

Market Edge versus Contract Edge

Negative-Expectancy Trading under an Idealized Evaluation Contract

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Abstract

Can a trading rule lose money on average in the market yet remain valuable to a participant who buys a trading-evaluation contract? In the model developed here, the answer is yes. The trading rule does not improve; the contract changes the participant's payoff. In the binary market benchmark, a successful market deployment earns U and an unsuccessful one loses D . These are market payoffs, not amounts paid by the provider. The baseline contract is explicitly a fictitious "50K" account with $U = \$3,000$ and $D = \$2,000$; the 50K label is nominal account size, not participant-deposited capital. In the idealized contract, the participant pays a fixed fee F , faces an evaluation-stage success probability p , and receives a payout W only after a second stage that succeeds with conditional probability q . Market expectancy is $pU - (1-p)D$, whereas participant contract expectancy is $pqsW - F - pA$, where s is the payout share and A is an activation fee. Under the baseline convention $q = p$, $s = 1$, and $A = 0$, the contract is positive expectancy when $p > \sqrt{F/W}$, while the market strategy is negative expectancy when $p < D/(U+D)$. With $U = \$3,000$, $D = \$2,000$, $F = \$40$, and $W = \$3,000$, the resulting interval is approximately $11.55\% < p < 40\%$. The analysis then varies three parameters: evaluation fee F , activation fee A , and participant share s ; an illustrative less-generous specification $(F, A, s) = (\$90, \$100, 90\%)$ raises the same-strategy break-even probability to 20.20% and gives expected participant cash flow of $-\$2$ at $p = q = 20\%$. Exact finite- N enumeration gives a strict-profit probability of 87.01% for 50 baseline cycles and 40.37% for the less-generous specification. The analysis is not an arbitrage claim and does not describe any identified firm. It is the deliberately simple starting point of a wider research program on contract rules, trader heterogeneity, strategic risk, population dynamics, payout governance, and multi-account dependence.

Keywords: retail trading evaluation; limited liability; contract design; negative expectancy; payout probability; target-to-drawdown ratio

JEL Classification: D81; D86; G11; G23

1. Introduction

The puzzle is easier to see once the cash flows are separated. Retail trading-evaluation contracts distinguish the participant's personal cash flow from the nominal profit and loss generated inside an evaluation account. A participant generally pays a fee to enter an

evaluation. The participant may then lose access after breaching a drawdown rule, but does not necessarily pay the nominal account loss from personal wealth. If the evaluation is completed and a later payout condition is met, the participant receives a share of a contractual payout. This structure creates an elementary but important distinction: a strategy can have one expected value in the market and a different expected value when placed inside a purchased contract.

The distinction is easy to state but often obscured in informal discussions. Three separate quantities are regularly treated as though they were the same:

1. expected market profit and loss per deployment of a trading strategy;
2. expected personal cash flow per purchased evaluation cycle; and
3. the probability of obtaining a positive realized result after a finite number of purchased cycles.

They are not the same quantity. They have different payoff functions, different costs, and potentially different time and capital bases. A high probability of at least one payout does not prove positive expected value. A positive expected value does not guarantee profit in a finite sample. Most importantly for the present paper, negative market expectancy does not automatically imply negative participant contract expectancy.

The analysis is deliberately foundational: it isolates that last result in the simplest useful model before introducing real-world rule complexity. The model uses a fictitious provider and deliberately permissive rules. An evaluation ends in one success or failure event. A successful evaluation proceeds to one payout-stage event. There are no minimum trading days, consistency rules, daily-loss rules, payout caps, waiting periods, trailing drawdown rules, commissions, slippage, or limits on the number of attempts. The success probabilities are parameters rather than estimates. These assumptions are not descriptions of an observed company. They define a benchmark against which the effect of fees and payoffs can be isolated before path-dependent rules are introduced.

This paper is also the entry point to a broader research program. The common question is how contract geometry, rule architecture, trader heterogeneity, and lifecycle behavior jointly determine selection, participant value, and provider sustainability. The present paper isolates the most favorable one-account, two-stage benchmark. Later work can make success probabilities path-dependent, add rule packages, allow traders to change risk, follow participants through re-entry and later payouts, study review and payment frictions, and model dependence across multiple accounts. None of those extensions is assumed here; keeping them outside the first model is what makes the threshold result transparent.

The central result is an existence result. Let a binary market deployment earn U with probability p and lose D otherwise. It has negative expected market profit when

$$p < \frac{D}{U + D}. \quad (1)$$

Now let a purchased evaluation cost F , and let a completed two-stage cycle produce gross payout W . If the same success probability applies to the evaluation and payout stages, and the participant receives the entire payout, contract expected value is $p^2 W - F$. It is positive when

$$p > \sqrt{\frac{F}{W}}. \quad (2)$$

Therefore, a strategy can have negative market expectancy but positive participant contract expectancy whenever

$$\sqrt{\frac{F}{W}} < p < \frac{D}{U+D}. \quad (3)$$

For the illustrative values used throughout the paper, the lower threshold is 11.55% and the upper threshold is 40%. At $p=20\%$, one market deployment has expected profit and loss of $-\$1,000$, but one purchased evaluation cycle has expected participant cash flow of $+\$80$.

This statement does not mean that a participant has found a risk-free trade. The participant can lose every fee. The model assumes payment certainty, unlimited availability, independence, and frictionless execution. It also holds the success probabilities fixed when the strategy is placed inside the contract. Real rules can invalidate every one of those assumptions. Moreover, if a participant has positive expected contractual value, the other side of the contract must finance that value through fees, trading revenue, hedging, cross-subsidy, restrictions, or some other source. Provider participation and endogenous rule choice are outside the baseline model.

The paper makes a specific three-way comparison. It places market expectancy, participant contract expectancy, and finite-sample realized-profit probability in the same two-stage model. That framework yields the sign-reversal condition, an exact treatment of activation fees across repeated attempts, and a clear account of what the Target-to-Drawdown Ratio can and cannot establish. The economic ingredients, including limited liability and nonlinear compensation, are familiar, and recent preprints already value trading-evaluation contracts. Within this research program, the benchmark remains necessary because it isolates the cash-flow sign reversal before later papers endogenize trader types, path rules, risk choice, and provider lifecycle states.

The rest of the paper proceeds as follows. Section 2 positions the model in the incentives and contract-design literature. Section 3 describes the institutional abstraction. Section 4 defines the market and contract models. Section 5 derives the main results. Section 6 studies multiple attempts and dependence. Section 7 gives the numerical illustration. Section 8 discusses economic interpretation and contract design. Section 9 discusses limitations and extensions. Section 10 concludes. Proofs and additional derivations appear in the Appendix.

2. Related Literature and Positioning

The closest theoretical foundation lies in principal–agent models with limited liability and incentive design. In the present setting, the provider writes the contract and the participant operates within it. The participant’s personal downside is limited to contractual charges rather than the nominal account loss, while fees, payout shares, and access conditions determine the payoff schedule. Jensen and Meckling (1976) provide the foundational agency framing. Innes (1990) studies incentive contracting under limited liability and hidden action. The present model is much simpler and is not an optimal-contract model.

