

Inflation is Economic Reflection of Portfolio Return.

Part II. Laspeyres Index Adjusted by Self-Inflation, Walsh's Problem, Market-Based and Basket-Based Variances, and Sharpe Ratio of Inflation

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We show that the description of a portfolio return and variance may model the Laspeyres index and inflation. Inflation is a tool to adjust prices of trades made at different moments to the same time. However, the current theory and the practice for the assessments of the Laspeyres index and inflation overlook this rule. We partially fix this flaw and obtain the equation for the adjusted inflation and derive its successive approximations. We resolve the century-old Walsh's problem of the different trade weights in the basket: *"Which weights, then, are the right ones—those of the first period? Or those of the second?"* (Walsh, 1921). We apply the transformation we developed for the solution of the similar problem of the portfolio theory and convert the trade weights during the current period so that they coincide with the trade weights of the past period or vice versa. We obtain the time series of values and volumes that model the trade with the basket as with a single commodity. That reveals the identity between the use of the Laspeyres and the elementary index. We introduce the random Laspeyres indices and inflation during the current period and derive market-based variance of inflation that describes the uncertainty of the current and future estimates of inflation caused by the random trades with the basket. We introduce basket-based variance of inflation to describe the uncertainty caused by the random distribution of elementary inflations of numerous goods in the basket. We propose Sharpe Ratio as a measure of the assurance of the current and future assessments of inflation. We refer to Markowitz's advice on portfolio selection and argue that economically effective fiscal and monetary policies, when they aim to manage inflation, should be implemented on the basis of the forecasts of inflation and its two variances.

Keywords : adjusted price indices, adjusted inflation, portfolio return, market-based and basket-based variances of inflation

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1. Introduction

This paper reveals the likeness of the description of the Laspeyres index and the portfolio returns. We show how the methods developed for the portfolio returns may be used for the description of inflation and price indices. We highlight that the randomness of values and volumes of the consecutive trades with portfolio securities at the exchange or with the goods and services of the consumer basket made during the averaging period is the main essential and irremovable source of risks of the deviations of the future actual returns and inflation from their current forecasts. Random trades cause random returns and random inflation. Any assessments and forecasts of the portfolio return or inflation that aim to be economically reasonable and valuable should be complemented by the corresponding estimates and predictions of their variances that describe the rates of the uncertainties of their mean appraisals. The fiscal and monetary policies, when they aim to manage inflation, should follow the same advice Markowitz (1952) gave for portfolio investors. To be economically effective and reliable, the policies and decisions taken to manage inflation should be based on the forecasts of both inflation and its variances.

We refer to Diewert (1988, 1995), who gave the perfect reviews of the history and methods of price indices that were developed for almost two centuries. The modeling of inflation is the main goal of the different price indices (Jevons, 1865; Edgeworth, 1888; Reinsdorf and Triplett, 2009; Rippy, 2014). Portfolio theory was developed a lot since the famous paper by Markowitz (1952) and followers (Merton, 1969; Pogue, 1970; Samuelson, 1970; Rubinstein, 2002; Boyd et al., 2024). The assessments of inflation are based on the use of the fixed consumer basket. The selection of the consumer basket generates the problem of appropriate measurements of its values, trade weights during the past and the current period, and the possible predictions of the basket value for one or more periods in the future (ILO et al., 2004; 2025). If the current $Q(t)$ and past $Q(t_0)$ (1.1) values of the consumer basket are determined by the total trade volumes or weights $U_{\Sigma 0}=(U_{\Sigma I}(t_0),.. U_{\Sigma N}(t_0))$ with $n=1,..N$ goods and services during the past period:

$$Q(t) = \sum_{n=1}^N p_n(t) U_{\Sigma n}(t_0) \quad ; \quad Q(t) = \sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t_0) \quad (1.1)$$

one assesses inflation $i(t, t_0)$ by the Laspeyres (1871) index $In_L(t, t_0)$ (1.2):

$$In_L(t, t_0) = 1 + i(t, t_0) = \frac{Q(t)}{Q(t_0)} = \frac{\sum_{n=1}^N p_n(t) U_{\Sigma n}(t_0)}{\sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t_0)} \quad (1.2)$$

When the values of the basket are determined by the weights $U_{\Sigma}=(U_{\Sigma I}(t),.. U_{\Sigma N}(t))$ of goods sold during the current period, one assesses inflation $i_p(t, t_0)$ by the Paasche (1874) index $In_P(t, t_0)$ (1.3):

$$In_p(t, t_0) = 1 + i_p(t, t_0) = \frac{Q(t)}{Q(t_0)} = \frac{\sum_{n=1}^N p_n(t) U_{\Sigma n}(t)}{\sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t)} \quad (1.3)$$

The prices $p_n(t)$ and $p_n(t_0)$ in (1.1-1.3) denote the mean prices of $n=1, \dots, N$ goods or services in the consumer basket averaged during the current and past periods. The averaging of the prices of each item in the consumer basket is a separate problem. Diewert (1988, 1995) presented many methods for averaging prices and price indices. We consider here the only definition: “*..the prices should be weighted according to the relative mass-quantities that were sold at them.*” (Walsh, 1901, p. 96; Diewert, 1995, p. 21) or “*..the average price found by dividing the total sum (or the full value) by the total quantities.*” (Walsh, 1921, p. 88). One may recognize that many decades later this definition was reproduced by Berkowitz et al. (1988) as volume weighed average price (VWAP) $p_n(t)$ (1.4) of the n -th good or service in the consumer basket:

$$p_n(t) = \frac{\sum_{j=1}^J p_n(t_j) U_n(t_j)}{\sum_{j=1}^J U_n(t_j)} = \frac{\sum_{j=1}^J p_n(t_j) U_n(t_j)}{U_{\Sigma n}(t)} = \frac{\sum_{j=1}^J C_n(t_j)}{U_{\Sigma n}(t)} = \frac{C_{\Sigma n}(t)}{U_{\Sigma n}(t)} ; \quad n = 1, \dots, N \quad (1.4)$$

In (1.4) $C_n(t_j)$, $U_n(t_j)$, and $p_n(t_j)$ (A.2) denote the value, volume, and price of trade at time t_j with the $n=1, \dots, N$ items of the consumer basket during the current period Δ (A.1). In (1.1-1.4) we denote the total volumes $U_{\Sigma n}(t)$ and values $C_{\Sigma n}(t)$ of trades made during the current period Δ (A.1) with the n -th good of the basket. Walsh (1901; p. 97) highlighted that the average price $p_n(t)$ (1.4) was proposed by Segnitz (1870) and refers to Heitz (1876), who, “*.. says this method is employed in the statistical bureaus of several German states – Wurtemberg, Bavaria, Gotha, and partly in Prussia and Hannover.*” (Walsh, 1901, p. 97). One should accept that the definition of VWAP (1.4) was proposed by Segnitz (1870), Walsh (1901; 1921), and the researchers of “*statistical bureaus of German states,*” but not by Berkowitz et al. (1988), who reproduced it a century later. We emphasize the economic identity of the selection of the fixed consumer basket in the past and the investor, who at time t_0 in the past composed his portfolio with securities traded at the exchange. It is assumed that since then the portfolio remains constant, and the investor doesn’t trade his securities and only observes the values and volumes of trades made at the exchange with his securities. If one focuses on the description of the portfolio’s gross returns determined only by the prices of trades made at the exchange with portfolio securities, one obtains almost exact identity with the description of the Laspeyres index, inflation, and fixed consumer basket.

It is evident that the number of goods in the consumer basket may be much more than the number of securities traded at the exchange, and the averaging period for estimating the Laspeyres index may be longer than the averaging periods that investors usually use for the assessment of portfolio

returns. However, these distinctions don't change the economic identity of the estimates of the Laspeyres index and the portfolio's gross return.

