

7517900-central-bank-digital-currency-versus-synthetic-cbdc-systemic-risk-and-policy-int

SUMMARY: This paper develops a dynamic stochastic general equilibrium (DSGE) model to compare the macro-financial implications of central bank digital currency (CBDC) and synthetic CBDC (sCBDC) regimes. The model incorporates endogenous portfolio allocation, bank balance-sheet constraints, and liquidity preference shocks. The analysis reveals that under a tiered-remuneration CBDC regime, households substitute deposits for central bank liabilities, reducing bank funding and amplifying disintermediation during liquidity stress episodes. Although remuneration penalties and CBDC holding ceilings mitigate these effects, they do not fully eliminate endogenous financial amplification. In contrast, the sCBDC regime generates smoother and more structurally bounded disintermediation. Digital liabilities are fully reserve-backed and institutionally separated from credit creation, resulting in weaker amplification effects and more predictable balance-sheet adjustments. However, the expansion of narrow-bank intermediation also reduces traditional bank funding, credit creation, output, consumption, and employment, highlighting a trade-off between financial stability and credit provision. The paper further examines the feasibility conditions of the sCBDC equilibrium and the role of narrow-bank participation and intermediation costs.

METHOD: The study uses a dynamic stochastic general equilibrium (DSGE) model with explicit banking frictions to compare the effects of CBDC and sCBDC architectures. The model incorporates endogenous portfolio allocation between deposits and digital money, bank balance-sheet constraints, and liquidity preference shocks inspired by the Diamond–Dybvig mechanism.

KEY FINDINGS:

- Under a tiered-remuneration CBDC regime, households substitute deposits for central bank liabilities, reducing bank funding and amplifying disintermediation during liquidity stress episodes.
- The sCBDC regime generates smoother and more structurally bounded disintermediation.
- Digital liabilities are fully reserve-backed and institutionally separated from credit creation, resulting in weaker amplification effects and more predictable balance-sheet adjustments.
- The expansion of narrow-bank intermediation also reduces traditional bank funding, credit creation, output, consumption, and employment.

IMPLICATION FOR TRADING: The findings suggest that synthetic CBDC arrangements may provide a more resilient monetary architecture by limiting endogenous instability and run-like dynamics, although potentially at the cost of weaker bank intermediation and lower endogenous credit creation. Practitioners should consider the implications of these findings when designing monetary policies and assessing the potential risks and benefits of CBDC and sCBDC architectures.

Central Bank Digital Currency versus Synthetic CBDC: Systemic Risk and Policy Interventions

Abstract¹

This paper develops a dynamic stochastic general equilibrium (DSGE) model with banking frictions to compare the macro-financial implications of central bank digital currency (CBDC) and synthetic CBDC (sCBDC) regimes. The framework incorporates endogenous portfolio allocation between deposits and digital money, bank balance-sheet constraints, and liquidity preference shocks inspired by the Diamond–Dybvig mechanism.

The analysis shows that digital money architecture critically shapes financial intermediation and the transmission of liquidity shocks. Under a tiered-remuneration CBDC regime, households substitute deposits toward central bank liabilities, reducing bank funding and amplifying disintermediation during liquidity stress episodes. Although remuneration penalties and CBDC holding ceilings mitigate these effects, they do not fully eliminate endogenous financial amplification.

By contrast, the sCBDC regime generates a smoother and more structurally bounded form of disintermediation. Because digital liabilities are fully reserve-backed and institutionally separated from credit creation, liquidity shocks produce weaker amplification effects and more predictable balance-sheet adjustments. However, the expansion of narrow-bank intermediation also reduces traditional bank funding, credit creation, output, consumption, and employment, highlighting a trade-off between financial stability and credit provision. The paper further characterizes the feasibility conditions of the sCBDC equilibrium and examines the role of narrow-bank participation and intermediation costs.

The results suggest that synthetic CBDC arrangements may provide a more resilient monetary architecture by limiting endogenous instability and run-like dynamics, although potentially at the cost of weaker bank intermediation and lower endogenous credit creation.

¹ Generative AI was used as an editorial and coding support tool during manuscript preparation. All outputs were reviewed and verified by the authors, who retain full responsibility for the content.

I. Introduction

The way money is designed matters. Beyond its role as a medium of exchange, the institutional architecture of monetary instruments shapes how liquidity flows through the financial system, how banks intermediate credit, and ultimately how shocks propagate to the real economy. This insight, long implicit in monetary theory, has gained renewed urgency with the global rise of central bank digital currencies (CBDCs) and their privately-issued counterparts, synthetic CBDCs (sCBDCs). As central banks worldwide move from conceptual exploration to active piloting, a fundamental question remains insufficiently answered: does the architecture of digital money — who issues it, how it is backed and how it remunerates holders — determine the economy's vulnerability to liquidity stress?

This paper addresses that question directly. We develop a dynamic stochastic general equilibrium (DSGE) model with explicit banking frictions to compare how CBDC and sCBDC architectures transmit liquidity preference shocks through the financial system and into the real economy. Our central argument is that architecture is not a detail, it is the mechanism.

The importance of addressing this issue reflects a growing transformation of monetary systems. Digital payment ecosystems have developed rapidly and are starting to replace bank deposits as the main form of liquid assets held by households. To keep up with these developments, central banks have explored two distinct institutional responses. The first is the retail CBDC, which represents a direct liability of the central bank, accessible to households and businesses, offering the security of sovereign currency in digital form. The second is the synthetic CBDC, which is a digital liability issued by authorised private intermediaries, narrow banks, and is fully backed by reserves held with the central bank. Whilst both architectures aim to provide a secure and liquid digital currency, they differ fundamentally in their implications for bank funding, credit creation and financial stability.