A second related branch studies nonlinear rewards and tournament incentives. Lazear and Rosen (1981) show how rank-order rewards can create incentives that differ from those under linear compensation. Brown, Harlow, and Starks (1996) and Chevalier and Ellison (1997) document how evaluation and flow incentives can affect mutual-fund risk choices. These settings differ from retail evaluation contracts: the participant in the present model purchases repeated access to a threshold contract rather than managing an existing mutual fund. The common economic feature is that evaluation architecture can matter independently of raw asset returns.

Entry-fee and costly-submission contests provide a closer analogue to the purchased-attempt feature. Duffy, Matros, and Valencia (2023) study contests with entry fees, while Whitmeyer (2023) studies submission costs in risk-taking contests. Their mechanisms are not the same as a two-stage retail trading evaluation, but they show that the price of participation can change entry and risk-taking incentives.

A third branch studies option-like or convex compensation. Carpenter (2000) demonstrates that the relation between option compensation and risk taking is more nuanced than the simple claim that convex pay always increases risk. Ross (2004) likewise separates the riskiness of a payoff schedule from the agent's effective risk aversion. Basak, Pavlova, and Shapiro (2007) derive a finite risk-shifting range when fund-flow rewards are convex in relative performance. Hodder and Jackwerth (2007) study dynamic risk taking under hedge fund incentive contracts and barriers. Dittmann, Yu, and Zhang (2017) show that optimal compensation can have different curvature in different outcome regions. These papers caution against an overly broad conclusion. The current paper does not claim that every convex contract rewards variance or that every participant should take maximum risk. It proves only that a specified binary contract can reverse the sign of expected personal cash flow relative to a specified binary market payoff.

Recent experimental evidence also reinforces the behavioral relevance of limited liability. Bosch-Rosa, Gietl, and Heinemann (2025) find that limited liability and moral hazard can affect both risk taking and beliefs. The present paper has no behavioral component: p and q are externally supplied parameters and the participant is assumed to calculate expected value correctly. Behavioral beliefs, probability miscalibration, and overconfidence are natural extensions.

Drawdown and stop-loss research is relevant to path-dependent extensions. Magdon-Ismael et al. (2004) derive maximum-drawdown results for Brownian motion, while Kaminski and Lo (2014) study stop-loss rules. The present model does not import those dynamics because p and q are parameters.

The directly related 2026 preprints now include explicit valuation work. Lim's *The Price of a Funded Account* (2026b) values a stylized challenge, Lim's *Phantom Generosity* (2026a) extends contract valuation to 31 contracts across 21 firms, and Tomàs Fernández (2026) derives a closed-form valuation and cross-firm break-even analysis. Maile (2026) provides an industry and policy discussion. Leo Ng's *Behavioral Biases in Prop Trading: Evidence from 1,000 AI Prop Traders* and *Pass-First-Pay-Later: A Structural and Comparative Analysis of the Deferred-Fee Prop Trading Model* (2026a, 2026b) examine behavioral evidence and deferred-fee design, while Gajon (2026) and Boutgajouft (2026) use evaluation settings to test trading

systems. Against that record, the narrower contribution here is the explicit sign comparison between the underlying binary market payoff and the participant's purchased-contract cash flow, followed by exact finite-attempt probabilities and activation-fee accounting.

3. Institutional Abstraction and Scope

Consider a fictitious company that sells access to a two-stage trading evaluation. The company is not intended to represent or identify any existing provider. All monetary values are hypothetical. The baseline product is explicitly labeled a 50K account. That label is nominal contract size; the model does not treat \$50,000 as cash deposited or personally at risk by the participant.

The participant first purchases one evaluation for fee F . The evaluation has two terminal barriers:

- success at nominal profit $+U$; and
- failure at nominal loss $-D$.

If the participant fails, the cycle ends. The participant loses the fee but does not personally pay the nominal drawdown D . If the participant passes, the cycle moves immediately to a payout stage. The payout stage again ends in success or failure. A success produces gross payout W , of which the participant receives share s . A failure produces no payout. An optional activation fee A is paid after passing the evaluation; $A=0$ in the baseline.

The probability of evaluation success is p . The conditional probability of payout-stage success after evaluation success is q . The baseline illustration sets $q=p$, which can be read as the same binary strategy being used under the same statistical success rate in both stages. This is a convention, not an empirical claim. The general model permits $q \neq p$.

The baseline makes the following restrictions:

1. p and q are exogenous.
2. Each stage has one terminal event.
3. The two stages are independent conditional on their probabilities.
4. Purchased cycles are independent.
5. A qualifying payout is paid with certainty.
6. There are no minimum days or waiting periods.
7. There are no consistency rules, trailing drawdown, daily-loss limits, payout caps, scaling rules, or account limits.
8. There are no commissions, slippage, taxes, latency, or execution errors.
9. Supply is unlimited at fee F .
10. The participant is risk neutral and maximizes expected personal cash flow.

These restrictions make the model intentionally favorable to analytical clarity. They do not imply that a real provider offers such a contract. In fact, additional rules are economically interesting precisely because they can change p , q , W , the cost base, the dependence across attempts, or the timing of cash flow.

Two units of analysis must also be distinguished. Market expectancy is defined per deployment of the binary market strategy. Contract expectancy is defined per purchased evaluation cycle. The paper compares their signs. It does not claim that the two objects use equal capital, equal duration, or equal notional exposure. Section 7.4 therefore adds a separate, explicitly illustrative equal-budget and equal-time comparison: \$10,000 of starting liquid capital, one month, own-capital returns of 5%, 10%, 20%, and 30%, and 50 contract cycles. Every return and the 50-cycle throughput are inputs, not empirical or typical-value claims.

3.1 Why these ideal-contract parameters are used

The numerical benchmark combines a \$3,000 target with a \$2,000 initial loss budget on a nominal 50K account. Those values are not chosen because they maximize the probability of passing. A lower target or a much larger loss budget would make target attainment easier, but would move the benchmark outside the conventional 50K evaluation geometry observed in the dated futures-product corpus assembled in *A Versioned Contract Atlas of Retail Futures Evaluations* (Matilla Serrano, 2026a). The \$3,000 target and \$2,000 drawdown recur across several publicly offered 50K routes. They are therefore recognizable market parameters rather than unrestricted hypothetical values.

The word *idealized* refers to the remaining contractual design. For each modeled dimension, the benchmark removes or relaxes a restriction that is commonly present in actual products: the lower boundary is static rather than trailing, no daily-loss limit is imposed, only one trading day is required, no best-day consistency test applies, no payout buffer delays withdrawal, and there is no payout cap below the stated gross amount. The resulting object is a component-wise favorable benchmark. Its individual ingredients are market-observable, but their combination is not represented as the current contract of any named provider.

This construction sets a disciplined upper benchmark for the participant under familiar headline geometry. It does not claim to be the mathematically most generous contract imaginable. Allowing the full nominal 50K label to be lost, using a \$10,000 or \$20,000 drawdown, or reducing the target toward zero would mechanically change the first-passage problem and would no longer resemble the conventional products being abstracted. Conversely, adding a trailing floor, daily stop, consistency threshold, minimum days, buffer or payout review would make the comparison less favorable and would mix the central transfer mechanism with the rule effects studied later in the program.

