

7537363-continuous-signed-weights-in-multi-factor-portfolio-design-a-factorial-design-an

SUMMARY: This paper develops a continuous signed-weight framework for multi-factor portfolio design, where positive and negative weights favor opposite directions, and zero removes a factor from the composite score. The study uses a three-level full factorial design with 243 points, including a non-discriminating market-average benchmark. The authors evaluate portfolio performance metrics using Chinese A-share data from 2005 to 2025. They find that positive ROE weights are prominent in high-return configurations, while negative ROE weights are common in low-volatility configurations. Negative MOM weights show strong return and risk-adjusted performance, consistent with reversal in the sample. The framework allows for the study of portfolio performance across a bounded continuous signed-weight region, providing a structured approach to examining the effects of factor weights and their interactions.

METHOD: The study employs a three-level full factorial design with 243 design points, including a non-discriminating market-average benchmark. The continuous signed-weight space is sampled through this design, with 242 active scoring rules defined by nonzero signed-weight points. Portfolio responses at the factorial design points are used to study the structure of the signed-weight performance surface, examining conditional performance differences and comparing recurring patterns.

KEY FINDINGS:

- Positive ROE weights are prominent in high-return configurations.
- Negative ROE weights are common in low-volatility configurations.
- Negative MOM weights show strong return and risk-adjusted performance.
- The effects of factor weights and their interactions vary with the performance objective and other weights.
- The framework provides a structured way to study portfolio performance across a bounded continuous signed-weight region.

IMPLICATION FOR TRADING: The continuous signed-weight framework offers a structured approach to studying multi-factor portfolio performance, allowing for the examination of factor weights and their interactions. This can help traders and portfolio managers understand how different weight specifications and factor combinations affect portfolio returns and risk. By identifying the optimal weight configurations, traders can potentially improve their investment strategies and enhance performance.

Continuous Signed Weights in Multi-Factor Portfolio Design: A Factorial Design and Response-Surface Approach

Abstract

Traditional multi-factor portfolios often assign factor directions in advance. This paper develops a continuous signed-weight framework in which positive and negative weights favor opposite directions, while zero removes a factor from the composite score. We study five factors—price-to-book ratio, return on equity (ROE), market value, Beta, and momentum (MOM)—using a three-level full factorial design with $3^5=243$ points. The all-zero point serves as a non-discriminating market-average benchmark; the other 242 points represent active scoring configurations.

Using Chinese A-share data from 2005 to 2025, we evaluate average return, volatility, Sharpe ratio, and maximum drawdown. Conditional comparisons, extreme-design-point analysis, and second-order response-surface models examine how performance varies with factor weights and their interactions.

Positive ROE weights are prominent among high-return configurations, while negative ROE weights are more common among low-volatility configurations. Negative MOM weights show strong return and risk-adjusted performance, consistent with reversal in this sample. The effects of market value, Beta, and price-to-book ratio depend more strongly on the investment objective and other weights. The framework provides a structured way to study portfolio performance across a bounded continuous signed-weight region.

Keywords: Multi-factor investing; Signed factor weights; Factor direction; Factor-weight; Response-surface methodology; Factorial design

1. Introduction

Multi-factor investing has become a central framework in empirical asset pricing and quantitative portfolio construction. A large literature documents systematic relations between firm characteristics and future stock returns, including size, valuation, profitability, investment, risk, and momentum. These findings have motivated increasingly broad factor models and practical scoring systems in which multiple characteristics are combined to rank securities and construct portfolios.

A common feature of such systems is that factor direction is usually specified before portfolio construction begins. Value-oriented strategies typically reward lower valuation ratios, profitability strategies favor stronger operating performance, momentum strategies favor continuation in prior returns, and defensive strategies favor lower risk. Once these directional conventions have been imposed, empirical studies generally examine alternative factor-weight specifications and their implications for portfolio performance.

This practice embeds an important restriction. The preferred direction of a characteristic need not be invariant across investment objectives or surrounding weight structures. A factor orientation associated with higher expected return may differ from one associated with lower volatility or smaller drawdown. Likewise, the effect of one factor weight may depend on the surrounding weight structure. If factor signs are fixed in advance, these possibilities cannot be examined within the empirical framework.

This paper addresses this issue through a continuous signed-weight framework for multi-factor portfolio design. Let

$$\mathbf{w} = (w_1, \dots, w_K), \quad w_k \in [-1, 1], \quad (1)$$

denote the factor-weight vector. For standardized factor values $X_{k,i,t}$, the composite score of stock i at time t is based on

$$S_{i,t}(\mathbf{w}) = \sum_{k=1}^K w_k X_{k,i,t}. \quad (2)$$

A positive weight favors higher values of a factor, a negative weight favors lower values, and a zero weight removes that factor from the score. The magnitude $|w_k|$ represents relative intensity within the scoring rule. Factor direction and factor intensity are therefore not treated as separate ex ante decisions; they are jointly represented within the signed-weight vector.

An important property of this framework is that the weights are relative rather than absolutely identified. Because portfolio formation depends on the ranking of composite scores, positive proportional rescaling of a weight vector preserves the stock ordering, apart from possible ties or additional implementation rules. Consequently, positively

proportional vectors may represent the same ranking rule and generate the same portfolio. The framework therefore admits scale-equivalent representations, and a numerically unique weight vector need not exist. Section 3 develops this property formally.

The continuous signed-weight space creates a substantial empirical challenge. Even with only five factors, portfolio performance may vary with individual weight directions, relative intensities, pairwise combinations, and the performance objective under consideration. Directly evaluating every point in a continuous five-dimensional region is infeasible. The present study therefore combines a continuous signed-weight framework with structured experimental design and response-surface methodology.

Specifically, the empirical analysis considers five factors: price-to-book ratio (PB), return on equity (ROE), market value (MV), Beta, and momentum (MOM). The continuous region $\mathbf{w} \in [-1, 1]^5$ is sampled through a three-level full factorial design with $w_k \in \{-1, 0, +1\}$.

This produces $3^5=243$ design points. Of these, 242 are nonzero signed-weight points that define active scoring rules. The center point, $\mathbf{w}=0$, assigns the same score to every stock and therefore provides no cross-sectional discrimination. It is not interpreted as an active factor-selection strategy. Nevertheless, it is retained as the factorial center and included in the 243-point response-surface regressions as a non-discriminating equal-weighted eligible-universe benchmark.

Portfolio responses at the factorial design points are used to study the structure of the signed-weight performance surface. The empirical analysis first examines conditional performance differences associated with negative, zero, and positive factor settings and then compares recurring structures among selected high- and low-performance design points. These descriptive analyses are complemented by second-order response-surface models containing linear, pairwise interaction, and quadratic terms. The fitted models provide parsimonious approximations to performance variation across the sampled signed-weight space and are subsequently examined over the bounded continuous region $[-1, 1]^5$.

The fitted response surfaces should be distinguished from the 243 directly evaluated factorial design points. They provide continuous empirical approximations to the observed portfolio-performance patterns rather than additional observed design points.

The empirical application uses Chinese A-share data from 2005 to 2025 and evaluates four portfolio-performance responses: average return, volatility, the Sharpe ratio, and maximum drawdown. China provides a particularly informative setting for examining signed factor structures within a major emerging equity market. The analysis therefore permits the preferred signed-weight structure to vary across performance objectives rather than imposing a universal factor orientation in advance.

The empirical results reveal several broad patterns. First, signed factor structure is strongly objective-dependent. The clearest example is ROE: positive ROE settings are strongly represented among high-return design points, whereas negative ROE settings are prominent among low-volatility design points. Thus, the preferred orientation of profitability changes materially with the portfolio objective.

Second, MOM displays pronounced directional asymmetry in the Chinese sample. Negative MOM settings are strongly associated with several return-oriented and risk-adjusted comparisons, indicating an important role for reversal-oriented structures. By contrast, PB, MV, and Beta exhibit more conditional patterns whose preferred settings vary with the objective and surrounding weight structure.

Third, the second-order response-surface models achieve high explanatory power for variation across the design-point responses, and several interaction and quadratic coefficients are statistically significant.

These results indicate that portfolio-performance variation is not summarized solely by isolated one-weight contrasts. The association between a factor weight and portfolio performance may depend on surrounding weights and on the broader location within the fitted surface. At the same time, interaction and quadratic coefficients are interpreted as features of the estimated response surface rather than direct identification of specific behavioral or institutional mechanisms.

The paper makes three main contributions. First, it formalizes factor direction as part of a continuous signed-weight framework rather than as a convention fixed before portfolio construction. Conventional weighted-scoring approaches typically specify factor signs in advance and then assign relative weights accordingly. The proposed framework allows both factor signs and relative weights to vary jointly within a bounded continuous signed-weight region.

Second, the paper integrates full factorial experimental design with continuous signed-weight response-surface analysis. The three-level factorial design provides a structured sample of the continuous signed-weight region, while the second-order response-surface models summarize the portfolio-performance patterns observed across the sampled design points.

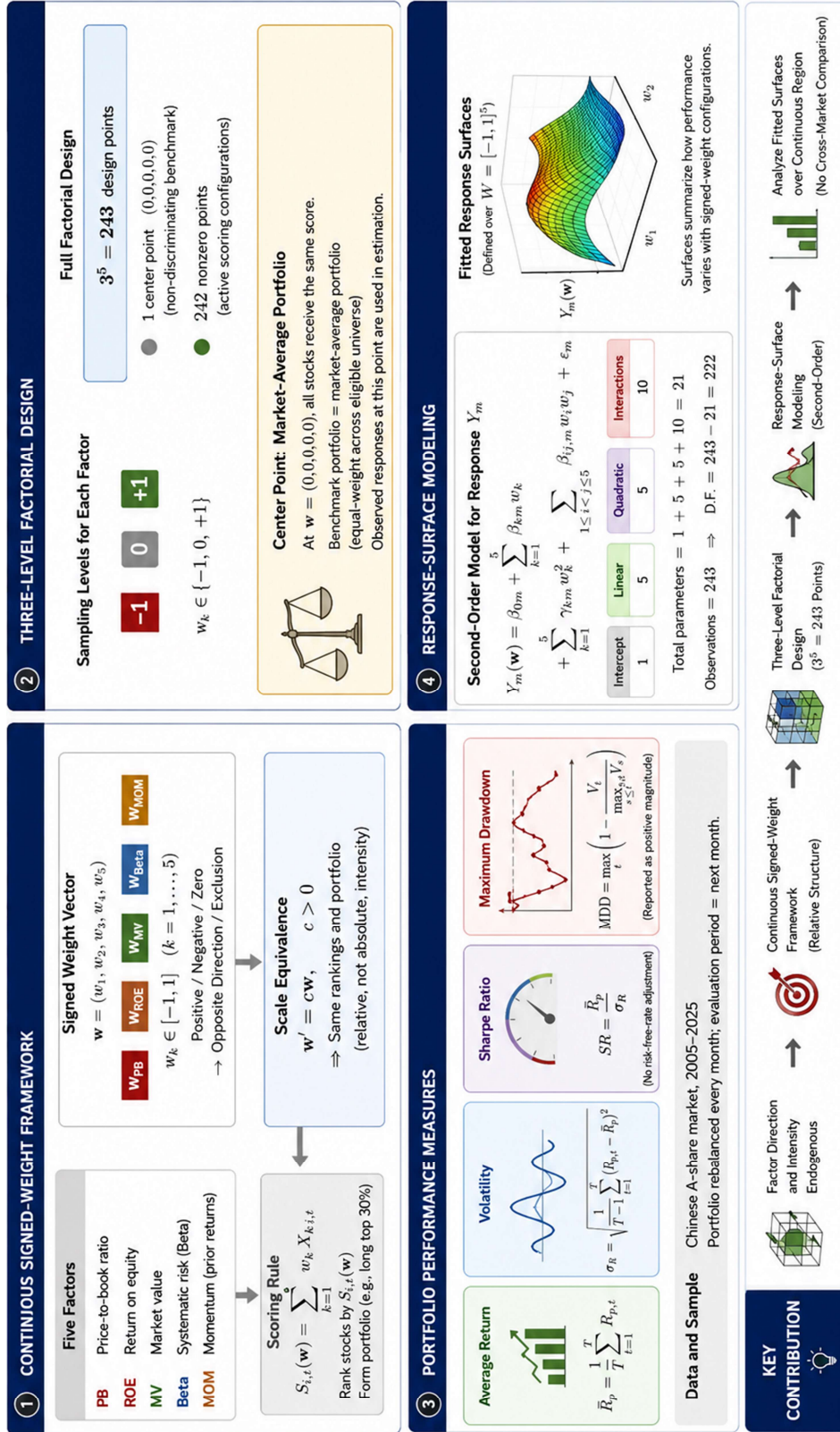
Third, the study shifts attention from isolated factor effects toward conditional multi-factor weight structures. By combining conditional design-point comparisons, extreme-point analysis, interaction terms, quadratic terms, and continuous signed-weight structures, the framework examines how portfolio performance varies with investment objective and surrounding weights.

Taken together, the analysis suggests that multi-factor portfolio construction should not be reduced to selecting factor magnitudes after fixing all factor signs in advance. A more general formulation treats factor signs and relative intensities as jointly variable and recognizes that preferred structures may differ across objectives

and surrounding weight structures. The contribution of the framework lies not in identifying a single universally preferred weight configuration, but in providing a structured framework for studying and approximating portfolio-performance variation over a continuous signed-weight space.

Figure 1 summarizes the overall research framework. The study begins by defining a continuous signed-weight space in which factor direction and relative intensity are jointly represented. A three-level full factorial design then samples this space at 243 design points using the levels -1 , 0 , and $+1$. For each design point, composite factor scores are constructed from PB, ROE, MV, Beta, and MOM, and portfolios are formed and evaluated using average return, volatility, the Sharpe ratio, and maximum drawdown. The resulting design-point responses are examined through descriptive comparisons and then used to estimate second-order response-surface models containing linear, pairwise interaction, and quadratic terms. Finally, the fitted surfaces are analyzed to characterize objective-dependent portfolio-performance variation and model-implied structures over the bounded continuous signed-weight region.

The remainder of the paper is organized as follows. Section 2 reviews the literature on factor investing, weighted scoring, factor direction, and response-surface analysis. Section 3 develops the continuous signed-weight framework and its scale-equivalence property. Section 4 describes the data, factorial design, portfolio construction, response measures, and second-order response-surface modeling. Section 5 presents the empirical results. Section 6 discusses their interpretation, methodological implications, and limitations. Section 7 concludes.



Note: The figure illustrates the overall research framework. All empirical results in this study are based solely on the Chinese A-share market (2005–2025).

Figure 1. Research Framework for Continuous Signed-Weight Multi-Factor Portfolio Analysis Using Factorial Design and Response-Surface Modeling

2. Literature Review

2.1 From Single-Factor Anomalies to Multi-Factor Portfolio Design

The modern literature on factor investing originates from evidence that firm characteristics are systematically related to the cross-section of stock returns. Early studies document return patterns associated with firm size, valuation, and related characteristics. Banz (1981) reports the size effect, Basu (1983) documents return differences associated with earnings-price ratios, and Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) provide early evidence on the role of book-to-market characteristics. Fama and French (1992) further show that size and book-to-market equity help account for cross-sectional variation in average stock returns.

The development of formal multi-factor models substantially broadened this literature. Fama and French (1993) augment the market factor with size and value factors, while Carhart (1997) adds momentum following the evidence of Jegadeesh and Titman (1993). Subsequent research identifies additional return-related characteristics and expands the set of candidate factors. Novy-Marx (2013) emphasizes profitability, and Fama and French (2015) incorporate profitability and investment factors into a five-factor model.