A growing literature has examined the macroeconomic implications of CBDC issuance. Barrdear and Kumhof (2022) use a DSGE framework to demonstrate that CBDC can improve welfare by enhancing payment efficiency and monetary transmission. Andolfatto (2021) shows that, although CBDCs may reduce banks' profitability and credit volumes, these effects can remain manageable through sound monetary policy. Keister and Sanches (2022) point out that the absence of holding constraints on CBDCs intended for retail users could exacerbate bank runs by offering households a frictionless exit from the banking system. In terms of design, Bindseil (2020) introduced the concept of tiered remuneration as a tool that would help limit

the excessive accumulation of CBDCs and reduce the risks of disintermediation. The neutrality of CBDC issuance was examined by Brunnermeier and Niepelt (2019), who propose a framework for compensatory adjustments to the central bank's balance sheet, while the interaction between digital currency and systemic fragility, in the vein of Gertler and Kiyotaki (2010), has been explored by Bitter (2020), amongst others.

Synthetic CBDC architectures have not yet been extensively studied in the specialist literature. Adrian and Mancini-Griffoli (2019) introduced the concept of sCBDC as a hybrid public-private currency and examined its impact. Chiu and Rivadeneyra (2021), for their part, demonstrated that transfers of liquidity to synthetic digital claims can weaken banking intermediation. More recently, Huang and Keister (2026) compared stablecoins and tokenised deposits in a blockchain-based economy. They found that their coexistence with reserve-backed instruments reduces bank deposits and lending. However, a unified DSGE framework directly comparing CBDCs and sCBDCs under conditions of liquidity stress is still largely lacking in the literature.

This paper fills that gap. Our framework incorporates three monetary regimes: a baseline banking economy without digital money, a tiered-remuneration CBDC regime, and a synthetic sCBDC regime with narrow-bank intermediation. We introduce a liquidity preference shock à la Diamond and Dybvig (1983) — an unanticipated increase in households' demand for immediately accessible, risk-free assets — to examine how each architecture shapes the propagation of financial stress.

Our results yield three main findings. First, under the CBDC regime, liquidity shocks generate endogenous amplification: as households substitute deposits toward central bank liabilities, bank funding contracts, credit supply falls, and the shock propagates nonlinearly to output, consumption, and employment. Tiered remuneration and holding ceilings partially stabilize this dynamic but cannot fully eliminate the underlying amplification mechanism. Second, under the sCBDC regime, the transmission mechanism is fundamentally different. Because digital liabilities are fully reserve-backed and institutionally separated from credit creation, portfolio reallocations between deposits and sCBDC occur within the narrow-banking segment without directly reducing commercial bank lending. Liquidity shocks are therefore absorbed structurally rather than amplified endogenously. Third, this stabilization is not without cost: as narrow-bank participation expands, traditional bank intermediation contracts, and the economy settles at a structurally lower level of credit creation, output, and employment.

Taken together, these results point to a central trade-off in digital monetary design: sCBDC arrangements offer greater resilience under liquidity stress but potentially at the cost of weaker long-run credit provision, while standard CBDC preserves higher steady-state intermediation but remains vulnerable to run-like amplification dynamics.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the unified model framework across the three regimes. Section 4 compares steady-state equilibria. Section 5 analyzes the transmission of liquidity shocks under each architecture. Section 6 discusses policy implications. Section 7 concludes.

II. Literature review

The emergence of central bank digital currencies has generated a rapidly expanding body of research at the intersection of monetary theory, banking stability, and financial innovation. Rather than surveying this literature exhaustively, we organize the relevant contributions around three thematic threads that directly motivate our analysis: the macroeconomic implications of CBDC design, the economics of synthetic CBDC and narrow banking, and the transmission of liquidity shocks in digitalized financial systems.

CBDC design and banking disintermediation. The macroeconomic consequences of CBDC issuance have been examined extensively within general equilibrium frameworks. Barrdear and Kumhof (2022) develop a large-scale DSGE model and find that CBDC issuance can raise output and improve welfare by reducing transaction costs and enhancing the efficiency of monetary transmission, though these gains come alongside a structural reduction in bank deposit funding. Andolfatto (2021) reaches a more measured conclusion, arguing that CBDC need not severely disintermediate banks provided that the central bank does not remunerate digital holdings at rates competitive with deposits. The competitive dynamics between deposits and digital money are further analyzed by Davoodalhosseini (2021), who shows that the degree of disintermediation depends critically on market structure and interest rate spreads between the two instruments.

A central design question concerns how remuneration policy shapes portfolio allocation. Bindseil (2020) introduced the concept of tiered remuneration for retail CBDC, under which holdings below a threshold earn a base return while excess balances are penalized, thereby limiting large-scale substitution away from bank deposits. This mechanism has been identified as a key instrument for preserving monetary transmission and financial stability (Niepelt, 2020). The neutrality of CBDC issuance under offsetting central bank balance sheet operations is

examined by Brunnermeier and Niepelt (2019), who show that equivalence between CBDC and deposit funding holds under specific conditions, but breaks down once banking frictions and balance sheet constraints are introduced. Our paper builds directly on this design literature by embedding tiered remuneration into a DSGE framework with endogenous bank leverage and portfolio substitution.