The distinction is important for interpretation. The 50K label is not participant-owned capital, the \$2,000 drawdown is a contractual failure distance, and the \$3,000 target is an evaluation threshold rather than the payout received. The benchmark asks whether a purchased, limited-liability contract can transform the sign of participant cash-flow expectancy under an observable target-to-drawdown geometry. It does not estimate the pass probability of a specific provider or assert that the component-wise bundle can be purchased as written.

4. Model

4.1 Binary market benchmark

Let X denote the terminal profit and loss from one deployment of the binary strategy:

$$X = \begin{cases} U, & \text{with probability } p, \\ -D, & \text{with probability } 1 - p, \end{cases}$$

where $U > 0$, $D > 0$, and $0 \leq p \leq 1$. Expected market profit and loss is

$$\mu_M = E[X] = pU - (1 - p)D.$$

The strategy has non-negative market expectancy if and only if

$$p \geq p_M^* = \frac{D}{U + D}.$$

Define the Target-to-Drawdown Ratio as

$$T = \frac{U}{D}.$$

Then

$$p_M^* = \frac{1}{1 + T}.$$

This equation is a break-even condition, not a model of the actual probability p . Increasing T while holding p fixed increases the success payoff relative to the failure loss and therefore lowers the probability required for non-negative market expectancy. It does not follow that the actual success probability stays fixed when a target is moved farther away. A stochastic price process or empirical model is required to map contract geometry into an actual p . The present model does not make that mapping.

4.2 Evaluation and payout contract

Let E be the evaluation-success indicator:

$$E = \begin{cases} 1, & \text{with probability } p, \\ 0, & \text{with probability } 1 - p. \end{cases}$$

Conditional on $E = 1$, let P be the payout-stage success indicator:

$$P = \begin{cases} 1, & \text{with conditional probability } q, \\ 0, & \text{with conditional probability } 1 - q. \end{cases}$$

The unconditional probability of a completed payout is therefore

$$r = P(E = 1, P = 1) = pq.$$

Participant net cash flow from one purchased cycle is

$$\Pi_C = sWEP - F - AE.$$

The participant always pays F , pays A only after passing the evaluation, and receives sW only after both stages succeed. Expected participant cash flow is

$$\mu_C = E[\Pi_C] = pq sW - F - pA.$$

For fixed q , positive contract expectancy requires

$$p(qsW - A) > F.$$

If $qsW > A$, the break-even evaluation probability is

$$p_C^*(q) = \frac{F}{qsW - A}.$$

If $qsW \leq A$, no positive value of p can make this contract profitable under the stated payoff definition.

4.3 Same-strategy baseline

The central illustration imposes

$$q = p, s = 1, A = 0.$$

Then

$$\mu_C = p^2 W - F$$

and

$$p_C^* = \sqrt{\frac{F}{W}}.$$

With a non-zero activation fee and $q = p$, the positive root of $sqWp^2 - Ap - F = 0$ is

$$p_C^*(A) = \frac{A + \sqrt{A^2 + 4sqWF}}{2sqW}.$$

The activation fee raises the required success probability. The baseline sets it to zero to isolate the fee-and-payout mechanism.

5. Main Results

Proposition 1. Market break-even threshold

The binary market strategy has negative expectancy if and only if

$$p < \frac{D}{U + D}.$$

This result follows directly from the sign of $pU - (1 - p)D$.

Proposition 2. Contract break-even threshold

Under $q = p$, $s = 1$, and $A = 0$, participant contract expectancy is positive if and only if

$$p > \sqrt{\frac{F}{W}}.$$

The threshold depends on the fee-to-payout ratio, not directly on the nominal drawdown. The reason is not that drawdown is irrelevant to success. Rather, drawdown affects the model

through the exogenous probability p , while the participant's personal contractual loss in the stated payoff function is the fee F .

Proposition 3. Contract edge without market edge

Under the same-strategy baseline, a non-empty set of probabilities produces positive participant contract expectancy and negative market expectancy if and only if

$$\sqrt{\frac{F}{W}} < \frac{D}{U+D}.$$

When the condition holds, the set is

$$C = \left(\sqrt{\frac{F}{W}}, \frac{D}{U+D} \right).$$

For any $p \in C$,

$$\mu_C > 0 \text{ and } \mu_M < 0.$$

Figure 1 displays this interval for the baseline values.

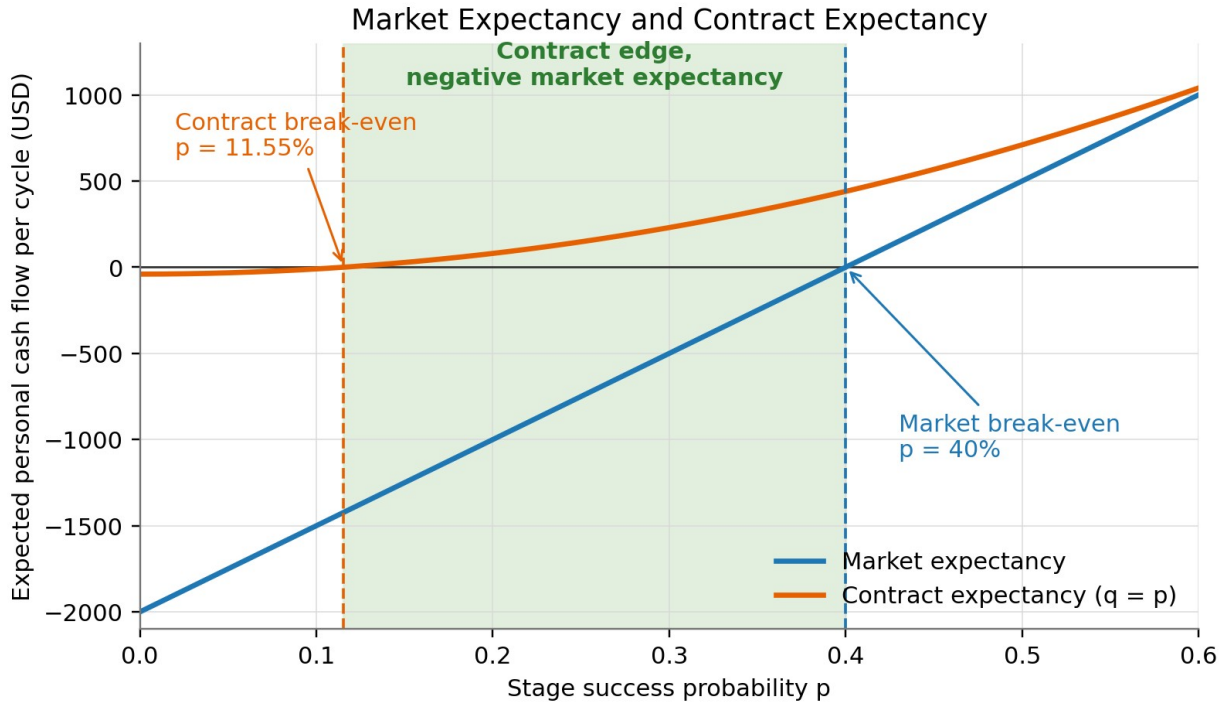


Figure 1. Market and contract expected-value thresholds. The shaded area is the same-strategy probability interval in which participant contract expectancy is positive while market expectancy is negative.