As the number of proposed return predictors increased, attention shifted from identifying individual anomalies toward evaluating larger sets of characteristics. Harvey, Liu, and Zhu (2016) emphasize the multiple-testing problem created by the rapid expansion of claimed factors, while Hou, Xue, and Zhang (2020) reassess a large body of published anomalies within a common empirical framework. This literature shows that the central problem in modern factor investing is no longer simply whether individual characteristics contain information, but how multiple signals should be selected, combined, and interpreted.

Practical multi-factor portfolio construction commonly addresses this problem through standardized scoring rules. Let $X_{k,i,t}$ denote the standardized value of characteristic k for stock i at time t . A general composite score can be written as

$$S_{i,t} = \sum_{k=1}^K w_k X_{k,i,t}, \quad (3)$$

where w_k determines the contribution of each factor to the ranking rule. This formulation provides a direct link between empirical factor research and portfolio design: factor signals are transformed into a common scale, combined through weights, and used to rank securities.

The resulting portfolio problem is inherently multivariate. Changing one factor weight alters the relative contribution of that characteristic to the composite score, while the effect of the change may depend on the remaining weights. Accordingly, multi-

factor portfolio construction involves not only factor discovery but also the structure of the weight vector used to combine signals.

2.2 Directional Ambiguity in Factor Investing

Although multi-factor methods allow signals to be combined flexibly, the sign of a factor is often specified before portfolio construction. Value strategies generally favor lower valuation ratios, profitability strategies favor stronger profitability, momentum strategies favor higher prior returns, and defensive strategies favor lower systematic risk. These conventions are economically interpretable and often empirically successful, but they also restrict the admissible weight space.

The valuation literature illustrates this issue. Traditional value research associates low relative prices with higher subsequent returns and interprets value premia through risk-based or behavioral mechanisms (Lakonishok, Shleifer, & Vishny, 1994; Fama & French, 1998). Yet high-valuation firms may also be favored under growth-oriented strategies when valuation multiples reflect expansion opportunities or other growth characteristics. Mohanram (2005), for example, develops a fundamentals-based approach specifically for growth stocks. International comparisons have also examined value versus growth alongside large-cap versus small-cap stocks (Bauman et al., 1998). The relevant portfolio question is therefore not exhausted by whether valuation matters; the preferred sign of a valuation-related signal may depend on the strategy and objective.

Momentum provides a clear example of horizon-dependent directional effects. Jegadeesh and Titman (1993) document continuation in past returns, whereas reversal has been documented at other horizons and in other settings (De Bondt & Thaler, 1985; Jegadeesh, 1990; Lehmann, 1990). Using Chinese A-share data, Kang et al. (2002) find significant abnormal profits for some short-horizon contrarian and intermediate-horizon momentum strategies, although they caution that the intermediate-horizon momentum profits are not distinct. These findings motivate evaluating both signs of a prior-return signal; they do not establish which sign performs better for the specific quarterly MOM measure and portfolio-formation rule used in this study.

Risk-related characteristics likewise admit competing orientations. Traditional asset-pricing intuition links greater systematic risk to greater expected return, whereas the low-volatility and betting-against-beta literatures document strong performance among lower-risk securities (Haugen & Heins, 1975; Blitz & van Vliet, 2007; Frazzini & Pedersen, 2014). Thus, positive and negative Beta weights may correspond to different legitimate portfolio objectives rather than to a universally correct and incorrect specification.

Similar ambiguity can arise for size and profitability. Small firms have long been associated with a size premium, but large firms may offer greater liquidity, stability, or defensive characteristics. High profitability is central to modern factor models (Novy-

Marx, 2013; Fama & French, 2015; Asness et al., 2019), yet the preferred signed weight on profitability in a multi-objective scoring problem need not be identical for return maximization, volatility minimization, Sharpe-ratio maximization, and drawdown control.

These examples imply a distinction between factor relevance and factor orientation. Evidence that a characteristic predicts returns does not by itself establish that one sign must be imposed under every objective or surrounding weight structure. Nevertheless, much of the literature evaluates factor significance, robustness, or combination after the directional interpretation of each signal has already been specified.

The unresolved issue is therefore not simply whether factors should be included, but whether their signs and relative intensities should be allowed to vary jointly within the portfolio-design problem.

2.3 Factor Combination, Factor Weighting, and Relative Scoring

Once multiple signals are admitted, portfolio construction requires a rule for combining them. The simplest approaches use equal weights, whereas more general methods assign different intensities according to predictive performance, risk, covariance structure, or an explicit portfolio objective. In a scoring framework, these choices can be represented by a weight vector $\mathbf{w}=(w_1,\dots,w_K)$. Quantitative portfolio-management treatments place signal combination and portfolio constraints within the broader construction process (Grinold & Kahn, 2000; Qian et al., 2007).

Most conventional formulations either restrict the weights to nonnegative values or impose directional preprocessing before portfolio construction. For example, a value characteristic may first be transformed so that a larger signal always represents greater presumed attractiveness; factor weights are then assigned to the pre-oriented signals to construct a composite score. This procedure is convenient, but it makes the resulting portfolio construction conditional on the chosen sign convention.

Allowing signed weights changes the problem. Under $w_k \in [-1,1]$, the sign of w_k determines which cross-sectional direction is favored, while $|w_k|$ determines relative intensity. A zero weight removes the factor from the scoring rule. Direction and intensity are therefore jointly represented by the same continuous variable rather than determined in separate stages.

Ranking-based scoring also creates an identification feature that differs from conventional budget-share formulations. Because positive proportional rescaling of a composite score preserves cross-sectional ordering, positively proportional feasible weight vectors may generate identical rankings and portfolios. The weights are therefore relative rather than uniquely identified absolute allocations. Section 3 formalizes this positive-scale equivalence property.

This property makes conventional simplex normalization problematic. In conventional mixture experiments, component proportions commonly satisfy (Cornell, 2011)

$$w_k \geq 0, \quad \sum_{k=1}^K w_k = 1. \quad (4)$$

Such restrictions identify mixture proportions naturally because the components are nonnegative shares of a whole. Signed factor scoring is different: positive and negative weights represent opposite ranking directions, and imposing a unit-sum constraint does not generally remove scale equivalence in an economically neutral way. The signed-weight problem therefore requires a design framework that can accommodate positive, negative, and zero coordinates without treating them as ordinary mixture proportions.

2.4 Experimental Design, Response Surfaces, and the Remaining Gap

Response-surface methodology extends this logic by fitting an approximate response function to the observed design-point outcomes. Although response surfaces are often used for optimization in engineering applications, they are used in the present study primarily to characterize and interpret portfolio-performance patterns across the sampled design points (Box & Wilson, 1951; Box & Draper, 1987; Myers, Montgomery, & Anderson-Cook, 2016).

These methods are well suited to multi-factor portfolio scoring because portfolio performance can be viewed as a response to the weight vector used to combine standardized signals. A second-order approximation takes the general form

$$Y(\mathbf{w}) = \beta_0 + \sum_{k=1}^K \beta_k w_k + \sum_{k=1}^K \gamma_k w_k^2 + \sum_{i < j} \beta_{ij} w_i w_j + \varepsilon, \quad (5)$$

where Y denotes a portfolio-performance measure. The linear terms describe first-order variation, the interaction terms allow the association of one weight with performance to depend on other weights, and the quadratic terms allow second-order variation within the fitted approximation.

Previous portfolio applications of experimental design and response-surface methods have shown that portfolio outcomes can be sensitive to factor-weight composition and that fitted response functions can be used to study multi-factor combinations. The mixture-design studies most directly related to the present paper examine portfolio performance over structured factor-weight designs and use fitted models to identify favorable weight combinations (Yeh et al., 2018; Yeh & Liu, 2020). These studies establish the usefulness of DOE and response-surface methods for portfolio construction.

However, the conventional mixture formulation imposes a different geometry from the present problem. Mixture weights are typically nonnegative and constrained to sum to one, whereas signed relative weights may be positive, negative, or zero. Moreover, when factor directions are specified in advance, the fitted response surface remains conditional on those predetermined orientations.

The remaining methodological gap is therefore the integration of three elements within one framework:

1. a **continuous signed relative-weight space** in which factor sign and relative intensity vary jointly;
2. a **structured experimental design** that samples this continuous space at positive, zero, and negative levels; and
3. a **response-surface model** that uses the sampled portfolio outcomes to approximate portfolio-performance responses over the bounded continuous region.

The present study addresses this gap by sampling a continuous five-dimensional signed-weight region through a three-level full factorial design and using the resulting portfolio responses to estimate second-order surfaces that describe performance variation across the sampled design points. The detailed distinction between the continuous admissible region, the 243 sampled design points, and the 242 active nonzero scoring strategies is developed in Section 3.

This formulation differs from both fixed-direction factor combination and conventional mixture optimization. It does not replace a continuous problem with a discrete one. Rather, the factorial design provides a structured finite sample of an underlying continuous signed-weight space, and the fitted response surface provides model-based approximations at intermediate weight coordinates.

3. Continuous Signed-Weight Framework

3.1 Problem Formulation

Consider K standardized firm characteristics. Let $X_{k,i,t}$ denote the standardized value of factor k for stock i at portfolio-formation date t . The portfolio-scoring rule is defined as

$$S_{i,t}(\mathbf{w}) = \sum_{k=1}^K w_k X_{k,i,t}, \quad (6)$$

where $\mathbf{w}=(w_1,\dots,w_K)$ is the factor-weight vector. The admissible weight region is $\mathbf{W}=[-1,1]^K$. Thus, for each factor, the feasible region is the K -dimensional hypercube $w_k \in [-1,1]$.

The sign and magnitude of w_k have distinct but jointly represented interpretations. If $w_k > 0$, higher values of factor k increase the composite score, holding other factor values fixed. If $w_k < 0$, lower values of factor k are favored through the scoring rule. If $w_k = 0$, factor k does not contribute to the composite score. The magnitude $|w_k|$ reflects the relative importance or sensitivity of factor k in determining the composite score.

The key methodological step is to represent factor direction and relative intensity jointly through a continuous signed weight, with zero corresponding to factor exclusion. This distinction is important because the empirical problem is not inherently discrete. The values -1 , 0 , and $+1$ used later in the experimental design are selected design levels within the continuous interval $[-1, 1]$; they do not define the entire admissible weight space.

For the five-factor application in this paper,

$$\mathbf{w} = (w_{\text{PB}}, w_{\text{ROE}}, w_{\text{MV}}, w_{\beta}, w_{\text{MOM}}) \in [-1, 1]^5.$$

Positive and negative weights therefore represent opposite cross-sectional orientations of the corresponding factor, while intermediate values represent continuous relative intensities.

3.2 Continuous Signed-Weight Representation

The scoring framework has an important invariance property. Portfolio formation is based on the cross-sectional ranking of composite scores rather than on their absolute numerical levels. For a signed-weight vector

$$\mathbf{w} = (w_1, \dots, w_K) \in \mathcal{W}, \quad \mathcal{W} = [-1, 1]^K, \quad (7)$$

the composite score of stock i at time t is

$$S_{i,t}(\mathbf{w}) = \sum_{k=1}^K w_k X_{k,i,t}. \quad (8)$$

For any scalar $c > 0$, consider the positively rescaled vector

$$\mathbf{w}' = c\mathbf{w}, \quad (9)$$

provided that both $\mathbf{w} \in \mathcal{W}$ and $c\mathbf{w} \in \mathcal{W}$. The corresponding score satisfies

$$S_{i,t}(c\mathbf{w}) = \sum_{k=1}^K cw_k X_{k,i,t} = cS_{i,t}(\mathbf{w}). \quad (10)$$

Because multiplication by a positive scalar preserves ordering,

$$S_{i,t}(\mathbf{w}) > S_{j,t}(\mathbf{w}) \quad (11)$$

if and only if

$$S_{i,t}(c\mathbf{w}) > S_{j,t}(c\mathbf{w}). \quad (12)$$

Therefore, apart from possible implementation differences associated with score ties or additional portfolio-formation rules,

$$\text{rank}[S_{i,t}(c\mathbf{w})] = \text{rank}[S_{i,t}(\mathbf{w})]. \quad (13)$$

Accordingly, positively proportional vectors are equivalent under the basic ranking rule:

$$\mathbf{w} \sim c\mathbf{w}, \quad c > 0, \quad (14)$$

whenever both vectors lie within the admissible region.

For example, (1, -1, 1, -1, 1) and (0.5, -0.5, 0.5, -0.5, 0.5) generate composite scores that differ only by a positive multiplicative constant and therefore produce the same cross-sectional ranking.

This equivalence does not imply that all distinct vectors in $\mathbf{W}$ are redundant. Positive rescaling preserves the signs and relative proportions of all coordinates. By contrast, changing one coordinate relative to another generally changes the composite-score ordering and therefore may change the selected portfolio. The economically relevant information is thus contained in the signed relative structure of the vector rather than in its absolute scale.

The weights should consequently be interpreted as relative signed weights, not as uniquely identified absolute allocations. The admissible hypercube $[-1,1]^K$ contains multiple numerical representations of some economically equivalent ranking rules. Consequently, positively proportional weight vectors may differ numerically while representing the same ranking rule and producing identical portfolio outcomes.

Conventional mixture designs assume nonnegative unit-sum proportions and are therefore inappropriate for signed-weight portfolio scoring.

The present study consequently retains the bounded signed-coordinate representation $\mathbf{W}=[-1,1]^K$ rather than imposing a normalization such as $\|\mathbf{w}\|_\infty=1$.

This choice preserves a simple continuous hypercube representation, accommodates positive, negative, and zero coordinates symmetrically, and permits direct use of the three-level factorial design developed below. It also allows the all-zero vector to remain the center point of the experimental design.

The consequence is that positively proportional vectors may represent the same underlying ranking rule while occupying different numerical locations in the coordinate region. This feature is treated as an inherent property of signed relative-weight scoring and is explicitly recognized when interpreting the fitted response surfaces. Consequently, numerical differences among positively proportional weight vectors

should not be interpreted as economically distinct portfolio specifications, since they induce identical ranking rules and identical portfolio outcomes.

For comparison, the absolute-sum constraint $|x|+|y|+|z|=1$ provides a scale-normalized representation of three nonzero signed weights. Figure 2 illustrates its octahedral surface, including the positive-octant facet corresponding to nonnegative unit-sum weights. This geometry is an alternative representation and is not the hypercube used for the factorial design in this study.

