

7529059-fewer-bets-than-pairs-a-one-number-dimensionality-gauge-and-the-diversification-

SUMMARY: This paper introduces D , a one-number gauge measuring the ratio of the volatility of an equal-weight dollar index to the mean volatility of its constituents. D is used to assess the diversification regime in currency portfolios. The study examines 6,117 trading days from 2007 to 2026 and finds that D tracks the first-eigenvalue share of the rolling correlation matrix at +0.88 and is nearly orthogonal to realized volatility at -0.09. The paper also reveals that the cross-pair correlation of daily strategy returns is higher in the high- D tercile for 86% of 88 published strategies, with a mean difference of +0.029 at $t = 10.6$. The research demonstrates that the rise in cross-pair correlation is due to reallocation rather than composition, with variance falling 30% on high- D days while covariance falls only 12%. The study concludes that D is a gauge of the diversification regime and not an alpha regime, as an injected informational edge earns the same in both regimes.

METHOD: The study uses hourly log returns of six currency pairs to compute the equal-weight dollar index return (Equation 1). The gauge D is then calculated as the ratio of the index volatility to the mean volatility of its constituents over a trailing 240-hour window (Equation 2). The paper analyzes 6,117 trading days from 2007 to 2026, with D having a mean of 0.730, standard deviation of 0.085, and a range of 0.437 to 0.921. The gauge's correlation with the first-eigenvalue share of the rolling correlation matrix is also examined.

KEY FINDINGS:

- D tracks the first-eigenvalue share of the rolling correlation matrix at +0.88.
- D is nearly orthogonal to realized volatility at -0.09.
- The cross-pair correlation of daily strategy returns is higher in the high- D tercile for 86% of 88 published strategies.
- On high- D days, variance falls 30% while covariance falls only 12%.
- D is a gauge of the diversification regime and not an alpha regime.

IMPLICATION FOR TRADING: The findings suggest that traders should consider the diversification regime when running strategies across currency pairs. The paper's gauge, D , can be used to assess the effectiveness of diversification and to identify strategies with higher cross-pair correlation. Practitioners can use this information to make informed decisions about their trading strategies and to optimize their portfolios for better risk management.

Fewer Bets Than Pairs: A One-Number Dimensionality Gauge and the Diversification Regime in Currency Portfolios

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ABSTRACT

A trader running one strategy across six currency pairs believes they hold six bets. This paper measures how many they actually hold, with a one-number gauge: D , the ratio of dollar-index volatility to the mean volatility of its constituents, computed on a trailing 240-hour window. Across 6,117 trading days from 2007 to 2026, D tracks the first-eigenvalue share of the rolling correlation matrix at +0.88 and is nearly orthogonal to volatility (−0.09). On 1,655,588 strategy-trades spanning 88 published entry-exit rules, the cross-pair correlation of daily strategy returns rises from 0.074 in the low- D tercile to 0.103 in the high- D tercile, a difference of +0.029 at $t = 10.6$, present in 86% of strategies; six pairs shrink from 4.4 to 4.0 effective bets. Decomposition shows genuine reallocation: variance falls 30% on high- D days while covariance falls 12%. Single-trade dispersion falls to 0.76 of its low- D value even as portfolio concentration rises, and an injected informational edge extracts the same expectancy in both regimes (−1.31 pips, $t = -0.47$). Dimensionality reprises the portfolio, not the signal.

Keywords: diversification; correlation regime; currency portfolio; dimensionality; foreign exchange; risk management

1. INTRODUCTION

Diversification across currency pairs is the multi-pair trader's free lunch, and like most free lunches it is priced by a quantity nobody watches. Six major dollar pairs are six instruments, but they are not six bets: every pair shares the dollar, and when the dollar moves as one, the portfolio quietly collapses toward a single position. This paper builds

the cheapest possible gauge of that collapse, one number per bar with no eigendecomposition, and asks what the gauge is worth to someone running strategies across pairs.