In Part I of this article, we discussed the similarity between modeling the elementary index and gross return of a single security traded at the exchange (Olkhov, 2026c). In Part II, we consider the likeness of the modeling of the Laspeyres index $In_L(t, t_0)$ (1.2) and inflation $i(t, t_0)$ (1.2) and the portfolio's gross return. The main text of the article is almost free from math. In Sec. 2, we describe the adjustment of trade prices $p_n(t_j)$ in (1.4) made at time t_j during the averaging period by Laspeyres inflation $i(t, t_0)$ (1.2) to the same time. In Sec. 3 we consider the century-old Walsh's problem: *"..Which weights, then, are the right ones—those of the first period? Or those of the second?"* (Walsh, 1921; p. 90; ILO, et al., 2025, p. 16). We show how one may transform the current *"weights of the commodities at the second period"* so that they will coincide with *"their weights at the first period"* or vice versa. Thus, we show that the Laspeyres index $In_L(t, t_0)$ (1.2) and the Paasche index $In_P(t, t_0)$ (1.3) may model the inflation of the consumer basket with identical *"weights of the commodities"* during both averaging periods. Moreover, we show that our transformation may model the trade with the consumer basket, which is composed of a great number of goods and services, as a trade with a single commodity. That reveals the identity between the use of the Laspeyres index $In_L(t, t_0)$ and the elementary index. In Sec. 4 we consider the Laspeyres index $In_L(t, t_0)$ and introduce the random Laspeyres index, random inflation, their market-based and basket-based variances. We suggest the Sharpe Ratio of inflation as a measure of their uncertainties for market-based, and basket-based variances. Conclusion in Sec. 5. In App. A, we use math to derive how one should adjust the prices of trades $p_n(t_j)$ in (1.4) made at time t_j with goods of the consumer basket by the Laspeyres inflation $i(t, t_0)$ (1.2) to the same time t *before* one performs the averaging of price $p_n(t)$ (1.4). In App. B, we present the transformation that resolves Walsh's problem and converts time series of values and volumes of trades with numerous goods of the consumer basket during the current period into the time series of trades, which have trade weights identical to trade weights during the past period, or vice versa. As well, this transformation helps obtain the time series of values and volumes that models the trades with the basket as with a single commodity. That reveals the identity between the modeling inflation via the Laspeyres index $In_L(t, t_0)$ and via the elementary index. In App. C, we introduce random Laspeyres indices and inflation similar to random elementary indices and determine their market-based and basket-based variances. In App. D, we introduce the Sharpe Ratio of inflation for market-based, and basket-

based variances. We refer to Part I of this paper as (Olkhov, 2026c), which describes the likeness between the elementary index and the gross return of a security.

2. Laspeyres Index Adjusted by self-inflation

To calculate the Laspeyres index $In_L(t, t_0)$ (1.2) and inflation $i(t, t_0)$, one should obtain the total values of trades $C_{\Sigma n}(t)$ (2.1) with $n=1, \dots, N$ items of the consumer basket during the current period:

$$C_{\Sigma n}(t) = \sum_{j=1}^J C_n(t_j) = \sum_{j=1}^J p_n(t_j) U_n(t_j) \quad (2.1)$$

However, at least since the classical Markowitz (1952) paper, it is generally accepted that the sums (2.1) of trade values $C_n(t_j)$ in (2.1) have correct economic meaning if and only if the prices $p_n(t_j)$ of the trades made at time t_j during the current period Δ (A.1) are adjusted to the same time t . Otherwise, the sum of two trade values has a rather dubious economic sense. In App. A, we consider the Laspeyres index $In_L(t, t_0)$ and inflation $i(t, t_0)$ (1.2) on a month-to-month basis and adjust prices of trades made at time t_j during the current period Δ (A.1) to time t . We emphasize that correct economic averaging of the trade prices $p_n(t_j)$ in (1.4) during a month-long period Δ (A.1) should be performed only after all trade prices $p_n(t_j)$ in (1.4) are adjusted by the inflation $i_a(t, t_0)$ (A.3) to the same time t . We denote $i_a(t, t_0)$ (A.3) adjusted inflation and highlight that it differs from $i(t, t_0)$ (1.2). As we know, such a mandatory adjustment of trade prices $p_n(t_j)$ in (1.4) during the period Δ (A.1) to the same time t is absent in the current theory and practice while making assessments of the Laspeyres index and inflation. To partially fix this flaw, in App. A, we derive the equation for the adjusted inflation $i_a(t, t_0)$ (A.13), obtain the successive approximations of the adjusted inflation $i_a(t, t_0)$ (A.3; A.13), adjusted trade prices $p_{an}(t_j)$ (A.7), and $p_{an}(t)$ (A.8). The accuracy of the successive approximations is determined by the use of unadjusted trade prices $p_n(t_j)$ during Δ_k (A.18) k periods in the past. In the first approximation (A.14-A.16), one may adjust trade prices $p_{an}(t_j)$ (A.7) during the current period Δ (A.1) and use unadjusted prices during the past period Δ_0 (A.5). In the first approximation the adjusted inflation takes the form $i_a(t, t_0)$ (A.14). The mean adjusted prices $p_{an}(t)$ and the adjusted current value $Q_a(t)$ of the basket are determined by (A.15; A.16). To derive the second approximation, one should use unadjusted prices two periods back, Δ_2 (A.18). That determines the adjusted value $Q_a(t_0)$ (A.17) of the consumer basket at time t_0 , which depends on the unadjusted value $Q(t_{-1})$ of the consumer basket during two periods back, Δ_2 (A.18). To derive the the adjusted inflation $i_a(t, t_0)$ during the current period in the second approximation one should substitute the adjusted value $Q_a(t_0)$ (A.17) into the equation (A.13).

3. Walsh problem and the transformation of trade weights

We recall the problem that was formulated by Walsh: *“There is a first period, and there is a second period which is compared with it. Price variations have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period. Which weights, then, are the right ones—those of the first period? Or those of the second?”* (Walsh, 1921, p. 90; ILO, et al., 2025, p. 16). Such general problems never exist in one field. Almost the same problem exists in portfolio theory. The investor who at time t_0 in the past has collected his portfolio of $n=1..N$ securities traded at the exchange meets the problem that is similar to Walsh’s. The attempt to model the return and variance of his portfolio meets the problem that the weights or volumes of the trades with portfolio securities made during the current period Δ (A.1) differ from the weights or number of shares of securities in his portfolio at time t_0 in the past. These distinctions are almost like Walsh’s remarks about the differences of the weights during the past and current periods. In Olkhov (2025a-2026b), we describe the transformation (App. B) of the time series of values and volumes of the consecutive trades made during the current period Δ (A.1) at the exchange with the portfolio’s securities, which transforms them into the time series of values and volumes that have the same weights as the securities in the portfolio at time t_0 in the past. That resolves the problem of the distinctions between the current and past trade volumes or weights of the securities in the portfolio. We use the same transformations (App. B) and show how one may convert the time series of values and volumes of trades with the $n=1..N$ goods of the consumer basket during the current period Δ (A.1) into the time series of values and volumes that have the trade weights identical to the trade weights during the past period. The same transformation may convert the weights of the past period to the weights of the current period.

Moreover, we show how to convert the time series of values and volumes of the consecutive trades with numerous goods of the consumer basket into the time series of values and volumes that model the trade with the basket as with a single commodity (App. B1). That reveals the identity between modeling the Laspeyres index and inflation using the consumer basket and modeling the elementary index and inflation as price changes of a single good.

The use of our transformation completely solves Walsh’s problem (Walsh, 1921). One may calculate the ratio of the values of the consumer basket during the current and past periods using time series of trades with $n=1..N$ items that have trade weights determined during the past period

and obtain the Laspeyres index. The ratio of the values of the consumer basket that have trade weights determined during the current period gives the Paasche index. Both indices may be presented as elementary indices composed of a single commodity. The choice between the Laspeyres or Paasche indices is determined by psychology, habits, and expectations, but not by the problem that was emphasized by Walsh (1921).

We outline that the values, volumes, and prices of trades with the $n=1,..N$ items of the consumer basket during the current month-long averaging period are highly irregular or random. Any assessments of the Laspeyres or Paasche indices give their mean values, which have economic sense only when one estimates their uncertainties or variances. Otherwise, the unknown uncertainty of the Laspeyres index and inflation makes their use rather insecure and precarious. In the next section we describe how one may use our transformation and derive the assessments of the market-based and basket-based variances of the Laspeyres index and inflation.

4. Random Laspeyres index and inflation, Market-Based and Basket-Based variances, and the Sharpe Ratio

We recall that we consider the trade prices $p_{an}(t_j)$ with the items of the consumer basket and the Laspeyres index $In_{La}(t, t_0)$ adjusted by its inflation $i_a(t, t_0)$ (App. A) but omit further index “ a ” for simplicity.

In Part I, we emphasized that the stochasticity of the Laspeyres index $In_L(t, t_0)$ and inflation $i(t, t_0)$ was studied a lot since Jevons (1865), Edgeworth (1888), and ILO et al. (2024; 2025). These studies focus on the impact of random distortions of econometric observations, gaps in the trade data, and any perturbations that accompany actual observations of such a complex problem as trades of a great number of goods and services that compose the consumer basket. We attract the readers’ attention to the effect of irregularity or randomness of trades with the basket during the averaging period. Even if one neglects all actual uncertainties and gaps of econometric observations, the random nature of economic and financial markets leaves no chance for “theoretical” deterministic derivation of inflation. The intrinsic and irremovable randomness of the economic markets determines the essential uncertainties of inflation. We describe how the randomness of real economic markets, exchanges, and trades causes stochasticity of inflation, even in the unrealistic assumption that all actual uncertainties and disturbances of econometric

observations are neglected. It is evident that the real uncertainty of econometric measurements will only increase the uncertainty and variances that we derive in this paper.