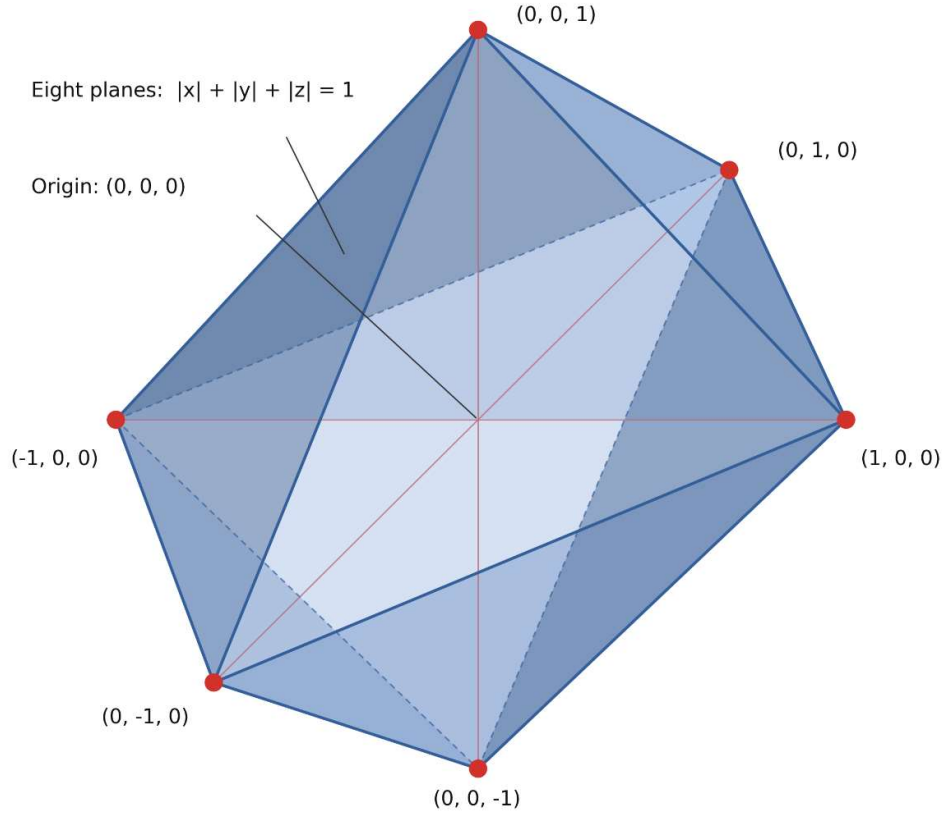


Figure 2. Three-Dimensional L1-Norm Weight Space under the Absolute-Sum Normalization Constraint Defined by $|x| + |y| + |z| = 1$ (Octahedron of the L1-Norm, Octant View)

3.3 Sampling the Continuous Signed-Weight Space

The admissible portfolio-design region is defined by the continuous signed-weight space $W=[-1,1]^K$, where each coordinate represents the signed relative weight assigned to one factor. The admissible weights are continuous, and any real value within the interval is allowed.

To obtain a structured set of observations for empirical analysis, this study samples the continuous region using a three-level full factorial design. Each coordinate is evaluated at the three experimental levels -1 , 0 , and $+1$. These values are sampling

levels used for experimental design rather than restrictions on the admissible weights. Intermediate values between -1 and $+1$ remain feasible and are considered later through response-surface analysis.

Figure 3 illustrates the fundamental geometric difference between a conventional mixture design and the three-level full factorial design adopted in this study. In a three-component mixture design, component proportions are nonnegative and constrained to sum to one, so the experimental region forms a simplex. The seven illustrated design points consist of the three vertices, three edge midpoints, and the centroid. By contrast, the proposed framework treats factor weights as independently varying signed coordinates rather than mixture proportions. In the three-factor illustration, sampling each factor at -1 , 0 , and $+1$ produces a cubic design space containing $3^3=27$ design points. The factorial geometry therefore accommodates positive, negative, and zero factor directions without imposing a nonnegative unit-sum constraint. The same structure generalizes directly to the five-factor empirical application.

With five factors and three sampling levels per factor, the factorial design contains $3^5=243$ design points. Among these, 242 nonzero points correspond to active scoring configurations, whereas the all-zero point $(0,0,0,0,0)$ contains no cross-sectional discrimination and serves as the center point of the experimental design. Although it is not an active portfolio-selection rule, its observed response is retained in the response-surface regressions, so that all 243 design points are used for model estimation.

This factorial design provides a structured and computationally manageable sample of the continuous signed-weight region for subsequent response-surface estimation.

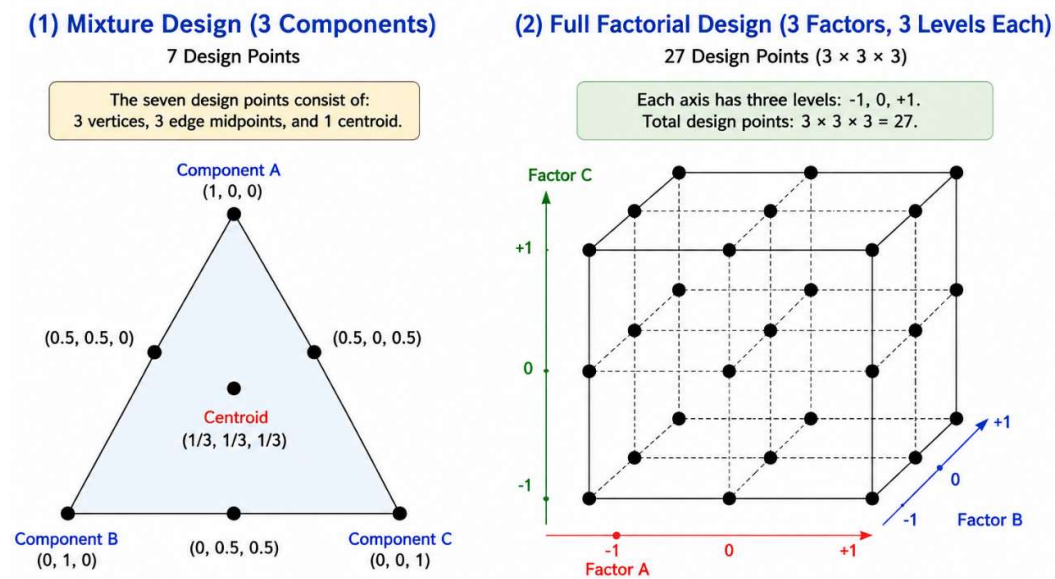


Figure 3. Comparison of Design Spaces: Mixture Design versus Three-Level Full Factorial Design

3.4 Second-Order Response Surface Model

Let $Y_m(\mathbf{w})$ denote portfolio-performance response m generated by the scoring rule associated with weight vector $\mathbf{w}$. In the empirical application, the responses include average return, volatility, the Sharpe ratio, and maximum drawdown.

Portfolio responses are evaluated at the factorial design points $\mathbf{w} \in D$. The resulting design-point observations are used to estimate a second-order response-surface approximation:

$$Y_m(\mathbf{w}) = \beta_{0m} + \sum_{k=1}^K \beta_{km} w_k + \sum_{k=1}^K \gamma_{km} w_k^2 + \sum_{i < j} \beta_{ij,m} w_i w_j + \varepsilon_m. \quad (15)$$

For the five-factor case, the specification contains: 5 linear terms, 5 quadratic terms, and

$$\binom{5}{2} = 10$$

pairwise interaction terms, in addition to the intercept. The full second-order model therefore contains $1+5+5+10=21$ parameters.

The linear coefficients summarize first-order variation in the fitted response with respect to the signed coordinates. The interaction coefficients allow the association between one weight coordinate and the fitted response to depend on another coordinate. The quadratic coefficients permit second-order variation along individual weight dimensions.

At the three-level factorial design points,

$$w_k^2 = \begin{cases} 1, & w_k = -1 \text{ or } +1, \\ 0, & w_k = 0. \end{cases} \quad (16)$$

Consequently, the quadratic coefficient should be interpreted as curvature of the fitted approximation rather than as a direct measure of economic nonlinearity.

Accordingly, within the sampled design, a quadratic term partly distinguishes nonzero factor activation from the zero setting. This feature should be recognized when interpreting the estimated quadratic coefficients. A significant quadratic coefficient is evidence of second-order structure in the fitted response model; it should not automatically be interpreted as direct evidence of a specific economic mechanism such as speculative amplification or stabilization.

Similarly, an interaction coefficient indicates that the fitted association of one weight coordinate with the response depends on another coordinate. It does not, by itself, identify the behavioral or institutional mechanism responsible for that dependence.

After estimation, the second-order model defines the fitted response function

$$\widehat{Y}_m(\mathbf{w}) = \widehat{\beta}_{0m} + \sum_{k=1}^K \widehat{\beta}_{km} w_k + \sum_{k=1}^K \widehat{\gamma}_{km} w_k^2 + \sum_{i < j} \widehat{\beta}_{ij,m} w_i w_j. \quad (17)$$

Unlike the original response observations, which are obtained at the 243 factorial design points, the fitted function can be evaluated throughout the continuous bounded region $\mathcal{W}=[-1,1]^5$.

3.5 Interpretation of the Response Surface

As established in Section 3.2, positive-scale equivalence applies to the underlying portfolio-ranking rule.

The second-order response-surface model serves a different purpose. It is an empirical approximation fitted to the observed responses at the sampled design points. Consequently, the fitted polynomial is not required to preserve the exact positive-scale equivalence of the underlying ranking system. Its coefficients summarize local variation in the observed responses over the experimental region rather than defining a unique portfolio-construction rule.

Accordingly, the fitted response surfaces should be interpreted as empirical approximations to the portfolio-performance patterns within the bounded signed-weight region. Throughout this study, the reported weight vectors therefore represent relative signed-weight structures rather than uniquely identified absolute factor allocations.

4. Data and Empirical Design

4.1 Sample Selection, Data Sources, and Information Timing

The primary empirical analysis uses quarterly data for Chinese A-share firms over the period 2005–2025. The investment universe is based on the CSI 300 Index. To avoid survivorship bias, the eligible universe at each portfolio-formation date is defined using the historical constituent membership prevailing at that date rather than a retrospectively fixed list of current constituents.

Market and accounting data are obtained primarily from the Wind Information Database and the China Stock Market & Accounting Research (CSMAR) database. Market variables include stock prices, returns, market capitalization, and information required for Beta and momentum construction. Accounting variables include the financial-statement information required to construct profitability and valuation measures.

Financial firms are excluded because their balance-sheet structures, leverage, and regulatory environments differ substantially from those of nonfinancial firms. Observations lacking information required for factor construction or subsequent

portfolio-performance measurement are removed. Continuous factor variables are winsorized cross-sectionally at the 1st and 99th percentiles to limit the influence of extreme observations.

The empirical design uses quarterly portfolio formation and separates factor-information measurement from subsequent portfolio-performance evaluation. Because the accounting variables are quarterly, a four-quarter reporting lag is imposed. Thus, accounting information used at a portfolio-formation date is restricted to information dated at least four quarters earlier. This conservative convention is applied to reduce the possibility that subsequently released financial information enters the portfolio-ranking procedure.

More generally, all factor values used for portfolio formation are aligned so that the information precedes the corresponding investment and performance-measurement periods. Historical index membership, factor measurement, portfolio formation, and subsequent return evaluation are therefore ordered sequentially.

4.2 Factor Construction

The empirical framework uses five firm characteristics representing valuation, profitability, size, systematic risk, and prior return performance: PB, ROE, MV, Beta, and MOM. These variables enter the scoring framework in their raw economic orientation before cross-sectional standardization. Their eventual positive or negative contribution to the composite score is determined by the signed weight w_k , not by imposing a preferred direction during variable construction.

4.2.1 Price-to-Book Ratio

Price-to-book ratio is defined as

$$PB_{i,t} = \frac{\text{Market Value}_{i,t}}{\text{Book Equity}_{i,t}}. \quad (18)$$

Higher PB indicates a higher market valuation relative to book equity, whereas lower PB indicates a lower relative valuation. The variable is not inverted before entering the signed-weight framework. Consequently, the orientation of valuation exposure is determined by the sign of w_{PB} .

A positive weight favors higher standardized PB values, whereas a negative weight favors lower standardized PB values.

4.2.2 Return on Equity

Return on equity is defined as quarterly ROE

$$ROE_{i,t} = \frac{\text{Net Income}_{i,t}}{\text{Shareholders' Equity}_{i,t}}. \quad (19)$$

ROE represents firm profitability. Its contribution to the composite score is determined by wROE.

Positive and negative weights favor higher and lower standardized ROE values, respectively. No profitability direction is imposed before estimation. Under different multi-factor weight structures, the portfolio implications associated with ROE may vary across performance objectives.

4.2.3 Market Value

Firm size is measured by market capitalization:

$$MV_{i,t} = \text{Market Capitalization}_{i,t}. \quad (20)$$

The signed weight wMV determines whether the scoring rule favors relatively larger or smaller firms within the eligible investment universe. Small-cap and large-cap orientations may be associated with different return and risk outcomes across objectives.

4.2.4 Beta

Beta measures systematic market exposure. For stock i , Beta is estimated from day stock and market returns over the preceding 250:

$$R_{i,\tau} = \alpha_i + \beta_i R_{M,\tau} + u_{i,\tau}. \quad (21)$$

The estimated coefficient $\hat{\beta}_i$ is used as the Beta characteristic. The sign of w_β determines whether the scoring rule favors higher- or lower-Beta stocks. Positive and negative Beta orientations may be associated with different return and risk profiles.

4.2.5 Momentum

Momentum is measured from prior stock-return performance over the study's specified historical ranking horizon:

Momentum is measured as the cumulative stock return over the last quarter.

$$MOM_{i,t} = R_{i,t}^{past}. \quad (22)$$

The variable is retained in its raw prior-return orientation. Therefore, wMOM>0 favors stocks with stronger prior returns, whereas wMOM<0 favors stocks with weaker prior returns.

This notation is important. The factor is consistently labeled **MOM**, while the sign of its weight determines whether the scoring rule is continuation-oriented or reversal-oriented. The paper therefore does not relabel the factor itself as REV when the estimated or selected weight is negative.

4.3 Cross-Sectional Standardization, Factorial Design Points, and Portfolio Formation

The empirical design samples the continuous signed-weight region $W=[-1, 1]^5$ at the three factorial levels $-1, 0$, and $+1$ in each coordinate. The resulting full factorial design contains 243 sampled points, of which 242 are nonzero active scoring configurations and one is the all-zero center point.

At each quarterly portfolio-formation date t , the five factor variables are standardized cross-sectionally within the eligible historical constituent universe. For factor k ,

$$X_{k,i,t} = \frac{Z_{k,i,t} - \mu_{k,t}}{\sigma_{k,t}}, \quad (23)$$

where $Z_{k,i,t}$ is the winsorized raw factor value, $\mu_{k,t}$ is the cross-sectional mean, and $\sigma_{k,t}$ is the cross-sectional standard deviation.

For a signed-weight vector

$$\mathbf{w} = (w_{PB}, w_{ROE}, w_{MV}, w_{\beta}, w_{MOM}), \quad (24)$$

the composite score is

$$S_{i,t}(\mathbf{w}) = w_{PB}X_{PB,i,t} + w_{ROE}X_{ROE,i,t} + w_{MV}X_{MV,i,t} + w_{\beta}X_{\beta,i,t} + w_{MOM}X_{MOM,i,t}. \quad (25)$$

The underlying admissible region is continuous:

$$\mathbf{w} \in [-1, 1]^5.$$

For experimental sampling, each coordinate is evaluated at the three factorial levels

$$w_k \in \{-1, 0, +1\}.$$

The complete design therefore contains

$$3^5 = 243$$

weight combinations.

For each nonzero design point,

$$\mathbf{w} \neq \mathbf{0},$$

stocks are ranked in descending order of their composite scores. The highest-ranked stocks are selected to form the corresponding long-only portfolio, and portfolio performance is subsequently measured according to the procedures described below. The same ranking and portfolio-formation rule is applied consistently across the 242 active nonzero design points.

