge toward the equally weighted portfolio as the number of assets increases. This result motivates the empirical focus on small portfolios, where Risk Parity allocations remain meaningfully different from the $1/N$ benchmark and may be more suitable for investors facing transaction costs, limited capital, and portfolio monitoring constraints. Secondly, we propose Risk Parity portfolio strategies based on coherent Gini-Type risk measures, which account for downside risk and incorporate investor preferences through a risk-aversion parameter. The use of Gini-Type Risk Parity strategies allows investors to adjust downside-risk exposure according to their degree of risk aversion. Finally, we empirically examine the impact of pre-selection in the practical implementation of Risk Parity portfolios, particularly when the investment universe is deliberately kept small according to different investors' risk aversion preferences.

Keywords: Risk Parity; Asset pre-selection; Portfolio optimization; Gini-Type risk measures; Risk aversion.

JEL classification: G11, C61, C58

1. Introduction

The allocation of risk across portfolio constituents is a central issue in financial economics. Portfolio construction does not only require deciding how much capital should be invested in each asset, but also how each asset contributes to the overall risk of the portfolio. This motivates the use of risk contribution methods, which aim to attribute total portfolio risk in a coherent and theoretically consistent way (Denault, 2001; Tasche, 1999; Tasche, 2007; Berkouch et al., 2022).

Within this framework, Risk Parity (RP) has become a widely used allocation principle. Unlike the mean-variance framework of Markowitz (1952), which depends on expected return estimates and is therefore highly sensitive to estimation risk (Merton, 1980; Michaud, 1989), RP focuses on balancing risk contributions across assets (Qian, 2011; Maillard et al., 2010; Roncalli, 2013). In this sense, RP provides an intermediate approach between naive diversification and fully optimized portfolios: it avoids relying directly on expected returns while still producing allocations that reflect the risk structure of the selected investment universe.

A key benchmark for RP is the equally weighted $1/N$ portfolio. The $1/N$ rule is simple, transparent, and relatively robust to parameter uncertainty (DeMiguel et al., 2009; Pflug et al., 2012). However, the practical interpretation of the $1/N$ portfolio depends on the size of the investment universe. When the number of assets is large, holding all securities may be costly or impractical for smaller investors because of transaction costs, rebalancing costs, monitoring requirements, and capital

constraints. In this context, Risk Parity portfolios based on a limited number of selected assets may represent a more feasible alternative, especially for investors who seek diversification and risk control without holding a very large number of securities (Pflug et al., 2012).

A theoretical motivation for focusing on relatively small portfolios comes from the convergence property of Risk Parity allocations. As the number of assets increases, RP weights tend to approach uniform allocation under positive homogeneous risk measures. Therefore, when the investment universe is large, RP becomes increasingly close to $1/N$. This suggests that the distinctive role of RP is more relevant in smaller portfolios, where the allocation remains sufficiently different from equal weighting and where the choice of the underlying assets becomes particularly important (Maillard et al., 2010; Roncalli 2013).

The focus on small portfolios naturally raises the issue of asset pre-selection. If only a limited number of assets are included in the portfolio, the selection rule may have a substantial effect on both wealth accumulation and risk exposure. To isolate this effect, the empirical analysis compares two alternative selection mechanisms. The first is a random-selection procedure, in which assets are randomly drawn from the Dow Jones Industrial Average universe at each rebalancing date. This procedure is repeated several times in order to assess the variability induced by random asset choice. The second is a Sharpe-ratio-based pre-selection rule, in which assets with the highest historical Sharpe ratios over the estimation window are selected. This second procedure introduces information about past risk-adjusted performance and allows us to evaluate how much an informed pre-selection rule improves the performance of RP-based strategies. This comparison is relevant because Risk Parity is not an asset-selection model by itself. RP determines how to allocate risk across a given set of assets, but it does not specify which assets should enter the portfolio. Therefore, the quality of the initial investment universe may strongly affect the outcome. The paper shows that random selection provides a conservative benchmark, while Sharpe-ratio pre-selection substantially improves terminal wealth, especially for portfolios with a small number of assets.

The paper also extends the classical RP framework by replacing standard deviation with coherent Gini-Type risk measures. These measures are particularly suitable for portfolio construction because they focus on downside outcomes and allow investor risk preferences to be incorporated through a risk-aversion parameter. Moreover, Gini-Type measures have an important computational advantage: their empirical estimators can be expressed in a tractable form and can be linearized, making the resulting optimization problem easier to solve numerically in a return-risk framework. This feature is especially relevant in repeated portfolio experiments, such as rolling window back tests and random-preselection analyses, where computational efficiency becomes important. By varying the risk-aversion parameter, we evaluate whether Gini-Type RP portfolios respond differently to adverse market conditions and whether more risk-averse specifications provide stronger downside-risk control.