Synthetic CBDC and narrow banking. The architecture of synthetic CBDC — privately issued digital liabilities fully backed by central bank reserves — has received comparatively less formal attention. Adrian and Mancini-Griffoli (2019) introduced the concept of sCBDC as a public-private hybrid in which private intermediaries provide the retail interface while the central bank supplies the underlying backing. This institutional arrangement preserves the safety properties of sovereign money while delegating operational functions to the private sector. Chiu and Rivadeneyra (2021) develop a general equilibrium framework showing that liquidity migration toward fully reserve-backed digital claims weakens traditional bank intermediation and may threaten banking sector stability, even when the digital instrument carries a lower return than deposits.

The most closely related contemporary contribution is Huang and Keister (2026), who compare stablecoins and tokenized deposits as alternative payment instruments within a blockchain-based economy. Their analysis shows that households optimally allocate liquidity between reserve-backed stablecoins and bank-issued tokenized deposits, and that the coexistence of these instruments reduces bank deposits, constrains credit creation, and lowers the volume of risky projects financed by banks. Importantly, their framework highlights how regulatory costs and institutional design govern the equilibrium allocation between inside and outside money. While Huang and Keister (2026) provide important insights into the portfolio mechanics of reserve-backed digital instruments, their analysis does not embed these dynamics within a DSGE environment with endogenous banking frictions, nor does it directly compare synthetic CBDC with standard retail CBDC under liquidity stress. Our paper addresses precisely this gap.

Liquidity shocks, run dynamics, and financial amplification. The fragility of bank-based financial systems under liquidity stress has been a cornerstone of monetary economics since Diamond and Dybvig (1983), who showed that the maturity mismatch inherent in fractional reserve banking exposes deposit-funded institutions to self-fulfilling run equilibria. The introduction of digital money reopens these classical questions in a new institutional context. Keister and Sanches (2022) demonstrate that retail CBDC, by providing households with a frictionless exit from the banking system, may lower the threshold for destabilizing runs.

Fernandez-Villaverde et al. (2021) show that CBDC issuance alters liquidity preferences in ways that reshape the credit environment and may amplify financial instability during stress episodes.

The stabilizing or destabilizing role of digital money under liquidity stress has been further examined by Souissi and Nabi (2023), who find that poorly regulated CBDC arrangements can accelerate deposit withdrawals and increase run risk, while policy tools such as holding limits can partially transform CBDC into a stabilizing mechanism. Ahner et al. (2024) similarly argue that CBDC design critically determines financial stability outcomes, with poorly calibrated access conditions increasing the probability of destabilizing runs during episodes of heightened liquidity demand. The interaction between digital currency holdings, bank net worth, and systemic fragility — in the tradition of Gertler and Kiyotaki (2010) — is explored by Bitter (2020), who finds that CBDC may weaken bank balance sheets during tranquil periods while simultaneously reducing run risk during crises, depending on design parameters.

This paper contributes to the above literature along three dimensions. First, to our knowledge, no existing study compares CBDC and sCBDC architectures within a unified DSGE framework that incorporates endogenous banking frictions, tiered remuneration, and explicit liquidity shock transmission. We provide a common analytical structure that allows direct comparison of the two architectures across steady states and shock responses. Second, we formally characterize the conditions under which sCBDC arrangements generate structurally bounded disintermediation — as opposed to the endogenous, potentially nonlinear amplification associated with standard CBDC — by deriving closed-form equilibrium expressions and comparative statics under liquidity stress. Third, we identify a fundamental asymmetry in shock transmission across the two regimes: while CBDC amplifies liquidity stress through endogenous deposit substitution and balance-sheet feedback, sCBDC insulates commercial bank lending from portfolio reallocations by separating payment liquidity from credit creation. This asymmetry, and the trade-off it implies between financial resilience and long-run credit provision, constitutes the central analytical contribution of the paper.

III. Baseline regime: banking economy without digital currency

1. Common framework

Households are infinitely-lived and maximize their utility over consumption C_t , supply labor to firms N_t , and deposit savings with banks. Their decisions depend on real wages, deposit interest rates and the expected return on savings. In regimes featuring digital money, households

additionally derive liquidity services from holding a portfolio of liquid assets. Preferences take the form:

$$\max_{\{C_t, N_t, D_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C^t - \chi \frac{N_t^{1+\phi}}{1+\phi} + \psi \mathcal{L}(\cdot) \right] \quad (1)$$

Where, $\beta \in (0,1)$ is the discount factor, $\chi > 0$ and $\phi > 0$ govern the disutility of labor, and $\mathcal{L}(\cdot)$ is a liquidity aggregator absent in the baseline regime ($\psi = 0$) and specified as a CES function over the relevant pair of liquid assets in the CBDC and sCBDC regimes. Liquidity services are derived using a CES aggregator of the general form:

$$\mathcal{L}(D_t, Z_t) = \left[\omega D_t^{\frac{\sigma-1}{\sigma}} + (1-\omega) Z_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (2)$$

Where (D_t) denotes deposits and (Z_t) the relevant digital asset (CBDC denoted M_t or sCBDC S_t , depending on the regime), $\omega \in (0,1)$ captures the household's structural preference for deposits, and $\sigma > 0$ governs the elasticity of substitution between the two instruments.

The household's period-by-period budget constraint is:

$$C_t^i + D_t^i + Z_t + E_t = W_t^i N_t^i + R_t^D D_{t-1} + R_t^Z Z_{t-1} + \Pi_t^i \quad (3)$$

$i = o, i = CBDC$ or $i = sCBDC$ depending on the regime.

Where, households are assumed to own the banking sector and therefore hold bank equity E_t as part of their financial wealth. The rest of the variables are: D_t stands for deposits in commercial banks chosen at time t , W_t is the real wage, R_t^D is the nominal gross interest rate on deposit and Π_t^B represents profits redistributed by banks to households.