For the all-zero center point, $\mathbf{w}=(0,0,0,0,0)$, all stocks receive the same composite score and no cross-sectional ranking is generated. To define an empirical response at

this design point, the portfolio is therefore represented by an equal-weighted market-average portfolio formed from the same eligible stock universe and evaluated over the same periods as the active scoring configurations. The resulting performance measures provide the observed center-point responses used in the response-surface regressions. Accordingly, the all-zero point is retained as a non-discriminating benchmark in model estimation but is excluded from active-strategy comparisons. Unlike the active design points, this benchmark holds the full eligible universe rather than the top-decile portfolio. It is a separately defined center response and is not the continuous limit of the active scoring rule as the weight vector approaches zero.

The distinction between the continuous admissible region and the factorial design is essential. The 243 points are experimental locations used to sample the performance response. They do not imply that economically admissible factor weights are restricted to integers or to the three values -1 , 0 , and $+1$. Intermediate values remain part of the continuous signed-weight region and enter the fitted response-surface analysis described in Section 4.6.

4.4 Portfolio Performance Measures

At each quarterly rebalancing date, the top 10% stocks are selected and combined using equal portfolio weights.

For each design point, portfolio performance is summarized using four response measures.

The mean portfolio return is

$$\bar{R}_p = \frac{1}{T} \sum_{t=1}^T R_{p,t}. \quad (26)$$

Portfolio-return volatility is measured by the sample standard deviation

$$\sigma_p = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (R_{p,t} - \bar{R}_p)^2}. \quad (27)$$

The Sharpe ratio is defined as

$$SR_p = \frac{\bar{R}_p}{\sigma_p}. \quad (28)$$

No risk-free-rate adjustment is applied.

Maximum drawdown is reported as a nonnegative magnitude. Let $V_{p,t}$ denote cumulative portfolio value and define the running peak as

$$P_{p,t} = \max_{s \leq t} V_{p,s}. \quad (29)$$

Then maximum drawdown is

$$MDD_p = \max_t \left(\frac{P_{p,t} - V_{p,t}}{P_{p,t}} \right) \geq 0. \quad (30)$$

Under this convention, a smaller value of MDD_p indicates less severe peak-to-trough loss.

The four responses represent distinct portfolio objectives. Mean return and the Sharpe ratio are maximized, whereas volatility and maximum drawdown magnitude are minimized. No single signed-weight configuration is therefore presumed to be uniformly preferred across all four criteria.

4.5 Descriptive Analysis of Design-Point Responses

The empirical analysis first examines performance variation among the directly evaluated factorial design points. These descriptive analyses are distinct from the subsequent response-surface estimation and response-surface estimation.

First, conditional design-point comparisons examine how portfolio performance differs across the three sampled settings of each factor coordinate: $w_k = -1, 0, +1$.

For a given factor k , response Y , and sampled level $a \in \{-1, 0, +1\}$, the conditional mean response is

$$\bar{Y}_k(a) = \frac{1}{N_{k,a}} \sum_{\mathbf{w} \in \mathcal{D}_A: w_k=a} Y(\mathbf{w}), \quad (31)$$

where $\mathcal{D}_A = \mathcal{D} \setminus \{0\}$ denotes the set of 242 active nonzero design points and $N_{k,a}$ is the number of active design points satisfying $w_k = a$.

The remaining all-zero point is not an active scoring configuration because it generates no cross-sectional ranking. Its response is defined by the market-average portfolio described in Section 4.3 and is retained only as the center-point observation in response-surface estimation.

The all-zero center point is excluded from these active-strategy conditional comparisons because it does not define a cross-sectional ranking strategy. This exclusion is specific to the descriptive comparison of active scoring rules and does not apply to the response-surface regressions, which use all 243 factorial design points.

Second, extreme-configuration analysis examines recurring signed-weight structures among selected high- and low-performance design points. For objectives to be maximized, such as average return and the Sharpe ratio, the analysis compares configurations near the upper and lower ends of the observed design-point distribution. For objectives to be minimized, such as volatility and maximum drawdown magnitude, favorable and unfavorable configurations are defined according to the corresponding minimization criterion.

These descriptive comparisons are intended to identify recurring directional patterns among directly evaluated configurations. They do not by themselves establish

marginal causal effects, uniquely preferred weight configurations, or the continuous response structure between sampled design points. The continuous structure of the signed-weight region is subsequently approximated using the second-order response-surface models described next.

4.6 Second-Order Response-Surface Estimation

For each performance response m , the observed factorial design-point responses are approximated by a second-order polynomial:

$$Y_m(\mathbf{w}) = \beta_{0m} + \sum_{k=1}^K \beta_{km} w_k + \sum_{k=1}^K \gamma_{km} w_k^2 + \sum_{k < \ell} \beta_{k\ell,m} w_k w_\ell + \varepsilon_m. \quad (32)$$

For $K=5$, the model contains

$$1 + 5 + 5 + \binom{5}{2} = 21$$

parameters: one intercept, five linear terms, five quadratic terms, and ten pairwise interaction terms.

The response-surface models are estimated from all $N=243$ factorial design-point responses, including the all-zero center point. The resulting residual degrees of freedom are therefore $243-21=222$.

The linear coefficients summarize first-order variation in the fitted response with respect to the signed weight coordinates. The interaction coefficients allow the fitted association between one coordinate and portfolio performance to depend on another coordinate. The quadratic coefficients capture second-order variation along individual weight dimensions.

Model adequacy is evaluated using standard regression diagnostics, including coefficient estimates, standard errors, t-statistics, overall F-statistics, R^2 , adjusted R^2 , and RMSE. Fitted values and residuals are also examined to assess how well the second-order approximation summarizes variation across the 243 sampled responses.

Interpretation of the polynomial terms requires caution. An interaction coefficient indicates conditional dependence within the fitted response surface: the association between one signed-weight coordinate and the fitted response varies with another coordinate. It does not, by itself, identify the behavioral or institutional mechanism responsible for that dependence.

The same caution applies to quadratic terms. At the sampled factorial levels,

$$w_k^2 = \begin{cases} 1, & w_k = -1 \text{ or } +1, \\ 0, & w_k = 0. \end{cases} \quad (33)$$

Within the factorial sample, the quadratic regressor therefore partly distinguishes nonzero settings from the zero setting. Its coefficient captures second-order structure in the fitted approximation, but it should not automatically be interpreted as evidence of amplification, stabilization, speculation, or any other specific economic mechanism.

After estimation, the fitted surface is

$$\hat{Y}_m(\mathbf{w}) = \hat{\beta}_{0m} + \sum_{k=1}^K \hat{\beta}_{km} w_k + \sum_{k=1}^K \hat{\gamma}_{km} w_k^2 + \sum_{k < \ell} \hat{\beta}_{k\ell, m} w_k w_\ell. \quad (34)$$

Although the empirical responses are directly observed only at the 243 factorial design points, the fitted polynomial can be evaluated throughout the continuous bounded region $\mathbf{W}=[-1,1]^5$.

Intermediate fitted values are therefore model-based interpolations generated by the second-order approximation. The empirical claim is correspondingly limited: the model assesses whether a second-order surface containing linear, interaction, and quadratic terms provides a useful approximation to performance variation across the sampled signed-weight design. It does not establish that the true economic response function is globally quadratic.

The second-order regression is used as an empirical approximation to the portfolio-performance variation observed across the 243 sampled factorial design points. Because portfolio formation is based on score rankings, positively proportional nonzero weight vectors may generate identical rankings and portfolio outcomes. The fitted polynomial is not imposed to satisfy this positive-scale equivalence exactly and is therefore not interpreted as a structural model over the entire continuous signed-weight space. Instead, it is used as an empirical approximation for describing the linear, interaction, and quadratic response patterns observed across the sampled factorial design points.

The all-zero point is retained as a non-discriminating equal-weighted eligible-universe benchmark. It does not represent an active factor-selection strategy, but its observed response provides the factorial center used in estimating the second-order approximation.

5. Empirical Results

5.1 Conditional Performance Comparisons Across Factor Levels

This section first examines the directly observed performance associated with the sampled factor-weight levels. The purpose is to determine whether positive exposure, negative exposure, and factor exclusion are associated with materially different portfolio outcomes. These comparisons are descriptive conditional comparisons across

the factorial design; they are not interpreted as causal effects or as conventional marginal factor premia.

Table 1 reports conditional mean portfolio performance across the 242 active nonzero scoring strategies. The all-zero center point is not included in this descriptive comparison because it does not generate a cross-sectional ranking rule. The results indicate that portfolio performance varies substantially with the signed setting of each factor coordinate. Thus, factor direction is an economically relevant part of the portfolio-design problem, rather than merely an implementation convention.

Table 1. Conditional Mean Portfolio Performance Across Sampled Factor Levels

Factor	Sampled Level	N	Mean Quarterly Return	Quarterly Return Volatility	Sharpe Ratio	Maximum Drawdown
PB	-1	81	6.75%	31.2%	0.213	59.3%
PB	0	80	6.96%	35.6%	0.191	62.4%
PB	+1	81	7.50%	39.9%	0.184	64.3%
ROE	-1	81	5.48%	28.5%	0.192	56.1%
ROE	0	80	7.01%	36.0%	0.194	62.8%
ROE	+1	81	8.72%	42.3%	0.203	67.1%
MV	-1	81	8.17%	37.1%	0.220	59.8%
MV	0	80	7.06%	36.8%	0.190	63.3%
MV	+1	81	5.97%	33.0%	0.179	62.9%
Beta	-1	81	8.03%	39.6%	0.201	61.6%
Beta	0	80	7.45%	36.7%	0.201	61.8%
Beta	+1	81	5.73%	30.5%	0.186	62.5%
MOM	-1	81	8.92%	39.7%	0.226	51.5%
MOM	0	80	7.04%	33.6%	0.208	58.7%
MOM	+1	81	5.24%	33.6%	0.154	75.7%

(1) Performance of PB

PB also exhibits directional heterogeneity. A negative PB setting corresponds to a low-PB value-oriented structure, whereas a positive PB setting corresponds to a high-PB growth-oriented valuation structure. The results indicate that neither valuation direction should be treated as universally preferred across all objectives.

This finding is relevant to the value-versus-growth debate. Traditional value-oriented interpretations emphasize low valuation ratios, while growth-oriented interpretations may favor firms with high valuation multiples when those multiples reflect expansion opportunities or investor demand for growth exposure.

The evidence in this study suggests that both low-PB and high-PB structures may appear in favorable regions of the configuration landscape. This does not reject value investing. Instead, it indicates that the preferred valuation direction may depend on the market environment, portfolio objective, and interaction with other factor-weight coordinates.

Accordingly, the valuation result should be interpreted as evidence of objective- and configuration-dependence, not as evidence that one valuation philosophy dominates under all conditions.

Figure 4 visualizes the PB-related landscape. Low-PB and high-PB settings both appear in economically relevant regions, although their roles differ by objective and surrounding configuration. The figure therefore supports the view that valuation exposure is better understood as part of a signed multi-factor structure than as a fixed one-direction rule.

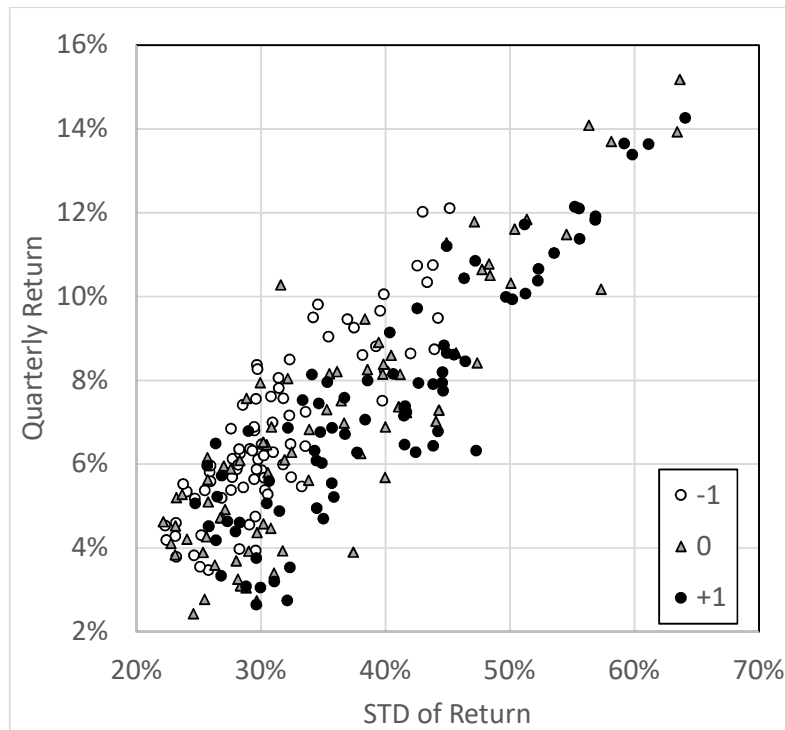


Figure 4. Directional Configuration Landscape under PB Structures

(2) Performance of ROE

Among the five factors, ROE displays the clearest directional contrast. Positive ROE settings are associated with stronger return-oriented performance, whereas

negative ROE settings appear more frequently in lower-risk regions. This pattern is consistent with the view that profitability is important for portfolio performance, but the evidence here is more specific: the role of profitability depends on the objective being considered and on the surrounding signed-weight configuration.

Figure 5 visualizes the 242 active design points under the three ROE settings. The horizontal axis reports portfolio-return volatility, and the vertical axis reports average quarterly return. The marker groups correspond to $wROE=-1$, $wROE=0$, and $wROE=+1$.

The figure shows that positive ROE settings tend to occupy higher-return regions of the return-volatility space, while negative ROE settings are more concentrated in lower-return and more stable regions. This pattern does not imply that ROE has a single universal role. Rather, ROE contributes differently depending on whether the portfolio objective emphasizes return generation, volatility reduction, or downside control.

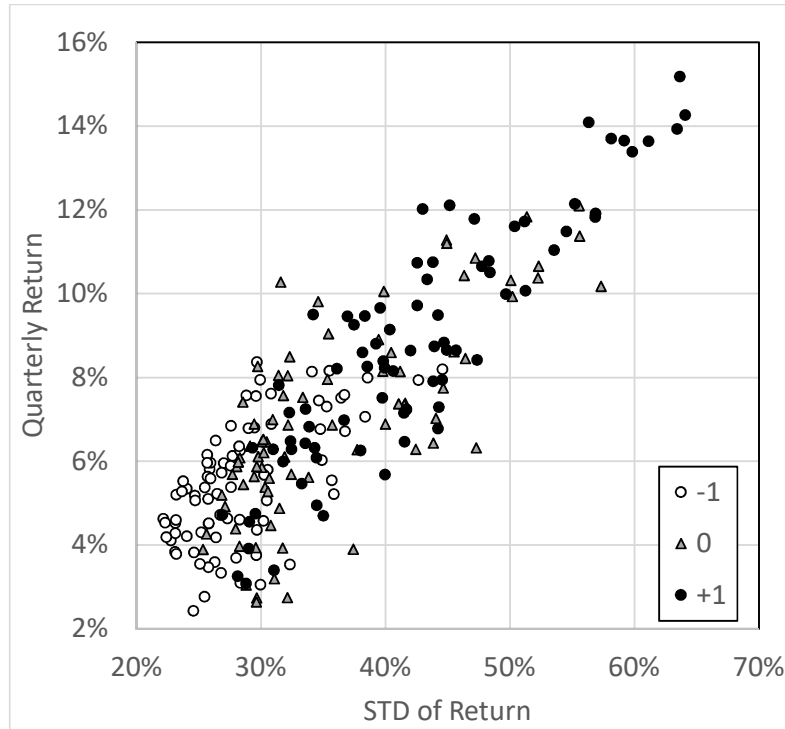


Figure 5. Directional Configuration Landscape under ROE Structures

(3) Performance of MV

MV also displays meaningful directional asymmetry. A negative MV setting corresponds to a small-cap orientation, whereas a positive MV setting corresponds to a large-cap orientation. In the observed design points, small-cap orientation is often associated with stronger return-oriented performance, while large-cap orientation is more closely related to stabilization-oriented structures.