In this paper, we first provide theoretical motivation for focusing on small Risk Parity portfolios. Since RP allocations converge toward the uniform $1/N$ allocation as the number of assets increases, the practical role of RP becomes more relevant when the portfolio is built on a limited set of assets. Second, we study the effect of asset pre-selection on RP-based strategies. We compare random asset selection, repeated across the different strategies, with Sharpe-ratio pre-selection. This allows

us to quantify how much of the performance of small RP portfolios depends on the quality of the selected investment universe.

Third, we extend classical Risk Parity by using coherent Gini-Type risk measures. This allows risk contributions to reflect downside risk and investor risk aversion. The linearizable structure of the Gini-Type estimator also provides a computationally efficient framework for repeated portfolio optimization.

2. Risk Parity, Uniform Diversification and Small Portfolios

Let $x = [x_1, x_2, \dots, x_N]^\top$ denotes the vector of portfolio weights and $r = [r_1, r_2, \dots, r_N]^\top$ the vector of asset returns. Assume no short sales are allowed, i.e., $x_i \geq 0$ for all i and $\sum_{i=1}^N x_i = 1$. The Risk Parity (RP) portfolio is defined as the allocation in which all assets contribute equally to the total portfolio risk. Let $\sigma(x)$ be a positively homogeneous risk measure associated with the portfolio x . According to Euler's theorem:

$$\sigma(x) = \sum_{i=1}^N x_i \cdot \partial \sigma(x) / \partial x_i \quad (1)$$

for the positively homogeneous risk measure $\sigma(x)$. In this context, the total risk contribution (TRC) of the i -th asset is:

$$TRC_i(x) = x_i \cdot \partial \sigma(x) / \partial x_i \quad (2)$$

Therefore, when the RP portfolio exists, it satisfies:

$$TRC_i(x) = TRC_j(x), \forall i, j \quad (3)$$

which ensures that each asset contributes equally to the overall portfolio risk. This condition forms the basis for deriving RP weights for positively homogeneous risk measures. Typically, to determine the Risk Parity portfolio for a given positively homogeneous risk measure $\sigma(x)$ we have to solve the optimization problem:

$$\begin{aligned} \min_x \sum_{i=1}^N \left(\frac{\sigma(x)}{N} - TRC_i(x) \right)^2 \\ \sum_{i=1}^N x_i = 1; x_i \geq 0; i = 1, \dots, N \end{aligned} \quad (4)$$

Moreover, according to the following Proposition, we get that the vector of RP portfolio weights converges to the uniform portfolio $x^{(U)} = [1/N, \dots, 1/N]^\top$, when the number of assets increases.

Proposition: Assume the RP portfolio exists for a given positively homogeneous risk measure σ ¹. Let $x^{(RP)}$ be the vector of portfolio weights of a Risk Parity portfolio with total risk $\sigma^{(RP)}$ and marginal contributions $\sigma_i^{(RP)} = \partial \sigma^{(RP)} / \partial x_i$. Assume the quantity $|(\sigma^{(RP)} / \sigma_i^{(RP)}) - 1|$ is uniformly bounded by a finite value M for all assets (i.e., $|(\sigma^{(RP)} / \sigma_i^{(RP)}) - 1| \leq M < +\infty$ for any admissible asset). Then the distance to the Uniform portfolio $x^{(U)}$ satisfies

¹For example, when the risk measure is the standard deviation and the variance covariance matrix is a positive definite matrix with all covariances between returns strictly positive (condition often satisfied in practice), then there exist a value M such that $|(\sigma^{(RP)} / \sigma_i^{(RP)}) - 1| \leq M < +\infty$ for any $i=1, \dots, N$.

$$\|x^{(RP)} - x^{(U)}\| \leq M/\sqrt{N}, \quad (5)$$

where N is the number of the assets. Thus, in a hypothetical "infinite market" (with infinite assets), where $|(\sigma^{(RP)}/\sigma_i^{(RP)}) - 1|$ is uniformly bounded, the RP strategy converges to the uniform one.

Proof: By Euler Theorem we know that $x_i^{(RP)} = \sigma^{(RP)}/(N\sigma_i^{(RP)})$. Therefore,

$$\|x^{(RP)} - x^{(U)}\| = \sqrt{\sum_{i=1}^N (\sigma^{(RP)}/N\sigma_i^{(RP)} - 1/N)^2} = \sqrt{\sum_{i=1}^N (1/N)^2 ((\sigma^{(RP)}/\sigma_i^{(RP)}) - 1)^2} \leq M/\sqrt{N}$$

Therefore, as M is finite for any N and N is large enough, the RP portfolio converges to the Uniform portfolio $x_i^{(RP)} \approx 1/N$.

The convergence result has a direct implication for empirical portfolio construction. If RP portfolios become close to the $1/N$ allocation when N is large, then using RP on a very broad universe may provide limited incremental allocation value relative to equal weighting. At the same time, the $1/N$ strategy over a broad universe may be feasible mainly for institutional investors or funds, because it requires sufficient capital, trading infrastructure, and regular rebalancing across many securities. For smaller investors or applications in which transaction costs and monitoring costs are relevant, a smaller RP portfolio may be more practical. However, when only a few assets are selected, the choice of those assets becomes crucial. A random selection may lead to unstable performance, because the portfolio is highly exposed to idiosyncratic risk. A pre-selection rule based on historical risk-adjusted performance can therefore improve the quality of the investment universe before the RP allocation is computed.

