

7546719-elementary-closed-form-equity-valuation-under-alternative-mean-reversion-paths

SUMMARY: This paper examines the conditions under which deterministic mean-reversion specifications allow for elementary closed-form equity valuation under complete earnings retention. The valuation is defined as the limit of discounted terminal book equity under an explicit terminal-realization convention. The key finding is that a strictly positive finite limit exists if and only if cumulative log-relative profitability converges. The paper derives and verifies five specifications of mean-reversion: geometric reversion, fast and slow components, rational decay, finite linear fading, and a nonlinear gross-return ratio. The paper also provides a vanishing-tail representation that serves as a constructive sufficient condition for elementary solvability. The findings clarify why distinct adjustment paths can generate identical limiting valuations. The paper compares mathematical simplicity and economic interpretation to favor geometric reversion as a benchmark and a two-component model when heterogeneous persistence is economically motivated.

METHOD: The study uses a complete-retention setting where all earnings remain within the firm and terminal book equity is realizable by shareholders at its book amount. The key variable is the logarithm of the gross return on equity (ROE) relative to the gross discount rate. The paper derives an exact convergence criterion: a strictly positive finite valuation exists precisely when cumulative log-relative profitability converges. The analysis is based on the object studied as the limit of discounted realizable terminal values under the explicit terminal-realization convention, which avoids treating an accounting accumulation identity as a complete market-pricing theory.

KEY FINDINGS:

- Cumulative log-relative profitability convergence is necessary for a strictly positive finite limit.
- Five specifications of mean-reversion are derived and verified: geometric reversion, fast and slow components, rational decay, finite linear fading, and a nonlinear gross-return ratio.
- A vanishing-tail representation provides a constructive sufficient condition for elementary solvability.
- Different mean-reversion paths can have the same cumulative log-relative profitability and therefore the same limiting valuation.
- Under common initial conditions and cumulative persistence greater than one, rational decay has a strictly larger absolute log-valuation error than geometric reversion at every integer horizon of at least two periods whenever the initial profitability deviation is nonzero.

IMPLICATION FOR TRADING: The findings provide a clear understanding of the conditions under which mean-reversion paths can be used for elementary closed-form equity valuation. This can help traders and analysts in making more precise and tractable models for equity valuation, especially when heterogeneous persistence is economically motivated. The paper's framework offers a tractable alternative without claiming an exhaustive classification of all elementary closed forms or an unconditional market-equilibrium valuation.

Elementary Closed-Form Equity Valuation under Alternative Mean-Reversion Paths

Abstract

This paper examines which deterministic mean-reversion specifications permit elementary closed-form equity valuation under complete earnings retention. Valuation is defined as the limit of discounted terminal book equity under an explicit terminal-realization convention. A strictly positive finite limit exists if and only if cumulative log-relative profitability converges; convergence of return on equity to the discount rate alone is insufficient. A vanishing-tail representation then provides a constructive sufficient condition for elementary solvability: each period's log-relative profitability is written as the difference between adjacent values of an elementary tail whose initial calibration is also elementary. Five specifications are derived and verified: geometric reversion, fast and slow components, rational decay, finite linear fading, and a nonlinear telescoping gross-return ratio. Exact finite-horizon errors and a persistence measure clarify why distinct adjustment paths can generate identical limiting valuations. With common initial conditions and cumulative persistence greater than one, rational decay has a strictly larger absolute log-valuation error than geometric reversion at every integer horizon of at least two periods whenever the initial profitability deviation is nonzero. Hypothetical convergence curves illustrate these differences without empirical estimation.

Comparing mathematical simplicity and economic interpretation favors geometric reversion as a benchmark and a two-component model when heterogeneous persistence is economically motivated. The framework generates tractable alternatives without claiming an exhaustive classification of all elementary closed forms or an unconditional market-equilibrium valuation.

Keywords: Equity valuation; mean reversion; elementary closed form; profitability persistence; telescoping sums; terminal valuation.

1. Introduction

Equity valuation depends on both profitability and the persistence of deviations from a benchmark. A compact formula clarifies this connection, but deriving it under one adjustment law does not make that law necessary for tractability. This paper asks which restrictions on deterministic mean reversion are sufficient to obtain a valuation expressed entirely through elementary functions.

Accounting-based valuation provides an established setting in which earnings, book equity, and distributions are linked explicitly. Ohlson (1995) and Feltham and Ohlson (1995) relate equity value to accounting information under specified accounting and information dynamics. Mean reversion supplies a separate restriction on the evolution of profitability. Fama and French (2000) document mean reversion while finding that a uniform partial-adjustment rate misses nonlinear

patterns. These strands motivate a comparison of alternative paths, but the existence of mean reversion does not by itself settle the mathematical convergence or elementary expressibility of a valuation.

The present analysis extends the complete-retention setting of the companion framework in Yeh (2026), referred to as Paper 15. All earnings remain within the firm, and, for each finite horizon, terminal book equity is assumed realizable by shareholders at its book amount. The object studied is the limit of those discounted realizable terminal values. Keeping this convention explicit avoids treating an accounting accumulation identity as a complete market-pricing theory. Dividend-policy comparisons are left to the companion paper; here the focus is the profitability path and its valuation consequences.

The key variable is the logarithm of the gross ROE relative to the gross discount rate. Its cumulative sum equals the logarithm of the valuation-to-initial-book-equity ratio. This observation yields an exact convergence criterion: a strictly positive finite valuation exists precisely when cumulative log-relative profitability converges. A path may approach the benchmark at every date and nevertheless accumulate an unbounded log-relative return. Thus, the speed and summability of adjustment matter in addition to the existence of an equilibrium target.

A second distinction concerns the meaning of a closed form. An infinite sum may converge without having an elementary expression in the model's primitive inputs. This paper uses a strict elementary-function requirement and excludes unevaluated infinite sums, infinite products, and special functions as final valuation formulas. Instead of attempting a universal symbolic classification, it constructs admissible paths from elementary vanishing tails. Adjacent tail differences define period-by-period log-relative profitability, and telescoping produces both finite-horizon values and their limits. The calibration to initial ROE must also be elementary; otherwise, apparent tractability could merely conceal an unsolved inverse problem.

The contribution is a modeling framework with explicit proofs, parameter domains, and economic comparisons. Standard logarithmic and telescoping identities organize five specifications under a common valuation convention: geometric adjustment, fast and slow components, rational decay, finite linear fading, and a nonlinear gross-return ratio. For each, the remaining tail measures finite-horizon valuation error exactly.

The comparison also exposes an identification issue. Different mean-reversion paths can have the same cumulative log-relative profitability and therefore the same limiting valuation. A common terminal value does not determine when the underlying profitability is earned. Beyond this identification observation, an analytical comparison establishes that, under common initial conditions and cumulative persistence greater than one, rational decay produces a strictly larger absolute log-valuation error than geometric reversion at every integer horizon of at least two periods when the initial deviation is nonzero. Hypothetical examples hold initial ROE and terminal value fixed while comparing convergence curves. Economic assessment then considers two primary criteria: simplicity of the assumed form and clarity of its economic meaning. The analysis

provides no empirical ranking, and the proposed interpretations are not substitutes for structural derivations or estimation.