Figure 6 illustrates this size-related landscape. Negative MV settings tend to appear in higher-return but more dispersed regions, whereas positive MV settings are more concentrated in lower-volatility regions. This pattern reinforces the broader conclusion that signed factor direction is an economically meaningful dimension of multi-factor portfolio design.

Taken together, the conditional comparisons show that factor direction is not a secondary implementation detail. Performance differs across positive, zero, and negative settings, and the preferred sign of a factor varies by objective. The appropriate unit of analysis is therefore not only the individual factor, but the signed configuration in which the factor operates.

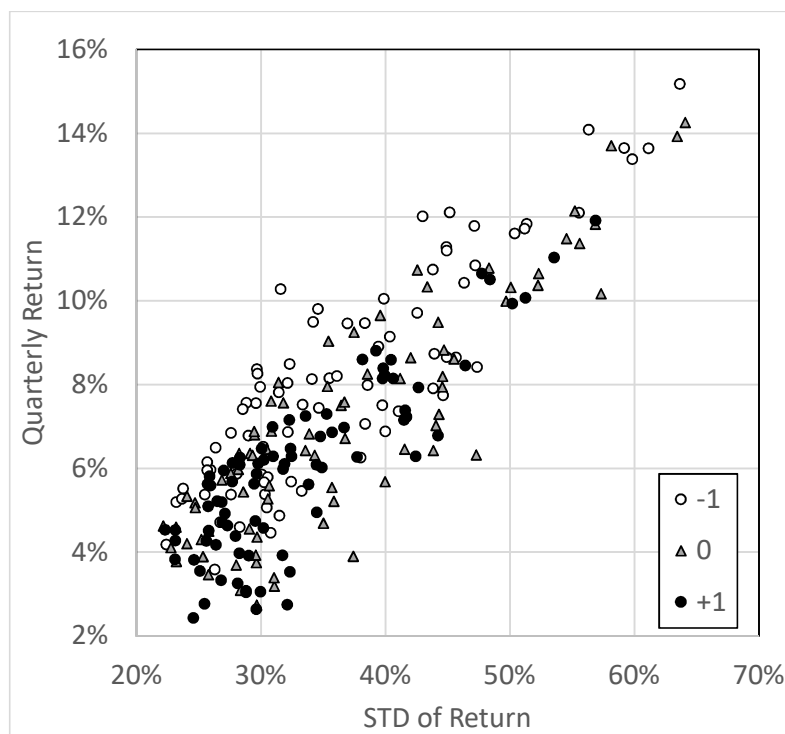


Figure 6. Directional Configuration Landscape under MV Structures

(4) Performance of Beta

Beta exhibits a more objective-dependent pattern. Some Beta settings are associated with return-oriented configurations, while other settings are more closely related to volatility or drawdown control. The results therefore caution against treating high-Beta and low-Beta orientations as universally superior or inferior. Their relevance depends on the portfolio objective and the surrounding signed-weight structure.

Figure 7 presents the performance landscape associated with Beta settings. Unlike the MOM results, the Beta patterns do not show a simple one-direction dominance. Instead, different Beta orientations occupy different regions of the return-volatility

space. This visual evidence supports the interpretation that systematic-risk exposure is objective-dependent within the broader signed-weight framework.

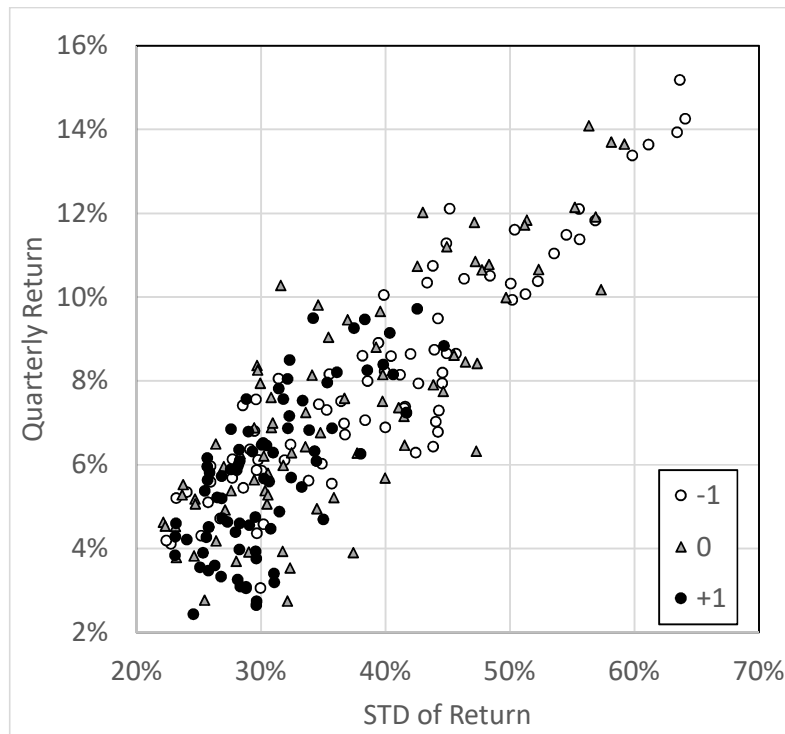


Figure 7. Directional Configuration Landscape under Beta Structures

(5) Performance of MOM

MOM also displays strong directional asymmetry. In the Chinese sample, negative MOM settings are generally associated with more favorable return-oriented and risk-adjusted outcomes than positive MOM settings. Because MOM is retained in its raw prior-return orientation, a negative MOM weight corresponds to a reversal-oriented ranking structure rather than to a relabeled reversal factor.

Figure 8 plots the 242 active design points according to average quarterly return and return volatility under different MOM settings. The distribution shows a clear separation between continuation-oriented and reversal-oriented structures. Negative MOM settings occupy more favorable regions in several parts of the performance landscape, suggesting that reversal-oriented structures are important in this sample.

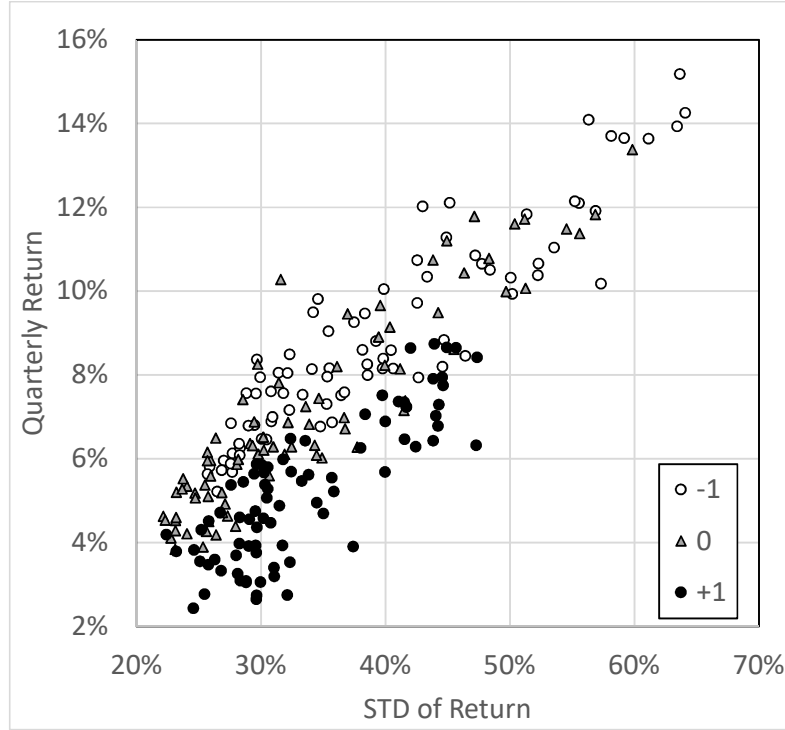


Figure 8. Directional Configuration Landscape under Momentum Structures

5.2 Extreme-Configuration Analysis

The previous section reports conditional comparisons by factor level. This section examines the signed-weight structures of extreme-performing active design points. The analysis considers the 20 highest-return portfolios, the 20 lowest-volatility portfolios, the 20 highest-Sharpe portfolios, and the 20 smallest-drawdown portfolios. Because these are directly observed active design points, the analysis is descriptive: it identifies recurring signed configurations, but it does not prove that any single sign independently causes the outcome.

5.2.1 High-Return Configurations

Table 2 reports the twenty active design points with the highest mean quarterly returns, and Table 3 summarizes the frequency of sampled factor levels within this group. Several recurring patterns are evident.

First, positive ROE settings appear frequently among the highest-return configurations. This result is consistent with the conditional comparisons in Section 5.1 and indicates that profitability is closely associated with return-oriented structures in the Chinese sample.

Second, negative MOM settings appear repeatedly among the highest-return configurations. This pattern again points to the importance of reversal-oriented structures in the observed Chinese design-point responses.

Third, negative MV settings are also common among the highest-return configurations, suggesting that small-cap orientation is often associated with return amplification. This association should be interpreted together with the risk measures reported later, because high-return structures may also involve greater volatility or drawdown exposure.

The valuation factor does not exhibit a single dominant interpretation. Both negative and positive PB settings can appear among high-return design points, indicating that both value-oriented and growth-oriented valuation structures may be relevant under different surrounding configurations.

This finding is important because it reframes the value-versus-growth issue as a configuration problem. Low-PB and high-PB structures need not be mutually exclusive investment philosophies; they may correspond to different regions of the signed-weight performance landscape.

The high-return evidence therefore suggests that aggressive return generation is associated with recurring combinations rather than isolated factor dominance.

In particular, favorable high-return structures often combine positive ROE settings, negative MOM settings, small-cap orientation, and valuation settings whose preferred direction depends on the surrounding configuration.

These patterns support the central argument that factor performance cannot be interpreted independently of signed-weight structure. The economic role of a factor may change depending on the other coordinates in the scoring rule.

5.2.2 Low-Volatility Configurations

Tables 4 and 5 examine the twenty lowest-volatility active design points. Their signed-weight structures differ substantially from those of the highest-return portfolios.

The lowest-volatility configurations are more stabilization-oriented. They tend to emphasize lower risk exposure, larger-cap orientation, and weaker speculative concentration than the high-return configurations.

- This contrast illustrates objective dependence within the configuration landscape.
- The signed-weight structures associated with higher return are not identical to those associated with lower volatility.
- Consequently, the evidence does not support a single universally preferred factor direction across all performance objectives.
- Instead, return amplification and volatility reduction appear to occupy different regions of the signed-weight design space.

This finding is central to the paper's argument that factor direction and relative intensity should be optimized jointly with the investment objective.

5.2.3 High-Sharpe Configurations

Tables 6 and 7 examine the twenty highest-Sharpe active design points. Compared with the pure return-maximization group, the highest-Sharpe configurations display more efficiency-oriented structures, balancing return generation against volatility.

Several patterns are noteworthy. Negative MOM settings remain common, indicating that reversal-oriented structures are not only associated with return performance but also with risk-adjusted performance in this sample.

Negative MV settings are also frequent among high-Sharpe configurations, suggesting that small-cap orientation contributes to return efficiency in several observed design points.

The role of ROE is less one-sided than in the high-return group. Positive ROE settings are important for return generation, but high-Sharpe configurations do not simply replicate the most aggressive profitability-oriented structures.

PB also becomes more objective-dependent. Value-oriented settings are more prominent in the Sharpe-ratio comparison than in the pure return comparison, suggesting that valuation direction depends on whether volatility is included in the objective.

The high-Sharpe evidence therefore shows that risk-adjusted efficiency is not merely a scaled version of return maximization. It corresponds to a distinct region of the signed-weight configuration landscape.

5.2.4 Low-Drawdown Configurations

Tables 8 and 9 examine the twenty active design points with the smallest maximum drawdown magnitudes. Because MDD is reported as a nonnegative magnitude, smaller values indicate less severe peak-to-trough loss.

The low-drawdown configurations differ from both the high-return and high-Sharpe groups. They are more strongly associated with downside-stabilization structures.

Negative MOM settings remain important in the low-drawdown group, suggesting that continuation-oriented momentum is relatively unfavorable for downside control in the Chinese sample.

ROE also changes role across objectives. Whereas positive ROE settings are strongly associated with high-return configurations, negative or non-aggressive ROE settings appear more frequently in low-drawdown structures.

This contrast indicates that profitability exposure may contribute to return generation while not necessarily being the preferred orientation for downside-risk control.

MV and PB again show configuration-dependent behavior. Some small-cap and valuation structures remain favorable, but the exact signed structure differs from the high-return case.

Overall, the low-drawdown evidence confirms that drawdown control represents a distinct dimension of portfolio performance. It should be analyzed separately from average return and the Sharpe ratio.

The four extreme-configuration analyses jointly show that different objectives select different regions of the signed-weight design. This objective dependence motivates the response-surface regressions in the next section.