The randomness of the values, volumes, and prices of the consecutive trades with the $n=1,..N$ goods of the consumer basket during the current period Δ (A.1) is the origin of the random Laspeyres index and inflation. To define the random Laspeyres index or inflation, one should apply the transformation (App. B) that converts the numerous time series of values and volumes of trades with $n=1,..N$ items of the basket into the time series that model the trade with the basket as with a single commodity (B.2; B.6-B.10). This transformation highlights the identity of the elementary index $In_e(t, t_0)$ and elementary inflation $i_e(t, t_0)$ (B.9) determined by the ratio of mean prices of a single good or service during the current and past periods and the Laspeyres index presented in the form of a single commodity. In simple words, we use our transformation to define the price per item of the consumer basket that helps consider the Laspeyres index, like the elementary index. Moreover, this transformation allows us to define the random Laspeyres index as a ratio of the random trade price per item of the basket at time t_j during the current period to the mean price per item of the consumer basket during the past period (C.1). This definition is almost identical to the definition of the random price per share and random gross return of the portfolio (Olkhov, 2025a-2026b). In App. C and C.1 we explain the similarity between modeling the Laspeyres index and portfolio gross return. The likeness of the random trade prices per share of the portfolio and the random trade prices per item of the consumer basket allows us to derive the market-based variance of the Laspeyres index (C.2-C10) that is like the economic reflection of the variance of portfolio return. The market-based variance of the Laspeyres index, or inflation, has its origin in random trades with the items of the basket and describes their deviations from their mean values.

Meanwhile, the huge number N of the goods in the consumer basket, which may exceed thousands, causes their elementary indices of $n=1,..N$ items to vary a lot. One may consider the values of these elementary indices as a random sample. One may consider the randomness of the values of $n=1,..N$ elementary indices of the items that compose the consumer basket as the origin of the basket-based variance of the Laspeyres index, like the random trades cause its market-based variance. That likeness helps us to introduce (App C.2) the basket-based variance (C.12-C.25) of the Laspeyres index and inflation as a measure of deviations of numerous elementary indices and inflations of the consumer basket from the basket Laspeyres index and inflation.

The market-based and basket-based variances of the Laspeyres index and inflation describe their uncertainties due to random trades with the basket's items during the current period and due to random distribution of elementary indices and inflations in the basket. One should consider both variances as mandatory tools that model the uncertainty of mean values of the Laspeyres index and inflation. To be economically reasonable, any predictions of inflation $i(t, t_0)$ at horizon T , as the forecasts of the mean values of a random variable, should be complemented by the forecasts of their market-based and basket-based variances.

The similarity between the variances of the Laspeyres index and portfolio return reveals an additional tool that may be rather useful for inflation modeling and predictions. Since Sharpe (1966) introduced his Ratio, it has become a widespread measure of portfolio risks. We believe that one may consider the Sharpe Ratio (App. D) of inflation as a measure of the uncertainty of the current assessments and of the forecasts of inflation. We propose market-based $Sp_M(i)$ (D.1) and basket-based $Sp_B(i)$ (D.1) Sharpe Ratios of inflation:

$$Sp_M(i) = \frac{i(t, t_0)}{\Theta^{1/2}(t, t_0)} = \frac{\ln_L(t, t_0) - 1}{\Theta^{1/2}(t, t_0)} \quad ; \quad Sp_B(i) = \frac{i(t, t_0)}{\Theta_B^{1/2}(t, t_0)} = \frac{\ln_L(t, t_0) - 1}{\Theta_B^{1/2}(t, t_0)}$$

The Sharpe Ratio is a reciprocal to the coefficient of variation of inflation and describes the relations between the mean inflation and its volatility. We recall that the square of volatility equals the variance. If the Sharpe Ratio of inflation $Sp_M(i) > 1$ is more than a unit, then the value of inflation is a fairly definite quantity. Otherwise, when $Sp_M(i) < 1$, the volatility of inflation is higher than its mean value, and one should account for such high uncertainty. The forecasts of inflation with low Sharpe Ratio $Sp_M(i) < 1$ are rather doubtful. The use of inflation's predictions with low $Sp_M(i) < 1$ generates high risks for macroeconomic modeling.

The complexity and the limitations of the forecasts of market-based and basket-based variances of inflation are like to the limitations of the predictions of the variance of portfolio returns and risks. The same economic and econometric factors prevent reliable predictions and limit the forecasting capacity of the variances of portfolio returns and variances of inflation (Olkhov, 2026b).

5. Conclusion

Inflation and portfolio returns are different factors of macroeconomics. We show that the description and forecasts of inflation and its variances may be almost identical to the modeling and predictions of a portfolio return and variance. The economic meaning of both issues, inflation and portfolio return, is the average values of random variables. Their assessments, predictions, and use

without assessments and forecasts of their variances have low economic value. The complexity and limitations of the modelling and predictions of inflation and its variances are almost identical to the limitations of the forecasting capacity of portfolio return and variance.

We show that the economic correctness of the definition of inflation requires adjustment of the trade prices with the items of the consumer basket to the same time. Meanwhile, this point is overlooked in the current theory and practice. We partially fill this gap and derive the equation for adjusted inflation and obtain successive approximations of adjusted prices, basket values, and adjusted inflation. We resolve the century-old Walsh problem that highlights the distinctions between the trade weights with the consumer basket during the past and current periods. We derive the transformation that converts the time series of the current values and volumes with the consumer basket into the time series that have the trade weights identical to trade weights during the past period or vice versa. That removes the question: *“Which weights, then, are the right ones—those of the first period? Or those of the second? Or should there be a combination of the two sets?”* (Walsh, 1921, p. 90; ILO, et al., 2025, p. 16). We believe that the selection between the Laspeyres or Paasche indices is more a psychological issue than a problem of economics or finance.

We obtain the time series of values and volumes that model the consecutive trades with the basket as with a single commodity. This reveals the economic identity between the use of the elementary and Laspeyres or Paasche indices.

The presentation of the trades with the consumer basket as trades with a single commodity allows us to define the price of consecutive trades per item of the basket, like we introduced price per share of the portfolio’s securities (Olkhov, 2025a-2026b). This leads to the derivation of the market-based variance of inflation. The distribution of the mean prices of numerous items that compose the consumer basket allows us to introduce the basket-based variance of inflation. These market-based and basket-based variances describe the uncertainty of inflation as the uncertainty of the average value due to the randomness of trades and due to the distribution of the numerous elementary inflations of the basket items during the current period. We propose the Sharpe Ratios of inflation that are determined by market-based and basket-based variances as measures of inflation’s uncertainties. The predictions of inflation with a low Sharpe Ratio or with high variances reveal their low economic value and deliver high risks for macroeconomic modeling. Practitioners should account for the forecasts of the inflation’s Sharpe ratios of market-based and basket-based variances.

The reliability of the forecasts of market-based and basket-based variances of inflation is the crucial problem for macroeconomics. However, the limitations of such predictions are almost identical to the limitations of the predictions of portfolio variances and risks (Olkhov, 2026b).

We argue that economically effective decisions, fiscal and monetary policies when they aim at inflation management, should be taken on the basis of the forecasts of inflation and both variances. Without the predictions of the market-based and basket-based variances, inflation management seems to be highly uncertain. Further studies of similarities between inflation and portfolio theory may benefit both problems.

Appendix A. Adjustment of Laspeyres Index by self-inflation

The economic laws justify the sum of two values $C_n(t_j)$ and $C_n(t_k)$ of trades made at time t_j and t_k with the n -th good or service of the consumer basket if and only if the prices of the trades are adjusted by inflation to the same time t . This simple rule is missed by the current theory and practice for estimating the Laspeyres index. To match the adjustment rule we derive the equation that determines the adjusted inflation $i_a(t, t_0)$ (A.3), which is determined by trade prices $p_{an}(t_j)$ adjusted to time t , and obtain its successive approximations similar to successive approximations of the adjusted elementary inflation (Part I). We consider a month-long averaging period Δ (A.1):

$$\Delta = [t_0; t] ; \quad t_0 = t - \Delta ; \quad t_j \in \Delta ; \quad t = t_j ; \quad j = 0, \dots, J \quad (\text{A.1})$$

We take t_0 and t as starting and final moments of Δ (A.1) and denote $C_n(t_j)$, $U_n(t_j)$, and $p_n(t_j)$ (A.2) the values, volumes, and prices of trades made at time $t_j, j=1, \dots, J$ with the n -th item, $n=1, \dots, N$ of the consumer basket that obey trivial equation (A.2):

$$C_n(t_j) = p_n(t_j) \cdot U_n(t_j) \quad (\text{A.2})$$

The trade values, volumes, and prices are rather irregular or random during Δ (A.1). However, there is no need to process each particular trade made in the market with items of the basket. For example, one may sum the values and volumes of trades made during half a day and consider these sums as a value and volume of a trade at time t_j . Then one obtains the time series of ~ 60 collective trades during a month-long period Δ (A.1), and that is sufficient for the reliable estimates of the first two statistical moments of trade values, volumes, and prices. For simplicity we assume that “half-a-day” trades are determined at the same times $t_j, j=1, \dots, J$ for all $n=1, \dots, N$ goods of the basket.