The empirical analysis below is designed to quantify this mechanism. We first examine random selection as a conservative benchmark and then compare it with a Sharpe-ratio pre-selection rule. The objective is not to replace Risk Parity with an asset-selection model, but to show that RP is most effective when applied to a sufficiently informative and manageable universe of candidate assets.

3. Gini-Type Risk Measures and Risk-Averse Risk Parity

To overcome the traditional limitation of RP as a "preference-free" approach based on symmetric variability, we extend the framework by employing coherent Gini-Type (GT) risk measures, in line with the literature on generalized deviation and downside-risk measures (Rockafellar et al., 2006; Ortobelli et al., 2013; Ben Hssain and Berkouch, 2024). Unlike standard deviation, Gini-Type measures focus specifically on downside tail risk and explicitly integrate investor risk aversion, making them suitable for portfolio selection problems involving asymmetric losses and worst-case risk considerations (Sehgal and Mehra, 2021).

For a portfolio payoff or return variable X , the continuous formulation of the Gini-Type measure is given by:

$$GT_{\beta,\alpha}(X) = -(\alpha - 2)(\alpha - 1) \int_0^\beta (\beta - u)^{\alpha-3} L_X(u) du / \beta^{(\alpha-1)} \quad (6)$$

where $L_x(u) = \int_0^u F_x^{-1}(t) dt$, $u \in [0,1]$ is the absolute Lorenz curve and $F_x^{-1}(t)$ is the t-quantile function of the random variable X , defined as $F_x^{-1}(t) = \inf \{ z \in \mathbb{R} : F_x(z) \geq t \}$.

Its flexibility and power stem from its dependence on two distinct parameters:

- Beta (β): This parameter defines the portion of the loss distribution considered in the risk calculation. Analogously to the quantile used in Conditional Value-at-Risk (CVAR), β sets the threshold for adverse outcomes.
- Alpha (α): This parameter acts as an index of risk aversion. Lower values of α correspond to higher risk aversion. By varying α , the sensitivity of the measure to extreme outcomes is adjusted, thereby allowing us to model different levels of investor preferences. A smaller α therefore corresponds to a more risk-averse investor.

To implement the Risk Parity framework, it is necessary to compute the marginal risk contributions. Following the approach of Rockafellar & Uryasev (2000) for tail-risk measures, the marginal risk contribution (*i.e.*, $\partial GT_{\beta,\alpha}(x'r)/(\partial x_i) = MRC_i(GT_{\beta,\alpha}(x'r))$) for the Gini-Type measure is derived as follows when return distributions are continuous:

$$MRC_i(GT_{\beta,\alpha}(x'r)) = -(\alpha - 2)(\alpha - 1)/(\beta^{\alpha-1}) \int_0^\beta (\beta - u)^{\alpha-3} u E(r_i | x'r \leq F_{x'r}^{-1}(u)) du \quad (7)$$

where $F_{x'r}^{-1}(u)$ is the u -th percentile of portfolio $x'r$. This formulation ensures that the risk allocation is consistent with the portfolio's sensitivity to extreme outcomes, as determined by the risk aversion parameter α .

The total risk contribution is then obtained by multiplying the marginal contribution by the portfolio weight:

$$TRC_i(GT_{\beta,\alpha}) = x_i MRC_i(GT_{\beta,\alpha}) \quad (8)$$

The Gini-Type RP portfolio is the allocation for which these Gini-based total risk contributions are equalized across the selected assets. A key practical advantage of Gini-Type risk measures is that their empirical estimators can be expressed in tractable finite sample forms. Given T historical portfolio returns observations and setting $\beta = s/T$ for $s < T$, the consistent empirical estimators for the Gini-Type measure and its marginal risk contributions are given by (see Ortobelli et al. (2013)):

$$\overline{GT}_{\beta,\alpha}(x'r) = \frac{-(\alpha - 2)(\alpha - 1)}{\beta^{\alpha-1}T} \sum_{p=1}^{s-1} \left(\frac{s-p}{T} \right)^{\alpha-3} \frac{1}{T} \sum_{k=1}^p (x'r)_{k:T} \quad (9)$$

$$\overline{MRC}_i(GT_{\beta,\alpha}) = \frac{-(\alpha - 2)(\alpha - 1)}{\beta^{\alpha-1}T} \sum_{p=1}^{s-1} \left(\frac{s-p}{T} \right)^{\alpha-3} \frac{1}{T} \sum_{k=1}^p r_{i,k} I[(x'r)_k \leq F_{x'r}^{-1}(\frac{p}{T})] \quad (10)$$

where $(x'r)_{(k:T)}$ denotes the k -th order statistic of portfolio returns and $\mathbb{I}[\cdot]$ is the indicator function. Therefore, to determine the Risk Parity portfolios using Gini-Type measure we have to solve optimization problem (4). However, Fassino et al., (2026) have recently proposed a very efficient method based on the fixed-point theorem that implements the classic solution and we use it in our analysis.