Section 2 reviews the relevant valuation, profitability, and symbolic-summation literature. Section 3 establishes the setting, convergence theorem, and elementary construction. Section 4 verifies five specifications. Section 5 compares their economic interpretations and convergence behavior, and Section 6 concludes. Table 3 summarizes the notation and parameter domains before the references.

2. Literature Review

2.1. Accounting-based equity valuation and terminal values

Ohlson (1995) develops a model linking equity value to earnings, book values, and dividends, with clean-surplus accounting and specified information dynamics providing central structure. Feltham and Ohlson (1995) distinguish operating and financial activities and show how accounting variables and abnormal earnings enter valuation. These studies establish that the relationship between accounting accumulation and value must be considered together with assumptions about future payoffs and dynamics. They do not imply that every mean-reverting accounting-return path has an elementary valuation.

The finite-horizon issue is especially relevant here. Penman and Sougiannis (1998) compare dividend, cash-flow, and earnings valuation approaches when forecasts are truncated, emphasizing the resulting terminal-value calculations. This concern motivates making terminal assumptions visible rather than leaving them embedded in a limiting expression. In the present model, realizability at book value for each finite horizon is stipulated from the outset. The limit of discounted terminal equity is then analyzed on its own stated terms. It is not obtained by asserting that permanently undistributed earnings, without a realization mechanism, necessarily constitute a positive market payoff.

The companion framework of Yeh (2026) analyzes alternative earnings-allocation mechanisms under common return dynamics. This paper retains its complete-retention accumulation rule and terminal-realization convention while varying the deterministic profitability path. The distinction in purpose is substantive: the companion framework compares allocation policies, whereas the present analysis studies convergence, elementary solvability, and economically interpretable alternatives to a single reversion law.

2.2. Profitability mean reversion and persistence

Fama and French (2000) provide evidence of profitability mean reversion and show that a simple model with a common adjustment rate does not capture all of its nonlinear behavior. Their findings motivate attention to the form of adjustment, rather than treating a single decay coefficient as an empirically universal description. They do not establish the specific deterministic paths

examined in this paper, nor do they imply that every firm's long-run ROE must equal an externally specified discount rate.

Nissim and Penman (2001) organize financial-statement analysis around equity valuation, combining profitability and growth analysis and distinguishing operating from financing activities. Their framework reinforces the importance of giving forecast assumptions a clear accounting and economic interpretation. In the present setting, this means distinguishing persistence in log-relative profitability from persistence in the raw ROE spread. It also means treating an assumed decomposition into fast and slow components as a model of adjustment, rather than an automatically valid decomposition of accounting earnings.

The models below are deliberately deterministic. They address which stylized paths are mathematically tractable and economically intelligible, not which stochastic process best describes observed profitability. A constant proportional adjustment is a transparent benchmark; multiple persistence components, a slowly decaying remainder, or an expiration date may be useful when the application gives a reason for them. Empirical profitability research motivates considering alternatives, but selecting among the five specifications would require evidence beyond the present theoretical comparison.

2.3. Closed-form valuation under specific return dynamics

Yeh (2025) develops a continuous differential-equation approach to equity valuation and derives a closed-form solution in which the decay of profitability affects value. Yeh and Chi (2025) examine stock valuation based on mean reversion of ROE. These studies provide a direct connection between profitability persistence and tractable valuation. They also motivate asking how much of the tractability is tied to one particular dynamic specification.

The present paper remains in discrete time and takes the first future-period return as its initial profitability input. It therefore derives all indexing and persistence coefficients within that convention rather than transferring a continuous-time coefficient or a differently dated initial condition unchanged. Geometric decay of log-relative profitability gives the benchmark solution. The extension is to show how additional paths can be produced and verified under the same accumulation and terminal-realization assumptions.

The resulting generalization is constructive. It does not assert that all conceivable mean-reverting functions admit elementary valuations, that the five examples exhaust the possibilities, or that a more elaborate path must be economically preferable. Instead, it identifies conditions that can be checked directly and shows how they deliver transparent formulas, admissible parameter domains, and exact truncation errors. This makes the comparison concern the assumptions themselves as well as the resulting expressions.

2.4. Symbolic summation and the meaning of an elementary solution

Finding a closed expression for a sum is a classical problem in symbolic computation. Gosper (1978) gives a decision procedure for indefinite hypergeometric summation within a specified class. Here, “hypergeometric” describes a summand whose consecutive-term ratio is rational in the index; it does not mean that a special hypergeometric function is accepted as the valuation output. Karr (1985) studies mathematical aspects of finding finite-sum formulas using difference-field methods. These contributions illustrate why statements about symbolic solvability must specify the permitted function class and the kind of expression being sought.

The present construction does not implement or extend those algorithms. It chooses a vanishing elementary tail first and then defines profitability by its first difference. The resulting sum telescopes, so elementary expressibility is established by an explicit formula rather than inferred from numerical convergence. The analytic converse—that every convergent series has a unique vanishing tail—does not imply that this tail is elementary. Likewise, requiring an elementary expression for every finite-horizon tail is stronger than requiring only an elementary expression for the infinite-horizon total.

These distinctions locate the paper’s contribution between valuation modeling and elementary discrete analysis. Accounting-based valuation supplies the economic setting, profitability research motivates examining alternative persistence patterns, and symbolic summation clarifies the limits of a closed-form claim. The paper combines these perspectives in a self-contained family of models with proofs and economic comparisons. Its convergence result is necessary and sufficient for a positive finite valuation; its elementary-tail condition is a sufficient construction rather than an exhaustive theorem about elementary summability.

3. Valuation, convergence, and elementary closed forms

3.1. Economic environment and the valuation object

Consider a deterministic firm that retains all accounting earnings. Let $B_0 > 0$ denote initial book equity, $R_t > -1$ the return on opening book equity in period t , and $r > -1$ a constant discount rate. There are no shareholder contributions, repurchases, or other direct adjustments to book equity. Consequently,

$$B_t = B_{t-1}(1 + R_t), \quad t = 1, 2, \dots \quad (3.1)$$

The valuation object follows the terminal-realization convention of the companion framework (Yeh, 2026). For each finite horizon T , shareholders are assumed able to realize terminal equity at its book amount B_T . Its discounted value is

$$V_T = \frac{B_T}{(1+r)^T} = B_0 \prod_{t=1}^T \frac{1+R_t}{1+r}. \quad (3.2)$$

One setting in which the convention is internally consistent is a deterministic firm whose net assets are measured at realizable values. Suppose that all earnings are retained, liabilities are fully recognized, and shareholders may terminate the firm at the chosen finite horizon without

liquidation costs or taxes. If termination realizes the recorded net assets, the distributable proceeds equal terminal book equity. The analysis then compares the discounted values associated with alternative hypothetical termination horizons. This setting is an idealized application of the convention; it does not assert that conventional book equity equals liquidation or market value for a general listed firm.

The accumulation rule also yields a residual-income representation of the same finite-horizon object.

