
Abstract

This paper examines whether volatility connectedness between Bitcoin and the S&P 500 changed after May 2023. Using monthly realized volatility constructed from daily returns over October 2014–December 2025, we estimate static connectedness measures based on a vector autoregression and generalized forecast-error variance decompositions. Total connectedness rises from about 1.6% before the breakpoint to 24.2% afterward, with cross-market shocks explaining nearly one quarter of forecast-error variance in each market. The findings indicate a marked strengthening of Bitcoin’s integration with U.S. equity-market volatility.

1. Introduction

Bitcoin has evolved from a relatively isolated digital asset into an increasingly important component of the global financial system. Its distinctive market structure and high volatility have motivated an extensive literature on the dynamics of cryptocurrency risk [6, 1]. An important related question is whether Bitcoin remains largely detached from traditional financial markets or whether its volatility has become increasingly connected with them. Early evidence suggested that cryptocurrencies exhibited relatively weak linkages with conventional financial assets [3]. More recent studies, however, document a substantial strengthening of the relationship between cryptocurrency and equity markets, particularly since the COVID-19 pandemic [5].

This paper examines whether the volatility connectedness between Bitcoin and the Standard & Poor’s 500 has undergone a structural change. We construct monthly realized volatility from daily returns for both markets over the period October 2014–December 2025 and compare their connectedness before and after May 2023. Volatility transmission is measured using the generalized forecast-error variance decomposition within the connectedness framework of Diebold and Yilmaz [4].

The results reveal a pronounced change in cross-market dependence. Before May 2023, volatility innovations are almost entirely market-specific, and total connectedness is approximately 1.6%. After the breakpoint, total connectedness rises to about 24.2%, with shocks in each market explaining close to one quarter of the forecast-error variance of the other. The increase is largely bilateral rather than strongly directional, although Bitcoin becomes a modest net transmitter. These findings indicate that Bitcoin volatility has become substantially more integrated with U.S. equity-market volatility.

2. Data and volatility construction

We use daily price data for Bitcoin and the S&P 500. Bitcoin is represented by BTC-USD and the S&P 500 by the GSPC index, with both series obtained

from Yahoo Finance. Adjusted closing prices are used throughout. Daily price data are collected from September 1, 2014 through January 1, 2026, while the final monthly sample extends from October 2014 through December 2025.

Daily continuously compounded returns for asset i are calculated as

$$r_{i,d} = \ln \left(\frac{P_{i,d}}{P_{i,d-1}} \right),$$

where $P_{i,d}$ denotes the adjusted closing price of asset i on day d . Importantly, daily returns are calculated separately using each asset's own observation calendar. Thus, Bitcoin returns retain its seven-day trading calendar, whereas S&P 500 returns are based only on U.S. stock-market trading days. The two series are merged after daily returns have been aggregated to the monthly frequency. This procedure avoids discarding Bitcoin weekend observations merely because no corresponding S&P 500 observation exists on those dates.

Monthly volatility is measured using realized volatility constructed from daily returns. More specifically, for each asset i and month m , monthly realized variance is defined as

$$RV_{i,m} = \sum_{d \in \mathcal{D}_{i,m}} r_{i,d}^2$$

where $\mathcal{D}_{i,m}$ is the set of daily observations available for asset i during month m . Monthly realized volatility is then obtained as the square root of realized variance,

$$RVol_{i,m} = \sqrt{\sum_{d \in \mathcal{D}_{i,m}} r_{i,d}^2}$$

This construction deserves some clarification. Monthly realized volatility is not calculated as the standard deviation of monthly returns, nor as the standard deviation of the daily returns observed within a given month. Instead, it aggregates the squared daily price movements occurring throughout the month and then takes their square root. Consequently, days with larger absolute returns make a larger contribution to monthly realized variance. Moreover, the squared returns are summed rather than averaged. This is the standard realized-variance aggregation principle, under which variation measured at a higher frequency is accumulated over the lower-frequency interval of interest.

Because Bitcoin trades continuously while the S&P 500 trades only on business days, the number of daily returns entering the monthly volatility calculation generally differs across the two assets. This does not require normalizing realized variance by the number of observations, since $RV_{i,m}$ is intended to measure the total price variation accumulated over month m , rather than average daily variation. The resulting measure is therefore a monthly, non-annualized realized volatility.