4. Empirical Analysis

The empirical analysis is conducted on the constituents of the Dow Jones Industrial Average (DJIA)² using daily price data from July 6, 2001, to July 6, 2026. Daily simple returns are computed from prices. Portfolio weights are estimated using a rolling window of 500 daily observations, approximately corresponding to two years of trading days. Portfolios are rebalanced monthly (21 trading days).

The analysis focuses on small portfolios with $N = 5, 10$, and 20 selected assets. The case $N = 5$ is particularly important because it represents a highly concentrated portfolio and therefore allows us to assess how sensitive RP strategies are to the asset-selection rule respect to pre-selection. In particular, we examine risk parity portfolios either randomly choosing the assets or preselecting them according to their best Sharpe ratio. For each selected number of assets, we compute the following allocation strategies:

- Classical Risk Parity (RP), based on the standard deviation as the risk measure.
- Gini-Type Risk Parity (Gini-RP), based on the Gini-Type risk measure with $\beta = 1\%$ and $\alpha = 3, 4$, and 5 where $\alpha = 3$ corresponds to highest aversion to risk, $\alpha = 4$ corresponds to medium risk aversion, and $\alpha = 5$ corresponds to smallest aversion to risk.
- The Dow Jones Industrial Average index, used as a market benchmark.

The first asset-selection procedure is random. At each rebalancing date, N assets ($N = 5, 10, 20$) are randomly selected from the available DJIA universe. The same selected subset is then used to compute all RP-based strategies at that rebalancing date. This ensures that differences across strategies are due to the allocation rules and not to different asset universes.

Because a single random draw can be highly influential, especially for $N = 5$, the random-selection procedure is repeated 100 times. The reported random-selection performance is summarized in two ways. First, we report average performance across the 100 “simulations”. Second, we report the best-case performance, defined as the simulation with the highest final wealth for each strategy and portfolio size. This second table is useful for assessing whether even the most favorable random-selection outcome can match the performance of an informed pre-selection rule.

The second asset-selection procedure is based on a preselection of the assets with the highest historical Sharpe ratios. At each rebalancing date, Sharpe ratios are computed over the same 500-day rolling estimation window used to estimate portfolio weights. The N assets with the highest historical Sharpe ratios are selected, and the same selected subset is used for classical RP and all Gini-RP measures.

The Sharpe-ratio rule introduces risk-adjusted performance information into the pre-selection stage. This allows us to evaluate whether a simple and transparent pre-selection rule can materially improve the performance of small RP portfolios relative to random selection.

² The constituents of the Dow Jones Industrial Average (DJIA) as of 6 July 2026 are: 3M, ALPHABET 'A', AMAZON.COM, AMERICAN EXPRESS, AMGEN, APPLE, BOEING, CATERPILLAR, CHEVRON, CISCO SYSTEMS, COCA COLA, GOLDMAN SACHS GP., HOME DEPOT, HONEYWELL INTL., INTERNATIONAL BUS.MCHS., JP MORGAN CHASE & CO., JOHNSON & JOHNSON, MCDONALDS, MERCK & COMPANY, MICROSOFT, NIKE 'B', NVIDIA, PROCTER & GAMBLE, SALESFORCE, SHERWIN-WILLIAMS, TRAVELERS COS., UNITEDHEALTH GROUP, VISA 'A', WALMART and WALT DISNEY.

The results of this empirical analysis are reported in Tables 1, and 2 where we evaluate the ex-post performance of each strategy using the following measures:

- Distance from 1/N: the Euclidean distance between the portfolio weight vector and the equally weighted allocation over the selected universe.
- Final Wealth: the final cumulative value of the ex-post wealth process.
- Maximum Drawdown: the largest peak-to-trough decline in cumulative wealth.
- Standard Deviation: the ex-post volatility of daily portfolio returns.
- Sharpe Ratio: the ratio between the average daily return and the standard deviation of daily returns.
- CVaR5%: the average loss in the worst 5% of daily return observations.

Table 1 reports the above measures for the best, average, and worst portfolio strategies (ordered for their final wealth) obtained by the random asset pre-selection procedure. Thus, Table 1 provides a comprehensive view of the outcomes generated by random asset selection, showing not only the average performance but also the range between the most favorable and least favorable, where the best and worst strategies are identified according to their final wealth.