Let us denote the adjusted inflation $i_a(t, t_0)$ (A.3) and adjusted Laspeyres index $In_{La}(t, t_0)$ (A.3) as the ratio of an adjusted value $Q_a(t)$ (A.4) of the basket at time t to the adjusted basket value $Q_a(t)$ (A.4) at time t_0 in the past:

$$In_{La}(t, t_0) = 1 + i_a(t, t_0) = \frac{Q_a(t)}{Q_a(t_0)} = \frac{\sum_{n=1}^N p_{an}(t) U_{\Sigma n}(t_0)}{\sum_{n=1}^N p_{an}(t_0) U_{\Sigma n}(t_0)} \quad (\text{A.3})$$

$$Q_a(t) = \sum_{n=1}^N p_{an}(t) U_{\Sigma n}(t_0) ; Q_a(t_0) = \sum_{n=1}^N p_{an}(t_0) U_{\Sigma n}(t_0) ; U_{\Sigma n}(t_0) = \sum_{j=1}^J U_n(t_j) \quad (\text{A.4})$$

The volumes $U_{\Sigma n}(t_0)$ in (A.4) denote the weights of the sales or the total volumes of trades made with item $n=1, \dots, N$ during the past period. Prices $p_{an}(t)$ (A.3; A.4) denote VWAP of prices $p_{an}(t_j)$ (A.7) of trades at time t_j with the item n of the consumer basket, adjusted by inflation $i_a(t, t_0)$ (A.3) to the same time t (A.1). Prices $p_{an}(t_0)$ denote VWAP of trade prices $p_n(t_j)$ at time t_j adjusted by the inflation $i_a(t_0, t_{-1})$ (A.6) during the past averaging period Δ_0 (A.5) to time t_0 .

$$\Delta_0 = [t_{-1}; t_0] ; t_{-1} = t_0 - \Delta ; t_j \in \Delta_0 ; t = t_j ; j = 0, \dots, J \quad (\text{A.5})$$

$$In_{La}(t_0, t_{-1}) = 1 + i_a(t_0, t_{-1}) = \frac{Q_a(t_0)}{Q_a(t_{-1})} \quad (\text{A.6})$$

The adjustment of the price $p_n(t_j)$ (A.2) of trade made at time t_j by inflation $i_a(t, t_0)$ (A.3) during Δ (A.1) to time t determines the adjusted trade price $p_{an}(t_j)$ (A.7):

$$p_{an}(t_j) = p_n(t_j) \cdot \left[1 + \left(1 - \frac{j}{J} \right) i_a(t, t_0) \right] \quad (\text{A.7})$$

The VWAP of the adjusted prices $p_{an}(t_j)$ (A.7) defines the adjusted VWAP $p_{an}(t)$ (A.8):

$$p_{an}(t) = \frac{1}{\sum_{j=1}^J U_n(t_j)} \sum_{j=1}^J p_n(t_j) \left[1 + \left(1 - \frac{j}{J} \right) i_a(t, t_0) \right] U_n(t_j) = p_n(t) + z_n(t) i_a(t, t_0)$$

It is evident that the interval $t - t_j = (J - j/J) \Delta$ is proportional to $(J - j/J)$, and that validates the above expressions. From above, obtain the adjusted VWAP $p_{an}(t)$ (A.8):

$$p_{an}(t) = p_n(t) + z_n(t) i_a(t, t_0) \quad (\text{A.8})$$

The functions $z_n(t)$ (A.9) define the growth of unadjusted VWAP $p_n(t)$ (1.4) by inflation $i_a(t, t_0) = I$:

$$z_n(t) = \frac{1}{\sum_{j=1}^J U(t_j)} \sum_{j=1}^J p_n(t_j) \left(1 - \frac{j}{J} \right) U_n(t_j) \quad (\text{A.9})$$

The adjusted value $Q_a(t)$ (A.4) of the consumer basket at time t takes the form:

$$Q_a(t) = \sum_{n=1}^N p_{an}(t) U_{\Sigma n}(t_0) = Q(t) + Z(t) i_a(t, t_0) \quad (\text{A.10})$$

$$Z(t) = \sum_{n=1}^N z_n(t) U_{\Sigma n}(t_0) \quad (\text{A.11})$$

From (A.10; A.11), obtain the expression of the adjusted past value $Q_a(t_0)$ (A.4):

$$Q_a(t_0) = Q(t_0) + Z(t_0) i_a(t_0, t_{-1}) \quad (\text{A.12})$$

From (A.3; A.10-A.12), obtain the equation for the adjusted inflation $i_a(t, t_0)$ (A.13):

$$In_{La}(t, t_0) = 1 + i_a(t, t_0) = \frac{Q_a(t)}{Q_a(t_0)} = \frac{Q(t) + Z(t)i_a(t, t_0)}{Q(t_0) + Z(t_0)i_a(t_0, t_{-1})}$$

$$i_a(t, t_0) = \frac{Q(t) - Q_a(t_0)}{Q_a(t_0) - Z(t)} \quad (A.13)$$

We highlight that the adjusted inflation $i_a(t, t_0)$ (A.13) during the current period Δ (A.1) depends on the adjusted value of the consumer basket $Q_a(t_0)$ during the previous period Δ_0 (A.5). That establishes the recurrent dependence of the current adjusted inflation $i_a(t, t_0)$ (A.13) on the values of the consumer basket during past periods. To derive the expression of the adjusted inflation $i_a(t, t_0)$ (A.13) in a closed form, one should, similar to Part I, at some stage approximate past adjusted value $Q_a(t_{-k})$ by its unadjusted value $Q(t_{-k})$.

As the first approximation of the adjusted inflation $i_a(t, t_0)$, one obtains:

$$Q_a(t_0) \sim Q(t_0) \Rightarrow i_a(t, t_0) = \frac{Q(t) - Q(t_0)}{Q(t_0) - Z(t)} = i(t, t_0) \frac{Q(t_0)}{Q(t_0) - Z(t)} \quad (A.14)$$

$$p_{an}(t) = p_n(t) + i(t, t_0) z_n(t) \frac{Q(t_0)}{Q(t_0) - Z(t)} \quad ; \quad i(t, t_0) = \frac{Q(t) - Q(t_0)}{Q(t_0)} \quad (A.15)$$

$$Q_a(t) = Q(t) + i(t, t_0) \frac{Q(t_0)Z(t)}{Q(t_0) - Z(t)} = \frac{Q(t) - Z(t)}{Q(t_0) - Z(t)} Q(t_0) \quad (A.16)$$

As the second approximation, one may assume:

$$Q_a(t_{-1}) \sim Q(t_{-1}) \Rightarrow i_a(t_0, t_{-1}) = i(t_0, t_{-1}) \frac{Q(t_{-1})}{Q(t_{-1}) - Z(t_0)} \quad ; \quad i(t_0, t_{-1}) = \frac{Q(t_0) - Q(t_{-1})}{Q(t_{-1})}$$

Then, obtain in the second approximation:

$$p_{an}(t_0) = p_n(t_0) + i(t_0, t_{-1}) z_n(t_0) \frac{Q(t_{-1})}{Q(t_{-1}) - Z(t_0)}$$

$$Q_a(t_0) = Q(t_0) + i(t_0, t_{-1}) \frac{Q(t_{-1})Z(t_0)}{Q(t_{-1}) - Z(t_0)} = \frac{Q(t_0) - Z(t_0)}{Q(t_{-1}) - Z(t_0)} Q(t_{-1}) \quad (A.17)$$

The substitution of $Q_a(t_0)$ (A.17) into the equation (A.13) determines the adjusted inflation $i_a(t, t_0)$ in the second approximation. To derive the k -th approximation, one should consider the k -th period in the past Δ_k :

$$\Delta_{-k} = [t_{-k}; t_{1-k}] \quad ; \quad t_{-k} = t_{1-k} - \Delta \quad ; \quad t_j \in \Delta_{-k} \quad ; \quad t = t_j \quad ; \quad j = 0, \dots, J \quad (A.18)$$

and assume:

$$Q_a(t_{1-k}) = Q(t_{1-k}) + i_a(t_{1-k}, t_{-k})Z(t_{1-k}) = Q(t_{1-k}) \Leftrightarrow i_a(t_{1-k}, t_{-k}) = 0$$

$$i_a(t_{1-k}, t_{-k}) = i(t_{1-k}, t_{-k}) \frac{Q(t_{-k})}{Q(t_{-k}) - Z(t_{1-k})} \quad ; \quad i(t_{1-k}, t_{-k}) = \frac{Q(t_{1-k})}{Q(t_{-k})} - 1$$

$$Q_a(t_{1-k}) = \frac{Q(t_{1-k}) - Z(t_{1-k})}{Q(t_{-k}) - Z(t_{1-k})} Q(t_{-k}) \quad (A.19)$$

The recurrent substitutions of $Q_a(t_{-k})$ (A.19) into the next period up to the current period Δ (A.1) determine the k -th successive approximation of the current adjusted inflation $i_a(t, t_0)$ (A.13).