For the empirical analysis, we use the logarithm of monthly realized volatility,

$$v_{i,m} = \ln(RVol_{i,m})$$

The logarithmic transformation reduces the positive skewness typically observed in volatility measures and expresses changes in volatility on a proportional rather than an absolute scale.

The structural breakpoint is May 2023. This date is not estimated in the present study but is taken from the structural-break analysis developed in [2]. Accordingly, the sample is divided into a pre-break period running from October 2014 through May 2023 and a post-break period running from June 2023 through December 2025. The complete sample contains 135 monthly observations, of which 104 belong to the pre-break period and 31 to the post-break period.

3. Statistical tests

We examine the stationarity properties of BTC and S&P 500 volatility using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit-root tests, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity test, and the Zivot-Andrews (ZA) unit-root test with an endogenous structural break. The ADF, PP, and ZA tests have a unit root as the null, whereas KPSS assumes level stationarity.

For the full sample, the ADF test fails to reject a unit root for BTC ($p = 0.191$) and S&P 500 volatility ($p = 0.086$) at the 5 level. KPSS does not reject level stationarity for either BTC ($p = 0.056$) or S&P 500 volatility ($p = 0.088$). By contrast, PP rejects a unit root for both series at the 1% level ($p < 0.01$).

Subperiod results reveal further differences. For BTC volatility, ADF fails to reject a unit root in both the pre-break ($p = 0.212$) and post-break ($p = 0.215$) periods, while KPSS does not reject level stationarity in either period ($p > 0.10$). For S&P 500 volatility, the pre-break ADF test rejects a unit root only at the 10% level ($p = 0.091$), while KPSS rejects level stationarity ($p = 0.020$). Post-break, ADF does not reject a unit root ($p = 0.435$), whereas KPSS does not reject level stationarity ($p > 0.10$). Thus, pre-break S&P 500 volatility shows evidence of instability.

Because conventional unit-root tests may be sensitive to structural change, we also apply ZA, allowing for an endogenous break in the intercept and trend. For BTC volatility, ZA does not reject a unit root even at the 10% level; for S&P 500 volatility, it does not exceed the relevant critical values in absolute magnitude.

Overall, the tests provide mixed evidence. PP strongly rejects unit roots, KPSS generally supports level stationarity except for pre-break S&P 500 volatility, and ADF and ZA provide weaker evidence against unit-root behavior. The results are consistent with persistent volatility processes affected by structural change rather than unambiguously stationary or unit-root behavior.

4. Static volatility connectedness

We examine the degree of volatility connectedness between Bitcoin and the S&P 500 using the Diebold–Yilmaz connectedness framework [4]. The analysis is based on a vector autoregression (VAR) for the logarithms of monthly realized volatility defined in Section 2 of the paper. Static connectedness measures are estimated separately for the pre-break and post-break samples in order to assess whether the relationship between the two volatility processes changed after May 2023.

4.1. VAR specification and lag selection

Let

$$\mathbf{v}_m = \begin{pmatrix} v_{\text{BTC},m} \\ v_{\text{SP500},m} \end{pmatrix}$$

denote the vector containing the logarithms of monthly realized volatility for Bitcoin and the S&P 500. For each sample, we estimate a VAR of order p ,

$$\mathbf{v}_m = \mathbf{c} + \sum_{\ell=1}^p A_{\ell} \mathbf{v}_{m-\ell} + \boldsymbol{\varepsilon}_m,$$

where A_{ℓ} denotes the matrix of autoregressive coefficients at lag ℓ , $\mathbf{c}$ is a vector of constants, $E(\boldsymbol{\varepsilon}_m) = 0$ and

$$E(\boldsymbol{\varepsilon}_m \boldsymbol{\varepsilon}_m') = \Sigma.$$

Lag selection is performed separately for the full sample, the pre-break sample, and the post-break sample, allowing a maximum of four monthly lags. We consider the Akaike information criterion (AIC), the Hannan–Quinn criterion (HQ), the Schwarz criterion (SC) and the final prediction error (FPE). The selected lag orders are reported in Table 1.