The gauge is D , the ratio of the volatility of an equal-weight dollar index to the mean volatility of its constituents. It is a closed-form function of average pairwise correlation, it tracks the first-eigenvalue share of the rolling correlation matrix at $+0.88$ over nineteen and a half years, and it is nearly orthogonal to realised volatility at -0.09 , so it isolates the correlation dimension of risk rather than restating its level. The orthogonality matters for inference. Forbes and Rigobon (2002) showed that correlations conditioned on high-volatility periods are mechanically biased upward, the artefact that felled a generation of contagion claims; conditioning on D , a volatility-orthogonal quantity, defuses that mechanism by construction, and the reallocation decomposition below confirms the rise is not a variance-composition effect. The related literature reaches the same territory from other directions: extreme and asymmetric equity correlations (Longin and Solnik, 2001; Ang and Chen, 2002), dynamic conditional correlation as a modelling device (Engle, 2002), the dollar factor in currency returns (Lustig, Roussanov and Verdelhan, 2011), and the systematic share of bilateral exchange-rate variation (Verdelhan, 2018). What none of these asks is the trade-ledger question: what does the comovement regime do to the realised risk of actual trading strategies, trade by trade and day by day. That is this paper's question.

The paper makes five contributions. It constructs and validates D as a per-bar dimensionality gauge over the full 2007 to 2026 history. It shows the diversification regime is real on 1,655,588 strategy-trades: cross-pair correlation of daily strategy returns is higher in the high- D tercile for 86% of 88 published strategies, a mean difference of $+0.029$ at $t = 10.6$, robust to the split convention and to the treatment of inactive days. It proves the rise is reallocation rather than composition: on high- D days variance falls 30% while covariance falls only 12%, a log-ratio of $+0.31$ at $t = 8.0$. It documents the single-trade paradox: per-trade dispersion falls by a quarter exactly when the portfolio concentrates, so the trader who watches trade-level risk sees safety while portfolio-level

risk pools. And it closes the extraction question: an injected informational edge earns the same in both regimes, so D is a risk regime, not an alpha regime.

The study closes an exit-census trilogy. Tolušić (2026a) holds entries fixed and varies exits; Tolušić (2026b) studies the catastrophic tail the exits are built for; this paper studies the portfolio dimension both inherit. The three share one engine, one dataset, and one theorem frame from the exit-invariant decomposition of Tolušić (2026c): machinery reprices risk and never creates expectancy. Exits, tails, and now dimensionality each confirm it in their own domain. The gauge itself descends from the flow-tomography programme (Tolušić, 2026d), where the same ratio proxies the dimensionality collapse of the dollar complex.

2. THE GAUGE

Let $r(p, t)$ be the hourly log return of pair p , signed so that a positive value is a dollar appreciation for every pair. The equal-weight dollar index return is

$$m(t) = (1/6) \sum r(p, t) \quad (1)$$

and the gauge is the ratio of index volatility to mean constituent volatility on a trailing 240-hour window,

$$D(t) = \sigma[m] / \text{mean } \sigma[r(p)] \quad (2)$$

For an equal-weight index of six assets with average pairwise correlation ρ , $D^2 = (1 + 5\rho)/6$, so D is a closed-form correlation index with a floor of 0.41 under independence; the same identity gives the effective number of independent bets,

$$N = 6 / (1 + 5\rho) = 1 / D^2 \quad (3)$$

Over 120,323 hourly observations from 2007 to 2026, D has mean 0.730, standard deviation 0.085, and range 0.437 to 0.921. Stepped weekly against the first-eigenvalue share of the 240-hour correlation matrix, D correlates at +0.88 in levels and +0.79 in changes, so the one-number gauge stands in for the eigendecomposition at a fraction of its cost. Its correlation with daily realised volatility is -0.091: D is a correlation gauge, not a volatility gauge in costume, which is the property the Forbes and Rigobon critique

requires. Figure 1 plots the daily series with the tercile boundaries at 0.696 and 0.775; throughout the paper, “regime” is shorthand for tercile conditioning on this continuous gauge, and no discreteness is claimed.