Table 2. Twenty Active Design Points with the Highest Mean Quarterly Returns

Strategy ID	PB	ROE	MV	BETA	MOM	Mean Quarterly Return	Quarterly Return Volatility	Sharpe Ratio	Maximum Drawdown
202	0	1	-1	-1	-1	15.2%	63.6%	0.238	55.5%
166	1	1	0	-1	-1	14.3%	64.1%	0.222	60.8%
110	0	1	-1	0	-1	14.1%	56.3%	0.250	48.2%
118	0	1	0	-1	-1	13.9%	63.4%	0.220	54.9%
36	0	1	0	0	-1	13.7%	58.1%	0.236	47.4%
150	1	1	-1	0	-1	13.6%	59.2%	0.231	52.4%
218	1	1	-1	-1	-1	13.6%	61.1%	0.223	59.3%
134	1	1	-1	-1	0	13.4%	59.8%	0.224	66.4%
68	1	1	0	0	-1	12.1%	55.2%	0.220	55.2%
234	-1	1	-1	-1	-1	12.1%	45.2%	0.268	48.0%
186	1	0	-1	-1	-1	12.1%	55.6%	0.218	51.7%
158	-1	1	-1	0	-1	12.0%	43.0%	0.280	45.6%
148	1	1	1	0	-1	11.9%	56.9%	0.209	58.8%
46	0	0	-1	0	-1	11.8%	51.4%	0.231	47.2%
60	1	1	0	-1	0	11.8%	56.9%	0.208	70.2%
28	0	1	-1	0	0	11.8%	47.2%	0.250	68.3%
52	1	1	-1	0	0	11.7%	51.2%	0.229	67.9%
102	0	1	-1	-1	0	11.6%	50.4%	0.230	62.5%
32	0	1	0	-1	0	11.5%	54.5%	0.211	70.5%
20	1	0	0	-1	0	11.4%	55.6%	0.204	63.2%

Table 3. Frequencies of Sampled Factor Levels Among the Twenty Highest-Return Active Design Points

Factor	-1	-1	0	0	+1	+1	Most Frequent Level
	Count	Share	Count	Share	Count	Share	
PB	2	10%	8	40%	10	50%	+1
ROE	0	0%	3	15%	17	85%	+1
MV	12	60%	7	35%	1	5%	-1
Beta	11	55%	9	45%	0	0%	-1
MOM	13	65%	7	35%	0	0%	-1

Table 4. Twenty Active Design Points with the Lowest Quarterly Return Volatility

Strategy ID	PB	ROE	MV	BETA	MOM	Mean Quarterly Return	Quarterly Return Volatility	Sharpe Ratio	Maximum Drawdown
4	0	-1	0	0	0	4.6%	22.2%	0.208	45.5%
57	-1	-1	1	0	0	4.5%	22.3%	0.203	49.6%
241	-1	-1	-1	-1	1	4.2%	22.4%	0.187	64.0%
34	0	-1	0	-1	0	4.1%	22.8%	0.180	55.2%
103	0	-1	1	1	0	3.8%	23.1%	0.166	49.9%
143	-1	-1	1	1	0	4.3%	23.1%	0.185	49.0%
29	0	-1	1	0	0	4.5%	23.2%	0.195	49.1%
65	-1	-1	0	1	0	4.6%	23.2%	0.198	50.4%
73	-1	-1	0	0	1	3.8%	23.2%	0.163	68.0%
106	0	-1	-1	-1	0	5.2%	23.2%	0.224	48.2%
30	0	-1	-1	0	0	5.3%	23.7%	0.223	44.6%
58	-1	-1	-1	0	0	5.5%	23.8%	0.232	44.3%
33	0	-1	0	1	0	4.2%	24.1%	0.175	54.0%
66	-1	-1	0	-1	0	5.3%	24.1%	0.222	49.3%
203	0	-1	1	1	1	2.4%	24.6%	0.099	65.8%
159	-1	-1	1	0	1	3.8%	24.7%	0.155	62.5%
14	-1	-1	0	0	0	5.2%	24.7%	0.210	46.6%
12	1	-1	0	0	0	5.1%	24.7%	0.205	45.2%
235	-1	-1	1	1	1	3.5%	25.1%	0.141	63.8%
177	-1	-1	0	-1	1	4.3%	25.2%	0.171	66.5%

Table 5. Frequencies of Sampled Factor Levels Among the Twenty Lowest-Volatility Active Design Points

Factor	-1 Count	-1 Share	0 Count	0 Share	+1 Count	+1 Share	Most Frequent Level
PB	11	55%	8	40%	1	5%	-1
ROE	20	100%	0	0%	0	0%	-1
MV	4	20%	9	45%	7	35%	0
Beta	5	25%	9	45%	6	30%	0
MOM	0	0%	14	70%	6	30%	0

Table 6. Twenty Active Design Points with the Highest Sharpe Ratios

Strategy ID	PB	ROE	MV	BETA	MOM	Mean Quarterly Return	Quarterly Return Volatility	Sharpe Ratio	Maximum Drawdown
6	0	0	-1	0	0	10.3%	31.6%	0.325	51.4%
90	-1	0	-1	0	-1	9.8%	34.6%	0.283	48.8%
162	-1	-1	-1	0	-1	8.4%	29.7%	0.282	46.6%
158	-1	1	-1	0	-1	12.0%	43.0%	0.280	45.6%
18	-1	0	-1	0	0	8.3%	29.8%	0.278	51.1%
232	-1	1	-1	1	-1	9.5%	34.2%	0.278	49.9%
234	-1	1	-1	-1	-1	12.1%	45.2%	0.268	48.0%
114	0	-1	-1	0	-1	7.9%	30.0%	0.265	49.8%
192	-1	0	-1	1	-1	8.5%	32.3%	0.263	60.8%
208	0	-1	-1	1	-1	7.6%	28.8%	0.262	53.5%
82	-1	0	-1	-1	0	7.4%	28.6%	0.259	53.9%
98	-1	0	0	-1	-1	8.1%	31.4%	0.256	44.4%
56	-1	1	-1	0	0	9.5%	37.0%	0.256	63.4%
242	-1	-1	-1	-1	-1	7.6%	29.6%	0.255	44.7%
26	-1	0	0	0	-1	9.0%	35.4%	0.255	48.1%
72	-1	1	0	0	-1	10.7%	42.6%	0.252	53.5%
194	-1	0	-1	-1	-1	10.0%	39.9%	0.252	42.8%
130	0	0	-1	-1	-1	11.3%	44.9%	0.251	48.4%
128	0	0	-1	1	-1	8.0%	32.1%	0.250	47.1%
110	0	1	-1	0	-1	14.1%	56.3%	0.250	48.2%

Table 7. Frequencies of Sampled Factor Levels Among the Twenty Highest-Sharpe Active Design Points

Factor	-1	-1	0	0	+1	+1	Most Frequent
	Count	Share	Count	Share	Count	Share	Level
PB	14	70%	6	30%	0	0%	-1
ROE	4	20%	10	50%	6	30%	0
MV	17	85%	3	15%	0	0%	-1
Beta	6	30%	10	50%	4	20%	0
MOM	16	80%	4	20%	0	0%	-1

Table 8. Twenty Active Design Points with the Smallest Maximum Drawdowns

Strategy ID	PB	ROE	MV	BETA	MOM	Mean Quarterly Return	Quarterly Return Volatility	Sharpe Ratio	Maximum Drawdown
178	-1	-1	0	-1	-1	6.8%	29.5%	0.231	39.2%
160	-1	-1	1	0	-1	6.3%	28.3%	0.221	41.0%
54	1	-1	-1	0	0	6.5%	26.4%	0.246	41.2%
210	0	-1	-1	-1	-1	8.2%	35.5%	0.230	41.5%
137	1	-1	-1	1	0	6.0%	25.7%	0.232	42.6%
194	-1	0	-1	-1	-1	10.0%	39.9%	0.252	42.8%
226	1	-1	-1	-1	-1	8.0%	38.6%	0.207	43.2%
122	0	-1	0	-1	-1	7.5%	36.5%	0.206	43.3%
105	0	-1	-1	1	0	6.2%	25.7%	0.239	43.7%
236	-1	-1	1	1	-1	5.8%	25.9%	0.224	44.2%
58	-1	-1	-1	0	0	5.5%	23.8%	0.232	44.3%
238	-1	-1	1	-1	-1	6.1%	27.7%	0.221	44.3%
98	-1	0	0	-1	-1	8.1%	31.4%	0.256	44.4%
206	0	-1	1	-1	-1	7.3%	35.3%	0.207	44.6%
30	0	-1	-1	0	0	5.3%	23.7%	0.223	44.6%
242	-1	-1	-1	-1	-1	7.6%	29.6%	0.255	44.7%
200	0	1	-1	1	-1	9.5%	38.4%	0.247	44.8%
12	1	-1	0	0	0	5.1%	24.7%	0.205	45.2%
4	0	-1	0	0	0	4.6%	22.2%	0.208	45.5%
158	-1	1	-1	0	-1	12.0%	43.0%	0.280	45.6%

Table 9. Frequencies of Sampled Factor Levels Among the Twenty Smallest-Drawdown Active Design Points

Factor	-1	-1	0	0	+1	+1	Most Frequent
	Count	Share	Count	Share	Count	Share	
PB	9	45%	7	35%	4	20%	-1
ROE	16	80%	2	10%	2	10%	-1
MV	11	55%	5	25%	4	20%	-1
Beta	9	45%	7	35%	4	20%	-1
MOM	13	65%	7	35%	0	0%	-1

5.3 Signed Quadratic Regression Analysis

To examine how portfolio-performance responses vary over the sampled signed-weight region, this section estimates second-order response-surface regressions. The purpose is to summarize systematic variation in the observed design-point responses and to capture first-order variation, pairwise interactions, and curvature that cannot be represented adequately by isolated one-factor comparisons.

For performance response Y_m , the estimated model is

$$Y_m(\mathbf{w}) = \beta_{0m} + \sum_{k=1}^5 \beta_{km} w_k + \sum_{k=1}^5 \gamma_{km} w_k^2 + \sum_{k < j} \beta_{kj,m} w_k w_j + \varepsilon_m. \quad (35)$$

Separate models are estimated for average return, volatility, the Sharpe ratio, and maximum drawdown. These regressions are response-surface models over the experimental signed-weight coordinates; they are not factor-pricing regressions and should not be interpreted as estimating stock-level risk premia or causal effects of firm characteristics.

5.3.1 Interpretation of the Second-Order Response-Surface Regression

The regression analysis uses all 243 factorial design points, including the all-zero center point. Thus, the regression sample includes the 242 active nonzero design points used in the active-strategy analyses together with the center point, whose empirically defined response is retained in model estimation.

The model contains one intercept, five linear terms, five quadratic terms, and ten pairwise interaction terms, for a total of 21 estimated parameters. Because all 243 factorial design points are included, the residual degrees of freedom are $243 - 21 = 222$.

The linear coefficients summarize first-order variation in the fitted response with respect to the signed-weight coordinates. Because the model also contains quadratic and interaction terms, a linear coefficient should not be interpreted as a globally constant marginal effect. The fitted slope with respect to coordinate w_k is

$$\frac{\partial \widehat{Y}_m}{\partial w_k} = \widehat{\beta}_{km} + 2\widehat{\gamma}_{km} w_k + \sum_{j \neq k} \widehat{\beta}_{kj,m} w_j. \quad (36)$$

Accordingly, the fitted association between a weight coordinate and the response may vary with both its own location and the levels of other coordinates.

For an interaction term,

$$\frac{\partial^2 \widehat{Y}_m}{\partial w_k \partial w_j} = \widehat{\beta}_{kj,m}. \quad (37)$$

A nonzero interaction coefficient therefore indicates that the fitted slope with respect to one coordinate varies with the level of another coordinate. Similarly, for a quadratic term,

$$\frac{\partial^2 \widehat{Y}_m}{\partial w_k^2} = 2\widehat{\gamma}_{km}, \quad (38)$$

so that it captures curvature of the fitted response along coordinate w_k . These coefficients describe features of the fitted second-order approximation and should not,

by themselves, be interpreted as direct evidence of specific behavioral, institutional, or causal mechanisms.

5.3.2 Overall Model Fit and Explanatory Power

Table 10 reports the goodness-of-fit statistics for the four second-order response-surface models. All four regressions are estimated with N=243 design-point observations, 20 non-intercept regressors, 21 total parameters, and 222 residual degrees of freedom.

The models exhibit high explanatory power across all four responses. Adjusted R² values are 0.903 for mean quarterly return, 0.868 for quarterly return volatility, 0.880 for the Sharpe ratio, and 0.883 for maximum drawdown. The corresponding F-statistics are large for all four models, indicating that the fitted second-order structures collectively explain substantial variation across the sampled design points.

These results support the use of a second-order response-surface approximation. They do not imply that the true economic response function is globally quadratic; rather, they show that linear, interaction, and quadratic terms summarize a large share of the observed variation across the 243 factorial design-point responses.

Table 10. Goodness-of-Fit Statistics for the Second-Order Response-Surface Models

Response	N	R ²	Adj-R ²	RMSE	F-stat	Model df	Residual df
Mean quarterly return	243	0.911	0.903	0.0082	113.3	20	222
Quarterly return volatility	243	0.879	0.868	0.0346	80.8	20	222
Sharpe ratio	243	0.889	0.880	0.0144	89.3	20	222
Maximum drawdown	243	0.893	0.883	0.0417	92.5	20	222

5.3.3 Average Return Regression

Table 11 reports the second-order response-surface estimates for mean quarterly return. The coefficient pattern indicates strong directional and interaction structure within the fitted return surface.

The largest linear pattern concerns ROE. The positive ROE coefficient (t=25.01) indicates that movement toward positive ROE settings is associated with higher fitted mean return, conditional on the remaining terms in the model.

MV and Beta both have significantly negative linear coefficients. Within the fitted return surface, this means that movement toward smaller-cap and lower-Beta settings is associated with higher mean return, holding the other response-surface terms constant.

MOM also has a strongly negative coefficient (t=-28.46), indicating that reversal-oriented MOM settings are associated with higher fitted mean return than continuation-oriented MOM settings in this sample.

Several interaction terms are significant, including PB×BETA, PB×MOM, ROE×MV, ROE×BETA, and ROE×MOM. These results indicate that the fitted

contribution of one coordinate depends on other coordinates; the return surface is therefore not adequately described by isolated one-factor signs alone.

For example, the ROE×BETA and ROE×MOM interactions imply that the fitted association of profitability with return depends on systematic-risk and momentum settings. The interaction terms should be interpreted as features of the response surface, not as direct evidence of causal mechanisms.

The valuation-related interactions further indicate that PB cannot be interpreted in isolation. Its fitted association with return changes with size, Beta, and MOM settings, which is consistent with the broader argument that value-oriented and growth-oriented structures are configuration-dependent.

The quadratic terms indicate additional curvature. In particular, the negative BETA² coefficient (t=-4.96) suggests that the fitted return response along the Beta coordinate is nonlinear rather than purely linear.

Overall, the return regression shows that high mean return is associated with a combination of signed relative-weight structure, interaction terms, and curvature within the fitted response surface.

Table 11. Second-Order Response-Surface Estimates for Mean Quarterly Return

Term	Estimate	Std. Error	t-stat	p-val	Term	Estimate	Std. Error	t-stat	p-val
Intercept	0.0722	0.0017	41.27	0.000	PB×ROE	0.0014	0.0008	1.72	0.086
PB	0.0038	0.0006	5.84	0.000	PB×MV	-0.0018	0.0008	-2.32	0.021
ROE	0.0162	0.0006	25.01	0.000	PB×BETA	-0.0044	0.0008	-5.52	0.000
MV	-0.0110	0.0006	-17.05	0.000	PB×MOM	-0.0026	0.0008	-3.25	0.001
BETA	-0.0115	0.0006	-17.77	0.000	ROE×MV	-0.0037	0.0008	-4.66	0.000
MOM	-0.0184	0.0006	-28.46	0.000	ROE×BETA	-0.0058	0.0008	-7.28	0.000
PB ²	0.0017	0.0011	1.52	0.131	ROE×MOM	-0.0048	0.0008	-6.04	0.000
ROE ²	0.0009	0.0011	0.84	0.404	MV×BETA	0.0003	0.0008	0.39	0.694
MV ²	0.0002	0.0011	0.14	0.886	MV×MOM	0.0006	0.0008	0.76	0.447
BETA ²	-0.0056	0.0011	-4.96	0.000	BETA×MOM	0.0004	0.0008	0.56	0.576
MOM ²	0.0005	0.0011	0.41	0.685					

5.3.4 Volatility Regression

Table 12 reports the second-order response-surface estimates for quarterly return volatility. Because volatility is a risk measure, lower fitted values are more favorable for this objective.

Positive PB and positive ROE settings are associated with higher fitted volatility. The PB coefficient (t=16.03) and ROE coefficient (t=25.30) indicate that growth-

oriented valuation and aggressive profitability settings tend to raise volatility within the fitted surface.

MV, Beta, and MOM have negative linear coefficients. At the center of the other weight coordinates, movement toward more positive settings on these coordinates is associated with lower fitted volatility. This coefficient pattern should be distinguished from the return and Sharpe-ratio results, in which negative MOM settings are more favorable.

The volatility regression also contains substantial interaction structure. The PB×BETA and ROE×BETA terms indicate that valuation and profitability settings interact with systematic-risk orientation in shaping fitted volatility.

ROE×MV and ROE×MOM are also significant, suggesting that profitability-related volatility depends on size and momentum settings. These coefficients reinforce the view that risk behavior is configuration-dependent.