RANDOM-SELECTION PERFORMANCE: BEST, AVERAGE AND WORST CASES								
# Assets	Strategy	Case	Distance from 1/N	Final Wealth	Maximum Drawdown	Standard Deviation	Sharpe Ratio	CVaR5%
5	Gini: $\alpha = 3$	Best	0.1153403	40.0796384	37.4815%	0.0123044	0.0561084	0.0283517
5	Gini: $\alpha = 3$	Average	0.1179546	18.2139273	47.8998%	0.0121745	0.0446715	0.0284016
5	Gini: $\alpha = 3$	Worst	0.1203686	6.4235873	45.2341%	0.0123992	0.0311946	0.0288088
5	Gini: $\alpha = 4$	Best	0.1194636	40.2877103	43.0505%	0.0123021	0.0562125	0.0283430
5	Gini: $\alpha = 4$	Average	0.1187955	18.0879103	47.9787%	0.0121797	0.0445885	0.0284045
5	Gini: $\alpha = 4$	Worst	0.1214844	6.6681330	45.7144%	0.0124406	0.0316317	0.0289078
5	Gini: $\alpha = 5$	Best	0.1172840	40.1546509	38.0265%	0.0123271	0.0560535	0.0283682
5	Gini: $\alpha = 5$	Average	0.1194570	17.9379037	48.1713%	0.0121867	0.0444891	0.0284334
5	Gini: $\alpha = 5$	Worst	0.1233893	6.6044881	46.3266%	0.0124204	0.0315347	0.0289197
5	RP	Best	0.1075063	36.1816720	40.1270%	0.0120531	0.0556264	0.0277341
5	RP	Average	0.1058821	17.6271381	46.5405%	0.0119531	0.0449175	0.0278981
5	RP	Worst	0.1074440	6.9409474	45.0918%	0.0122210	0.0325247	0.0283879
10	Gini: $\alpha = 3$	Best	0.0948929	37.8133458	43.7461%	0.0113249	0.0590835	0.0263584
10	Gini: $\alpha = 3$	Average	0.0945670	18.6632214	46.6858%	0.0112379	0.0484642	0.0264241
10	Gini: $\alpha = 3$	Worst	0.0958139	8.1308550	48.1761%	0.0111443	0.0368845	0.0263920
10	Gini: $\alpha = 4$	Best	0.0959775	38.1713668	43.6584%	0.0113305	0.0591979	0.0263645
10	Gini: $\alpha = 4$	Average	0.0946774	18.5946083	46.6748%	0.0112499	0.0483512	0.0264596
10	Gini: $\alpha = 4$	Worst	0.0964097	7.9619004	47.8305%	0.0111936	0.0364592	0.0265883
10	Gini: $\alpha = 5$	Best	0.0957082	38.7653989	43.5560%	0.0113443	0.0593667	0.0264177
10	Gini: $\alpha = 5$	Average	0.0949095	18.4450615	46.7997%	0.0112599	0.0482199	0.0264879
10	Gini: $\alpha = 5$	Worst	0.0961167	8.2283039	47.9567%	0.0111501	0.0370492	0.0264009
10	RP	Best	0.0856277	36.8642460	41.4100%	0.0112179	0.0591603	0.0260793
10	RP	Average	0.0864617	18.3176124	45.4170%	0.0111043	0.0486988	0.0261115
10	RP	Worst	0.0859779	8.8605288	46.3870%	0.0109886	0.0385540	0.0260277
20	Gini: $\alpha = 3$	Best	0.0742241	25.7291382	44.3767%	0.0108202	0.0553958	0.0255869
20	Gini: $\alpha = 3$	Average	0.0741532	17.8043469	45.8608%	0.0107562	0.0498202	0.0255049

20	Gini: $\alpha = 3$	Worst	0.0743612	11.6067750	48.7011%	0.0107170	0.0434592	0.0255532
20	Gini: $\alpha = 4$	Best	0.0742130	26.3489884	44.2295%	0.0108274	0.0557323	0.0255852
20	Gini: $\alpha = 4$	Average	0.0741412	17.7783233	45.8101%	0.0107703	0.0497391	0.0255433
20	Gini: $\alpha = 4$	Worst	0.0740845	11.9521889	48.3377%	0.0107498	0.0438140	0.0256158
20	Gini: $\alpha = 5$	Best	0.0741680	26.5769176	43.4867%	0.0108353	0.0558318	0.0256205
20	Gini: $\alpha = 5$	Average	0.0741952	17.7600735	45.7157%	0.0107796	0.0496993	0.0255697
20	Gini: $\alpha = 5$	Worst	0.0746094	11.7549925	48.7740%	0.0107352	0.0436008	0.0255878
20	RP	Best	0.0659564	24.8186411	43.9879%	0.0107282	0.0552182	0.0253400
20	RP	Average	0.0663437	17.6884130	44.3696%	0.0106677	0.0500694	0.0252628
20	RP	Worst	0.0660651	11.7816306	47.3304%	0.0106400	0.0439297	0.0252996
30	DJIA	\\	9.8827492	50.6325%	0.0112597	0.0395186	0.0270753	

Table 1. Performance of Random-Selection Strategies: Best, Average, and Worst Cases

Almost all Risk Parity-based strategies (except the worst ones with $N=5, 10$) achieve higher average final wealth than the DJIA benchmark. Moreover, when the asset universe is selected randomly, the risk-allocation mechanism contributes positively to long-term portfolio performance. In this case, the DJIA exhibits the highest maximum drawdown, indicating that RP-based strategies maintain better downside control even in the random-selection setting. The average results also highlight the role of the risk-aversion parameter α in the Gini-Type framework.