First-order conditions at date t yield the Euler equation:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} R_{t+1}^D \right] \quad (4)$$

Firms operate under perfect competition. They produce a homogeneous good using a Cobb-Douglas technology with fixed capital normalized to one, and employing labor supplied by households. Wages are borrowed from banks and perfect competition implies zero profits at equilibrium.

$$Y_t = AN_t^{1-\alpha} \quad (5)$$

Firms borrow from banks to finance the wage bill within the period, so that credit demand equals labor income, $B_t = W_t N_t$. Zero profits in equilibrium pin down the lending rate at:

$$R^L = \frac{1}{1-\alpha}, \text{ independent of the monetary regime.}$$

Commercial banks intermediate household savings into firm loans subject to a regulatory leverage constraint, $B_t \leq \varphi E_t$, where $\varphi > 1$ is the maximum permissible leverage ratio, treated as an exogenous prudential parameter following Gerali et al. (2010). Bank equity $\left(E_t = \frac{B_t}{\varphi}\right)$ adjusts to support the equilibrium level of credit, with short-run rigidity and long-run adjustment via retained earnings. This mechanism becomes important once digital money competes with deposits for household savings.

The central bank sets the policy rate $R_t = R^D = \frac{1}{\beta}$ in all regimes and, where applicable, an additional digital money remuneration schedule. This common architecture allows the three regimes to be nested within a single equilibrium structure, differing only in the assets available to households and the resulting funding constraint facing commercial banks.

In the baseline regime, deposits are the unique liquid asset and $\psi = 0$ in equation (1). Household savings are fully intermediated through bank deposits, and the leverage constraint binds, $B_t = \varphi E_t$.

Solving the household's Euler equation, the firm's labor demand condition, and the bank's balance sheet jointly yields a closed-form steady state.

Proposition 1

Given the structural parameters $\{A, \alpha, \beta, \chi, \phi, \varphi\}$

The steady-state equilibrium of the economy is determined by

$\{\bar{N}, \bar{Y}, \bar{C}, \bar{W}, \bar{B}, \bar{D}, \bar{E}, R^D, R^L, \bar{\mu}\}$:

the set of the following six variables:

$$\bar{Y} = \bar{C} = A \left(\frac{1-\alpha}{\chi} \right)^{\frac{1-\alpha}{1+\phi}}$$

$$\bar{D} = \left(1 - \frac{1}{\varphi}\right)(1 - \alpha)A \left(\frac{1 - \alpha}{\chi}\right)^{\frac{1 - \alpha}{1 + \phi}}$$

$$\bar{N} = \left(\frac{1 - \alpha}{\chi}\right)^{\frac{1}{1 + \phi}}$$

$$\bar{W} = (1 - \alpha)A \left(\frac{1 - \alpha}{\chi}\right)^{\frac{-\alpha}{1 + \phi}}$$

$$\bar{B} = (1 - \alpha)A \left(\frac{1 - \alpha}{\chi}\right)^{\frac{1 - \alpha}{1 + \phi}}$$

$$\bar{E} = \frac{\bar{B}}{\varphi} = \frac{1 - \alpha}{\varphi}A \left(\frac{1 - \alpha}{\chi}\right)^{\frac{1 - \alpha}{1 + \phi}}$$

$$R^D = \frac{1}{\beta}$$

$$R^L = \frac{1}{1 - \alpha}$$

$$\bar{\mu} = \frac{1}{1 - \alpha} - \frac{1}{\beta}$$

With $\varphi > 1$ and $\mu > 0$

which requires: $\frac{1}{1 - \alpha} > \frac{1}{\beta}$ or equivalently: $\beta > 1 - \alpha$

Proof: See appendix A.

This baseline equilibrium is entirely pinned down by real-side fundamentals — technology, labor supply preferences, and bank leverage — since no alternative liquid asset competes with deposits.

In this regime, the equilibrium level of bank credit $B = \varphi E$ is entirely determined by real side fundamentals. Since no CBDC is available, deposits remain the unique savings instrument, ensuring that the banking sector fully intermediates household resources.

An increase in the leverage ratio φ or equity E amplifies credit supply and, implies higher levels of output, consumption, and employment, but lower equilibrium wage rates.

The baseline system presents a transparent and analytically tractable foundation for understanding a banking system running without digital monetary instruments. This structure allows for a comparison with other regimes for which the issuance of CBDC or sCBDC can change the behaviour of depositors and disrupt intermediation.

IV. Regime 2: Central Bank Digital Currency with tiered Remuneration

The CBDC regime introduces a central bank liability M_t alongside deposits, with the liquidity aggregator (2) now defined over $((D_t, M_t))$. CBDC remuneration follows a tiered schedule, following Bindseil (2020): balances up to a threshold $\bar{M}$ earn a base rate R_1^M , while balances in excess of $\bar{M}$ earn a penalized rate $R_2^M < R_1^M$:

$$R_t^M(M_{t-1}) = \begin{cases} R_1^M & \text{if } M_{t-1} \leq \bar{M} \\ R_2^M < R_1^M & \text{if } M_{t-1} > \bar{M} \end{cases}$$

Households optimally allocate savings between the two assets according to the relative return differential and the marginal liquidity service of each instrument. When $(R_1^M = R^D = 1/\beta)$, deposits and CBDC are perfect substitutes for $M \leq \bar{M}$. Households are indifferent between the two assets. For $M > \bar{M}$ and $(R_2^M < R^D)$: deposits dominate CBDC.