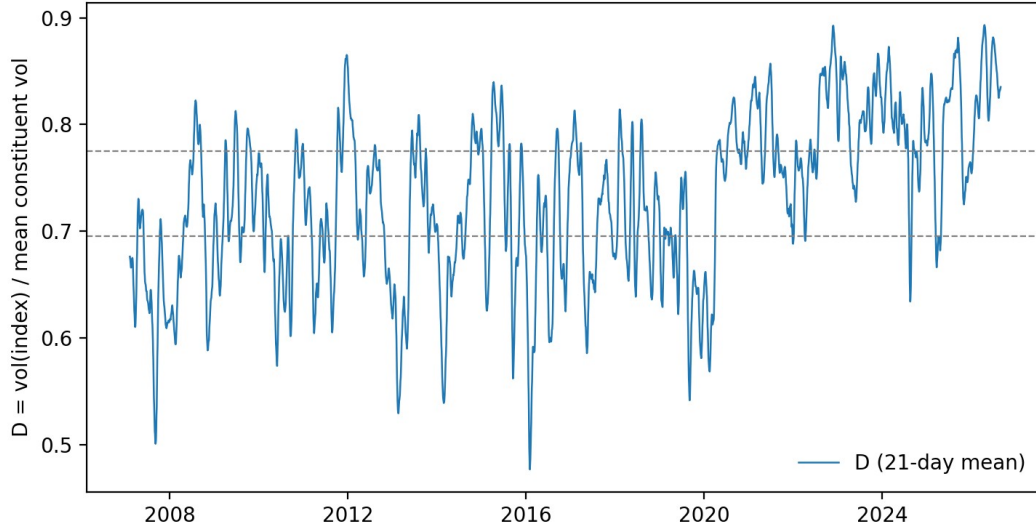


Figure 1. The dimensionality gauge D , 21-day mean, 2007 to 2026. Dashed lines mark the tercile boundaries used throughout.

3. DATA AND TRADE UNIVERSE

Prices are free HistData.com minute bars for EURUSD, GBPUSD, USDJPY, USDCHF, AUDUSD and USDCAD, aggregated to hourly bars from January 2007 to September 2026. The trade universe is inherited verbatim from the companion census of Tolušić (2026a): the same eight canonical entry rules, the same eleven exit policies, the same fills, cooldowns, ATR convention and declared spreads, executed from the census engine itself rather than reimplemented. The counts reconcile as follows. The census generates 150,508 entry events across the six pairs; each event is run under all eleven exit policies, giving 1,655,588 strategy-trades; a strategy is one entry-exit combination, $8 \times 11 = 88$ in all. The three-year pilot that pre-specified this study’s targets covered 66 of the 88, the subset with both regimes populated in its shorter window.

Daily strategy returns are built by booking each trade’s net pips to its entry date, giving a six-pair daily panel per strategy over the 6,117 days with a defined gauge; days without a

trade are zeros, and Section 4 shows the results survive restricting to co-active days. Day-level tests condition on the day's mean D, a description of the day rather than a trading signal; the one trade-level test, the dispersion split of Section 6, conditions on D at the entry hour, so every trade-level conditioning variable is measurable at entry.

4. THE DIVERSIFICATION REGIME

Table 1 is the paper's central fact. For each of the 88 strategies, the mean pairwise correlation of daily returns across the fifteen pair combinations is computed separately over low-tercile and high-tercile days. The strategy-mean correlation is 0.074 in the low regime and 0.103 in the high regime, a difference of +0.029 with a cross-strategy t of 10.60, and the difference is positive for 86.4% of strategies. Figure 2 plots every strategy against the diagonal. The result is convention-proof: a median split gives +0.020 at $t = 9.86$, restricting each pairwise correlation to days on which both pairs actually traded gives +0.025 at $t = 3.61$ across all 88 strategies, and the final three years alone give +0.022 at $t = 2.96$.

Table 1. Cross-pair correlation of daily strategy returns by D regime, mean over 88 strategies, with robustness rows.

Specification	Low D	High D	Difference	t	Share positive
Tercile split (main)	0.074	0.103	+0.029	10.60	86.4%
Median split			+0.020	9.86	
Co-active days only			+0.025	3.61	
Final three years			+0.022	2.96	

The pilot on three years of data recorded levels of 0.164 and 0.221 with a difference of +0.057 at $t = 7.77$ across its 66 strategies. The full history reproduces the sign, the significance and the breadth, and halves the levels, for a reason the gauge itself explains: the pilot window was a high-D era, with mean D of 0.854 against the full history's 0.730,

so the pilot sampled the top of the very regime this paper maps. Levels are era-dependent; the regime effect is not.