Table 1: VAR lag selection

Criterion	Full sample	Pre-break	Post-break
AIC	2	2	1
HQ	2	1	1
SC	1	1	1
FPE	2	2	1

The criteria therefore provide some support for two lags in the full and pre-break samples, particularly according to the AIC and FPE, whereas the SC selects one lag in all three samples. In the post-break period, all four criteria unanimously select one lag. Given the relatively small size of the post-break sample and the greater penalty imposed by the SC on additional parameters, we adopt a VAR(1) specification throughout the connectedness analysis. Using the same lag order across subsamples facilitates comparison of the connectedness measures by ensuring that differences between periods do not arise mechanically from different specifications.

4.2. Diebold–Yilmaz connectedness measures

For a stable VAR, the model can be represented in moving-average form as

$$\mathbf{v}_m = \sum_{h=0}^{\infty} \Psi_h \boldsymbol{\varepsilon}_{m-h},$$

where Ψ_h contains the dynamic responses of the variables to innovations in the system.

Connectedness is calculated from the generalized forecast-error variance decomposition (GFEVD) of Pesaran and Shin [7], as implemented in the Diebold–Yilmaz framework [4]. The generalized approach is convenient because the resulting variance decompositions are invariant to the ordering of Bitcoin and the S&P 500 in the VAR.

For an H -month forecast horizon, the contribution of shocks to variable j to the forecast-error variance of variable i is

$$\theta_{ij}^g(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (\mathbf{e}_i' \Psi_h \Sigma \mathbf{e}_j)^2}{\sum_{h=0}^{H-1} (\mathbf{e}_i' \Psi_h \Sigma \Psi_h' \mathbf{e}_i)},$$

where $\mathbf{e}_i$ is a selection vector with unity in its i th position and zero elsewhere, and σ_{jj} is the j th diagonal element of Σ .

Because generalized forecast-error variance decompositions do not necessarily sum to one across shocks, the variance shares are normalized row by row:

$$\tilde{\theta}_{ij}^g(H) = \frac{\theta_{ij}^g(H)}{\sum_{j=1}^N \theta_{ij}^g(H)}.$$

Consequently,

$$\sum_{j=1}^N \tilde{\theta}_{ij}^g(H) = 1.$$

In our application, $N = 2$ and the forecast horizon is set to $H = 12$ months. Since no rolling estimation window is used, a single VAR is estimated over each subsample and a single connectedness matrix is obtained for each regime. The resulting measures should therefore be interpreted as static summaries of average predictive volatility connectedness within each subsample rather than as measures of its evolution over time.

The connectedness matrix is defined as

$$C(H) = 100 \begin{pmatrix} \tilde{\theta}_{11}^g(H) & \tilde{\theta}_{12}^g(H) \\ \tilde{\theta}_{21}^g(H) & \tilde{\theta}_{22}^g(H) \end{pmatrix}.$$

Rows correspond to the variable whose forecast-error variance is being explained, whereas columns identify the source of the innovation. Thus, an off-diagonal element C_{ij} measures the percentage of the H -month-ahead forecast-error variance of variable i attributable to innovations in variable j . The diagonal elements measure the contribution of each variable's own innovations.

Directional connectedness *from* all other variables to variable i is

$$C_{i\leftarrow\bullet}(H) = 100 \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\theta}_{ij}^g(H),$$

whereas directional connectedness *to* all other variables from variable i is

$$C_{\bullet\leftarrow i}(H) = 100 \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\theta}_{ji}^g(H).$$

Net directional connectedness is then

$$C_i^{\text{NET}}(H) = C_{\bullet\leftarrow i}(H) - C_{i\leftarrow\bullet}(H).$$

A positive value indicates that variable i is a net transmitter of volatility connectedness, whereas a negative value indicates that it is a net receiver.

Finally, the standard Total Connectedness Index (TCI) is

$$TCI(H) = \frac{100}{N} \sum_{\substack{i,j=1 \\ i \neq j}}^N \tilde{\theta}_{ij}^g(H).$$

The TCI therefore measures the average proportion of forecast-error variance attributable to cross-variable innovations rather than to each variable's own innovations.