Appendix B. Walsh problem and the transformation of the basket weights

Correa Walsh formulated a problem of different trade weights of the consumer basket sold during two periods: *"Which weights, then, are the right ones—those of the first period? Or those of the second? Or should there be a combination of the two sets?"* (Walsh, 1921, p. 90; ILO, et al., 2025, p. 16). We respond to this century-old Walsh's remark and show how one may transform the time series of trades made with the commodities of the consumer basket during the current period so that the weights of the transformed time series of trades will exactly match the weight during the past period. That completely validates the use of the Laspeyres index. It is obvious that one may perform the similar transformation and convert the past trade time series so that they will match the trade weights during the current period. That will properly validate the Paasche index.

We use the notations of the App. A and consider the weights of the sales or the total volumes $U_{\Sigma n}(t_0)$ (A.4) of trades made with the basket items $n=I,...N$ during the past period Δ_0 (A.5). The total volumes $U_{\Sigma n}(t)$ (B.1) during the current period Δ (A.1) may be different from the total volumes $U_{\Sigma n}(t_0)$ (A.4) during the past period Δ_0 (A.5):

$U_{\Sigma n}(t_0) = \sum_{j=1}^J U_n(t_j) ; t_j \in \Delta_0 ; U_{\Sigma n}(t_0) \neq U_{\Sigma n}(t) = \sum_{j=1}^J U_n(t_j) ; C_{\Sigma n}(t) = \sum_{j=1}^J C_n(t_j) ; t_j \in \Delta$ (B.1)
 $C_{\Sigma n}(t)$ in (B.1) denotes the total value of the n -th good or service in the consumer basket sold during the current period. Let us consider the time series of values $C_n(t_j)$ and volumes $U_n(t_j)$ of trades made at time t_j during the current period Δ (A.1) with the n -th item of the consumer basket. For each $n=I,...N$, let us transform the values $C_n(t_j)$ and volumes $U_n(t_j)$ of trades into the time series of the values $c_n(t_j)$ and volumes $u_n(t_j)$ as follows:

$$u_n(t_j) = \frac{U_{\Sigma n}(t_0)}{U_{\Sigma n}(t)} U_n(t_j) ; \quad c_n(t_j) = \frac{U_{\Sigma n}(t_0)}{U_{\Sigma n}(t)} C_n(t_j) ; \quad c_n(t_j) = p_n(t_j) u_n(t_j) \quad (B.2)$$

The converted time series of values $c_n(t_j)$ and volumes $u_n(t_j)$ obey the equations (B.2) that determine the same prices $p_n(t_j)$ as (A.2). Meanwhile, the total volumes $u_{\Sigma n}(t)$ (B.3) of the converted volumes $u_n(t_j)$ (B.2) of trades made during the current period equal the total volumes $U_{\Sigma n}(t_0)$ (A.4) of trades during the past period:

$$u_{\Sigma n}(t) = \sum_{j=1}^J u_n(t_j) = U_{\Sigma n}(t_0) ; \quad c_{\Sigma n}(t) = p_n(t) u_{\Sigma n}(t) = p_n(t) U_{\Sigma n}(t_0) \quad (B.3)$$

The transformation (B.2) was derived for the description of the variance of portfolio returns (Olkhov, 2025a-2026b). It defines the trade volumes or weights $u_{\Sigma n}(t)$ of the n -th item of the basket during the current period, which equal the weights $U_{\Sigma n}(t_0)$ during the past period. It is obvious that

the transformation (B.2) for each item $n=1,..N$ determines the same VWAP $p_n(t)$ (1.4) or $p_{na}(t)$ (A.8) and hence the same values of the Laspeyres (1871) index $In_L(t, t_0)$ (1.2) or $In_{La}(t, t_0)$ (A.3). Thus, the transformation (B.2) resolves the question: “ Which weights, then, are the right ones ? ” (Walsh, 1921, p. 90). One may transform the current trade weights to the weights of the past period or vice versa. That makes the choice between the Laspeyres index $In_L(t, t_0)$ (1.2) or the Paasche index $In_P(t, t_0)$ (1.3) completely a matter of psychology. One may prefer to assess how the value of the past consumer basket went up since the past period (Laspeyres index) or may prefer to know how cheaper the value of the current consumer basket was in the past (Paasche index).

B.1. The consumer basket as a single commodity

In Part I, we highlighted that the elementary index $In_e(t, t_0)$ (1.5) and mean gross return $R(t, t_0)$ of a single security have the same expressions. The same is valid for the Laspeyres index $In_L(t, t_0)$ (1.2) and a mean gross return of a portfolio.

We consider the portfolio that was collected by the investor at time t_0 in the past of $n=1,..N$ securities traded at the exchange and since remains unchanged. During the current period Δ (A.1), the investor doesn't trade shares of his portfolio but only observes the trades made at the exchange with securities of his portfolio and models the portfolio's current return and variance. We recall that we account for the returns that are determined by the prices of trades with securities made at the exchange and don't account for the returns determined by the payoff of the dividends. Indeed, the trades at the exchange with the portfolio securities are made each working day, and the payoff occurs once a year. It is obvious that one may consider their impact on the portfolio returns independently. The portfolio's mean gross return $R(t, t_0)$ (B.4) has the same expression as the Laspeyres index $In_L(t, t_0)$ (1.2).

$$In_L(t, t_0) = \frac{Q(t)}{Q(t_0)} = \frac{\sum_{n=1}^N p_n(t) U_{\Sigma n}(t_0)}{\sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t_0)} = \sum_{n=1}^N \frac{p_n(t)}{p_n(t_0)} \frac{p_n(t_0) U_{\Sigma n}(t_0)}{Q(t_0)} = R(t, t_0)$$

$$In_{Ln}(t, t_0) = 1 + i_n(t, t_0) = \frac{p_n(t)}{p_n(t_0)} = R_n(t, t_0) = In_{en}(t, t_0) = 1 + i_{en}(t, t_0)$$

At the same time, the above relations and (B.4) emphasize the identity of the elementary index $In_{en}(t, t_0)$ (Part I, 1.5) of the n -th item of the consumer basket to the mean return $R_n(t, t_0)$ of the n -th security in the portfolio and to the Laspeyres index $In_{Ln}(t, t_0)$. Actually, (B.4) presents the distribution of the Laspeyres index $In_L(t, t_0)$ by the Laspeyres indices $In_{Ln}(t, t_0)$ or elementary indices $In_{en}(t, t_0)$ by the mean prices $p_n(t)$ over the $n=1,..N$ items of the consumer basket:

$$In_L(t, t_0) = \sum_{n=1}^N In_{Ln}(t, t_0) X_n(t_0) = \sum_{n=1}^N In_{en}(t, t_0) X_n(t_0) = \sum_{n=1}^N R_n(t, t_0) X_n(t_0) = R(t, t_0) \quad (B.4)$$

Since the famous Markowitz (1952) paper, the equation in the right side of (B.4) determines the dependence of the portfolio mean gross return $R(t, t_0)$ on mean returns $R_n(t, t_0)$ (B.5) of its $n=1, \dots, N$ securities, and $X_n(t_0)$ (B.5) have the meaning of the relative amounts invested into security n at time t_0 in the past.

$$R_n(t, t_0) = \frac{p_n(t)}{p_n(t_0)} ; X_n(t_0) = \frac{p_n(t_0)U_{\Sigma n}(t_0)}{Q(t_0)} = \frac{Q_n(t_0)}{Q(t_0)} = \frac{p_n(t_0)U_{\Sigma n}(t_0)}{\sum_{n=1}^N p_n(t_0)U_{\Sigma n}(t_0)} ; \sum_{n=1}^N X_n(t_0) = 1 \quad (\text{B.5})$$

In terms of indices, one may consider the Laspeyres index $In_L(t, t_0)$ (B.4) as an average of a random Laspeyres index $In_{Ln}(t, t_0)$ or random elementary indices $In_{en}(t, t_0)$ weighted by the relative shares $X_n(t_0)$ (B.5) of the values $Q_n(t_0)$ of the items $n=1, \dots, N$ in the total value $Q(t_0)$ of the consumer basket at time t_0 in the past.