The result can also be stated without the same-strategy restriction. With $A=0$, fixed q , and payout share s , positive contract expectancy requires

$$p > \frac{F}{qsW}.$$

The contract-edge region is non-empty if

$$\frac{F}{qsW} < \frac{D}{U+D},$$

or equivalently

$$F(U+D) < qsWD.$$

Figure 2 shows the joint p - q region. The same-strategy case is the 45-degree line.

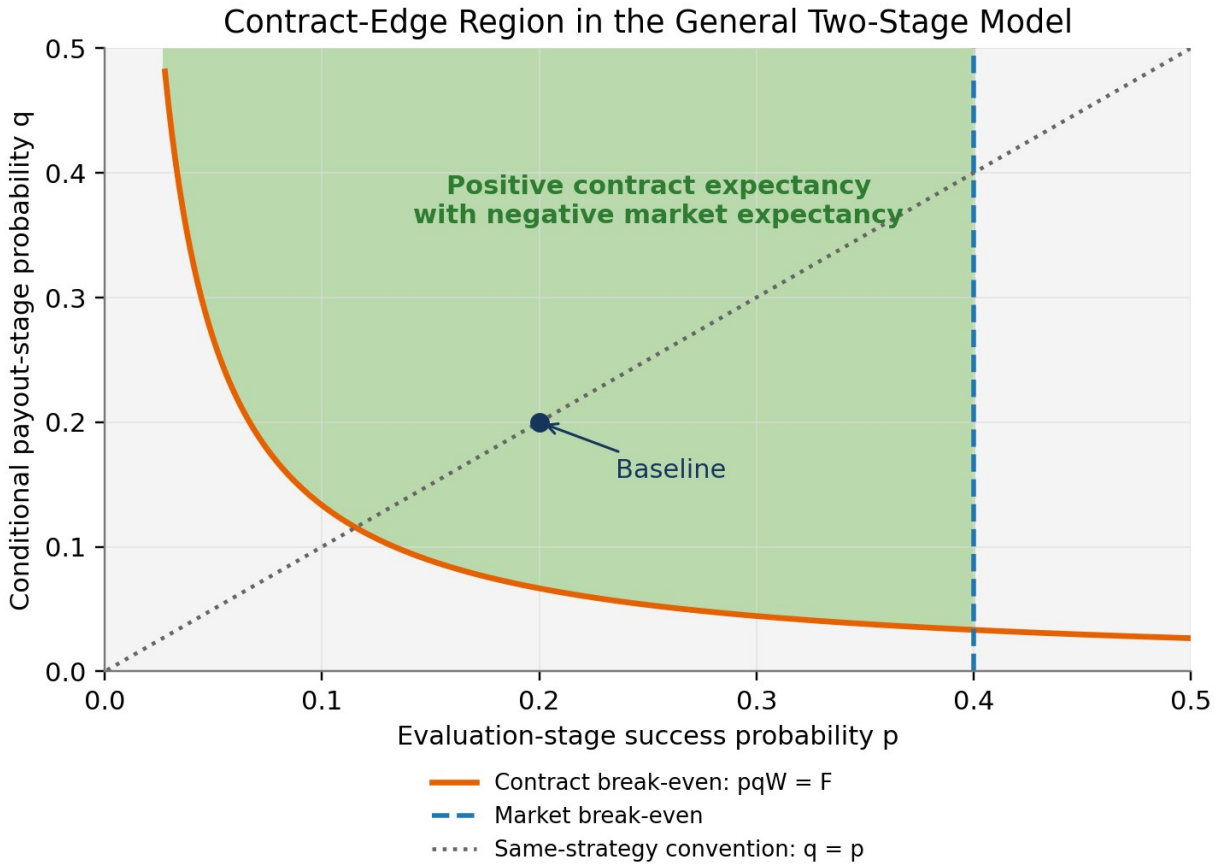


Figure 2. General contract-edge region. The shaded area satisfies positive participant contract expectancy and negative market expectancy for the baseline monetary values.

5.1 Comparative statics

The baseline contract threshold

$$p_C^* = \sqrt{\frac{F}{sW}}$$

has intuitive comparative statics:

- a higher evaluation fee F raises the required probability;
- a larger gross payout W lowers it;
- a larger participant share s lowers it; and
- a positive activation fee A raises it.

For the general two-stage model, the product $p q$ is central. A high evaluation pass rate is not sufficient if the conditional payout rate is low. Likewise, a low evaluation pass rate can be compatible with positive contract expectancy if the conditional payout probability, payout size, or payout share is sufficiently high relative to the fee.

The target U and drawdown D enter the market threshold directly, but do not enter the baseline contract threshold mechanically. This separation is a modeling choice. In any realistic extension, U , D , and the path rules will influence p and q . The current model keeps those probabilities visible rather than hiding them inside an untested price process.

5.2 Target-to-Drawdown Ratio

The Target-to-Drawdown Ratio is

$$T = \frac{U}{D}.$$

It is useful because it summarizes the asymmetry between the nominal success barrier and failure barrier. In the binary market benchmark,

$$p_M^* = \frac{1}{1 + T}.$$

This is the success rate required to make the binary market payoff break even. It is not a forecast of the probability of hitting the profit target. For the baseline $U = \$3,000$ and $D = \$2,000$, $T = 1.5$, so a success rate of 40% produces zero expected market profit and loss.

Table 1. Public account geometries used as TDR examples

Nominal example	Target / drawdown	$T = U / D$	Success rate required for market break-even
25K	\$1,500 / \$1,000	1.50	40.00%
50K	\$3,000 / \$2,000	1.50	40.00%
100K	\$6,000 / \$3,000	2.00	33.33%
150K	\$9,000 / \$4,000	2.25	30.77%

Note: Target-to-Drawdown Ratio (TDR) equals the stated target divided by the stated drawdown. The entries summarize public program terms and do not constitute endorsements.

Source: Public Apex Trader Funding end-of-day and intraday evaluation tables. The company modeled in this paper remains fictitious; the public table is used only to show that these geometries exist.

A higher T lowers the required break-even success rate because each successful market deployment earns more relative to the loss attached to a failure. In the 150K example, a

success earns \$9,000 and a failure loses \$4,000, so one success offsets more failures than in the 50K example. A 30.77% success rate is therefore sufficient for zero expected market profit and loss.

That arithmetic does not make the 150K target easier to reach. Holding the trading process fixed, moving the success barrier farther away relative to the failure barrier can reduce the actual probability of passing. The comparison therefore contains two different quantities: the success rate required for market break-even and the actual probability of reaching the target. The TDR determines the first; a price-process model or empirical estimate is needed for the second.

The same distinction applies to the first payout. In the present contract model, T does not mechanically determine the evaluation-stage probability p or the conditional payout-stage probability q . A higher TDR therefore cannot establish that the first payout is easier to obtain. Table 1 ranks payoff geometry only. Figure 3 plots that break-even geometry while keeping the contract threshold fixed at its baseline value.