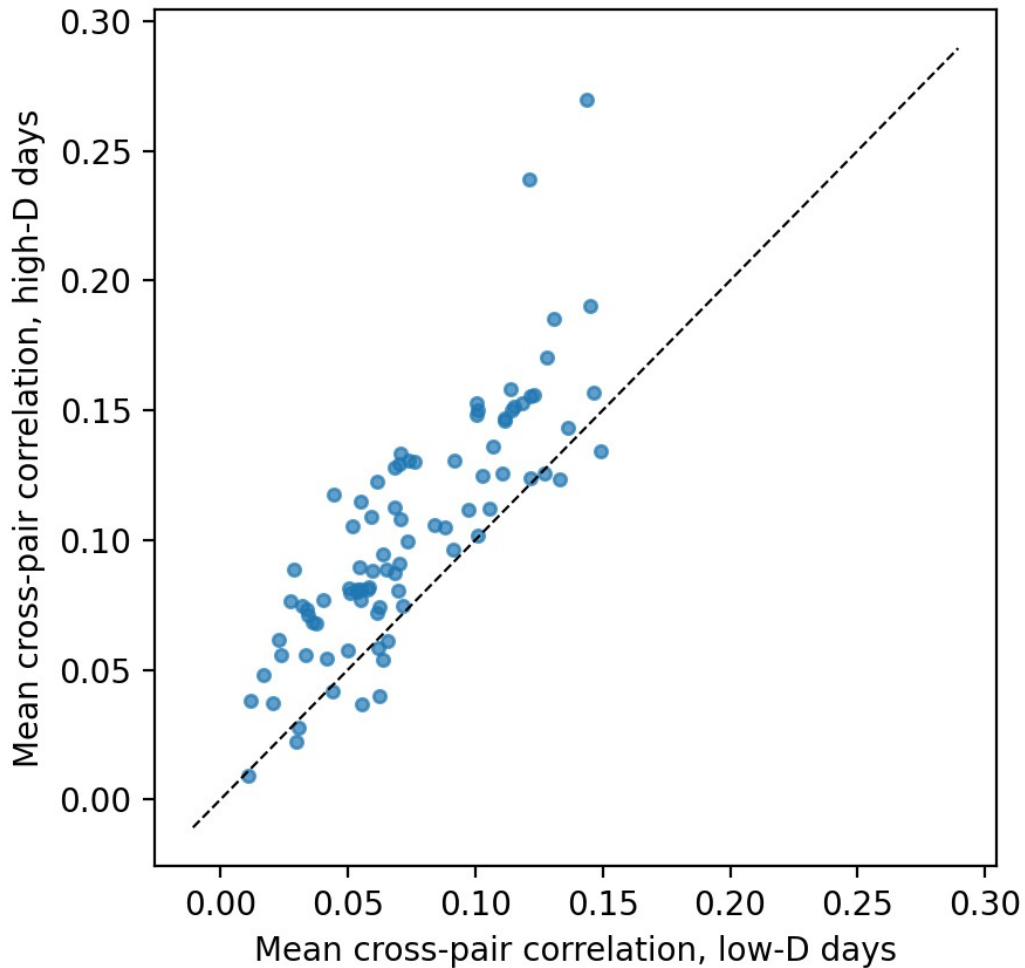


Figure 2. Mean cross-pair correlation per strategy, high-D against low-D days. Points above the diagonal lose diversification in the high regime.

5. REALLOCATION, NOT COMPOSITION

A correlation that rises when some conditioning variable is high invites the Forbes and Rigobon objection: condition on anything correlated with variance and the correlation estimator moves mechanically. Two facts close the objection here. D is nearly orthogonal to realised volatility, at -0.09 , so the conditioning variable is not a variance proxy. And

the decomposition runs the other way from the artefact: define, per strategy, the mean pairwise covariance and mean variance of daily returns in each regime, and the statistic

$$\Delta = \ln(\text{cov high} / \text{cov low}) - \ln(\text{var high} / \text{var low}) \quad (4)$$

which is zero if correlation moves only through variance composition. Across the 88 strategies, all with positive mean covariance in both regimes, Δ averages +0.308 at $t = 7.95$. The median variance ratio across regimes is 0.695 and the median covariance ratio 0.884: on high-D days variance falls roughly 30% while covariance falls only 12%. Idiosyncratic risk shrinks disproportionately and the risk that remains is shared. The correlation rise is genuine reallocation of the risk budget toward the common component, which is exactly what a dimensionality collapse means at the ledger level.