The quadratic terms provide evidence of curvature in the volatility surface. The negative MV² coefficient suggests nonlinear size-related stabilization, while the positive MOM² coefficient indicates nonlinear volatility amplification associated with the MOM coordinate.

This result should be interpreted as curvature in the fitted response surface, not as proof of a specific speculative mechanism.

The positive BETA×MOM interaction further indicates that the joint configuration of systematic-risk and momentum settings is relevant for volatility.

Taken together, the volatility regression demonstrates that risk minimization corresponds to a different fitted region from return maximization.

Table 12. Second-Order Response-Surface Estimates for Quarterly Return Volatility

Term	Estimate	Std. Error	t-stat	p-val	Term	Estimate	Std. Error	t-stat	p-val
Intercept	0.3615	0.0074	49.16	0.000	PB×ROE	0.0091	0.0033	2.73	0.007
PB	0.0435	0.0027	16.03	0.000	PB×MV	-0.0015	0.0033	-0.45	0.653
ROE	0.0687	0.0027	25.30	0.000	PB×BETA	-0.0256	0.0033	-7.68	0.000
MV	-0.0205	0.0027	-7.56	0.000	PB×MOM	-0.0108	0.0033	-3.25	0.001
BETA	-0.0455	0.0027	-16.77	0.000	ROE×MV	-0.0155	0.0033	-4.65	0.000
MOM	-0.0306	0.0027	-11.27	0.000	ROE×BETA	-0.0220	0.0033	-6.63	0.000
PB ²	-0.0001	0.0047	-0.03	0.978	ROE×MOM	-0.0173	0.0033	-5.21	0.000
ROE ²	-0.0059	0.0047	-1.26	0.209	MV×BETA	0.0021	0.0033	0.62	0.533
MV ²	-0.0173	0.0047	-3.69	0.000	MV×MOM	-0.0031	0.0033	-0.93	0.354
BETA ²	-0.0155	0.0047	-3.29	0.001	BETA×MOM	0.0128	0.0033	3.86	0.000
MOM ²	0.0306	0.0047	6.50	0.000					

5.3.5 Sharpe-Ratio Regression

Table 13 reports the second-order response-surface estimates for the Sharpe ratio. Because the Sharpe ratio incorporates both return and volatility, this model describes risk-adjusted performance rather than pure return generation.

Several directional patterns are evident.

First, the negative PB coefficient ($t=-12.92$) indicates that movement toward low-PB, value-oriented settings is associated with higher fitted Sharpe ratios. This differs from the return-only comparison, where both PB directions can appear in favorable high-return configurations.

Second, the negative MV coefficient ($t=-18.08$) indicates that smaller-cap settings are associated with higher fitted Sharpe ratios in this sample.

Third, the negative MOM coefficient ($t=-31.65$) indicates that reversal-oriented MOM settings are strongly associated with higher fitted risk-adjusted performance.

ROE remains positive but is less dominant in the Sharpe-ratio model than in the return model. This suggests that profitability-oriented structures contribute to return but may also introduce volatility or downside exposure that reduces risk-adjusted efficiency.

The interaction terms show that Sharpe-ratio performance is also configuration-dependent. The PB×MV and MV×BETA terms indicate that valuation, size, and systematic-risk settings jointly affect fitted efficiency.

The BETA×MOM interaction is especially important, with a strongly negative coefficient ($t=-7.64$). This indicates that the fitted efficiency effect of MOM depends strongly on the systematic-risk coordinate.

As before, this interaction should be interpreted as a response-surface feature rather than as a directly identified behavioral mechanism.

The quadratic terms reveal additional curvature. The positive PB² and MV² coefficients indicate nonlinear fitted relationships for valuation and size, while the negative MOM² coefficient indicates nonlinear deterioration in fitted Sharpe-ratio performance along the MOM coordinate.

This curvature implies that favorable Sharpe-ratio regions need not lie along a single simple sign rule; they may depend on combinations of signed coordinates.

Overall, the Sharpe-ratio regression demonstrates that risk-adjusted efficiency is a distinct objective with its own signed-weight geometry.

The structures associated with maximum return are therefore not identical to those associated with maximum return efficiency.

Table 13. Second-Order Response-Surface Estimates for the Sharpe Ratio

Term	Estimate	Std. Error	t-stat	p-val	Term	Estimate	Std. Error	t-stat	p-val
Intercept	0.1995	0.0031	65.08	0.000	PB×ROE	-0.0008	0.0014	-0.57	0.568
PB	-0.0146	0.0011	-12.92	0.000	PB×MV	-0.0037	0.0014	-2.70	0.007
ROE	0.0054	0.0011	4.78	0.000	PB×BETA	0.0016	0.0014	1.18	0.237
MV	-0.0205	0.0011	-18.08	0.000	PB×MOM	0.0007	0.0014	0.53	0.598
BETA	-0.0076	0.0011	-6.75	0.000	ROE×MV	0.0016	0.0014	1.16	0.248
MOM	-0.0358	0.0011	-31.65	0.000	ROE×BETA	-0.0023	0.0014	-1.64	0.102
PB ²	0.0074	0.0020	3.77	0.000	ROE×MOM	0.0023	0.0014	1.63	0.105
ROE ²	0.0038	0.0020	1.92	0.056	MV×BETA	-0.0033	0.0014	-2.38	0.018
MV ²	0.0097	0.0020	4.96	0.000	MV×MOM	0.0002	0.0014	0.11	0.914
BETA ²	-0.0078	0.0020	-3.96	0.000	BETA×MOM	-0.0106	0.0014	-7.64	0.000
MOM ²	-0.0181	0.0020	-9.22	0.000					

5.3.6 Maximum Drawdown Regression

Table 14 reports the second-order response-surface estimates for maximum drawdown magnitude. Because MDD is defined as a nonnegative magnitude, smaller fitted values are more favorable.

Several directional patterns are evident.

The MOM coefficient is strongly positive ($t=36.82$), indicating that movement toward positive MOM, or continuation-oriented momentum, is associated with larger fitted drawdown magnitude.

Conversely, negative MOM settings are more favorable for drawdown control in the fitted surface, consistent with the earlier volatility and Sharpe-ratio results.

Positive PB and positive ROE settings are also associated with larger fitted drawdowns. The PB coefficient ($t=7.62$) and ROE coefficient ($t=16.75$) suggest that growth-oriented valuation and aggressive profitability settings may increase downside exposure even when they are favorable for return generation.

This contrast again shows objective dependence: a signed coordinate that is favorable for return may be less favorable for drawdown control.

The MDD model also contains important interaction terms. The positive PB×MV coefficient indicates that valuation and size jointly affect the fitted downside-risk surface.

The negative PB×BETA coefficient indicates that valuation and systematic-risk settings interact in determining fitted drawdown magnitude.

The positive ROE×MOM interaction ($t=4.17$) further indicates that profitability and momentum settings jointly affect downside exposure.

The quadratic terms reveal additional curvature. The negative ROE² and MV² coefficients indicate nonlinear fitted stabilization along those coordinates, whereas the positive MOM² coefficient indicates nonlinear drawdown amplification along the MOM coordinate.

These quadratic results are interpreted as response-surface curvature, not as direct identification of the underlying economic mechanism.

Taken together, the MDD regression shows that downside-risk control corresponds to a distinct region within the signed-weight performance landscape.

Across the four regressions, the evidence supports the central argument of the paper: portfolio performance is configuration-dependent. Outcomes vary with signed factor direction, relative intensity, interaction terms, curvature, and the chosen performance objective. The regressions use all 243 factorial design points, including the all-zero center point, and the fitted response surfaces summarize the portfolio-performance patterns observed across the sampled factorial design points.

Table 14. Second-Order Response-Surface Estimates for Maximum Drawdown

Term	Estimate	Std. Error	t-stat	p-val	Term	Estimate	Std. Error	t-stat	p-val
Intercept	0.6103	0.0089	68.79	0.000	PB×ROE	-0.0005	0.0040	-0.13	0.898
PB	0.0250	0.0033	7.62	0.000	PB×MV	0.0114	0.0040	2.83	0.005
ROE	0.0549	0.0033	16.75	0.000	PB×BETA	-0.0121	0.0040	-3.03	0.003
MV	0.0155	0.0033	4.74	0.000	PB×MOM	0.0024	0.0040	0.61	0.544
BETA	0.0042	0.0033	1.28	0.201	ROE×MV	0.0004	0.0040	0.09	0.927
MOM	0.1206	0.0033	36.82	0.000	ROE×BETA	-0.0024	0.0040	-0.61	0.546
PB ²	-0.0062	0.0057	-1.09	0.275	ROE×MOM	0.0167	0.0040	4.17	0.000
ROE ²	-0.0119	0.0057	-2.10	0.037	MV×BETA	-0.0021	0.0040	-0.52	0.606
MV ²	-0.0192	0.0057	-3.38	0.001	MV×MOM	-0.0074	0.0040	-1.84	0.067
BETA ²	0.0026	0.0057	0.45	0.653	BETA×MOM	-0.0041	0.0040	-1.03	0.304
MOM ²	0.0489	0.0057	8.62	0.000					

6. Discussion

6.1 Beyond Factor Inclusion: Direction and Relative Intensity as Joint Design Variables

The central implication of the empirical analysis is that multi-factor portfolio design cannot always be reduced to a binary decision about whether a characteristic should be included. Once weights are allowed to take positive, negative, and zero values, three related choices enter the same design problem: factor inclusion, factor orientation, and relative intensity.

This perspective differs from conventional procedures in which a characteristic is first transformed into a predetermined favorable direction and only then assigned a nonnegative weight. Under such procedures, the relative weight of a pre-oriented signal may vary, but the factor orientation is specified in advance and remains unchanged throughout the analysis. The present framework instead treats the signed coordinate $w_k \in [-1, 1]$ as the joint representation of direction and relative intensity.

The empirical evidence indicates that this additional flexibility is economically relevant. The preferred sampled settings differ across factors and performance criteria, while the fitted response surfaces contain interaction and second-order structure that cannot be summarized by a single fixed sign convention.

The broader implication is not that established factor directions are generally incorrect. Many conventional orientations remain economically well motivated and empirically successful. Rather, the results show that fixing all orientations before examining the joint portfolio problem imposes a restriction that may be consequential when the objective, surrounding factor structure, or market environment changes.

Accordingly, the contribution of the signed-weight framework is best understood as an expansion of the admissible portfolio-design space. It allows conventional orientations to emerge when supported by the objective and data, while also permitting opposite or near-zero orientations when those settings are favored within the fitted structure.

6.2 Objective Dependence and the Absence of a Universal Weight Structure

A second implication concerns the role of the portfolio objective. The empirical analyses show that weight structures associated with high average return differ from those associated with low volatility, high Sharpe ratio, or favorable drawdown behavior.

This result is economically important because factor discussions often refer to “the preferred factor,” “the preferred direction,” or “the preferred factor combination” without making the underlying performance objective sufficiently explicit. In a multi-response portfolio setting, such statements are incomplete. A signed-weight structure can only be interpreted relative to a specified performance criterion.

The contrast between return-oriented and volatility-oriented structures illustrates this point. The active-design analysis shows that profitability orientation occupies different positions across these objectives. More generally, the recurring patterns among extreme design points change when the response criterion changes.

This objective dependence should not be interpreted as a contradiction among the empirical results. It reflects the fact that

$$\bar{R}, \quad \sigma_R, \quad SR, \quad MDD$$

represent different properties of the portfolio-return process. A structure that increases average return may also increase variability or downside exposure, while a structure that reduces volatility need not minimize maximum drawdown.

The appropriate interpretation is therefore multi-objective rather than universal. Rather than identifying a single preferred factor-weight structure, the framework characterizes different regions of the signed-weight space according to different performance criteria.

This distinction also limits the interpretation of the fitted response surfaces. Their interpretation depends on:

- the selected performance objective,
- the estimated response surface,
- the bounded signed-weight region $[-1,1]^5$,
- the empirical sample.

Accordingly, the fitted response surfaces should be interpreted as objective-specific empirical approximations to portfolio-performance patterns rather than as universally applicable factor-allocation rules.

6.3 Interaction Effects, Second-Order Structure, and the Limits of Mechanistic Interpretation

The response-surface analysis indicates that portfolio performance varies jointly with the signed-weight coordinates. This joint dependence is represented through pairwise interactions and quadratic terms.

For two coordinates w_i and w_j , an interaction term

$$\beta_{ij}w_iw_j$$

allows the fitted association between one coordinate and the response to depend on the other. In this sense, the framework moves beyond an additive view in which each factor contributes independently to portfolio performance.

This result has an important implication for factor interpretation. A factor orientation that appears favorable in one surrounding weight structure may be less favorable, neutral, or unfavorable in another. Consequently, isolated statements about the effect of one factor may provide an incomplete description of the joint portfolio problem.

At the same time, interaction coefficients should not be assigned stronger structural interpretations than the empirical design supports. A statistically significant term such as $w_{ROE}w_{\beta}$ shows conditional dependence within the fitted second-order approximation. It does not by itself identify investor overreaction, speculative trading, information diffusion, institutional constraints, or any other specific economic mechanism.

The same caution applies to quadratic terms. Because the experimental levels are $-1, 0, +1$, the squared coordinate satisfies

$$w_k^2 = \begin{cases} 1, & w_k = \pm 1, \\ 0, & w_k = 0. \end{cases} \quad (39)$$

Within the factorial sample, the quadratic regressor therefore partly distinguishes nonzero settings from the zero setting. Its coefficient captures second-order structure in the fitted approximation, but it should not automatically be labeled an amplification, stabilization, or speculative mechanism.

This point is particularly important for the interpretation of a second-order response surface outside the directly observed factorial points. The fitted polynomial provides a continuous approximation over $[-1,1]^5$ but the empirical responses themselves are observed only at the design points. Intermediate fitted values are model-based interpolations, and the estimated curvature depends on the adequacy of the second-order specification.

The empirical contribution should therefore be stated precisely: the results provide evidence that a second-order approximation containing interaction and quadratic terms summarizes substantial variation across the sampled signed-weight design. They do not establish that the true economic response function is globally quadratic.

6.4 Relative Weights, Positive-Scale Equivalence, and Coordinate Dependence of the Fitted Surface

A distinctive property of the ranking-based scoring rule is that the numerical weight vector is not generally identified as a unique vector of absolute allocations. For a given signed-weight vector $\mathbf{w} \in W$, consider a positively rescaled vector $c\mathbf{w}$, $c > 0$, such that $c\mathbf{w} \in W$.

The corresponding stock score satisfies

$$S_{i,t}(c\mathbf{w}) = \sum_{k=1}^K cw_k X_{k,i,t} = cS_{i,t}(\mathbf{w}). \quad (40)$$

Because multiplication by a positive scalar preserves cross-sectional ordering,

$$\text{rank}[S_{i,t}(c\mathbf{w})] = \text{rank}[S_{i,t}(\mathbf{w})], \quad (41)$$

apart from implementation issues associated with ties or additional portfolio-formation rules. Accordingly, whenever both vectors are feasible,

$$\mathbf{w} \sim c\mathbf{w}, \quad c > 0, \quad (42)$$

under the ranking rule.

This property implies that the coordinates should be interpreted as relative signed weights rather than uniquely identified absolute allocations. For example, (1, -1, 1, -1, 1) and (0.5, -0.5, 0.5, -0.5, 0.5) generate the same score ranking under the basic scoring rule, even though they occupy different numerical locations in the bounded coordinate region.