The transformation (B.2) that converts the trade weights of the consumer basket during the current period into the trade weights during the past period allows us to present the trade with the basket composed of numerous different commodities as a trade with a single commodity. Let us define the values $Q(t_j)$ and volumes $W(t_j)$ as the sums of the converted values $c_n(t_j)$ and volumes $u_n(t_j)$:

$$Q(t_j) = \sum_{n=1}^N c_n(t_j) ; W(t_j) = \sum_{n=1}^N u_n(t_j) ; Q(t_j) = s(t_j)W(t_j) \quad (\text{B.6})$$

The function $W(t_j)$ describes the volume of a trade at time t_j with all commodities of the basket, and $Q(t_j)$ describes its trade value. Consequently, the equation (B.6) denotes the price $s(t_j)$ per single piece of the commodity of the basket sold at time t_j . We recall that the transformation (B.2) converts the current trade weights of the basket into trade weights during the past period. Hence, the total volume $W_{\Sigma}(t)$ (B.7) of trades with the basket during Δ (A.1) equals the total volume of trades $W_{\Sigma}(t_0)$ during the past period Δ_0 (A.5).

$$W_{\Sigma}(t) = \sum_{j=1}^J W(t_j) = \sum_{j=1}^J \sum_{n=1}^N u_n(t_j) = \sum_{n=1}^N \sum_{j=1}^J u_n(t_j) = \sum_{n=1}^N U_{\Sigma n}(t_0) = W_{\Sigma}(t_0) \quad (\text{B.7})$$

Consequently, the sum of trade values $Q(t_j)$ defines the total value $Q(t)$ (B.8) of the basket during the current period Δ :

$$Q(t) = \sum_{j=1}^J Q(t_j) = \sum_{j=1}^J \sum_{n=1}^N c_n(t_j) = \sum_{n=1}^N \frac{U_{\Sigma n}(t_0)}{U_{\Sigma n}(t)} \sum_{j=1}^J c_n(t_j) \quad (\text{B.8})$$

The relations (B.6) define the time series that model values $Q(t_j)$, volumes $W(t_j)$, and prices $s(t_j)$ of trades made at time t_j with all goods of the consumer basket as trades with a single commodity. The use of (B.7; B.8) converts the mean Laspeyres index $In_L(t, t_0)$ (1.2) and inflation $i(t, t_0)$ (B.9) into the elementary index $In_e(t, t_0)$ and inflation $i_e(t, t_0)$ determined by the ratio of prices $s(t)$ and $s(t_0)$ (B.10) during the current and past periods:

$$In_L(t, t_0) = 1 + i(t, t_0) = \frac{Q(t)}{Q(t_0)} = \frac{Q(t)/W_\Sigma(t_0)}{Q(t_0)/W_\Sigma(t_0)} = \frac{s(t)}{s(t_0)} = In_e(t, t_0) = 1 + i_e(t, t_0) \quad (B.9)$$

In (B.9), $s(t_0)$ denotes the price per piece of the single commodity of the basket averaged during past period Δ_0 , and $s(t)$ (B.10) is the price per piece averaged during the current period Δ :

$$s(t) = \frac{Q(t)}{W_\Sigma(t)} = \frac{1}{W_\Sigma(t_0)} \sum_{j=1}^J s(t_j) W(t_j) \quad ; \quad s(t_0) = \frac{Q(t_0)}{W_\Sigma(t_0)} \quad (B.10)$$

The relations (B.9) reveal the identity between the economic meaning of the Laspeyres index $In_L(t, t_0)$ and inflation $i(t, t_0)$ (B.9) on one hand and the elementary index $In_e(t, t_0)$ and elementary inflation $i_e(t, t_0)$ (B.9) determined by the single commodity in the consumer basket.

Appendix C. Random Laspeyres Indices, Random Inflation, and Variances

This Appendix describes the parallels between the portfolio random returns as the result of random trades at the exchange with its securities and the randomness of inflation as the result of random trades with the consumer basket. Indeed, the values $C_n(t_j)$, volumes, $U_n(t_j)$, and prices $p_n(t_j)$ of trades at time t_j with the n -th commodity of the consumer basket during the current period Δ (A.1) are rather irregular or random. One may consider the variances of their fluctuations as a measure of the randomness of price indices and inflation. The transformation (B.2) and relations (B.6-B.8) convert the random trade values $C_n(t_j)$ and volumes $U_n(t_j)$ of numerous goods and services of the consumer basket into the random time series that models the values $Q(t_j)$ and volumes $W(t_j)$ of trades with the basket as with a single commodity. That conversion emphasizes the likeness with the description via the values $Q(t_j)$ and volumes $W(t_j)$ that model the trades with the portfolio as with a single security (Olkhov, 2025a-2026b). We recall that all prices are adjusted by the adjusted inflation $i_a(t, t_0)$ (A.13), but for simplicity, we don't use the index " a ."

Let us consider the identity of the Laspeyres $In_L(t_j, t_0)$ and elementary $In_e(t_j, t_0)$ (B.9) price indices. Let us account for the random prices $s(t_j)$ (B.6) of trades at time t_j during the current period Δ that are determined by the random values $Q(t_j)$ and volumes $W(t_j)$, which model the deals with the consumer basket as with a single commodity. The dependence of the mean price $s(t)$ (B.10) on the random prices $s(t_j)$ (B.6) allows us to define random Laspeyres $In_L(t_j, t_0)$ indices and random inflation $i(t_j, t_0)$ (C.1) similar to the definition of random elementary $In_e(t_j, t_0)$ indices (Part I, B.14).

$$In_L(t_j, t_0) = 1 + i(t_j, t_0) = \frac{s(t_j)}{s(t_0)} = In_e(t_j, t_0) = 1 + i_e(t_j, t_0) \quad (C.1)$$

The VWAP $s(t)$ (B.10) determines the mean Laspeyres $In_L(t, t_0)$ index.

C.1 Market-Based Variance of Laspeyres Indices and Inflation

The randomness of the prices $s(t_j)$ (B.6) during the current period Δ (A.1) determines the stochasticity of Laspeyres $In_L(t_j, t_0)$ indices and inflation $i(t_j, t_0)$ (C.1) and causes that they may deviate from their mean values. One may describe the rate of such deviations by their variance. It is obvious that the Laspeyres $In_L(t_j, t_0)$ indices and inflation $i(t_j, t_0)$ (C.1) have the same variance.

We emphasize that the randomness of the price $s(t_j)$ (B.6) is determined by random values $Q(t_j)$ and volumes $W(t_j)$ (B.6) that model the consecutive trades as with a single commodity during Δ (A.1). This dependence allows us to call variance of the random price $s(t_j)$ (B.6) market-based variance that accounts for the impact of random volumes $W(t_j)$ (B.6) of consecutive trades.

We gave the derivation and justification of market-based variance $\Phi(t)$ (C.2) of the random price $s(t_j)$ in Part I and (Olkhov, 2022; 2023; 2025a-2026b) and refer there for details:

$$\Phi(t) = E[(s(t_j) - s(t))^2] = \frac{\psi^2(t) - 2\varphi(t) + \chi^2(t)}{1 + \chi^2(t)} s^2(t) \quad (C.2)$$

We denote $\psi(t)$ and $\chi(t)$ (C.3-C.5) the coefficients of variation of values $Q(t_i)$ and volumes $W(t_i)$:

$$\psi^2(t) = \frac{\text{cov}\{Q(t), Q(t)\}}{Q^2(t;1)} = \frac{\Psi_Q(t)}{Q^2(t;1)} \quad ; \quad \chi^2(t) = \frac{\text{cov}\{W(t), W(t)\}}{W^2(t;1)} = \frac{\Psi_W(t)}{W^2(t;1)} \quad (C.3)$$

We denote the mean values $Q(t;1)$ and volumes $W(t;1)$ (C.4):

$$Q(t;1) = E[Q(t_i)] = \frac{1}{J} \sum_{j=1}^J Q(t_j) \quad ; \quad W(t;1) = E[W(t_i)] = \frac{1}{J} \sum_{j=1}^J W(t_j) \quad (C.4)$$

$$\text{cov}\{Q(t), Q(t)\} = \frac{1}{J} \sum_{j=1}^J [Q(t_j) - Q(t;1)]^2 \quad ; \quad \text{cov}\{W(t), W(t)\} = \frac{1}{J} \sum_{j=1}^J [W(t_j) - W(t;1)]^2 \quad (C.5)$$

$\varphi(t)$ in (C.2) denotes the ratio (C.6) of the covariance of values $Q(t_i)$ and volumes $W(t_i)$ to their mean values $Q(t;1)$ and $W(t;1)$ (C.4):

$$\varphi(t) = \frac{\text{cov}\{Q(t), W(t)\}}{Q(t;1)W(t;1)} \quad ; \quad \text{cov}\{Q(t), W(t)\} = \frac{1}{J} \sum_{j=1}^J [Q(t_j) - Q(t;1)] [W(t_j) - W(t;1)] \quad (C.6)$$