4.3. Pre- and post-break connectedness

For the period from October 2014 through May 2023, the estimated connectedness matrix is

$$C_{\text{pre}} = \begin{pmatrix} 98.39 & 1.61 \\ 1.60 & 98.40 \end{pmatrix},$$

where the first row and column correspond to Bitcoin volatility and the second row and column to S&P 500 volatility.

The matrix reveals very limited cross-market volatility connectedness during the pre-break period. Bitcoin's own innovations explain approximately 98.39% of its 10-month-ahead forecast-error variance, while innovations in S&P 500 volatility account for only 1.61%. Similarly, S&P 500 volatility innovations explain 98.40% of its own forecast-error variance, whereas Bitcoin volatility innovations account for only 1.60%.

The Total Connectedness Index is

$$TCI_{\text{pre}} = \frac{1.61 + 1.60}{2} \simeq 1.60\%.$$

Thus, only about 1.6% of the forecast-error variance in the two-variable system is attributable to cross-market volatility innovations.

Directional connectedness is also almost perfectly symmetric. Bitcoin transmits 1.595% to S&P 500 volatility and receives 1.611% from it, resulting in net connectedness of

$$C_{\text{BTC,pre}}^{\text{NET}} = 1.595 - 1.611 \simeq -0.015\%.$$

The corresponding net measure for the S&P 500 is +0.015%. Economically, these values are negligible and indicate that neither market can be regarded as a net transmitter of volatility during the pre-break period.

The results change markedly in the post-break period, from June 2023 through December 2025. The estimated connectedness matrix becomes

$$C_{\text{post}} = \begin{pmatrix} 76.50 & 23.50 \\ 24.80 & 75.20 \end{pmatrix}.$$

The own-variance shares fall substantially for both variables. Bitcoin innovations now account for 76.50% of Bitcoin's forecast-error variance, while S&P 500 volatility innovations account for 23.50%. In the opposite direction, Bitcoin volatility innovations explain 24.80% of the forecast-error variance of S&P 500 volatility, compared with an own contribution of 75.20%.

The post-break Total Connectedness Index is therefore

$$TCI_{\text{post}} = \frac{23.50 + 24.80}{2} \simeq 24.15\%.$$

Relative to the pre-break value of 1.60%, the TCI increases by approximately 22.55%. This result indicates a pronounced increase in the predictive interdependence of Bitcoin and equity-market volatility after May 2023.

The directional measures show that Bitcoin becomes a modest net transmitter of volatility connectedness in the post-break period. Bitcoin transmits 24.797% to S&P 500 volatility while receiving 23.501%, yielding

$$C_{\text{BTC,post}}^{\text{NET}} = 24.797 - 23.501 = 1.296\%.$$

Conversely, the S&P 500 has net connectedness of -1.296% and therefore acts as a net receiver.

The magnitude of the net measure should, however, be considered in relation to the larger gross directional connectedness in both directions. The difference between the two directional effects is only about 1.3%, whereas approximately one quarter of each market's forecast-error variance is attributable to innovations originating in the other market. Hence, the main post-break result is not a strongly asymmetric transmission mechanism from one market to the other, but rather a substantial strengthening of bilateral volatility connectedness.

The comparison of the two regimes suggests a structural change in the relationship between Bitcoin and U.S. equity-market volatility. Before May 2023, the two volatility processes are almost entirely driven by their own innovations. After the breakpoint, cross-market innovations account for approximately one quarter of forecast uncertainty in each market. At the same time, Bitcoin moves from being a neutral participant in the connectedness system to being a modest net transmitter.

5. Conclusion

This paper provides evidence of a substantial change in the volatility connectedness between Bitcoin and the Standard & Poor's 500 after May 2023. While volatility innovations were largely market-specific before the breakpoint, cross-market shocks explain approximately one quarter of forecast-error variance in the post-break period. The increase in connectedness is predominantly bilateral, although Bitcoin emerges as a modest net transmitter of volatility. These findings suggest that Bitcoin has become more closely integrated with U.S. equity-market dynamics. This greater interdependence may have important implications for portfolio diversification, risk management, and the role of Bitcoin within the broader financial system.

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