Connection to Residual-Income Valuation

The accumulation rule also yields a residual-income representation of the same finite-horizon object. In particular,

$$\frac{B_t}{(1+r)^t} - \frac{B_{t-1}}{(1+r)^{t-1}} = \frac{(R_t - r)B_{t-1}}{(1+r)^t}.$$

Summing over periods gives

$$V_T = B_0 + \sum_{t=1}^T \frac{(R_t - r)B_{t-1}}{(1+r)^t}.$$

This identity relates the present accumulation model to the familiar residual-income decomposition. Under the stated terminal-realization convention, both expressions describe the same finite-horizon value. The contribution developed below concerns the construction of profitability paths with elementary finite-horizon values and explicit remaining-error formulas, rather than a new residual-income identity or a market-equilibrium pricing result.

The infinite-horizon object is the limit of this sequence of finite-horizon realizable values. It is not a cash payment literally made at infinity. Nor does the analysis establish that a security with no distributions and no realization mechanism has a positive fundamental price. All valuation statements below are conditional on the stated terminal-realization convention. Establishing a market-equilibrium price would require additional assumptions beyond this deterministic accumulation model.

Define the relative gross return and its logarithm by

$$X_t = \frac{1+R_t}{1+r} > 0, \quad x_t = \log X_t, \quad Q = X_1. \quad (3.3)$$

We call x_t log-relative profitability. It measures profitability relative to the discount-rate benchmark, on a logarithmic gross-return scale. It is not identical to the accounting spread $R_t - r$. Their exact and local relationships are

$$R_t = (1+r)e^{x_t} - 1, \quad x_t = \log \left(1 + \frac{R_t - r}{1+r} \right) \sim \frac{R_t - r}{1+r} \quad (R_t \rightarrow r). \quad (3.4)$$

In this paper, mean reversion refers to a deterministic path approaching the fixed benchmark r . It does not refer to the conditional expectation of a stochastic diffusion. Whether a firm's long-run ROE should equal its discount rate is an economic modeling assumption, not a consequence of accounting alone.

Writing $S_T = \sum_{t=1}^T x_t$ transforms the valuation product into

$$V_T = B_0 e^{S_T}. \quad (3.5)$$

This transformation separates two questions: whether the valuation limit exists as a positive finite number, and whether it can be written using only elementary functions.

3.2. An exact convergence condition

Theorem 3.1 (positive finite valuation). Under the assumptions in Section 3.1, the sequence V_T converges to a limit $V_\infty \in (0, \infty)$ if and only if the series $\sum_{t=1}^\infty x_t$ converges to a finite real number S . In that case,

$$V_\infty = B_0 e^S. \quad (3.6)$$

Proof. If $S_T \rightarrow S \in \mathbb{R}$, continuity of the exponential in (3.5) yields (3.6). Conversely, if $V_T \rightarrow V_\infty \in (0, \infty)$, continuity of the logarithm gives $S_T = \log(V_T/B_0) \rightarrow \log(V_\infty/B_0)$. Thus the series converges. ▀

Corollary 3.1 (mean reversion is necessary but insufficient). A positive finite valuation implies $x_t \rightarrow 0$ and hence $R_t \rightarrow r$. The converse does not hold.

Proof. Convergence of a series requires its terms to approach zero. Equation (3.4) then gives $R_t \rightarrow r$. For the converse, choose $x_t = c/t$ with $c \neq 0$. This path approaches zero monotonically in magnitude, but $S_T = c \sum_{t=1}^T 1/t$. Therefore $V_T \rightarrow \infty$ if $c > 0$, and $V_T \rightarrow 0$ if $c < 0$. Neither case gives a strictly positive finite limit. ▀

The qualification “positive finite” matters. A limit of zero is mathematically a finite limit, but is excluded from Theorem 3.1 because the intended valuation retains a positive scale relative to initial book equity. Similarly, a convergent series need not be absolutely convergent if its terms change sign. For an eventually one-sided mean-reversion path, however, convergence and absolute convergence are equivalent: the tail contains no cancellation. The five principal models below are one-sided and absolutely summable.

A useful boundary example is $x_t = c/t^p$. For $c \neq 0$ and $p > 0$, mean reversion holds, whereas a positive finite valuation requires $p > 1$. This example is only a convergence diagnostic. It does not provide an elementary closed-form valuation for general p and is not included among the elementary models below.

3.3. Tail potentials and constructive elementary solvability

Definition 3.1 (elementary closed form). A valuation is elementary in a stated set of primitive inputs if it is expressible by a finite composition of algebraic operations and algebraic functions, exponentials, logarithms, and other elementary functions on an explicitly admissible real domain. An unevaluated infinite sum or product, a special function such as Gamma or a hypergeometric function, or a new parameter merely defined to equal the unknown sum does not meet this definition. All examples below use only arithmetic, powers, exponentials, and logarithms. A predetermined finite horizon may be handled by a finite piecewise specification.

The qualification concerning primitive inputs prevents a circular argument. Writing $V_\infty = B_0 e^S$ does not establish an elementary solution if S remains an unevaluated infinite sum. The following representation makes the required constructive step explicit.

Theorem 3.2 (vanishing-tail representation). The following statements are equivalent:

- (a) $\sum_{t=1}^{\infty} x_t$ converges in $\mathbb{R}$.
- (b) There is a real sequence H_t such that

$$x_t = H_t - H_{t+1}, \quad H_t \rightarrow 0 \quad (t \rightarrow \infty). \quad (3.7)$$

The sequence in (b) is unique. For every finite horizon and at the limit,

$$V_T = B_0 \exp(H_1 - H_{T+1}), \quad V_\infty = B_0 e^{H_1}. \quad (3.8)$$

Proof. Under (a), define $H_t = \sum_{s=t}^{\infty} x_s$. Convergence gives $H_t \rightarrow 0$, and subtraction of adjacent tails yields (3.7). Under (b), telescoping gives $S_T = H_1 - H_{T+1} \rightarrow H_1$, proving (a) and (3.8). If H_t and $\tilde{H}_t$ both satisfy (b), their difference is constant in t because its first difference vanishes. Both sequences tend to zero, so this constant is zero. $\square$

Theorem 3.2 is an exact analytic characterization of convergence. It does not assert that the unique tail is elementary. That additional property is supplied by the next result.

Proposition 3.1 (an elementary construction). Let $H(t; \theta)$ be an explicitly given elementary function of the integer time index and a finite vector of primitive parameters, or a finite piecewise elementary function. Suppose that, on a stated parameter domain, it is real for all $t \geq 1$, tends to zero, and the desired initial condition can be imposed by an explicit elementary calibration satisfying

$$H(1; \theta) - H(2; \theta) = \log Q. \quad (3.9)$$

Then the return path defined by

$$R_t = (1 + r) \exp[H(t; \theta) - H(t + 1; \theta)] - 1 \quad (3.10)$$

has a positive finite valuation, and both V_T and V_∞ are elementary in the calibrated primitive inputs.