From (C.1; C.2), obtain market-based variance $\Theta(t, t_0)$ (C.7) of random inflation $i(t_j, t_0)$:

$$\begin{aligned} \Theta(t, t_0) &= \frac{\Phi(t)}{s^2(t_0)} = E \left[\left(\frac{s(t_j)}{s(t_0)} - \frac{s(t)}{s(t_0)} \right)^2 \right] = E[(i(t_j, t_0) - i(t, t_0))^2] \\ \Theta(t, t_0) &= \frac{\psi^2(t) - 2\varphi(t) + \chi^2(t)}{1 + \chi^2(t)} In_L^2(t, t_0) \end{aligned} \quad (C.7)$$

We underline that market-based variance $\Theta(t, t_0)$ (C.7) of the random Laspeyres index $In_L(t_j, t_0)$ coincides with market-based variance of random inflation $i(t_j, t_0)$ (C.1). The economic meaning of the market-based variance $\Phi(t)$ (C.2) of price $s(t_j)$ (B.6) and market-based variance $\Theta(t, t_0)$ (C.7) of the inflation $i(t_j, t_0)$ (C.1) is as follows: they, as well as VWAP $s(t)$ (B.10), account for the impact of random volumes $W(t_i)$ of consecutive trades with the consumer basket. If during the current

period Δ (A.1) all volumes $W(t_i)=W$ of consecutive trades are assumed constant, then the VWAP $s(t)$ (B.10), $\Phi(t)$ (C.2), and variance $\Theta(t, t_0)$ (C.7) of inflation take the form:

$$\text{if } W(t_j) = W = \text{const} \Rightarrow s(t) = \frac{1}{W_{\Sigma}(t_0)} \sum_{j=1}^J s(t_j) W(t_j) \rightarrow \pi(t) = \frac{1}{J} \sum_{j=1}^J s(t_j) \quad (\text{C.8})$$

$$\text{if } W(t_j) = W \Rightarrow \chi^2(t) = \varphi(t) = 0 \ ; \ \Phi(t) \rightarrow \Phi_{\pi}(t) = \frac{1}{J} \sum_{j=1}^J \left(s(t_j) - \pi(t) \right)^2 \quad (\text{C.9})$$

$$\Theta(t, t_0) \rightarrow \Theta_{\pi}(t, t_0) = \frac{\Phi_{\pi}(t)}{\pi^2(t_0)} = \frac{1}{J} \sum_{j=1}^J \left(i(t_j, t_0) - i_{\pi}(t, t_0) \right)^2 \quad (\text{C.10})$$

$$In_{L\pi}(t, t_0) = 1 + i_{\pi}(t, t_0) = \frac{\pi(t)}{\pi(t_0)} = In_{e\pi}(t, t_0) = 1 + i_{e\pi}(t, t_0) \quad (\text{C.11})$$

In (C.11) we remind that the approximation of the Laspeyres index $In_{L\pi}(t, t_0)$ and inflation $i_{\pi}(t, t_0)$ to the case of constant trade volumes equals the similar approximation of the elementary index $In_{e\pi}(t, t_0)$ and inflation $i_{e\pi}(t, t_0)$.

The practitioners should decide what approximation of market trades they may use during the current period. To do that, the practitioners should assess the coefficient of variation $\chi(t)$ (C.3-C.5) of random volumes $W(t_j)$ (B.6) of consecutive trades. If $\chi(t) < I$, then the variance of random volumes of consecutive trades is small to compare with the mean trade volume $W(t; I)$ during Δ , and one may use the approximation of constant trade volumes (C.8-C.11). However, when $\chi(t) \sim I$ or $\chi(t) > I$, then practitioners should account for the effects of random trade volumes $W(t_j)$ on the means and variances of random prices $s(t_j)$ (B.6), the Laspeyres index, and inflation. In this case practitioners should use market-based variance $\Theta(t, t_0)$ (C.7) of the Laspeyres index and inflation (Olkhov, 2025a-2025c).

C.2 Basket-Based Variance of Laspeyres Index and Inflation

In C.1 we described market-based variance $\Theta(t, t_0)$ (C.7) of the Laspeyres index and inflation that accounts for the random volumes $W(t_j)$ (B.6), which model consecutive trades made in the market during the current period Δ (A.1) with the consumer basket as with a single commodity. That market-based variance $\Theta(t, t_0)$ (C.7) assesses the rate of deviations at time t_j of random inflation $i(t_j, t_0)$ (C.1) from its mean value during period Δ (A.1) as the cause of random trades in the market. Meanwhile, the classical expression of the Laspeyres index (1.2; B.9) and the identity of the expressions of the Laspeyres index and portfolio mean return $R(t, t_0)$ (B.4; B.5) present the complementary decomposition of the VWAP price $s(t)$ (B.10) by the $n=1, \dots, N$ items of the basket:

$$s(t) = \frac{1}{W_{\Sigma}(t_0)} \sum_{n=1}^N p_n(t) U_{\Sigma n}(t_0) = \frac{1}{W_{\Sigma}(t_0)} \sum_{n=1}^N p_n(t) u_{\Sigma n}(t) \ ; \ In_L(t, t_0) = \frac{s(t)}{s(t_0)} \quad (\text{C.12})$$

One may substitute random volumes $W(t_j)$ by $U_{\Sigma n}(t_0)$ and random prices $s(t_j)$ by $p_n(t)$ and consider relations (C.12) as VWAP $s(t)$ averaged by random prices $p_n(t)$ and random trade volumes $U_{\Sigma n}(t_0)$ over $n=1,...N$ goods of the basket. Similar to the equation (B.6) on random values $Q(t_j)$ and volumes $W(t_j)$ that determine random prices $s(t_j)$ for $j=1,...J$ trades at time j made during period Δ (A.1), one may consider the equation C(13; C.15) as the definition of random values $c_{\Sigma n}(t)$ and volumes $U_{\Sigma n}(t_0)$ (B.1-B.3) of different goods $n=1,...N$ of the consumer basket:

$$c_{\Sigma n}(t) = \sum_{j=1}^J c_n(t_j) = \sum_{j=1}^J p_n(t_j) u_n(t_j) = \left[\frac{1}{U_{\Sigma n}(t)} \sum_{j=1}^J p_n(t_j) u_n(t_j) \right] U_{\Sigma n}(t) = p_n(t) U_{\Sigma n}(t_0) \quad (C.13)$$

VWAP $p_n(t)$ (1.4; B.1; B.2) has the form:

$$p_n(t) = \frac{1}{U_{\Sigma n}(t)} \sum_{j=1}^J p_n(t_j) U_n(t_j) = \frac{1}{U_{\Sigma n}(t_0)} \sum_{j=1}^J p_n(t_j) u_n(t_j) \quad (C.14)$$

VWAP $p_n(t)$ (C.14) is averaged in time during the current period Δ (A.1). However, one may consider $p_n(t)$ (C.13; C.14) as a random variable over $n=1,...N$ goods of the consumer basket. It is obvious that the trade weights or total volumes $U_{\Sigma n}(t_0)$ (B.1) of the n -th item of the consumer basket during the past period Δ_0 (A.2) differ a lot from each other, and one may consider these weights $U_{\Sigma n}(t_0)$ as a random sample for $n=1,...N$. During the current period Δ , the series of total converted values $c_{\Sigma n}(t)$ (C.13) and VWAP $p_n(t)$ (1.4; C.14) for $n=1,...N$ also present highly irregular or random samples. Currently, the number N of the goods and services in the consumer baskets equals many thousands (ILO et al., 2025), and one may consider the series of values $c_{\Sigma n}(t)$, volumes $U_{\Sigma n}(t_0)$ (B.1), and prices $p_n(t)$ as random variables.

$$c_{\Sigma n}(t) = p_n(t) U_{\Sigma n}(t_0) \quad (C.15)$$

We emphasize that there is no difference between the consideration of the random sample of $j=1,...J$ values $C_n(t_j)$, volumes $U_n(t_j)$, and prices $p_n(t_j)$ of trades made at time t_j during a month-long period Δ (A.1) with each item n of the consumer basket, which obey equations (A.2), and the consideration of an irregular or random sample composed of $n=1,...N$ values $c_{\Sigma n}(t)$, volumes $U_{\Sigma n}(t_0)$ (C.13), and prices $p_n(t)$ (C.14), which obey the equation (C.15) that is similar to (A.2).