Upon the introduction of CBDC, Bank equity remains temporarily fixed at its baseline level $E_t = E = \frac{\bar{B}}{\varphi}$. This is consistent with Gerali et al (2010), who hypothesise precisely this mechanism: banking capital adjusts slowly via retained earnings. In the short term E_t is rigid, in the long term it adjusts.

Households optimally allocate savings between the two assets according to the relative return differential and the marginal liquidity service of each instrument, yielding the portfolio condition:

$$\frac{M}{D^{CBDC}} = \left(\frac{1 - \omega}{\omega} \right)^\sigma \left(\frac{\beta - R^D}{\beta - R^M} \right)^\sigma = \Omega \left(\frac{\beta - R^D}{\beta - R^M} \right)^\sigma \equiv \Lambda \quad (6)$$

Proof : See appendix B

Proposition 2

The new steady state equilibrium of the economy with CBDC is determined by the following variables

The interior solution: $\Lambda = \Omega$ when $R^M = R^D$ (case 1)

Or

$\Lambda = \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma < \Omega$ when $R^{2M} < R^D$ and $\Omega \leq 1$
(case 2a)

$$\begin{aligned} R^D &= \frac{1}{\beta} \\ M &= \frac{\Lambda}{1 + \Lambda} \cdot \bar{D} \\ D^{CBDC} &= \left(\frac{1}{1 + \Lambda} \right) \bar{D} \\ B^{CBDC} &= \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \cdot \bar{B} \\ Y^{CBDC} = C^{CBDC} &= \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \cdot \bar{C} \\ N^{CBDC} &= \left(\frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \right)^{\frac{1}{1-\alpha}} \cdot \bar{N} \\ W^{CBDC} &= \left(\frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \right)^{\frac{-\alpha}{1-\alpha}} \cdot \bar{W} \end{aligned}$$

The corner solution: $R^{2M} < R^D, \Omega > 1$
(case 2b)

$$\begin{aligned} R^D &= \frac{1}{\beta} \\ M &= \bar{M} \\ D^{CBDC} &= \bar{D} - \bar{M} \\ B^{CBDC} &= \bar{B} - \bar{M} \\ Y^{CBDC} = C^{CBDC} &= \bar{C} \cdot \left(1 - \frac{\bar{M}}{\bar{B}} \right) \\ N^{CBDC} &= \bar{N} \cdot \left(1 - \frac{\bar{M}}{\bar{B}} \right)^{\frac{1}{1-\alpha}} \\ W^{CBDC} &= \bar{W} \cdot \left(1 - \frac{\bar{M}}{\bar{B}} \right)^{\frac{-\alpha}{1-\alpha}} \end{aligned}$$

Proof : See appendix B

The second proposition identifies two separate equilibrium regimes characterized by the interaction between household liquidity preferences and the central bank's tiered remuneration policy. In the first case, disintermediation contracts credit, consumption, and employment relative to the baseline while pushing wages up — and a lower penalized rate attenuates this effect, confirming the stabilizing role of tiered remuneration. In the second case, the central bank instead caps disintermediation directly through the holding ceiling, which becomes the binding constraint regardless of household preferences; as this ceiling approaches zero, the economy converges back to its baseline equilibrium.

These findings are aligned with Chang et al. (2023) but the underlying mechanism differs. In their model disintermediation stems from household heterogeneity and access costs, but in our model it stems from the central bank's remuneration design within a DSGE framework featuring

endogenous bank leverage. In our setting, leverage adjusts at the end of the period following CBDC's introduction, yielding a new equilibrium expression for bank equity in each case.

Its new expression at equilibrium becomes:

$$E^{CBDC} = E \cdot \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \text{ in cases 1 and 2a}$$

$$E^{CBDC} = E \cdot \left(1 - \frac{\bar{M}}{B}\right), \text{ in case 2b}$$

Policy corollary (steady state):

Under the tiered remuneration plan $(R^M, \bar{M})$ adopted by the central bank, the tiered remuneration rule

$$R^M = \begin{cases} R^M = R^D & \text{if } M \leq \bar{M} \\ R^{2M} < R^D & \text{if } M > \bar{M} \end{cases}$$

directly affects the equilibrium demand for CBDC.

A **lower penalized rate R_2^M or a tighter ceiling $\bar{M}$** reduces the attractiveness of large CBDC balances and thus attenuates disintermediation. When $\Omega \leq 1$ households naturally self-limit their CBDC holdings below $\bar{M}$. **The bidding instrument is then R^{2M} not $\bar{M}$.**

V. Regime 3: Synthetic Central Bank Digital Currency – private money backed by central bank reserves

This section examines a third monetary regime in which synthetic central bank digital currencies (sCBDCs) coexist with traditional bank deposits. Unlike the CBDC regime, where digital money is issued directly to the public by the central bank, synthetic CBDC are instruments launched by private-sector that replicate the features of public money under strict regulatory and collateral frameworks. Specifically, they are issued by licensed private intermediaries, such as non-bank financial institutions and are fully backed by reserves held at the central bank.

In this regime, a fraction $\lambda \in (0,1)$ of financial intermediaries exits commercial banking to become narrow banks, issuing fully reserve-backed digital liabilities S_t one-for-one against central bank reserves H_t^C :

$$S_t = H_t^C \tag{7}$$

This ensures full convertibility and eliminates default risk at the level of the issuer. The reserves are acquired endogenously from the banking sector whenever households choose to convert their deposits into sCBDC.