The risk (Standard deviation and CVaR5%) decreases, increasing the number of the selected assets N and, for the average strategies, it increases when the aversion to risk decreases (i.e. increasing α). In this context of random selection, the highest variability implies that the most risk-averse average Gini-Type strategy (corresponding to $\alpha = 3$) presents higher wealth and lower risk than those with lower risk aversion, appearing more robust in average terms. This suggests that less risk-averse specifications are more exposed to the instability generated by random asset selection.

The comparison between Gini-Type Risk Parity and classical Risk Parity shows a clear trade-off. Gini-Type RP often achieves slightly higher final wealth, whereas classical RP reports higher Sharpe ratios because it keeps the allocation closer to the $1/N$ portfolio and controls volatility and downside risk more effectively. In this sense, classical RP provides a more defensive risk-adjusted profile, while Gini-Type RP allows investors to introduce downside-risk preferences through the α parameter. The best and worst cases confirm that random selection is highly unstable especially for small portfolios. As the number of selected assets increases, the gap between best and worst outcomes becomes smaller. However, for $N = 5$ and $N = 10$, the gap between the best and worst final wealth values is very large, showing that performance strongly depends on the specific assets randomly selected at each rebalancing date. A favorable random draw can generate very strong terminal wealth, while an unfavorable draw can lead to much weaker outcomes. Portfolio size plays an important stabilizing role. The distance from the $1/N$ portfolio also decreases as N increases, consistently with the theoretical convergence result. This means that Risk Parity allocations remain more distinct from equal weighting in small portfolios, while they become progressively closer to uniform allocation as the asset universe expands.

Overall, Table 1 shows that random asset selection can generate reasonable average results and in favorable cases very strong outcomes. However, random selection also produces substantial instability especially for small portfolios. This evidence confirms that small portfolios are the setting

in which Risk Parity remains most different from 1/N, but also the setting in which asset selection matters the most. For this reason, the random-selection results motivate the introduction of an informative pre-selection rule before applying the Risk Parity allocation.

Table 2 reports the results obtained by applying a Sharpe-ratio pre-selection rule before the Risk Parity allocation. This table is directly comparable with Tables 1, because it reports the same performance indicators: portfolio sizes, risk measures, rebalancing frequency, etc.

PRE-SELECTION SHARPE RATIO							
# Assets	Strategy	Distance from 1/N	Final Wealth	Maximum Drawdown	Standard Deviation	Sharpe Ratio	CVaR5%
5	Gini: $\alpha=3$	0.1085176	59.5915028	44.8100608%	0.0127936	0.0596105	0.0295521
5	Gini: $\alpha=4$	0.1093008	56.7456442	44.7601993%	0.0127930	0.0589754	0.0295965
5	Gini: $\alpha=5$	0.1099396	59.0588008	44.7769947%	0.0128248	0.0593795	0.0296751
5	RP	0.1079889	56.5750532	42.1252679%	0.0125389	0.0598728	0.0289720
10	Gini: $\alpha=3$	0.0949335	22.6155531	47.4273355%	0.0114413	0.0511213	0.0272182
10	Gini: $\alpha=4$	0.0956473	22.1663713	47.4872403%	0.0114461	0.0508122	0.0272245
10	Gini: $\alpha=5$	0.0946610	22.5312955	47.5291154%	0.0114702	0.0509667	0.0272854
10	RP	0.0861147	21.7351578	43.5392638%	0.0112280	0.0512862	0.0266605
20	Gini: $\alpha=3$	0.0800409	19.2086564	47.1122763%	0.0106799	0.0514270	0.0253768
20	Gini: $\alpha=4$	0.0792518	18.9058939	47.0820266%	0.0106874	0.0511514	0.0254101
20	Gini: $\alpha=5$	0.0798765	19.0020013	47.3572606%	0.0106945	0.0512034	0.0254354
20	RP	0.0677455	20.6674237	44.7246391%	0.0105879	0.0529325	0.0251217
	DJIA	\\	9.8827492	50.6325374%	0.0112597	0.0395186	0.0270753

Table 2. Performance under Sharpe-ratio pre-selection.