Proof. Equation (3.10) ensures $R_t > -1$ and $x_t = H(t; \theta) - H(t + 1; \theta)$. Apply Theorem 3.2. Evaluating H at 1 and $T + 1$, subtracting, and exponentiating preserves elementary expressibility; the assumed calibration does so as well. ▀

This proposition is a sufficient construction, not a classification theorem for every elementary terminal value. In particular, requiring an elementary expression for the entire tail is stronger than requiring only an elementary total at $t = 1$. A terminal value may simplify without furnishing an elementary finite-horizon expression. Accordingly, the five models in Section 4 illustrate a broad construction rather than exhaust all possible models. Finite linear combinations of elementary vanishing tails already generate arbitrarily many additional forms; further nonlinear choices remain possible.

3.4. Restrictions that give a monotone economic path

Elementary solvability alone does not guarantee an economically interpretable mean-reversion path. An oscillating or temporarily explosive path could still telescope. The following restrictions give a transparent sufficient condition for monotone adjustment.

Proposition 3.2 (one-sided monotone reversion). Suppose $x_1 \neq 0$, let $s = \text{sign}(x_1)$, and write $G_t = sH_t$. If $G_t \rightarrow 0$ and

$$G_t \geq G_{t+1} \geq 0, \quad G_t - 2G_{t+1} + G_{t+2} \geq 0, \quad (3.11)$$

then x_t has the sign of x_1 or is zero, $|x_t|$ is nonincreasing, and R_t approaches r monotonically. Moreover, V_T is nondecreasing when $x_1 > 0$ and nonincreasing when $x_1 < 0$.

Proof. The first inequality gives $sx_t = G_t - G_{t+1} \geq 0$. The second gives $sx_t \geq sx_{t+1}$, proving monotonicity in magnitude. Since $H_t \rightarrow 0$, its first differences tend to zero. The map $x \mapsto (1 + r)e^x - 1$ is strictly increasing, so the same direction of adjustment applies to ROE. Finally, $V_T/V_{T-1} = e^{x_T}$ determines the direction of the valuation sequence. ▀

For $x_1 = 0$, the equilibrium path $x_t = 0$ gives $R_t = r$ and $V_T = B_0$ for every horizon. Each model below includes this case. The treatment of negative x_1 describes gradual restoration of below-benchmark profitability. Such restoration is an assumption; insolvency, exit, and financing frictions are outside the model.

3.5. Persistence, truncation error, and observational equivalence

A particularly interpretable subclass separates the initial deviation from its time profile:

$$x_t = x_1 g_t, \quad g_1 = 1, \quad M = \sum_{t=1}^{\infty} g_t < \infty. \quad (3.12)$$

When $g_t \geq 0$, the coefficient $M \geq 1$ is the equivalent duration of initial log-relative profitability: the cumulative log-relative return equals M periods of the first-period deviation. It is an area measure, not necessarily the date on which an advantage disappears. Equation (3.6) becomes

$$V_{\infty} = B_0 Q^M. \quad (3.13)$$

For the separable subclass, holding the profile fixed gives

$$\frac{\partial \log V_\infty}{\partial R_1} = \frac{M}{1+R_1}, \quad \frac{\partial \log V_\infty}{\partial M} = \log Q. \quad (3.14)$$

Thus greater persistence raises value above the benchmark and lowers it below the benchmark. Two distinct profiles with the same M produce the same terminal value for the same B_0 , r , and R_1 . A single valuation observation therefore cannot identify the shape of mean reversion, even within this deterministic model.

The tail also provides an exact finite-horizon error measure:

$$\log \frac{V_\infty}{V_T} = H_{T+1}, \quad \frac{V_\infty - V_T}{V_\infty} = 1 - e^{-H_{T+1}}. \quad (3.15)$$

In particular, $|H_{T+1}| \leq \varepsilon$ guarantees $e^{-\varepsilon} \leq V_T/V_\infty \leq e^\varepsilon$. Equal terminal values need not imply equal truncation errors at a practical forecast horizon.

4. Five elementary mean-reversion specifications

Throughout this section, B_0 , r , and R_1 are primitive inputs satisfying Section 3.1, and $x_1 = \log Q$. Each proposition supplies an admissible domain, an explicit vanishing tail, and a verification. All reported valuations are conditional on the terminal-realization convention.

4.1. Geometric reversion: a constant proportional adjustment

Proposition 4.1. For $0 < a < 1$, let

$$x_t = x_1 a^{t-1}, \quad H_t = \frac{x_1 a^{t-1}}{1-a}. \quad (4.1)$$

This specification satisfies Proposition 3.1 and yields

$$V_T = B_0 Q^{(1-a^T)/(1-a)}, \quad V_\infty = B_0 Q^{1/(1-a)}. \quad (4.2)$$

Proof. Subtraction gives $H_t - H_{t+1} = x_1 a^{t-1}$, and $a^{t-1} \rightarrow 0$. At $t = 1$ the difference is x_1 , as required. The valuation follows from (3.8). Because $0 < a < 1$, the deviation has constant sign and decreases geometrically in magnitude. ▀

The recursion is $x_{t+1} = ax_t$, or equivalently $X_{t+1} = X_t^a$. This is the benchmark specification in the companion framework (Yeh, 2026). Every period removes the fraction $1 - a$ of the remaining log-relative profitability. Its half-life on that scale is $\log(1/2)/\log a$, and its persistence measure is $M = 1/(1 - a)$. The exact tail after horizon T is $x_1 a^T/(1 - a)$.

The economic interpretation is a constant rate of competitive erosion or operational adjustment. It is particularly simple because the same proportional mechanism operates at every date. The restriction concerns log-relative profitability, not an exactly constant proportional decline in the accounting spread $R_t - r$; the latter interpretation is only a local approximation near equilibrium.

4.2. Fast and slow components: heterogeneous persistence

Proposition 4.2. Let $0 < a < b < 1$ and $0 \leq w \leq 1$. Define

$$x_t = x_1[wa^{t-1} + (1-w)b^{t-1}], \quad (4.3)$$

$$H_t = x_1 \left[\frac{wa^{t-1}}{1-a} + \frac{(1-w)b^{t-1}}{1-b} \right]. \quad (4.4)$$

Then

$$V_T = B_0 Q^{w(1-a^T)/(1-a) + (1-w)(1-b^T)/(1-b)}, \quad (4.5)$$

$$V_\infty = B_0 Q^M, \quad M = \frac{w}{1-a} + \frac{1-w}{1-b}. \quad (4.6)$$

Proof. Subtract adjacent values in (4.4). The two differences are $x_1 wa^{t-1}$ and $x_1(1-w)b^{t-1}$, which sum to (4.3). Both tail components vanish; their initial differences sum to x_1 . Theorem 3.2 gives (4.5)–(4.6). A nonnegative weighted sum of decreasing positive geometric profiles is decreasing and positive, establishing monotone reversion. ▀

The coefficient a describes the fast component and b the slow component. Temporary demand or operating conditions may motivate the former, while a more durable organizational or competitive advantage may motivate the latter. These are interpretations of components of log-relative profitability, not an accounting decomposition of ROE into additive business-unit returns.