Remaining commercial banks continue to intermediate deposits into firm loans, but their balance sheet capacity is now structurally reduced:

$$B_t^{sCBDC} = (1 - \lambda)\bar{B} \quad (8)$$

Household deposits are demand-determined and result from households' optimal portfolio allocation between bank deposits and sCBDC. In steady-state, deposits are jointly determined with sCBDC holdings through the portfolio arbitrage.

$$D_t^{sCBDC} = \bar{D} - \lambda\bar{B} \quad (9)$$

For bank deposits to remain positive in the sCBDC regime, the fraction of narrow banks should not exceed the limit:

$$\lambda < \frac{\varphi - 1}{\varphi}$$

As sCBDC issuance is restricted to narrow banks, their supply is constrained by the fraction of banks choosing to be narrow banks. This implies:

$$S_t = \lambda\tilde{S} \quad (10)$$

Where $\tilde{S}$ is the unconstrained demand for sCBDC by households:

$$\tilde{S} = \Omega \left(\frac{\beta - R^D}{\beta - R^D + \kappa} \right)^\sigma \cdot D^{sCBDC} \quad (11)$$

(proof in appendix C)

Households allocate savings between deposits and sCBDC according to a portfolio condition analogous to (4), but with the sCBDC return reflecting an intermediation cost spread $\kappa \geq 0$ borne by narrow banks, $R_t^S = R_t - \kappa$:

$$\frac{S}{D^{sCBDC}} = \Gamma = \left(\frac{1 - \omega}{\omega} \right)^\sigma \left(\frac{\beta - R^D}{\beta - R^D + \kappa} \right)^\sigma \quad (12)$$

(Proof: See appendix D).

The derivations show that a larger fraction λ of narrow banks reduces both D^{sCBDC} and S . This highlights the structural disintermediation induced by the conversion of commercial banks into narrow banks. Moreover, a higher intermediation cost κ reduces Γ , making sCBDC less attractive compared to deposits.

Proposition 3:

The new equilibrium of the economy with sCBDC is determined by the set of the following eight variables which are fully determined from the model's structural parameters as follows:

$$\begin{aligned}
R^D &= 1/\beta \\
R^S &= R^D - \kappa = 1/\beta - \kappa \\
N^{sCBDC} &= (1 - \lambda)^{\frac{1}{1-\alpha}} \bar{N} \\
Y^{sCBDC} &= C^{sCBDC} = (1 - \lambda) \bar{C} \\
W^{sCBDC} &= (1 - \lambda)^{\frac{-\alpha}{1-\alpha}} \bar{W} \\
B^{sCBDC} &= (1 - \lambda) \bar{B} \\
D^{sCBDC} &= \frac{1}{1 + \Gamma} \cdot \frac{\varphi(1 - \lambda) - 1}{\varphi} \bar{B} \\
S &= \frac{\Gamma}{1 + \Gamma} \cdot \frac{\varphi(1 - \lambda) - 1}{\varphi} \bar{B} \\
E^{sCBDC} &= \frac{(1 - \lambda) \bar{B}}{\varphi}
\end{aligned}$$

Proposition 3 reveals that sCBDC generates a two-tier contraction of the economy. First, all the real variables (output, consumption, employment, credit and bank equity) contract proportionally to $(1 - \lambda)$. This reflects the structural reduction of the traditional banking sector. Second, the deposit base also contracts through portfolio reallocation toward sCBDC which is governed by Γ . We also conclude that since $\kappa > 0$, we have $\Gamma < \Omega$ which means the synthetic CBDC regime generates less financial disintermediation than a regime with CBDC at equal households preferences.

Calibration values for all structural parameters across the three regimes are reported in Appendix E.

VI. The impacts of liquidity shock: amplification versus stabilization

1. Modelling the liquidity shock

Following Diamond and Dybvig (1983), we model a liquidity preference shock as an unanticipated increase in households' valuation of immediately accessible, risk-free assets. Formally, we introduce a shock parameter $\theta \in [0,1]$ that reduces the relative weight placed on bank deposits within the liquidity aggregator. For the CBDC regime, the aggregator (2) becomes:

$$\mathcal{L}(D_t, Z_t) = \left[(1 - \theta_t) \omega D_t^{\frac{\sigma-1}{\sigma}} + (1 - (1 - \theta_t) \omega) Z_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (13)$$

Where $Z_t = (M_t, S_t)$

When $\theta_t = 0$, the aggregator collapses to its steady-state form; as $\theta \mapsto 1$, households' valuation of deposits vanishes entirely, capturing an extreme flight-to-quality episode. This formulation is deliberately parsimonious: the shock operates identically across both regimes, isolating the architecture of digital money as the source of any divergence in transmission.

2. CBDC: endogenous amplification

Under the CBDC regime, the shock raises the relative attractiveness of CBDC, and the portfolio ratio (6) becomes a function of θ_t :

$$\Lambda(\theta_t) = \left(\frac{1 - (1 - \theta_t) \omega}{(1 - \theta_t) \omega} \right)^{\sigma} \left(\frac{\beta - R^D}{\beta - R^M} \right)^{\sigma} \quad (14)$$

Proposition 4. CBDC and amplification of liquidity shock:

Consider a liquidity preference shock $\theta_t \in [0,1]$ that increases households' desire to hold liquid assets. Given the balance sheet identity: $B_t^{CBDC*} = D_t^{CBDC*} + E_t^{CBDC*}$, with

$E_t^{CBDC*} = \frac{B_t^{CBDC*}}{\varphi}$ and the CBDC holdings $M^* = \frac{\Lambda(\theta_t)}{1 + \Lambda(\theta_t)} \cdot \bar{D}$, the following relationships hold:

$$\frac{\partial M^*}{\partial \theta_t} > 0; \frac{\partial B_t^{CBDC*}}{\partial \theta_t} < 0; \frac{\partial N_t^{CBDC*}}{\partial \theta_t} < 0; \frac{\partial W_t^{CBDC*}}{\partial \theta_t} > 0$$

Therefore, a positive liquidity shock increases $\Lambda(\theta_t)$ which raises M^* which induces a proportional contraction in bank credit supply B_t^{CBDC*} . This reduction in lending translates into the labor market through the credit demand channel: lower credit availability reduces firms' capacity to finance the wage bill, thereby lowering labor demand and

employment N^{CBDC} while real wages W^{CBDC} increase slightly due to reduced labor demand. The magnitude of the contraction is determined by the parameters σ and ω .