6. THE SINGLE-TRADE PARADOX

The trader who inspects individual trades will never see the regime, because single-trade risk moves the wrong way. Labelling each of the 1,655,588 trades with D at its entry hour and splitting into quintiles, per-trade dispersion falls monotonically with D: the standard deviation runs from 94.54 pips in the lowest quintile to 71.89 in the highest, a ratio of 0.760, and the 1% tail from -292.50 to -225.84 pips, a ratio of 0.772. Table 2 gives the ladder. Single-trade risk falls by a quarter precisely when the pairs stop being separate bets, so trade-level risk statistics and portfolio-level concentration give opposite readings of the same days. A risk report built from per-trade quantities is structurally blind to the regime that matters.

Table 2. Per-trade dispersion by entry-hour D quintile, all strategy-trades.

D quintile at entry	Per-trade sd (pips)	1% tail (pips)
1	94.54	-292.50
2	88.44	-290.51
3	77.95	-243.00
4	78.82	-259.40
5	71.89	-225.84

7. EFFECTIVE BETS

Equation (3) turns the regime into the number the practitioner needs. At the low-tercile mean correlation of 0.074, six pairs supply 4.38 effective bets; at the high-tercile mean of 0.103, they supply 3.96. In the pilot's high-D era the same identity gives 3.3 falling to 2.9. The gauge is measurable per bar from prices alone, so a portfolio that targets risk per effective bet, rather than per pair, can hold its true diversification constant across regimes by scaling gross exposure with $1/D^2$. This is sizing, not signal: Section 8 shows the regime contains no expectancy information, so the correct use of D is the denominator of the risk budget, never the sign of a trade.

8. EDGE EXTRACTION IS REGIME-NEUTRAL

Does the regime change what information is worth? The pilot's one untested number said perhaps: an injected edge earned +3.95 pips in low D against +2.38 in high D, a single run with no error bars. The full-history test settles it. Random entries, 2,000 per pair with direction set to the sign of the next 24-hour move and flipped with 35% probability, produce 11,982 trades carrying a genuine informational edge through the standard engine. The edge earns +14.99 pips per trade on low-tercile days and +13.69 on high-tercile days: a difference of -1.31 pips at $t = -0.47$ on month-block errors, null. Figure 3 shows both the dispersion ladder and the extraction bars. The pilot's suggestive gap does not survive its own test, and the trilogy's theme closes: exit machinery, the catastrophic tail, and the dimensionality regime each reprice risk, and none of them touches expectancy.

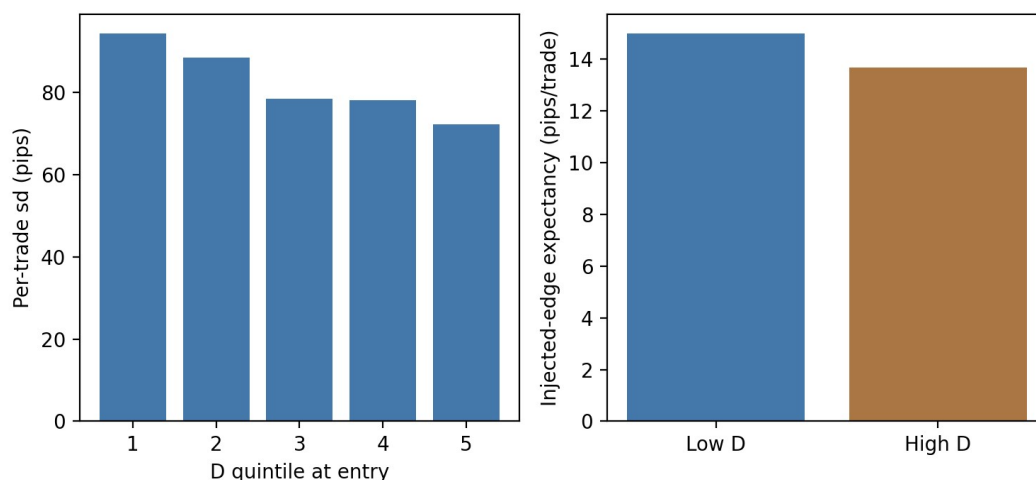


Figure 3. Left: per-trade dispersion by D quintile. Right: injected-edge expectancy by D regime; the difference is -1.31 pips at $t = -0.47$.