To emphasize that the randomness of values $c_{\Sigma n}(t)$ and volumes $U_{\Sigma n}(t_0)$ (C.13) has its origin in the diversity of numerous $N \gg 1$ goods and services of the consumer basket, we call it basket-based randomness. We call the variance of prices $p_n(t)$ (C.14) that obeys the equations (B.3; C.12-C.15) the basket-based variance $\Phi_B(t)$ (C.16) that completely reproduces the expressions of the market-based variance $\Phi(t)$ (C.2-C.6):

$$\Phi_B(t) = E_B[(p_n(t) - s(t))^2] = \frac{\psi_B^2(t) - 2\varphi_B(t, t_0) + \chi_B^2(t_0)}{1 + \chi_B^2(t_0)} s^2(t) \quad (C.16)$$

$E_B[\dots]$ in (C.16) denotes mathematical expectation over the random distribution of $n=1, \dots, N$ of values $c_{\Sigma n}(t)$, volumes $U_{\Sigma n}(t_0)$, and prices $p_n(t)$ of trades with the items of the basket. We denote $\psi_B(t)$ and $\chi_B(t_0)$ (C.17-C.20) the coefficients of variation of values $c_{\Sigma n}(t)$ and volumes $U_{\Sigma n}(t_0)$:

$$\psi_B^2(t) = \frac{cov\{c_{\Sigma}(t), c_{\Sigma}(t)\}}{c_{\Sigma}^2(t;1)} = \frac{\Psi_{c_{\Sigma}}(t)}{c_{\Sigma}^2(t;1)} \quad ; \quad \chi_B^2(t_0) = \frac{cov\{U_{\Sigma}(t_0), U_{\Sigma}(t_0)\}}{U_{\Sigma}^2(t;1)} = \frac{\Psi_{U_{\Sigma}}(t)}{U_{\Sigma}^2(t;1)} \quad (C.17)$$

We denote the mean values $c_{\Sigma}(t;1)$ and volumes $U_{\Sigma}(t_0;1)$ (C.18):

$$c_{\Sigma}(t;1) = E_B[c_{\Sigma n}(t)] = \frac{1}{N} \sum_{i=1}^N c_{\Sigma n}(t) \quad ; \quad U_{\Sigma}(t_0;1) = E_B[U_{\Sigma n}(t_0)] = \frac{1}{N} \sum_{i=1}^N U_{\Sigma n}(t_0) \quad (C.18)$$

$$cov\{c_{\Sigma}(t), c_{\Sigma}(t)\} = \frac{1}{N} \sum_{n=1}^N [c_{\Sigma n}(t) - c_{\Sigma}(t, 1)]^2 \quad (C.19)$$

$$cov\{U_{\Sigma}(t_0), U_{\Sigma}(t_0)\} = \frac{1}{N} \sum_{n=1}^N [U_{\Sigma n}(t_0) - U_{\Sigma}(t_0, 1)]^2 \quad (C.20)$$

$\varphi_B(t, t_0)$ in (C.16) denotes the ratio (C.21) of the covariance of values $c_{\Sigma n}(t)$ and volumes $U_{\Sigma n}(t_0)$ to their mean values $c_{\Sigma}(t;1)$ and $U_{\Sigma}(t_0;1)$ (C.18):

$$\varphi_B(t, t_0) = \frac{cov\{c_{\Sigma}(t), U_{\Sigma}(t_0)\}}{c_{\Sigma}(t;1)U_{\Sigma}(t_0;1)} \quad (C.21)$$

$$cov\{c_{\Sigma}(t), U_{\Sigma}(t_0)\} = \frac{1}{N} \sum_{n=1}^N [c_{\Sigma n}(t) - c_{\Sigma}(t, 1)] [U_{\Sigma n}(t_0) - U_{\Sigma}(t_0, 1)] \quad (C.22)$$

To derive the basket-based variance, let us recall that the decomposition of the price $s(t)$ (B.10) by random prices $s(t_i)$ of trades with the consumer basket, as with a single commodity, results in the definition of the *market-based random Laspeyres* index $In_L(t, t_0)$ as (C.1). The similar decomposition of the price $s(t)$ (C.12) by the random prices $p_n(t)$ of the $n=1, \dots, N$ items of the consumer basket results in the definition of the *basket-based random Laspeyres* index $In_{Ln}(t, t_0)$ as:

$$In_{Ln}(t, t_0) = 1 + i_n(t, t_0) = \frac{p_n(t)}{s(t_0)} = In_{en}(t, t_0) = 1 + i_{en}(t, t_0) \quad (C.23)$$

We highlight the identity (C.23) of the definitions of the *basket-based random Laspeyres* index $In_{Ln}(t, t_0)$ and *inflation* $i_n(t, t_0)$ and the *basket-based random elementary index* $In_{en}(t, t_0)$ and elementary inflation $i_{en}(t, t_0)$ related to the n -th item of the consumer basket. From basket-based variance $\Phi_B(t)$ (C.16) and (C.23) of random prices $p_n(t)$ of the $n=1, \dots, N$ items, obtain basket-based variance $\Theta_B(t, t_0)$ (C.24; C.25) of the Laspeyres index $In_L(t, t_0)$ or inflation $i(t, t_0)$:

$$\Theta_B(t, t_0) = E_B[(In_{Ln}(t, t_0) - In_L(t, t_0))^2] = E_B[(i_n(t, t_0) - i(t, t_0))^2] = \frac{\Phi_B(t)}{s^2(t_0)} \quad (C.24)$$

$$\Theta_B(t, t_0) = \frac{\psi_B^2(t) - 2\varphi_B(t, t_0) + \chi_B^2(t_0)}{1 + \chi_B^2(t_0)} In_L^2(t, t_0) \quad (C.25)$$

The expressions of market-based variance $\Theta(t, t_0)$ (C.7) and basket-based variance $\Theta_B(t, t_0)$ (C.24; C.25) assess the deviations from mean inflation $i(t, t_0)$ due to random trades with the basket as with a single commodity and due to the random distribution over the goods of the basket.

The basket-based variance $\Theta_B(t, t_0)$ (C.24; C.25) of the Laspeyres index accounts for the impact of the highly irregular or random weights of trade volumes $U_{\Sigma n}(t_0)$ of $n=1, \dots, N$ items of the consumer basket that have been sold in the past period. Similar to the simplified market-based case when all volumes $W(t_i)=W$ of consecutive trades with the consumer basket are assumed constant, one may consider the simplified basket-based case when all trade volumes $U_{\Sigma n}(t_0)=U$ with $n=1, \dots, N$ items of the consumer basket during the past period are assumed constant. In this simple approximation the basket-based variance $\Theta_B(t, t_0)$ (C.24; C.25) takes the form similar to (C.8-C.11).

Appendix D. Sharpe Ratio of Inflation

Sharpe (1966) introduced his Sharpe Ratio as a measure of the risks or uncertainties of a portfolio's return. We suggest the similar assessment to measure the current and predicted uncertainty of the Laspeyres index $In_L(t, t_0)$ or inflation $i(t, t_0)$ (1.2). The Sharpe Ratio of portfolio returns equals the reciprocal of the coefficient of variation of returns. In App. C, we introduced market-based $\Theta(t, t_0)$ (C.7) and basket-based $\Theta_B(t, t_0)$ (C.24; C.25) variances of inflation. We define the market-based Sharpe Ratio $Sp_M(i)$ (D.1) and basket-based Sharpe Ratio $Sp_B(i)$ (D.1) of inflation in similar forms:

$$Sp_M(i) = \frac{i(t, t_0)}{\Theta^{1/2}(t, t_0)} = \frac{In_L(t, t_0) - 1}{\Theta^{1/2}(t, t_0)} \quad ; \quad Sp_B(i) = \frac{i(t, t_0)}{\Theta_B^{1/2}(t, t_0)} = \frac{In_L(t, t_0) - 1}{\Theta_B^{1/2}(t, t_0)} \quad (D.1)$$

$$Sp_M(i) = \frac{i(t, t_0)}{In_L(t, t_0)} \left[\frac{1 + \chi^2(t)}{\psi^2(t) - 2 \varphi(t) + \chi^2(t)} \right]^{1/2} \quad ; \quad Sp_B(i) = \frac{i(t, t_0)}{In_L(t, t_0)} \left[\frac{1 + \chi_B^2(t_0)}{\psi_B^2(t) - 2 \varphi_B(t, t_0) + \chi_B^2(t_0)} \right]^{1/2}$$

We recall that market-based $Sp_M(i)$ (D.1) and basket-based Sharpe Ratios $Sp_B(i)$ (D.1) of inflation determine the uncertainties of a different nature. $Sp_M(i)$ (D.1) determines the uncertainty generated by random values $Q(t_j)$ and volumes $W(t_j)$ that model the trades with the consumer basket as with a single commodity during the current period Δ . $Sp_B(i)$ (D.1) determines the deviations of elementary inflations $i_n(t, t_0)$ of $n=1, \dots, N$ items of the consumer basket from its mean value $i(t, t_0)$.

To be economically reasonable, any prediction of inflation $i(t, t_0)$ at horizon T should be accompanied by the forecasts of market-based $\Theta(t, t_0)$ and basket-based $\Theta_B(t, t_0)$ variances, which measure the rate of uncertainty of inflation's predictions. The predictions of the inflation without corresponding forecasts of its variances don't make much economic sense.

A detailed description of the market-based Sharpe Ratio is given in Olkhov (2025b; 2026a).

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