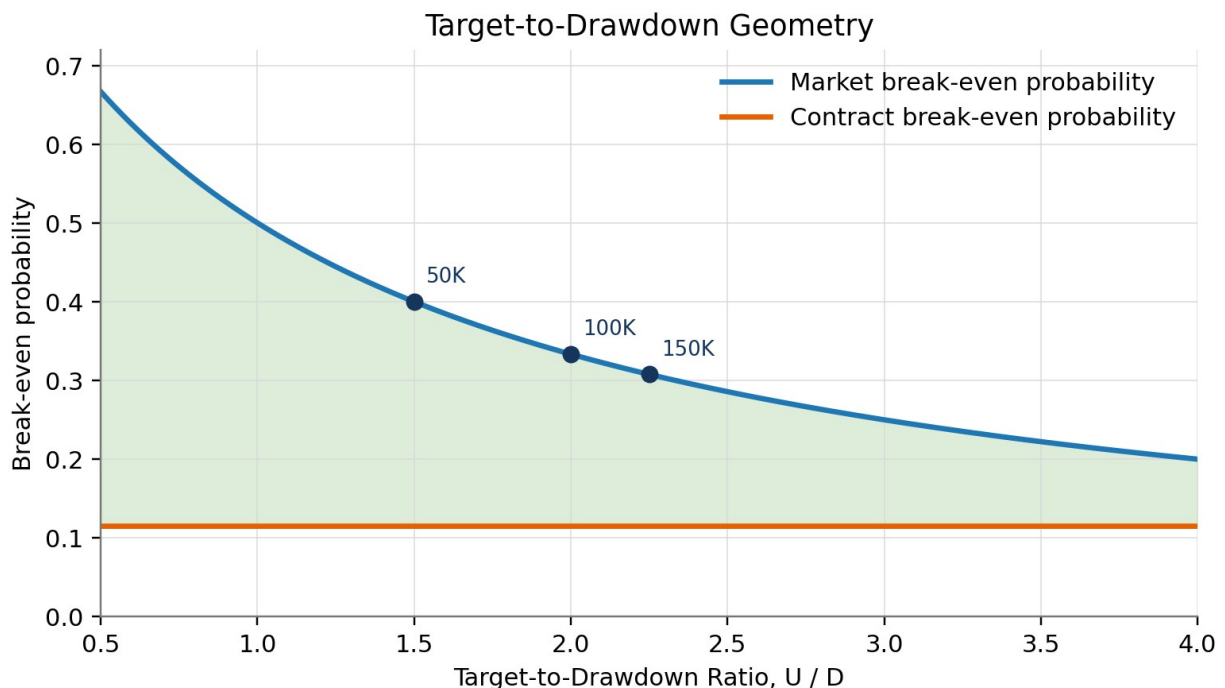


Figure 3. Target-to-Drawdown geometry. The blue line is the market break-even probability $1 / (1 + T)$; the orange line is the baseline same-strategy contract threshold.

The shaded distance between the lines is a probability-space interval, not a direct forecast of pass rates. A price-process extension could map different targets, drawdowns, volatility levels, and time limits into endogenous pass probabilities.

6. Multiple Evaluation Attempts

The one-cycle result concerns expected value. A participant considering many attempts also cares about the distribution of realized cash flow.

Suppose the participant purchases N independent cycles. Let $Y_i = E_i P_i$ equal one if cycle i completes a payout and zero otherwise. Each Y_i is Bernoulli with success probability

$$r = p q.$$

The number of completed payouts is

$$K = \sum_{i=1}^N Y_i.$$

Under independence,

$$K \sim \text{Binomial}(N, r).$$

With $A = 0$ and $s = 1$, aggregate participant cash flow is

$$\Pi_N = W K - N F.$$

Its expected value is

$$E[\Pi_N] = W N r - N F = N(p q W - F) = N \mu_C.$$

Increasing N scales expected value but does not change its sign. Buying more negative-expectancy contracts does not make their expected value positive. Buying more positive-expectancy contracts does not eliminate the possibility of loss.

6.1 Probability of at least one payout

The probability of at least one completed payout is

$$P(K \geq 1) = 1 - (1 - r)^N.$$

This quantity is sometimes attractive because it rises with N . It is not generally the probability of aggregate profit. One payout may be smaller than the total fees paid across all attempts.

6.2 Probability of positive realized cash flow

Strictly positive aggregate cash flow requires

$$W K - N F > 0.$$

The minimum number of payouts is therefore

$$K_{\min} = \text{floor}\left(\frac{N F}{W}\right) + 1.$$

Hence

$$P(\Pi_N > 0) = P(K \geq K_{\min}) = \sum_{k=K_{\min}}^N \binom{N}{k} r^k (1 - r)^{N-k}.$$

The floor function creates visible jumps. With $F = \$40$ and $W = \$3,000$, one payout produces strict aggregate profit for $N \leq 74$. At $N = 75$, total fees equal exactly $\$3,000$, so one payout gives zero and two are required for strict profit. The same threshold changes again at multiples of 75. Figure 4 shows the resulting sawtooth pattern.

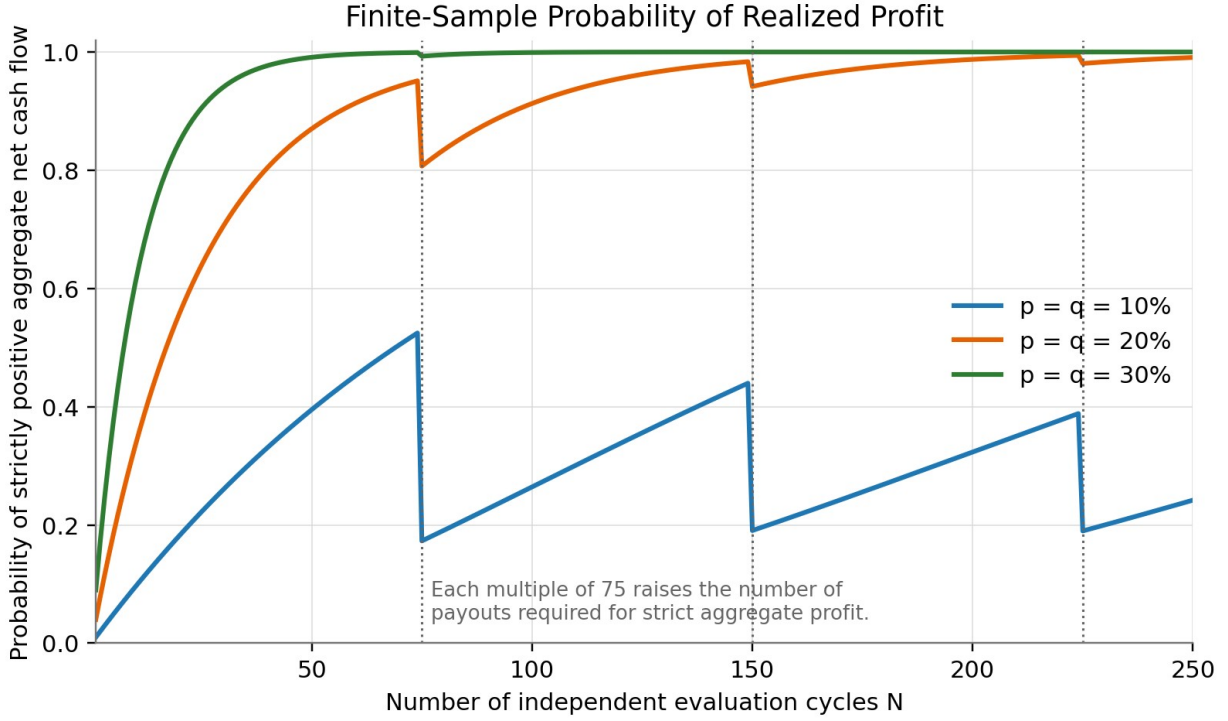


Figure 4. Probability of strictly positive aggregate participant cash flow for independent cycles. The horizontal axis now extends to $N = 250$, displaying threshold changes at 75, 150, and 225 cycles.