In Table 2, the investable universe is no longer selected randomly. At each rebalancing date, the portfolio is constructed by selecting the assets with the highest historical Sharpe ratios over the estimation window. The purpose of this table is therefore to evaluate whether an informative asset-selection criterion improves the performance of classical Risk Parity and Gini-Type Risk Parity strategies. The results show a clear improvement relative to the random-selection experiment reported in Table 1. The effect is particularly strong for $N = 5$, where both Gini-Type Risk Parity and classical Risk Parity achieve much higher final wealth than under average random selection. This confirms that, when the portfolio contains only a small number of assets, the quality of the selected universe becomes crucial. In small portfolios, each asset has a stronger impact on the outcome, and an informative pre-selection rule can substantially improve the performance. For $N = 5$, the Gini-Type strategies achieve the highest final wealth values. In particular, the specifications with $\alpha = 3$ and $\alpha = 5$ reach very similar terminal wealth, close to 59, and outperform classical Risk Parity in terms of wealth accumulation. For $N = 10$, Gini-Type RP also remains slightly ahead of classical RP in final wealth, while for $N = 20$ classical Risk Parity achieves the highest terminal value. This suggests that Gini-Type RP is especially effective when the selected universe is small and favorable, whereas the advantage becomes less clear as the number of assets increases. The differences between $\alpha = 3$, $\alpha = 4$, and $\alpha = 5$ are not extreme, but they show that the degree of risk aversion affects the risk-return profile of the Gini-Type strategies. As we expect, the risk (standard deviation and CVaR5%) increases when the aversion to risk decreases (i.e. alpha increases). Classical Risk Parity maintains a more defensive profile. It generally reports lower maximum drawdown, lower standard deviation, and lower CVaR5% than the Gini-Type strategies. It also achieves the highest Sharpe ratio for each portfolio size. This result is consistent with the interpretation given in Table 1: classical RP is more conservative, keeps the portfolio closer to the $1/N$ allocation, and provides a strong balance between return and risk control. Gini-Type RP, instead, can generate higher final wealth in smaller selected universes, accepting slightly higher downside-risk exposure. Portfolio size also affects the impact of Sharpe-ratio pre-selection. The strongest improvement is observed for $N = 5$, while the advantage becomes more moderate for $N = 10$ and $N = 20$. This is consistent with the idea that asset pre-selection matters most when the portfolio is small. As the number of selected assets increases, diversification reduces the impact of individual asset choices and the allocation becomes closer to the uniform $1/N$ portfolio.

Overall, Table 2 shows that Sharpe-ratio pre-selection substantially improves the performance of Risk Parity-based strategies, especially for small portfolios. The results confirm that Risk Parity should not be interpreted as a stock-selection rule, but as a risk-allocation rule applied after the investment universe has been defined. The quality of the pre-selected asset universe is therefore fundamental: when the selected assets are more favorable. Both classical RP and Gini-Type RP can produce stronger and more stable results.