Relative to the benchmark, the model uses three shape parameters instead of one. Restricting $a < b$ fixes component labels, but does not eliminate weak identification when the rates are similar or a weight is near zero. At $w = 0$ or $w = 1$, the model collapses to a single geometric component. The slow component eventually dominates whenever its weight is positive. This specification offers a clear economic extension when there is a substantive reason to distinguish transitory and durable influences.

4.3. Rational decay: a persistent but summable tail

Proposition 4.3. For $c > 0$, define

$$x_t = x_1 \frac{c(c+1)}{(t+c-1)(t+c)}, \quad H_t = x_1 \frac{c(c+1)}{t+c-1}. \quad (4.7)$$

Then the path has elementary valuations

$$V_T = B_0 Q^{(c+1)T/(T+c)}, \quad V_\infty = B_0 Q^{c+1}. \quad (4.8)$$

Proof. The identity

$$\frac{1}{t+c-1} - \frac{1}{t+c} = \frac{1}{(t+c-1)(t+c)} \quad (4.9)$$

shows that $H_t - H_{t+1} = x_t$. Also $H_t \rightarrow 0$ and $x_t = x_1$ at $t = 1$. Substitution of $H_1 = x_1(c + 1)$ and $H_{T+1} = x_1c(c + 1)/(T + c)$ into (3.8) gives (4.8). The profile is positive and decreasing because its denominator increases. ▀

The proportional persistence of the deviation changes with time:

$$\frac{x_{t+1}}{x_t} = \frac{t+c-1}{t+c+1} \quad (x_1 \neq 0). \quad (4.10)$$

The fraction eliminated each period is $2/(t + c + 1)$, which declines over time. This can represent a setting in which easily eroded sources of advantage disappear first, leaving a more resistant remainder. That account is an economic interpretation, not a derived microfoundation.

The construction deliberately distinguishes a slowly declining tail from a nonsummable path. Here x_t declines at order t^{-2} , so cumulative profitability remains finite even though relative adjustment becomes progressively slower. The remaining log-valuation tail declines only at order T^{-1} . The single parameter has the simple interpretation $M = c + 1$, but short forecast horizons can substantially understate the eventual valuation adjustment.

4.4. Finite linear fading: a known expiration horizon

Proposition 4.4. Let $N \geq 1$ be an integer. Define

$$\begin{aligned} x_t &= x_1(N - t + 1)/N \quad (1 \leq t \leq N), \\ x_t &= 0 \quad (t \geq N + 1). \end{aligned} \quad (4.11)$$

and the finite piecewise elementary tail

$$\begin{aligned} H_t &= x_1(N - t + 1)(N - t + 2)/(2N) \quad (1 \leq t \leq N), \\ H_t &= 0 \quad (t \geq N + 1). \end{aligned} \quad (4.12)$$

Then, writing $m = \min(T, N)$,

$$V_T = B_0 Q^{m(2N-m+1)/(2N)}, \quad V_\infty = B_0 Q^{(N+1)/2}. \quad (4.13)$$

Proof. For $1 \leq t < N$, subtracting the quadratic numerators in (4.12) gives $2(N - t + 1)$, which produces (4.11). At $t = N$, the difference is x_1/N ; after N , both tails are zero. Thus the representation holds at the boundary as well as within each regime. The initial tail is $x_1(N + 1)/2$, and the sum through m is the arithmetic-series expression in (4.13). The magnitude falls by $|x_1|/N$ per period until it reaches zero. ▀

This model assumes that above- or below-benchmark profitability has a predetermined end date. An expiring contract, license, or protected business opportunity may make a finite horizon economically relevant. However, a known expiration date does not itself imply linear fading: the constant absolute decline on the log-relative scale is an additional assumption.

The model is transparent and reaches its terminal valuation exactly at $T = N$. Its limitation is the change of regime after the final nonzero deviation. It also restricts M to the values $(N + 1)/2$

for integer N , rather than allowing every real persistence level. If $N = 1$, only the first-period deviation is nonzero, giving $V_\infty = B_0 Q$.

4.5. A telescoping gross-return ratio: nonlinear adjustment

Proposition 4.5. Let $0 < q < 1$ and $d > -1$. Specify the relative gross return directly as

$$X_t = \frac{1+dq^{t-1}}{1+dq^t}, \quad H_t = \log(1 + dq^{t-1}). \quad (4.14)$$

Every numerator and denominator is positive, and

$$V_T = B_0 \frac{1+d}{1+dq^T}, \quad V_\infty = B_0(1+d). \quad (4.15)$$

Given an initial relative gross return Q , elementary calibration requires

$$d = \frac{Q-1}{1-qQ}, \quad 0 < Q < \frac{1}{q}. \quad (4.16)$$

On this domain,

$$V_\infty = B_0 \frac{Q(1-q)}{1-qQ}. \quad (4.17)$$

Proof. If $d \geq 0$, positivity is immediate. If $-1 < d < 0$, then $dq^{t-1} \geq d > -1$, so positivity also holds. Taking logarithms in (4.14) gives $x_t = H_t - H_{t+1}$, and $H_t \rightarrow \log 1 = 0$. Theorem 3.2 gives (4.15), equivalently obtained by canceling adjacent factors in the finite product. Solving $Q = (1+d)/(1+dq)$ gives (4.16). The map from $d \in (-1, \infty)$ to Q is strictly increasing with range $(0, 1/q)$; this establishes the exact admissible domain. Substitution yields (4.17). ▀

For completeness, this model also has one-sided monotone reversion. Put $u_t = dq^{t-1}$ and $f(u) = (1+u)/(1+qu)$. On the admissible path, $f'(u) = (1-q)/(1+qu)^2 > 0$. Since u_t approaches zero monotonically, $X_t = f(u_t)$ approaches one monotonically. The logarithm preserves this ordering, giving the required behavior of x_t and ROE.

The implied recursion is

$$X_{t+1} = \frac{1}{1+q-qX_t}. \quad (4.18)$$

Its derivative at the equilibrium $X = 1$ equals q , so q measures local persistence. Away from equilibrium, adjustment is nonlinear. The tail is an elementary logarithmic function of a geometrically declining state, which is the source of telescoping solvability. A direct economic mechanism for this particular nonlinear mapping would require additional modeling.

This model is not globally separable as $x_t = x_1 g_t$ with a profile independent of x_1 . For example, at fixed q ,

$$\frac{\partial \log(V_\infty/B_0)}{\partial \log Q} = \frac{1}{1-qQ}. \quad (4.19)$$

The sensitivity rises sharply as qQ approaches one from below. Thus the admissibility restriction is economically material, not merely technical. For $Q > 1$, the choice of q is bounded above by $1/Q$. When $Q = 1$, calibration gives $d = 0$ and the equilibrium value is B_0 .

5. Economic interpretation and model comparison

5.1. What constitutes a useful specification?

An elementary formula is valuable because it makes assumptions and comparative statics inspectable. It does not, by itself, make a model economically persuasive. We evaluate the five specifications using two primary criteria: simplicity of the mathematical form and clarity of its economic interpretation. Parameter admissibility, sensitivity, and convergence speed provide secondary checks that determine whether this simplicity is useful in practice.