The formula reveals a second important distinction. At $p = q = 10\%$, one cycle has negative expected value of $-\$10$, yet 50 attempts have a 39.50% probability of a positive realized result because one payout exceeds the $\$2,000$ total fee bill. A profitable sample therefore does not prove positive expectancy. Conversely, at $p = q = 20\%$, the expected value across 50 attempts is $+\$4,000$, but the probability of strict profit is 87.01%, not 100%.

6.3 Exact finite- N probability with an activation fee

When $A > 0$, realized cost depends on the number of evaluations passed, so the single binomial payout count K is insufficient. Let

$$J \sim \text{Binomial}(N, p)$$

be the number of passed evaluations. Conditional on $J = j$,

$$K \mid J = j \sim \text{Binomial}(j, q).$$

Aggregate participant cash flow is

$$\Pi_N = s W K - N F - A J.$$

For a realized pass count j , strict profit requires at least

$$k_{\min}(j) = \text{floor}\left(\frac{N F + j A}{s W}\right) + 1$$

payouts. Therefore,

$$P(\Pi_N > 0) = \sum_{j=0}^N P(J=j) \left[1 - F_{\text{Binomial}(j,q)}(k_{\min}(j) - 1) \right],$$

where the bracketed term is zero when $k_{\min}(j) > j$.

Table 2. Exact finite-N results for 50 cycles with p = q = 20%

Specification	Contract inputs $F / A / s$	EV per cycle	$E[\Pi_{50}]$	$P(\Pi_{50} > 0)$
Baseline ideal	\$40 / \$0 / 100%	+\$80	+\$4,000	87.01%
Less-generous illustration	\$90 / \$100 / 90%	-\$2	-\$100	40.37%

Note: Results use 50 independent cycles with evaluation and funded-stage success probabilities both fixed at 20%. Probabilities are exact binomial calculations, not Monte Carlo estimates.

The second row changes prices and payout share only. It does not introduce minimum days, trailing drawdown, consistency rules, or other path restrictions. The calculation therefore remains an ideal contract, but one with less participant-favorable parameters.

6.4 A limited correlation extension

Independence is strong. Attempts placed at similar times, in similar assets, or under the same strategy may share common shocks. Diversifying across time and assets can reduce dependence, but it does not prove independence.

As a limited robustness result, suppose the payout indicators Y_i are exchangeable, have common success probability r , and have common pairwise correlation ρ . Then

$$\text{Var}(K) = N r (1 - r) [1 + (N - 1) \rho]$$

and

$$\text{Var}(\Pi_N) = W^2 N r (1 - r) [1 + (N - 1) \rho].$$

Positive correlation increases aggregate variance; negative feasible correlation reduces it. Expected value remains

$$E[\Pi_N] = N (W r - F).$$

This identity does not determine the full distribution of K . Pairwise correlation alone is insufficient to calculate an exact probability of positive profit or at least one payout. Such probabilities require a complete joint model, such as a common-factor specification, a copula, or a beta-binomial mixture, which is not selected or calibrated here.

7. Numerical Illustration

7.1 Fictitious 50K ideal baseline

The baseline values are shown in Table 3.

Table 3. Baseline parameters

Parameter	Symbol	Value	Interpretation
Nominal account size	B_{nom}	\$50,000	Contract label; not deposited participant capital
Profit target	U	\$3,000	Positive binary market payoff
Maximum drawdown	D	\$2,000	Negative binary market payoff
Evaluation fee	F	\$40	Personal cost per purchased cycle
Gross payout	W	\$3,000	Payout after both stages succeed
Payout share	s	100%	Participant share in the baseline
Activation fee	A	\$0	Excluded from the baseline
Evaluation success probability	p	parameter	Exogenous
Conditional payout probability	q	$q = p$	Same-strategy convention

Note: The Ideal Basic case isolates the contract mechanism. The Ideal Realistic case changes fees, activation cost, and payout share while retaining the same success probabilities.

The Target-to-Drawdown Ratio is

$$T = \frac{3000}{2000} = 1.5.$$

The market break-even probability is

$$p_M^* = \frac{2000}{3000 + 2000} = 40\%.$$

The contract break-even probability is

$$p_C^* = \sqrt{\frac{40}{3000}} \approx 11.55\%.$$

The baseline therefore satisfies the existence condition, and the contract-edge interval is

$$11.55\% < p < 40\%.$$

Table 4 reports the required probability scenarios.

Table 4. Market and participant contract expectancy by success probability

$p = q$	Market EV	Contract EV	$E[\Pi_{50}]$	$P(\Pi_{50} > 0)$
10%	−\$1,500	−\$10	−\$500	39.50%
20%	−\$1,000	+\$80	+\$4,000	87.01%
30%	−\$500	+\$230	+\$11,500	99.10%
40%	\$0	+\$440	+\$22,000	99.98%
50%	+\$500	+\$710	+\$35,500	100.00%

Note: Market expectancy records nominal trading P&L; participant contract expectancy records only personal contract cash flows under the stated terms.

At $p = 20\%$, market expected value is

$$0.20(3000) - 0.80(2000) = -1000.$$

The same success rate in both contract stages gives a 4% unconditional payout probability:

$$pq = 0.20 \times 0.20 = 0.04.$$

Contract expected value is

$$0.04(3000) - 40 = 80.$$

For 50 independent purchased cycles,

$$E[K] = 50(0.04) = 2,$$

$$E[\Pi_{50}] = 3000(2) - 40(50) = 4000,$$

and because one payout exceeds the total fee bill,

$$P(\Pi_{50} > 0) = P(K \geq 1) = 1 - (1 - 0.04)^{50} \approx 87.01\%.$$

The expected net return on fees in this illustration is

$$\frac{\mu_C}{F} = \frac{80}{40} = 200\%.$$

That percentage should not be interpreted as a conventional unlevered investment return. The denominator includes only evaluation fees and omits time, labor, infrastructure, taxes, execution costs, account limits, payment risk, and any other personal resources required by a real process.

7.2 Same ideal contract, less-generous parameters

The next step changes F , A , and s without changing any path rule. This creates an idealized but more realistic parameter bridge.

Table 5. Parameter bridge at W = \$3,000

Scenario	Inputs $p; F; A; s$	Participant EV	Provider gross EV	Break-even p
Baseline ideal	20%; \$40; \$0; 100%	+\$80	−\$80	11.55%
Higher evaluation fee	20%; \$90; \$0; 100%	+\$30	−\$30	17.32%
Activation fee only	20%; \$40; \$100; 100%	+\$60	−\$60	13.33%
Lower payout share	20%; \$40; \$0; 90%	+\$68	−\$68	12.17%
Combined illustration	20%; \$90; \$100; 90%	−\$2	+\$2	20.20%
Combined, $p = q = 30\%$	30%; \$90; \$100; 90%	+\$123	−\$123	20.20%

Note: Thresholds use a \$3,000 gross payout and the monetary assumptions shown in each row. The two columns answer different break-even questions.