9. THE PRE-SPECIFIED PILOT SCORECARD

The three-year pilot recorded its numbers before this study ran, and the full history is graded against them. Reproduced: the correlation split (sign, significance and breadth, with levels halved by the era effect of Section 4); the reallocation decomposition ($+0.308$ at $t = 7.95$ against the pilot's $+0.544$ at $t = 5.48$); and the single-trade paradox (dispersion ratios 0.760 and 0.772 against the pilot's 0.706 and 0.692). Closed as null: the extraction gap, whose pilot value of -1.57 pips was recorded untested and fails its test here at $t = -0.47$. New to the full history: the effective-bets translation and the volatility-orthogonality disclosure. One pilot claim is thereby retired and none is contradicted.

10. LIMITATIONS AND OVERTURN TESTS

Four named tests could overturn the conclusions. The universe is six major dollar pairs with an equal-weight index; a recompute of D on tick-volume weights, which the tomography programme found near-identical for the majors, or on a wider sixteen-pair universe with crosses, would overturn the regime claim if the correlation difference reversed sign at two standard errors. The gauge is one construction; substituting the rolling first-eigenvalue share directly for D in every test would overturn the gauge's sufficiency if the conclusions changed. The paper describes contemporaneous structure

and claims no forecast; a predictive design regressing next-period portfolio concentration on lagged D would extend the result, and a finding that lagged D carries no information about next-day concentration would confine the gauge to nowcasting without overturning the regime fact. And the extraction test injects one edge at one strength and horizon; a grid of injection designs finding a regime gap at two block standard errors would overturn the neutrality claim. Booking conventions are a fifth, smaller exposure: results survive the co-active restriction, but exit-date booking remains untested.

11. CONCLUSIONS

The finding: on 1,655,588 trades from 88 published strategies over 2007 to 2026, the cross-pair correlation of daily strategy returns rises from 0.074 to 0.103 between low and high terciles of the dimensionality gauge D , $+0.029$ at $t = 10.60$, in 86.4% of strategies; six pairs shrink from 4.38 to 3.96 effective bets.

The mechanism: reallocation, not composition. On high- D days variance falls to 0.695 of its low- D level while covariance falls only to 0.884, a log-ratio of $+0.308$ at $t = 7.95$, with D orthogonal to volatility at -0.091 , so the Forbes and Rigobon artefact is excluded by construction and by decomposition.

The practice: the regime is invisible at trade level, where dispersion falls to 0.760 of its low- D value, and it carries no expectancy, with an injected edge earning the same in both regimes at $t = -0.47$. D therefore belongs in the denominator of the risk budget, scaling gross exposure by $1/D^2$ to hold effective bets constant, and nowhere near the signal.

The research: the trilogy closes on one sentence said three ways. Exit machinery manufactures options whose premium is paid in realised outcomes; the catastrophic tail belongs to the process and defeats every filter; and the dimensionality regime repackages the same risk into fewer bets without changing what any signal earns. Machinery and regimes reprice risk. Only information prices expectancy.

DATA AND MATERIALS AVAILABILITY

All source data are publicly available minute-bar histories from HistData.com, 2007 to 2026, six major USD pairs, aggregated to hourly bars by the author. A replication

package accompanies the manuscript with the gauge construction, the trade-panel builder run from the companion census engine, the analysis script, frozen result tables, the figures, and a pointer script that retrieves the source data from the public archive without redistributing it. The companion packages are available at SSRN (abstracts 7527998 and 7528560).

DECLARATION OF GENERATIVE AI IN SCIENTIFIC WRITING

During the preparation of this work the author used Anthropic's Claude in order to implement the analysis under the author's direction and to assist with drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

FUNDING AND COMPETING INTERESTS

This research received no funding. The author declares no competing interests.

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