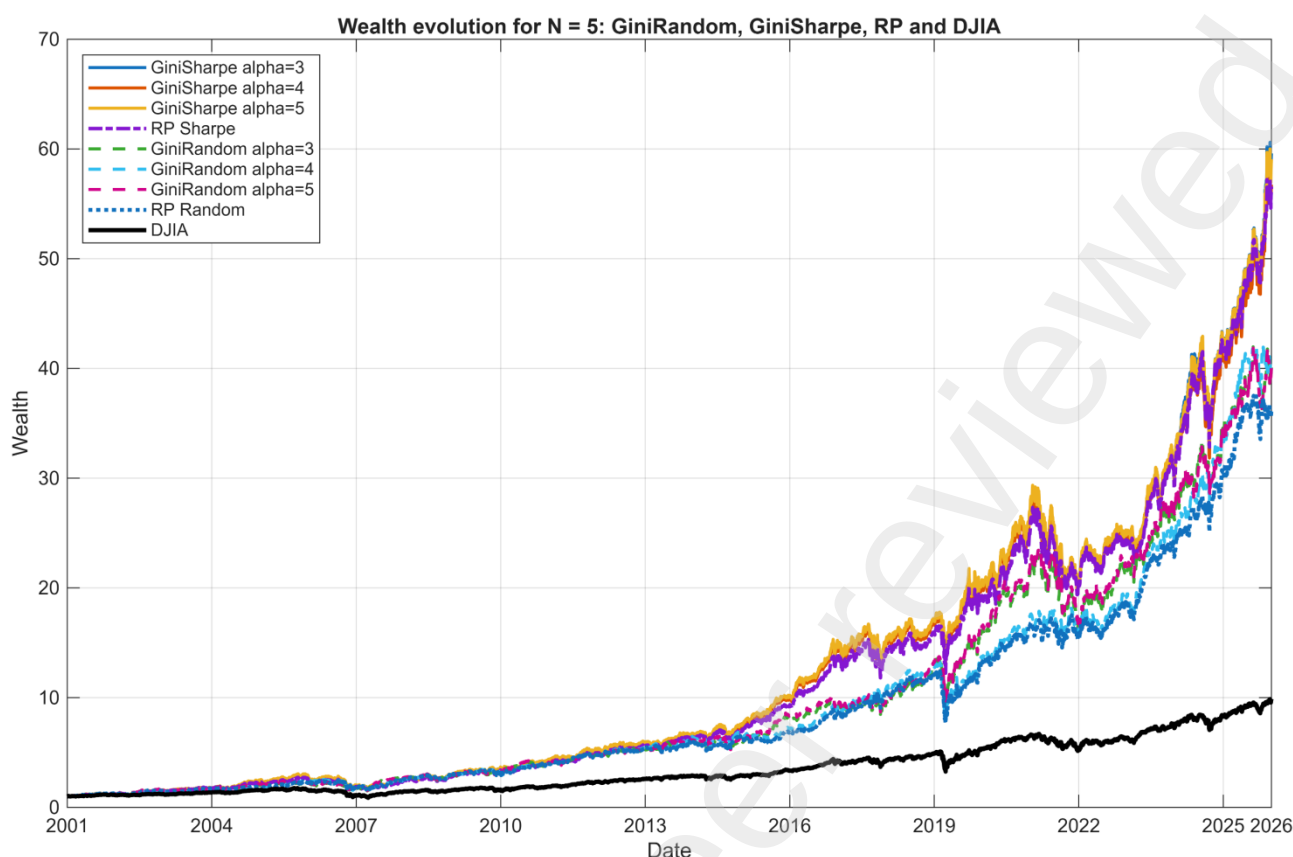


Figure 1. Wealth evolution for $N = 5$: best random-selection versus Sharpe-ratio pre-selection.

Figure 1 reports the wealth evolution of the strategies for $N = 5$: a) the best strategies obtained in the random-selection experiment; b) the Sharpe-based strategies constructed after applying the Sharpe ratio pre-selection rule and; c) the market benchmark represented by the DJIA index. The figure shows that the Sharpe-based strategies generate higher wealth accumulation than the best random-selection cases. This comparison is particularly informative because the Sharpe-based strategies are not compared with average random outcomes, but with the most favorable random pre-selection. Therefore, the use of an informative pre-selection rule leads to superior performance. The difference becomes especially visible in the second part of the sample, where the Sharpe-based strategies grow more than the random-selection ones. The DJIA remains below the Risk Parity-based strategies for most of the period, confirming the advantage of combining asset pre-selection with risk-based allocation.

Overall, Figure 1 provides visual evidence of the main empirical result: in small portfolios, the effectiveness of Risk Parity depends strongly on the quality of the selected asset universe. Sharpe-ratio pre-selection strengthens the allocation process by identifying a more favorable set of assets before applying the Risk Parity rule.

5. Conclusion

This paper investigates Risk Parity strategies by focusing on two related aspects: the role of asset pre-selection and the incorporation of different degrees of risk aversion through Gini-Type risk measures. The analysis clarifies that Risk Parity should be interpreted as a risk-allocation rule rather than as an asset-selection procedure. For this reason, the quality of the investment universe selected before the optimization step plays a crucial role in the final performance of the strategies. From a theoretical perspective, we prove that Risk Parity allocations tend to move closer to the uniform $1/N$ portfolio as the number of assets increases. This result motivates the empirical focus on relatively small portfolios. In these cases, Risk Parity remains meaningfully different from equal weighting and is more relevant for investors facing capital constraints, transaction costs, and monitoring costs. The empirical evidence confirms this intuition. Under random asset selection, Risk Parity-based strategies achieve higher final wealth than the DJIA benchmark and generally display a more controlled downside-risk profile. However, the comparison between best, average, and worst random-selection outcomes shows that random selection is highly unstable, especially for small portfolios. In these cases the portfolio performance depends strongly on the specific assets selected at each rebalancing date. This confirms that small portfolios are the setting in which Risk Parity is most distinct from $1/N$, but also the setting in which asset selection matters the most. The introduction of Sharpe-ratio pre-selection substantially improves the performance of portfolio strategies. When the selected universe is constructed using an informative criterion, both classical Risk Parity and Gini-Type Risk Parity achieve stronger performance than under average random selection. This confirms the main empirical finding of the paper: Risk Parity can be effective in small portfolios, but its performance depends critically on the quality of the assets selected before the risk-allocation rule is applied. The comparison between classical Risk Parity and Gini-Type Risk Parity highlights a clear trade-off. Classical Risk Parity is generally more defensive, with stronger control of volatility, drawdown, and tail risk, and often delivers competitive Sharpe ratios. Gini-Type Risk Parity, instead, allows downside-risk preferences to be incorporated through the α parameter and can achieve higher final wealth, particularly when the selected asset universe is favorable. The α parameter therefore provides a useful way to model different degrees of risk aversion within the Risk Parity framework.

Overall, the results suggest that small Risk Parity portfolios combined with an informative pre-selection rule can represent a practical alternative to broad equally weighted portfolios. The contribution of the paper is to show that asset pre-selection is not a secondary step, but a central component in the practical implementation of Risk Parity strategies. At the same time, the Gini-Type extension enriches the framework by introducing downside risk and investor risk aversion into the allocation process.

Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Declaration of Competing Interest

The authors declare that they have no competing interests.

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Declaration of Generative AI and AI-Assisted Technologies in the Manuscript Preparation Process

During the preparation of this work, the authors used ChatGPT (OpenAI) and Gemini to refine the English language, improve clarity, and support manuscript editing. After using these tools, the authors reviewed and edited the content as needed and took full responsibility for the content of the manuscript.

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