Simplicity has at least two dimensions. A formula may be short while requiring a difficult calibration, or it may use several transparent parameters with a longer formula. Likewise, an economically suggestive narrative is weaker than a structural derivation. The interpretations offered here are mechanisms compatible with the assumed paths; none is empirically established or uniquely implied by the closed form.

Table 1 records the mathematical comparison. Its final column is the remaining log-valuation tail H_{T+1} , which determines finite-horizon error exactly through (3.15). The first four models admit an initial-deviation-independent persistence measure M . The last instead gives a nonlinear valuation in Q at fixed q .

Table 1. Elementary valuations and remaining log-valuation tails. All models share B_0 , r , and R_1 . The parameter column lists additional shape inputs. In the finite model, the displayed tail applies for $T < N$ and is zero for $T \geq N$.

Specification	Shape inputs	Terminal ratio V_∞/B_0	Remaining tail H_{T+1}
Geometric	$0 < a < 1$	$Q^{1/(1-a)}$	$x_1 a^T / (1 - a)$
Fast-slow	$0 < a < b < 1;$ $0 \leq w \leq 1$	$Q^{w/(1-a)+(1-w)/(1-b)}$	$x_1 [wa^T / (1 - a) + (1 - w)b^T / (1 - b)]$
Rational	$c > 0$	Q^{c+1}	$x_1 c(c + 1) / (T + c)$
Finite linear	Integer $N \geq 1$	$Q^{(N+1)/2}$	$x_1 (N - T)(N - T + 1) / (2N)$
Gross-return ratio	$0 < q < 1;$ $0 < Q < 1/q$	$Q(1 - q)/(1 - qQ)$	$\log(1 + dq^T),$ with $d = (Q - 1)/(1 - qQ)$

5.2. Strengths, limitations, and preferred uses

Table 2 summarizes the economic interpretations, strengths, and limitations of the five specifications.

Table 2. Qualitative comparison of economic meaning and simplicity. These are analytical judgments about the specifications, not empirical rankings.

Model	Main economic interpretation	Main strength	Main limitation
Geometric	A constant fraction of the remaining log-relative profitability disappears each period.	One shape parameter; direct recursion and half-life.	A single adjustment rate may be too restrictive.
Fast–slow	Transitory and durable components disappear at different rates.	Clear extension when two sources of persistence are plausible.	Three shape parameters; components may be weakly identified.
Rational	The remaining advantage becomes progressively harder to erode.	One parameter; permits a summable, long-lived tail.	Long-horizon value can be missed by short forecasts; mechanism is assumed.
Finite linear	Profitability fades toward a predetermined expiration date.	Transparent horizon; exact finite termination.	Linear fading and an abrupt end of adjustment need justification.
Gross-return ratio	A geometrically fading latent state drives nonlinear gross-return adjustment.	Compact finite and limiting valuation formulas.	Less direct economic foundation; initial-return domain and high sensitivity near its boundary.

Among these alternatives, geometric reversion is the most natural benchmark when the aim is to minimize assumptions and keep the economic message immediately visible. Its parameter answers a direct question: what fraction of the remaining log-relative advantage persists into the next period? This interpretation is equally clear for gradual recovery from an initial disadvantage, subject to the firm-survival assumption.

The fast–slow model is the preferred extension when explanatory richness matters more than using a single parameter. It distinguishes two economically recognizable persistence scales without sacrificing elementary solutions. The interpretation is strongest when the application identifies a reason for two components independently of the valuation equation. Otherwise, the extra flexibility risks becoming a convenient fit rather than a substantive explanation.

Rational decay is useful when persistent remnants of an initial advantage are central to the argument. Its decreasing erosion fraction makes the long tail explicit. Relative to geometric

reversion with the same total log-profitability, it assigns more valuation weight to distant dates, as the calibrated illustration below demonstrates. This feature should be treated as an economically consequential assumption because the present setup excludes the uncertainty and survival risks that often become more important at long horizons.

Finite linear fading is compelling when a relevant terminal date can be defended. The horizon parameter is easy to communicate, but the mathematical convenience of a linear profile should not be mistaken for an implication of contract expiration. The final specification is particularly useful for demonstrating that nonlinear gross-return dynamics can also admit elementary solutions. Its economic interpretation is less immediate than that of the benchmark or fast–slow model, and its domain restriction makes it a less attractive default.

These comparisons support a conditional recommendation: use geometric reversion as the baseline; use fast–slow reversion when two persistence mechanisms can be articulated; use rational or finite fading when the relevant tail behavior or expiration horizon is central to the economic setting. The nonlinear ratio model is a useful analytical alternative, but its structural motivation should be supplied before it is adopted as the principal specification.

5.3. Convergence curves under a common terminal valuation

A controlled numerical illustration separates terminal value from adjustment shape. It uses hypothetical parameters only; it is neither an empirical test nor an estimated calibration. Set

$$B_0 = 100, \quad r = 0.08, \quad R_1 = 0.20, \quad Q = \frac{10}{9}, \quad K = Q^5. \quad (5.1)$$

For the geometric model choose $a = 0.8$. For the fast–slow model choose $a = 0.5$, $b = 0.9$, and $w = 5/8$. For rational decay use $c = 4$, and for finite linear fading use $N = 9$. These four choices all give $M = 5$. For the gross-return ratio model choose

$$d = K - 1, \quad q = \frac{K-Q}{Q(K-1)}. \quad (5.2)$$

Because $1 < Q < K$, this choice satisfies $0 < q < 1/Q$ and reproduces both the same initial return and the same terminal ratio K . Numerically, $q \approx 0.755806$ and all five specifications give $V_\infty \approx 169.3509$.