The combined illustration is deliberately labeled illustrative, not typical. Official provider pages confirm that non-zero evaluation prices, activation charges, and participant shares below 100% exist, but product structures and prices vary. The point of Table 5 is comparative statics: less favorable prices can remove the large participant edge even before adding trading-day, consistency, or trailing-drawdown rules.

7.3 How the provider can earn within the simplified system

Before operating costs and other revenue, the modeled transfer is zero-sum:

$$\mu_P = F + pA - pq s W = -\mu_C.$$

Under the combined illustration and $p = q = 20\%$, provider gross EV is only +\$2 per cycle. This is near break-even, not a complete business model. It excludes staff, technology, market-data, payment, hedging, fraud, and compliance costs.

A simple heterogeneity vignette nevertheless shows the cross-subsidy mechanism. Under $F = \$90$, $A = \$100$, $s = 90\%$, and $W = \$3,000$, a participant with $p = q = 10\%$ has EV −\$73, while one with $p = q = 30\%$ has EV +\$123. If 80% of cycles come from the first type and 20% from the second, average participant EV is −\$33.80 and provider gross EV is +\$33.80. The provider remains gross-positive while the higher type's share is below approximately 37.24%. These weights are an illustration, not an empirical estimate of the trader population.

7.4 Equal-time and equal-budget illustration

Table 6 asks a different question from the sign comparison: what happens if both routes are placed on a common one-month and \$10,000 liquid-capital base? The own-capital route is shown at four monthly-return assumptions rather than at a single headline rate. The contract routes assume 50 evaluation-to-payout cycles finish within the month. All five assumptions are editable scenario inputs, not factual estimates.

Table 6. One-month illustration with \$10,000 starting liquid capital

Route	Fees (evaluation + activation)	Gross P&L or payout	Net P&L	Ending capital / return
Own capital at illustrative 5%	\$0 +\$0	\$500	\$500	\$10,500 / 5%
Own capital at illustrative 10%	\$0 +\$0	\$1,000	\$1,000	\$11,000 / 10%
Own capital at illustrative 20%	\$0 +\$0	\$2,000	\$2,000	\$12,000 / 20%
Own capital at illustrative 30%	\$0 +\$0	\$3,000	\$3,000	\$13,000 / 30%
50 ideal contract cycles	\$2,000 +\$0	\$6,000	\$4,000	\$14,000 / 40%
50 combined-parameter cycles	\$4,500 +\$1,000	\$5,400	-\$100	\$9,900 / - 1 %

Note: Values are one-month illustrations, not forecasts. Evaluation and activation costs are stated separately; \$5,500 is the expected combined pre-funding requirement when 20% of 50 evaluations pass.

The ideal contract requires \$2,000 to cover all evaluation fees. In the combined scenario, the table uses expected activation incidence: 50 evaluations cost \$4,500 and the expected 10 passes cost another \$1,000, for an expected total of \$5,500. A maximum-liquidity reserve that assumed all 50 evaluations passed would instead be \$9,500, but that is not the basis used in Table 6. Neither amount is equivalent to the 50K nominal account label. The comparison is useful because it makes capital and time assumptions visible; it should not be read as a forecast of achievable monthly return or operational throughput.

8. Economic Interpretation and Contract Design

8.1 Why the sign can reverse

The sign reversal is produced by payoff transformation. In the market benchmark, a failure costs D . In the participant contract, the same nominal failure ends the evaluation, but personal cash loss is capped at the purchased fee F . The success path can still produce payout W . The participant therefore purchases a nonlinear claim on a threshold event.

The structure is option-like in the limited sense that downside is capped and upside is conditional. It is not necessarily a legally or economically traded option, and the paper does not price it with no-arbitrage methods. There may be no replicating portfolio, no transferable claim, no continuously traded underlying contract, and substantial counterparty and rule risk.

8.2 Market edge and contract edge

A market edge means positive expected market profit under the specified strategy and payoff. A contract edge means positive expected personal cash flow under the specified contract. The participant in the contract-edge region does not “beat the market” according to the binary market benchmark. Instead, the purchased contract changes which losses are borne personally and which successes produce cash flow.

This distinction also prevents a common mistake: a strategy’s win rate does not identify its quality without its payoff sizes. A 10% success probability can accompany a positive, zero, or negative market expectancy depending on U , D , and costs. In the baseline $U=\$3,000$ and $D=\$2,000$, a 10% success probability is strongly negative in the market. The contract can nevertheless be closer to break-even because the personal failure cash flow is $-F$, not $-D$.

8.3 Why the result is not arbitrage

The model contains risk. A participant may lose all fees, and a positive expected value can be accompanied by a substantial probability of realized loss. The model also assumes rather than proves that p , q , F , W , and payment certainty remain stable over repeated attempts.

Real providers can change prices or rules, limit account supply, require minimum days, enforce consistency, delay withdrawals, cap payouts, impose activation or data fees, apply trailing drawdown, restrict strategies, or terminate relationships. Market execution can add slippage, commissions, latency, and model error. Any of these changes can reduce p , reduce q , reduce W , raise total cost, or create dependence across attempts.

Positive model expectancy is thus a conditional mathematical statement:

$$\text{if the assumptions and parameters hold, then } E[\Pi_C] > 0.$$

It is not evidence that the assumptions hold in a real contract.

8.4 The provider side

If the provider receives F , receives A after evaluation success, and pays sW after both stages succeed, its gross expected cash flow from the modeled transfer is

$$\mu_P = F + pA - pq sW = -\mu_C.$$

This identity is before operating costs and other revenue. A provider would not voluntarily supply an unlimited, unmodified contract with persistently negative gross expected transfer unless it earned offsetting revenue or reduced payout exposure elsewhere. Section 7.3 shows how prices and a mixture of lower- and higher-probability participants can generate a positive gross transfer even while the higher-probability group earns positive expected participant cash flow.

This observation gives economic meaning to realistic restrictions. A rule can alter selection, success probabilities, timing, dependence, or payout size. The rules need not exist only to make more participants fail; they may also control provider risk, reduce fraud, satisfy operational constraints, or screen for a desired trading process. The effect of each rule is an empirical and theoretical question, not a conclusion built into the baseline.

8.5 Evaluation as selection

The model also motivates a broader research question: does an evaluation select trading skill, or can it select high-variance paths that happen to cross a target before a drawdown? The present analysis cannot answer that question because p is exogenous and strategy choice is absent. Answering it requires allowing the participant to choose a distribution, variance, or stopping policy and asking which behavior maximizes contract value. That extension would connect the retail setting more directly to tournament and risk-shifting models.

9. Limitations and Extensions

The foundational model has substantial limitations.

First, success probabilities are assumed rather than estimated. The model is therefore a conditional theorem and numerical illustration, not an empirical claim about pass rates. This simplification keeps the mechanism transparent but limits external validity.