Proof in Appendix F.

The mechanism is direct and self-reinforcing. As θ_t rises, households convert deposits into CBDC holdings, which mechanically tightens the bank's funding base and, through the binding leverage constraint, forces a proportional contraction in lending. This reduction in credit propagates to the labor market: firms facing tighter working-capital financing reduce labor demand, lowering employment, while the resulting rise in the marginal product of the (smaller) employed workforce pushes wages. The asymmetry between deposits (contingent on bank solvency) and CBDC (a risk-free central bank liability) is precisely what makes this substitution attractive during stress episodes, and precisely what generates the amplification.

The tiered remuneration scheme plays a crucial stabilizing role during liquidity shocks because it directly shapes the volume of deposits needed to fund bank credit. When $\bar{M}$ is sufficiently low, the incentive for households to convert large amounts of deposits into CBDC is weakened which keeps deposits from collapsing. This parameter acts as a policy lever to contain the flight-to-quality effect observed during stress episodes.

Formally, since $M^* = \frac{\Lambda(\theta_t)}{1+\Lambda(\theta_t)} \cdot \bar{D}$, a lower penalized rate R^{2M} reduces $\Lambda(\theta_t)$ and therefore the sensitivity of M^* to shock θ_t . Similarly, in case 2b ($\Omega(\theta_t) > 1$), a tighter ceiling $\bar{M}$ directly caps the flight-to-quality effect, preventing a full collapse of the deposit base.

This result highlights that the design of CBDC remuneration and its quantitative limit can serve as a macroprudential instrument for the central bank in order to ensure financial stability during episodes of liquidity stress.

3. sCBDC: structural stabilization

Under the sCBDC regime, the same shock specification yields a portfolio condition:

$$\Gamma(\theta_t) = \left(\frac{1 - (1 - \theta_t)\omega}{(1 - \theta_t)\omega} \right)^\sigma \left(\frac{\beta - R^D}{\beta - R^D + \kappa} \right)^\sigma \quad (15)$$

Where $\Gamma(\theta_t)$ is strictly increasing in θ_t . We can simply conclude that if $\theta_t = 0$, then $\Gamma(\theta_t) = 0$ which corresponds to the steady-state. In the other hand, if $\theta_t = 1$, it corresponds to complete flight-to-sCBDC episode, an extreme case in which deposits are fully substituted by synthetic CBDCs.

Proposition 5:

Under no interventions from the central bank and a positive liquidity shock $\theta_t \in [0,1]$. The equilibrium sCBDC holdings and deposits under the shock are:

$$S^* = \frac{\Gamma(\theta_t)}{(1 + \Gamma(\theta_t))} \frac{(\varphi - 1)(1 - \lambda)}{\varphi} \bar{B} \quad (16)$$

$$D^{sCBDC*} = \frac{1}{(1 + \Gamma(\theta_t))} \frac{(\varphi - 1)(1 - \lambda)}{\varphi} \bar{B} \quad (17)$$

The following relationships hold:

$$\frac{\partial S^*}{\partial \theta_t} > 0, \frac{\partial D^{sCBDC*}}{\partial \theta_t} < 0, \frac{\partial B^{sCBDC*}}{\partial \theta_t} = 0$$

Proof: See appendix G.

The result $\frac{\partial B^{sCBDC*}}{\partial \theta_t} = 0$ is the central analytical finding of this paper. Because traditional bank credit is structurally pinned at $B^{sCBDC} = (1 - \lambda)\bar{B}$, the liquidity shock cannot transmit to commercial bank lending through the deposit channel. When households reallocate toward sCBDC under stress, this reallocation takes place entirely within the pool of already-diverted liquidity $\left(\frac{(\varphi-1)(1-\lambda)}{\varphi}\right) \bar{B}$, leaving the funding available to traditional banks untouched. The flight-to-quality is absorbed within the narrow-banking segment rather than propagated to the credit channel.

This constitutes a fundamental architectural distinction: where CBDC allows liquidity stress to migrate directly out of the banking system and into the central bank's balance sheet, sCBDC channels the same behavioral response into a sub-segment of the financial system that was, by construction, already separated from credit creation.

4. Real economy transmission

The divergence in credit transmission translates directly into a divergence in real economic outcomes. Under the CBDC regime, the contraction in B_t^{CBDC} documented in Proposition 4 propagates through the firm's working-capital constraint to consumption and employment: both decline nonlinearly as θ_t rises, with the contraction accelerating once liquidity preference intensifies sufficiently to trigger large-scale substitution toward CBDC. Wages rise only marginally, reflecting reduced labor demand rather than improved household welfare.