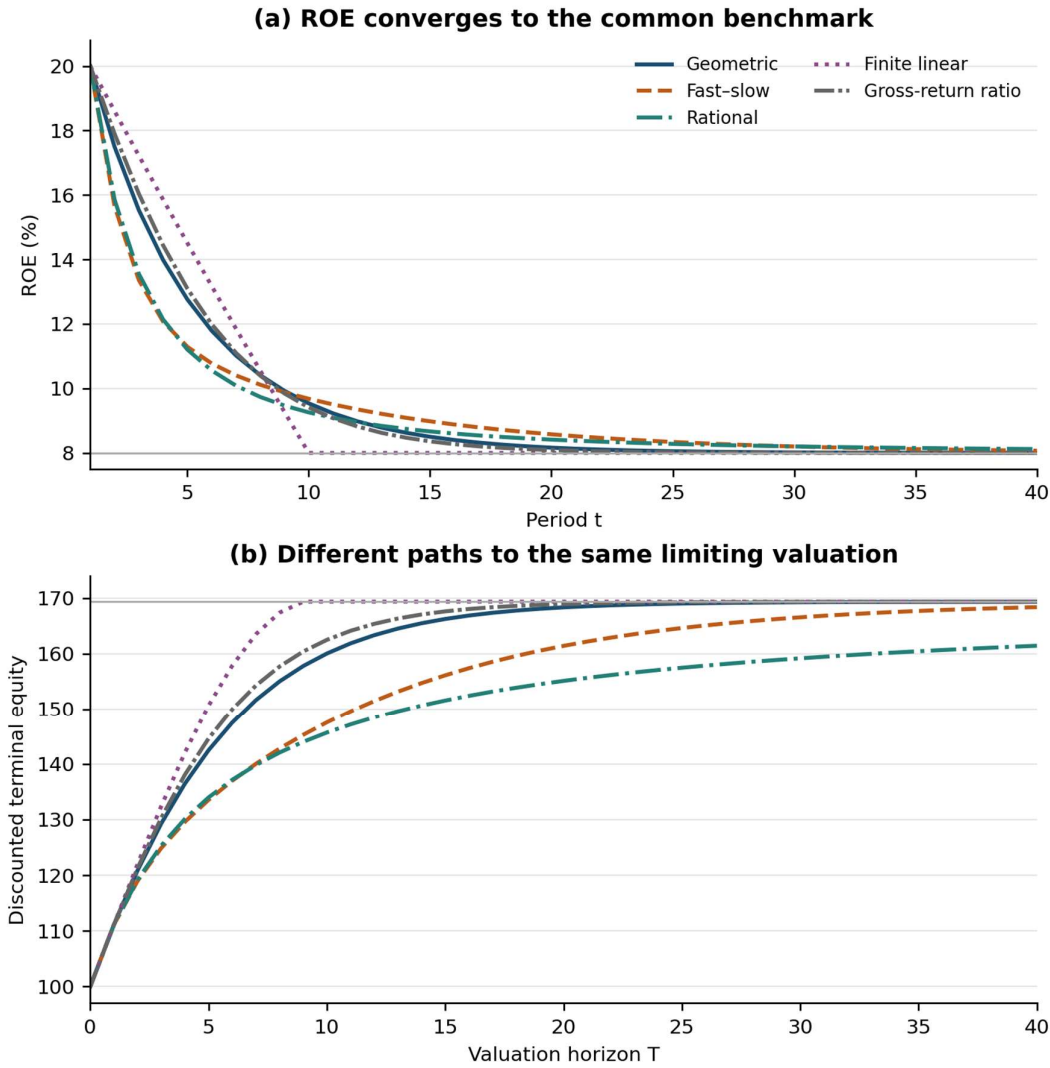


Figure 1. Alternative paths with the same initial ROE and terminal value. Panel (a) shows annual ROE; the horizontal line is the 8% benchmark. Panel (b) shows discounted terminal equity for successive horizons; the horizontal line is the common limiting value. Lines connect discrete-period observations. All inputs are hypothetical and specified in (5.1)–(5.2).

Figure 1 shows that the common initial and terminal conditions do not determine a unique intervening path. The finite model reaches its limiting value after nine periods, while every other model continues to adjust. Rational decay leaves a visibly larger gap at longer horizons despite sharing the same terminal value. The fast–slow model initially sheds its transitory component and subsequently adjusts mainly at the slower rate.

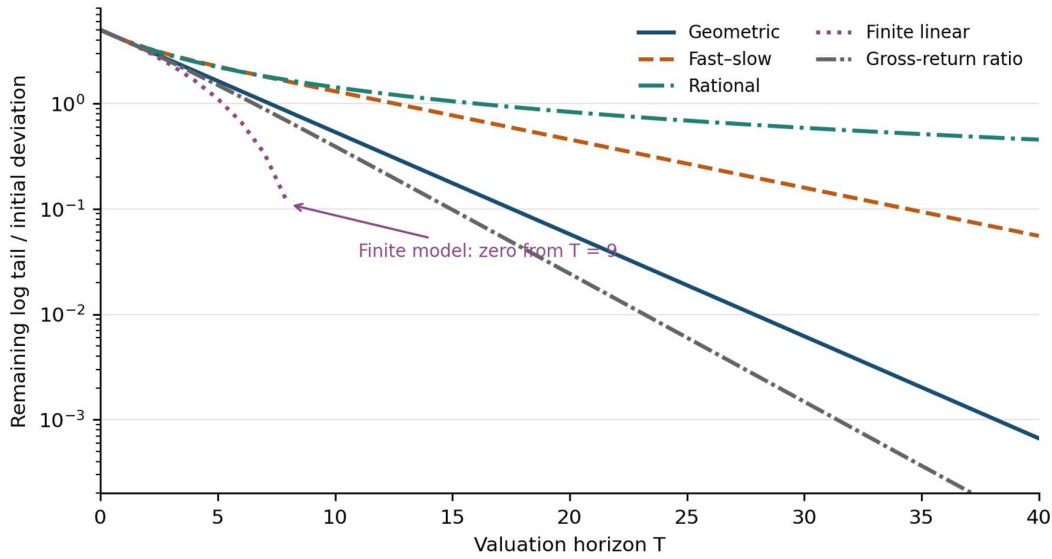


Figure 2. Remaining log-valuation tail divided by the initial log-relative profitability. The vertical axis is logarithmic. A smaller ordinate means a smaller remaining cumulative deviation and hence a smaller log-valuation error. The finite model is shown only while its tail is positive and has exactly zero error for all horizons $T \geq 9$. Parameters match Figure 1.

Figure 2 isolates the error that a finite forecast horizon would leave. Geometric tails decline exponentially, the rational tail declines algebraically, and the finite tail disappears entirely at the specified horizon. The nonlinear model's tail is logarithmic in a geometric state and is asymptotically proportional to q^T for a nonzero initial deviation. In the fast-slow model, the slow component controls the eventual rate whenever its weight is positive.

For example, at $T = 10$, the rational model leaves $H_{11}/x_1 = 20/14 \approx 1.4286$, whereas the geometric benchmark leaves $5(0.8)^{10} \approx 0.5369$. Equation (3.15) translates these into shortfalls of approximately 14.0% and 5.5%, respectively, relative to the common terminal valuation. The difference reflects the assumed timing of profitability rather than a difference in total long-run log-profitability.

Proposition (finite-horizon error under equal cumulative persistence). Fix the same initial book equity, discount rate, initial log-relative profitability and cumulative persistence for the geometric and rational specifications. For any real cumulative persistence greater than one, choose the parameters as follows:

$$M > 1, \quad a = \frac{M-1}{M}, \quad c = M-1.$$

Both specifications then have the same limiting value. Their remaining log tails are

$$V_\infty = B_0 \exp(Mx_1),$$

$$H_{T+1}^{\text{geo}} = x_1 M \left(\frac{M-1}{M} \right)^T, \quad H_{T+1}^{\text{rat}} = x_1 \frac{M(M-1)}{T+M-1}.$$

For every integer horizon at least two, if initial log-relative profitability is nonzero, the rational specification has a strictly larger absolute log valuation error:

$$\left| \log \frac{V_\infty}{V_T^{\text{rat}}} \right| > \left| \log \frac{V_\infty}{V_T^{\text{geo}}} \right|, \quad T \geq 2, \quad x_1 \neq 0.$$

Proof. Bernoulli's inequality gives

$$\left(1 + \frac{1}{M-1} \right)^T \geq 1 + \frac{T}{M-1}.$$

Taking reciprocals of these positive quantities yields

$$\left(\frac{M-1}{M} \right)^T \leq \frac{M-1}{T+M-1}.$$

The inequality is strict for integer horizons at least two. Multiplying by the positive magnitude of the common initial log-relative profitability and by the common cumulative persistence, and using the exact log-error identity, proves the claim. The tails coincide at horizons zero and one. If initial profitability equals the benchmark, both values equal initial book equity at every horizon.