Second, the binary payoff omits the path. Real evaluations are path dependent. Trailing drawdown, daily loss limits, end-of-day calculations, open-equity rules, and time limits can change success probabilities even when terminal profit and loss are unchanged.

Third, the baseline assumes immediate transition and payout. Minimum trading days, profitable-day requirements, consistency rules, waiting periods, and payout caps introduce additional states and can make q very different from p .

Fourth, attempts are independent. Multiple accounts running the same strategy can be highly correlated. Temporal staggering and multiple assets do not guarantee independence because common factors and common strategy logic can remain.

Fifth, costs are incomplete. Fees for data, platforms, activation, resets, commissions, slippage, taxes, and labor can materially change expected value.

Sixth, payment is assumed certain. Counterparty risk, rule interpretation, compliance review, and contract termination can change the probability and timing of realized cash flow.

Seventh, supply is unlimited. Account caps, identity restrictions, strategy restrictions, and provider discretion can prevent linear scaling in N .

Eighth, the participant is risk neutral. A risk-averse participant can reject a positive-expectancy contract because the distribution is highly skewed. A capital-constrained participant can also face ruin before the long-run expectation is realized.

Ninth, provider behavior is not endogenous. An economically viable provider chooses fees, rules, payout shares, and monitoring in response to participant behavior and competitive pressure.

Several extensions follow directly from these limitations. Endogenizing strategy choice would permit a test of whether evaluation rules select skill or variance seeking. Other extensions include:

- alternative activation-fee and reset-fee schedules;
- minimum trading days;

- consistency and profitable-day rules;
- payout delays and periodic caps;
- static and trailing drawdown;
- daily loss limits;
- account and strategy limits;
- correlated accounts and common shocks; and
- provider-side optimal contract design.

A separate empirical analysis could encode actual historical rules, estimate or simulate pass and payout probabilities using reproducible data, and compare predicted with observed outcomes where data are available. Separating the ideal theorem from realistic calibration prevents institutional complexity from obscuring the foundational mechanism.

10. Conclusion

This paper separates market edge from contract edge. In the binary market benchmark, expected profit is $pU - (1-p)D$. In the idealized two-stage evaluation contract, expected participant cash flow is $pqsW - F - pA$. These objects need not have the same sign because the purchased contract changes the participant's payoff: nominal market drawdown can terminate an account without becoming an equal personal cash loss, while successful paths retain a contractual payout.

Under the baseline $q=p$, $s=1$, and $A=0$, positive contract expectancy requires $p > \sqrt{F/W}$, whereas negative market expectancy requires $p < D/(U+D)$. With a \$3,000 target, \$2,000 drawdown, \$40 fee, and \$3,000 payout, the fictitious 50K baseline interval is approximately $11.55\% < p < 40\%$. At $p=20\%$, the market strategy loses \$1,000 per deployment in expectation while the purchased contract earns \$80 per cycle in expectation.

That large ideal edge is parameter-sensitive. With illustrative values $F=\$90$, $A=\$100$, and $s=90\%$, the break-even probability rises to 20.20% and participant EV at $p=q=20\%$ becomes $-\$2$. For 50 cycles, strict-profit probability falls from 87.01% to 40.37%. A separate equal-budget/equal-time table shows own-capital returns from 5% to 30%, making clear why capital, cycle throughput, and return assumptions must be stated before comparing the contract route with trading \$10,000 of personal capital.

Multiple attempts do not change the sign of expected value, but they change the distribution of realized outcomes. The exact probability of aggregate profit depends on the number of payouts required to cover total fees. Dependence can materially increase variance, and pairwise correlation alone does not determine finite-sample success probabilities.

The result is deliberately narrow. It is not an arbitrage claim, a recommendation, or a description of an identified provider. It is a base case. Its value is that additional rules can be introduced one by one and evaluated against a clear benchmark. This makes it possible to examine when a contract rewards genuine market skill, when it rewards variance or repeated trials, and how provider rules reshape the boundary between the two.

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Appendix A. Proofs

A.1 Proof of Proposition 1

Market expectancy is

$$\mu_M = pU - (1 - p)D.$$

Therefore,

$$\mu_M < 0$$

if and only if

$$pU - (1 - p)D < 0.$$

Rearranging,

$$p(U + D) < D,$$

so

$$p < \frac{D}{U + D}.$$

□

A.2 Proof of Proposition 2

Under $q = p$, $s = 1$, and $A = 0$,

$$\mu_C = p^2 W - F.$$

Because $W > 0$ and $p \geq 0$,

$$\mu_C > 0$$

if and only if

$$p^2 > \frac{F}{W},$$

which is equivalent to

$$p > \sqrt{\frac{F}{W}}.$$

□

A.3 Proof of Proposition 3

By Proposition 2, contract expectancy is positive when

$$p > \sqrt{\frac{F}{W}}.$$

By Proposition 1, market expectancy is negative when

$$p < \frac{D}{U + D}.$$

Both inequalities hold for at least one p if and only if

$$\sqrt{\frac{F}{W}} < \frac{D}{U + D}.$$

When this condition holds, their intersection is

$$\left(\sqrt{\frac{F}{W}}, \frac{D}{U + D} \right).$$

□

A.4 Finite-N distribution

For independent payout indicators $Y_i \sim \text{Bernoulli}(r)$,

$$K = \sum_{i=1}^N Y_i \sim \text{Binomial}(N, r).$$

Aggregate cash flow is positive when

$$W K - N F > 0,$$

or

$$K > \frac{N F}{W}.$$

The smallest integer satisfying the strict inequality is

$$K_{\min} = \text{floor}\left(\frac{N F}{W}\right) + 1.$$

The binomial upper tail therefore gives

$$P(\Pi_N > 0) = \sum_{k=K_{\min}}^N \binom{N}{k} r^k (1-r)^{N-k}.$$

A.5 Finite-N distribution with activation fees

Let J count evaluation passes and K count completed payouts. Under stage and cycle independence,

$$J \sim \text{Binomial}(N, p), K \mid J = j \sim \text{Binomial}(j, q).$$

Because

$$\Pi_N = s W K - N F - A J,$$

strict profit conditional on $J = j$ requires

$$K \geq \text{floor}\left(\frac{N F + j A}{s W}\right) + 1 = k_{\min}(j).$$

The law of total probability gives

$$P(\Pi_N > 0) = \sum_{j=0}^N \binom{N}{j} p^j (1-p)^{N-j} \sum_{k=k_{\min}(j)}^j \binom{j}{k} q^k (1-q)^{j-k},$$

with the inner sum defined as zero if $k_{\min}(j) > j$. This is the enumeration used for Table 2. The corresponding implementation is provided in the Supplementary Material.

A.6 Correlation identity

Let Y_i have mean r , variance $r(1-r)$, and common pairwise correlation ρ . Then

$$\text{Cov}(Y_i, Y_j) = \rho r(1-r) (i \neq j).$$

Therefore,

$$\text{Var}(K) = N r (1 - r) + N (N - 1) \rho r (1 - r) = N r (1 - r) [1 + (N - 1) \rho].$$

Because $\Pi_N = W K - N F$,

$$\text{Var}(\Pi_N) = W^2 \text{Var}(K) = W^2 N r (1 - r) [1 + (N - 1) \rho].$$

Declarations

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