The equivalence is, however, a property of the underlying ranking rule and should be distinguished from the fitted response-surface approximation. The second-order model is estimated as

$$\widehat{Y}_m(\mathbf{w}) = \widehat{\beta}_{0m} + \sum_{k=1}^K \widehat{\beta}_{km} w_k + \sum_{k=1}^K \widehat{\gamma}_{km} w_k^2 + \sum_{k < \ell} \widehat{\beta}_{k\ell, m} w_k w_\ell. \quad (43)$$

This polynomial specification does not impose the restriction

$$\widehat{Y}_m(c\mathbf{w}) = \widehat{Y}_m(\mathbf{w}). \quad (44)$$

Indeed, positive rescaling generally changes the linear and quadratic components of the fitted polynomial. Consequently, two positively proportional vectors that induce the same stock ranking may receive different fitted response values from the unconstrained second-order approximation.

This distinction has two implications. First, positive-scale equivalence limits the interpretation of raw weight magnitudes in the underlying portfolio rule. Numerically different vectors may correspond to the same ranking-based strategy. Second, the fitted response surface should be interpreted as an empirical approximation within the chosen bounded coordinate representation. Its shape and implied performance patterns therefore depend on both the adopted parameterization and the estimated second-order approximation.

The reported weight vectors should therefore not be interpreted as uniquely identified absolute factor allocations. Instead, they should be viewed as relative signed-weight structures within the parameterization $\mathbf{W} = [-1, 1]^K$, where different numerical vectors may represent economically equivalent ranking rules.

This interpretation also explains why conventional mixture normalization is not imposed. A simplex restriction such as

$$w_k \geq 0, \quad \sum_{k=1}^K w_k = 1 \quad (45)$$

is appropriate when weights represent nonnegative component shares. In the present framework, however, positive and negative coordinates encode opposite cross-sectional orientations. A conventional simplex would therefore exclude a central feature of the signed-weight problem.

Positive-scale equivalence is thus both a structural property and a limitation of the

present parameterization. It clarifies why raw numerical weights should not be interpreted as absolute allocations, while also showing that an unconstrained polynomial surrogate need not preserve every invariance property of the underlying ranking rule.

6.5 Limitations

Several limitations define the appropriate scope of the conclusions.

First, the response surface is an empirical approximation estimated from a finite factorial design. The underlying admissible region is continuous, $[-1,1]^5$, whereas observed portfolio responses are available only at the three-level design points $\{-1,0,+1\}^5$. Consequently, the fitted second-order polynomial should be interpreted as an approximation to the portfolio-performance patterns within the sampled design region rather than as an exact representation of the entire continuous signed-weight space.

Second, the scoring framework exhibits positive-scale equivalence. Because positively proportional weight vectors generate identical rankings, numerical weights are not uniquely identified as absolute allocations. The reported signed-weight structures should therefore be interpreted as relative representations of portfolio-construction rules rather than unique numerical solutions.

Third, the empirical analysis is conditional on the selected factor set (PB, ROE, MV, Beta, and MOM). Although these variables represent important dimensions of valuation, profitability, size, systematic risk, and prior returns, they do not span the full factor literature. Extending the framework to additional characteristics would substantially increase the dimensionality of the design space, since a three-level full factorial design requires 3^K experimental design points.

Fourth, the estimated response surfaces are sample dependent. The empirical evidence is based on Chinese A-share data over 2005–2025 and may vary across historical periods, market regimes, or future samples. The present analysis therefore does not establish the temporal invariance of the estimated performance surfaces.

Fifth, the empirical portfolio responses may depend on implementation choices, including portfolio size, weighting schemes, rebalancing frequency, transaction costs, turnover, liquidity constraints, and the timing of factor information. These practical considerations should be taken into account when interpreting the empirical results.

The long-only portfolio rule is also a construction constraint; portfolio constraints can affect how investment signals translate into realized portfolio positions (Clarke et al., 2002).

6.6 Directions for Future Research

The proposed framework may be extended in several directions.

First, more efficient experimental designs. The present study adopts a three-level full factorial design because it provides complete and balanced coverage of the signed-weight space. As the number of factors increases, however, the required number of design points grows exponentially. Future research may investigate fractional factorial designs, Latin hypercube sampling, or other space-filling designs that reduce computational cost while preserving adequate coverage of the continuous signed-weight region.

Second, more flexible response-surface models. The present study employs second-order polynomial response surfaces because they are simple, interpretable, and widely used in response-surface methodology. Future studies may compare alternative surrogate models, such as Gaussian-process regression, spline surfaces, random forests, or neural networks, to examine whether they provide more accurate approximations of portfolio-performance patterns.

Third, temporal and cross-market validation. The framework should be evaluated over different historical periods, market regimes, and international equity markets to examine the stability and generalizability of the estimated response surfaces. Rolling-window estimation, subperiod analysis, and out-of-sample validation would provide additional evidence regarding the robustness of the observed signed-weight structures.

Fourth, broader applications of the signed-weight framework. Although the present study focuses on equity factor investing, the proposed signed-weight methodology may also be applied to other asset classes, multi-asset allocation problems, and machine-learning-based portfolio scoring systems. Extending the framework to these settings would help assess its general applicability beyond the Chinese equity market.

7. Conclusion

This paper develops a continuous signed-weight framework for multi-factor portfolio design. Instead of specifying factor directions in advance, the framework allows each factor weight to vary over $w_k \in [-1, 1]$, so that positive and negative values represent opposite cross-sectional orientations, zero removes the factor from the scoring rule, and absolute magnitude represents relative intensity. The framework therefore treats factor direction and relative intensity as joint design variables.

The empirical application considers five characteristics—price-to-book ratio (PB), return on equity (ROE), market value (MV), Beta, and momentum (MOM). The continuous signed-weight region is sampled at three factorial levels, -1 , 0 , and $+1$, in each coordinate through a full three-level factorial design, $w_k \in \{-1, 0, +1\}$, yielding $3^5=243$ design points. The 242 nonzero points represent active scoring configurations,

while the all-zero center point is retained in response modeling as a non-discriminating scoring point with an empirically defined portfolio response. Portfolio responses at the design points are used to estimate second-order response surfaces, which provide continuous empirical approximations to the portfolio-performance patterns within the bounded signed-weight region $[-1,1]^5$.

The empirical evidence shows that preferred signed-weight structures depend materially on the performance objective. In the Chinese sample, positive ROE settings are strongly associated with return-oriented structures, whereas negative ROE settings are more prominent in low-volatility structures. Negative MOM settings exhibit strong performance in several return-oriented and risk-adjusted comparisons, indicating an important reversal-oriented pattern. PB, MV, and Beta display more conditional behavior across objectives and surrounding weight structures.

The response-surface analysis further indicates that performance variation is not adequately summarized by isolated one-factor comparisons alone. Interaction and quadratic terms contribute to the fitted second-order structure, implying that the association between one weight coordinate and portfolio performance may depend on other coordinates and on location within the fitted surface. These terms are interpreted as features of the empirical approximation rather than as direct identification of specific behavioral or institutional mechanisms.

The paper makes three main contributions.

First, it extends conventional factor-weight frameworks by allowing factor direction and relative intensity to vary jointly within a continuous signed-weight region.

Second, it integrates three-level factorial experimental design with second-order response-surface modeling to characterize portfolio-performance patterns across sampled design points.

Third, it distinguishes the underlying continuous signed-weight space, the finite factorial design used to sample that space, and the fitted response surfaces that provide empirical approximations to the observed portfolio-performance patterns.

A further implication concerns identification. Because portfolio formation is based on score rankings, positively proportional weight vectors may generate identical stock rankings and portfolios:

$$\mathbf{w} \sim c\mathbf{w}, \quad c > 0.$$

The resulting weights are therefore relative rather than uniquely identified absolute allocations, and numerically different vectors may represent the same ranking rule. Recognizing this positive-scale equivalence is important when interpreting fitted response surfaces and comparing signed-weight structures.

Overall, the findings suggest that multi-factor portfolio construction should not be viewed solely as a problem of selecting factor magnitudes after imposing fixed signs. A more general formulation allows signed relative weights to vary jointly and

recognizes that preferred structures may depend on the investment objective, surrounding factor configuration, and market environment. Rather than seeking a universally preferred weight vector, the proposed framework provides a systematic approach for sampling, characterizing, and approximating portfolio-performance patterns over a continuous signed-weight space while explicitly recognizing the limits of numerical uniqueness and economic interpretation.

List of Symbols and Abbreviations

Symbol / Abbreviation	Definition
i	Stock index
j	Alternative stock index used in pairwise ranking comparisons
t	Portfolio-formation or evaluation period index
τ	Historical return-period index used in parameter estimation
k	Factor index, $k=1, \dots, K$
M	Portfolio-performance response index
K	Number of factors; $K=5$ in the empirical application
$Z_{k,i,t}$	Winsorized raw value of factor k for stock i at time t
$X_{k,i,t}$	Cross-sectionally standardized value of factor k for stock i at time t
$\mu_{k,t}$	Cross-sectional mean of factor k at time t
$\sigma_{k,t}$	Cross-sectional standard deviation of factor k at time t
w_k	Continuous signed relative weight of factor k
$\mathbf{w}$	Signed factor-weight vector, $(w_1, \dots, w_K)$
$\mathbf{W}$	Continuous admissible signed-weight region, $[-1, 1]^K$
$\mathbf{D}$	Three-level full factorial design set, $\{-1, 0, +1\}^K$
$\mathbf{0}$	All-zero center point, $(0, \dots, 0)$
c	Positive scalar used in positive-scale equivalence, $c > 0$
$S_{i,t}(\mathbf{w})$	Composite score of stock i at time t under weight vector $\mathbf{w}$
$Y_m(\mathbf{w})$	Observed portfolio-performance response m associated with design point $\mathbf{w}$
β_{0m}	Intercept of response-surface model m
β_{km}	Linear coefficient for factor-weight coordinate k in model m
γ_{km}	Quadratic coefficient for coordinate k in model m

Symbol / Abbreviation	Definition
$\beta_{ij,m}$	Pairwise interaction coefficient between coordinates i and j in model m
ε_m	Approximation error term in response-surface model m
$R_{i,t}$	Return of stock i in period t
$R_{p,t}$	Portfolio return in period t
$R_{M,t}$	Market return in period t
σ_R	Standard deviation of portfolio returns
SR	Sharpe ratio
V_t	Cumulative portfolio value at time t
MDD	Maximum drawdown magnitude
T	Number of portfolio-performance periods
N	Number of observations used in a reported analysis
$N_{k,a}$	Number of active design points with coordinate k fixed at sampled level a
a	Sampled factor level, $a \in \{-1, 0, +1\}$
PB	Price-to-book ratio
ROE	Return on equity
MV	Market value / market capitalization
Beta	Estimated systematic market-risk characteristic
MOM	Momentum characteristic based on prior stock returns
DOE	Design of experiments
RSM	Response-surface methodology
RMSE	Root mean squared error
CSI 300	CSI 300 Index

Note: The symbols i and j may denote stock indices or, where explicitly stated in interaction terms, factor-coordinate indices. To avoid ambiguity in the response-surface specification, pairwise factor interactions are preferably indexed by $k < \ell$ rather than $i < j$.

References

Asness, C. S., Frazzini, A., & Pedersen, L. H. (2019). Quality minus junk. *Review of Accounting Studies*, 24(1), 34–112.

- Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics*, 9(1), 3–18.
- Basu, S. (1983). The relationship between earnings yield, market value and return for NYSE common stocks: Further evidence. *Journal of Financial Economics*, 12(1), 129–156.
- Bauman, W. S., Conover, C. M., & Miller, R. E. (1998). Growth versus value and large-cap versus small-cap stocks in international markets. *Financial Analysts Journal*, 54(2), 75–89.
- Blitz, D., & van Vliet, P. (2007). The volatility effect: Lower risk without lower return. *Journal of Portfolio Management*, 34(1), 102–113.
- Box, G. E. P., & Draper, N. R. (1987). *Empirical model-building and response surfaces*. Wiley.
- Box, G. E. P., & Wilson, K. B. (1951). On the experimental attainment of optimum conditions. *Journal of the Royal Statistical Society: Series B (Methodological)*, 13(1), 1–38.
- Carhart, M. M. (1997). On persistence in mutual fund performance. *Journal of Finance*, 52(1), 57–82.
- Clarke, R., de Silva, H., & Thorley, S. (2002). Portfolio constraints and the fundamental law of active management. *Financial Analysts Journal*, 58(5), 48–66.
- Cornell, J. A. (2011). *Experiments with mixtures: Designs, models, and the analysis of mixture data* (3rd ed.). Wiley.
- De Bondt, W. F. M., & Thaler, R. (1985). Does the stock market overreact? *Journal of Finance*, 40(3), 793–805.
- Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *Journal of Finance*, 47(2), 427–465.
- Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3–56.
- Fama, E. F., & French, K. R. (1998). Value versus growth: The international evidence. *Journal of Finance*, 53(6), 1975–1999.
- Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116(1), 1–22.
- Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, 111(1), 1–25.
- Grinold, R. C., & Kahn, R. N. (2000). *Active portfolio management* (2nd ed.). McGraw-Hill.
- Harvey, C. R., Liu, Y., & Zhu, H. (2016). ... and the cross-section of expected returns. *Review of Financial Studies*, 29(1), 5–68.

- Haugen, R. A., & Heins, A. J. (1975). Risk and the rate of return on financial assets: Some old wine in new bottles. *Journal of Financial and Quantitative Analysis*, 10(5), 775–784.
- Hou, K., Xue, C., & Zhang, L. (2020). Replicating anomalies. *The Review of Financial Studies*, 33(5), 2019–2133.
- Jegadeesh, N. (1990). Evidence of predictable behavior of security returns. *The Journal of Finance*, 45(3), 881–898.
- Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48(1), 65–91.
- Kang, J., Liu, M. H., & Ni, S. X. (2002). Contrarian and momentum strategies in the China stock market: 1993–2000. *Pacific-Basin Finance Journal*, 10(3), 243–265.
- Lakonishok, J., Shleifer, A., & Vishny, R. W. (1994). Contrarian investment, extrapolation, and risk. *Journal of Finance*, 49(5), 1541–1578.
- Lehmann, B. N. (1990). Fads, martingales, and market efficiency. *The Quarterly Journal of Economics*, 105(1), 1–28.
- Mohanram, P. S. (2005). Separating winners from losers among low book-to-market stocks using financial statement analysis. *Review of Accounting Studies*, 10(2–3), 133–170.
- Myers, R. H., Montgomery, D. C., & Anderson-Cook, C. M. (2016). *Response surface methodology: Process and product optimization using designed experiments* (4th ed.). Wiley.
- Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, 108(1), 1–28.
- Qian, E., Hua, R., & Sorensen, E. (2007). *Quantitative equity portfolio management: Modern techniques and applications*. Chapman & Hall/CRC.
- Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *Journal of Portfolio Management*, 11(3), 9–16.
- Stattman, D. (1980). Book values and stock returns. *The Chicago MBA: A Journal of Selected Papers*, 4, 25–45.
- Yeh, I. C., Hsu, T. K., Yen, J. X., & Tsai, C. C. (2018). Building multi-factor stock selection system using mixture design and neural networks. *Journal of Data Analysis*, 13(4), 1–27.
- Yeh, I. C., & Liu, Y. C. (2020). Discovering optimal weights in weighted-scoring stock-picking models: A mixture design approach. *Financial Innovation*, 6(1), 41.