5.4. Scope of inference and implications for model choice

Three distinctions delimit the contribution. First, convergence is not the same as elementary expressibility. The summability theorem answers the former question exactly, while the tail construction gives a verifiable sufficient answer to the latter. Second, an elementary price formula does not identify a unique economic adjustment process. Even the deliberately restricted examples in Section 5.3 agree on terminal value while disagreeing on the time profile. Third, the interpretability of a path is separate from the validity of the terminal-realization convention used to value it.

The deterministic assumption also matters. If profitability were random, the discounted expected terminal value would involve the expectation of an exponential of cumulative log-relative returns. In general, this cannot be obtained by simply inserting expected log-relative returns into the deterministic formula. Moment existence and interchange of expectations and limits would need separate justification. The present theorems therefore do not claim stochastic valuation formulas.

These limitations point to a disciplined use of closed forms. Select a path because its adjustment mechanism fits the economic question, verify its admissible parameter domain, and then use the elementary expression to examine persistence and truncation effects. Choosing a specification solely because a product cancels would reverse that order. Conversely, economic flexibility need not eliminate tractability: elementary vanishing tails and their finite combinations offer a systematic way to build additional interpretable models.

6. Conclusion

This paper develops a constructive framework for elementary equity valuation under complete earnings retention and a specified terminal-realization convention. The basic convergence requirement is exact: discounted terminal book equity has a strictly positive finite limit if and only if cumulative log-relative profitability converges. Mean reversion of ROE toward the discount rate is necessary, but is not sufficient without adequate cumulative decay.

The vanishing-tail representation converts this requirement into a practical modeling device. Expressing each period's log-relative profitability as the difference between adjacent values of a vanishing tail makes the valuation telescope. When that tail and its calibration are elementary, both the finite-horizon valuation and its limit are elementary. This construction supplies a broad sufficient family while keeping separate the unresolved, more general task of classifying all elementary terminal expressions.

Five specifications demonstrate the framework: geometric adjustment, fast and slow components, rational decay, finite linear fading, and a nonlinear telescoping gross-return ratio. Each has an explicit admissible domain, a direct proof, and an exact expression for remaining valuation error. Together, they show that elementary solvability does not require a unique mean-reversion law.

The comparison also yields a general result beyond the hypothetical calibration. Holding initial book equity, the discount rate, initial log-relative profitability, and cumulative persistence fixed, with cumulative persistence greater than one, rational decay leaves a strictly larger absolute log-valuation error than geometric reversion at every integer horizon of at least two periods whenever the initial deviation is nonzero. Identical limiting valuations therefore need not imply equally accurate finite-horizon valuations.

The economic comparison favors geometric reversion as the simplest benchmark and the fast-slow specification as a transparent extension when heterogeneous persistence is substantively motivated. Rational and finite-horizon specifications offer different economically meaningful tail assumptions, while the nonlinear ratio model illustrates additional mathematical flexibility with a less immediate structural interpretation. Equal terminal values across these models can conceal materially different adjustment paths. Model choice should therefore be guided jointly by elementary tractability, an explicit economic mechanism, and the implications of the assumed tail for finite-horizon valuation.

Notation

Table 3. Symbols and parameter domains. Time is discrete. The same symbol a denotes the geometric persistence coefficient in Section 4.1 and the fast persistence coefficient in Section 4.2; model-specific parameters are not used simultaneously across specifications.

Symbol	Definition or interpretation	Domain or restriction
t, s	Period index; s is also a dummy summation index.	Positive integers in sums.
T	Finite valuation horizon; $V_0 = B_0$ in the illustration.	Integer $T \geq 1$; $T = 0$ when displaying the initial value.
B_0, B_t	Initial and end-of-period book equity.	$B_0 > 0$; $B_t = B_{t-1}(1 + R_t) > 0$.
R_t, R_1	ROE on opening book equity; its initial future-period value.	$R_t > -1$.
r	Constant discount rate and assumed long-run ROE benchmark.	$r > -1$.
X_t	Gross ROE relative to the gross discount rate.	$X_t = (1 + R_t)/(1 + r) > 0$.
x_t, x_1	Log-relative profitability and its initial value.	$x_t = \log X_t$; real.
Q	Initial relative gross return.	$Q = X_1 = e^{x_1} > 0$.
V_T, V_∞	Discounted realizable terminal equity and its limiting value.	$V_T = B_T/(1 + r)^T$; a positive finite limit requires Theorem 3.1.
S_T, S	Cumulative log-relative profitability and its sum when convergent.	$S_T = \sum_{t=1}^T x_t$; $S \in \mathbb{R}$ when it exists.
H_t	Unique vanishing tail of cumulative log-relative profitability.	$x_t = H_t - H_{t+1}$; $H_t \rightarrow 0$.
$H(t; \theta), \theta$	Elementary tail function and its finite parameter vector.	Real admissible domain; calibration satisfies (3.9).
s, G_t	Sign of the initial deviation and its sign-adjusted tail in Proposition 3.2.	Here $s = \text{sign}(x_1)$ and $G_t = sH_t$; $x_1 \neq 0$.
g_t	Normalized time profile in the separable subclass.	$x_t = x_1 g_t$; $g_1 = 1$. The compared profiles are nonnegative.
M	Equivalent duration of initial log-relative profitability.	$M = \sum_{t=1}^\infty g_t$; finite, and $M \geq 1$ for nonnegative profiles.

Symbol	Definition or interpretation	Domain or restriction
ε	Bound on the magnitude of the remaining log-valuation tail.	$\varepsilon \geq 0$.
a	Geometric persistence; alternatively, the fast-component persistence.	$0 < a < 1$; in Section 4.2, $0 < a < b < 1$.
b, w	Slow-component persistence and weight on the fast component.	$a < b < 1$; $0 \leq w \leq 1$.
c	Shape parameter of rational decay in Section 4.3.	$c > 0$; $M = c + 1$. Elsewhere, c is a nonzero constant in convergence counterexamples.
p	Power in the convergence diagnostic $x_t = c/t^p$.	$p > 0$; for $c \neq 0$, a positive finite valuation requires $p > 1$.
N, m	Last period with a potentially nonzero deviation in finite fading; truncated horizon.	Integer $N \geq 1$; $m = \min(T, N)$.
q	Geometric state persistence in the nonlinear gross-return ratio model.	$0 < q < 1$; initial calibration also requires $qQ < 1$.
d	Initial latent-state deviation in the gross-return ratio model.	$d > -1$; $d = (Q - 1)/(1 - qQ)$.
$u_t, f(u)$	Latent state and gross-return mapping used to establish monotonicity.	$u_t = dq^{t-1}$; $f(u) = (1 + u)/(1 + qu)$ on the admissible path.
K	Common terminal valuation-to-book ratio in Section 5.3.	In that illustration, $K = Q^5$.

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