Under the sCBDC regime, by contrast, Proposition 5 implies that output, consumption, and employment are invariant to the liquidity shock, since none of these variables depend on θ_t once $B^{sCBDC} = (1 - \lambda)\bar{B}$ is fixed by the structural parameter λ . The shock alters the composition of household liquid wealth — more sCBDC, fewer deposits — without altering aggregate real activity. This is a striking implication: in our framework, the sCBDC architecture renders the real economy fully insulated from Diamond-Dybvig-type liquidity preference shocks, a property that does not hold under standard CBDC at any non-trivial value of $\Lambda(\theta_t)$.

This sharp contrast should be interpreted with the model's stylization in mind: the stabilization result follows directly from the assumption that narrow-bank reserve backing is frictionless and that λ is fixed rather than itself responsive to stress (e.g., through endogenous bank conversion).

5. Portfolio dynamics

The underlying portfolio reallocation differs in both magnitude and character across regimes. Under CBDC, the substitution toward M_t accelerates as θ_t rises, reflecting the unconstrained, frictionless nature of the deposit-CBDC trade-off: digital currency behaves as a near-perfect substitute for deposits precisely when households need it to. Under sCBDC, the substitution toward S_t follows a smoother trajectory, dampened by the intermediation cost κ which continues to discourage large-scale conversion even under stress.

The qualitative pattern is shared across both regimes: deposits falling, digital holdings rising; what differs is whether this reallocation occurs within a segment of the financial system that feeds back into credit creation (CBDC) or one that has already been structurally separated from it (sCBDC). This distinction, formalized in Propositions 6 and 7, is what ultimately separates an architecture that amplifies liquidity shocks from one that absorbs them.

VII. Analytical discussion of the equilibrium in the three regimes

Having characterized how the two digital architectures transmit liquidity shocks, it is now useful to step back and compare their steady-state properties directly.

Three observations emerge. First, the baseline regime generates the highest level of bank intermediation and real activity among the three architectures. With deposits as the sole liquid asset, household savings are fully channeled into bank funding, and credit creation

is constrained solely by the leverage ratio, $B = \varphi E$. Any digital alternative to deposits, by construction, diverts some fraction of household liquidity away from this channel.

Second, the two digital regimes generate disintermediation through structurally different channels, governed respectively by Λ (CBDC) and by the pair (Γ, λ) (sCBDC). Under CBDC, the entire disintermediation effect operates through the portfolio ratio Λ : deposits contract from D to $D^{CBDC} = \frac{\bar{D}}{1+\Lambda}$, and this contraction transmits one-for-one to bank credit via the leverage constraint. Under sCBDC, disintermediation operates through two compounding channels: a structural channel, governed by the narrow-bank participation rate λ , which mechanically removes a fraction of banks from credit intermediation regardless of household preferences; and a portfolio channel, governed by Γ , which further reallocates the remaining liquidity pool between deposits and sCBDC.

Third, the magnitude of portfolio-driven disintermediation differs systematically between the two digital architectures. Since $\Gamma < \Omega$ whenever the intermediation cost $\kappa > 0$ — a direct consequence of Proposition 4 — the sCBDC regime generates strictly less portfolio reallocation toward digital money than the CBDC regime would, for identical household preferences (ω, σ) . The intermediation cost κ thus functions as a natural brake on synthetic digital money adoption that has no counterpart under direct CBDC issuance, where the central bank can in principle set R^M arbitrarily close to R^D and thereby push Λ toward Ω or beyond.

Taken together, the steady-state and shock-response comparisons reveal a consistent pattern: the same architectural features that attenuate disintermediation in normal times under sCBDC are precisely those that insulate commercial bank lending from liquidity shocks under stress. Conversely, the CBDC regime's steady-state disintermediation channel is identical to the channel through which liquidity shocks propagate and amplify. Architecture, in other words, determines both the level and the volatility of intermediation simultaneously.

VIII. Policy implications

The results derived in Sections 4 and 5 point to a structural trade-off that should inform the design of digital monetary architectures: the same institutional features that make sCBDC resilient to liquidity shocks also limit its capacity to support credit creation, while the same features that allow CBDC to preserve higher steady-state intermediation also expose it to endogenous amplification under stress.

The limits of remuneration-based stabilization under CBDC. Tiered remuneration and holding ceilings are the central bank's primary tools for containing CBDC-driven disintermediation, and our results confirm that both instruments work in the intended direction: a lower penalized rate R_2^M or a tighter ceiling $\bar{M}$ reduces $\Lambda(\theta_t)$ and thereby dampens the flight-to-quality response to a liquidity shock. However, these instruments attenuate rather than eliminate amplification. As long as CBDC remains a risk-free substitute for deposits, sensitivity of bank credit to liquidity preference shocks is unavoidable. This suggests that remuneration design should be understood as a dial that trades off CBDC's usefulness as a payment instrument against its destabilizing potential during stress, not as a mechanism capable of fully decoupling the two.

The narrow-bank participation threshold under sCBDC. The property in Proposition 5 ($\frac{\partial B^{sCBDC*}}{\partial \theta_t} = 0$) depends on the premise that traditional bank credit is structurally fixed at $(1 - \lambda)B$, independent of household behavior. This is a desirable stability property, but it is purchased at the cost of a permanently smaller banking sector: a higher λ improves resilience to liquidity shocks in the sense that more of the financial system is removed from leveraged credit creation and placed into full-reserve narrow banking, but it also permanently lowers steady-state credit, output, and employment, as shown in Proposition 5. There is therefore an implicit optimal range for λ : too low, and the narrow-banking segment is too small to meaningfully absorb liquidity stress; too high, and the cost in foregone credit creation may outweigh the stability benefit.

Intermediation costs as an implicit policy lever. A further implication concerns the role of κ , the intermediation cost spread borne by narrow banks. Because Γ is decreasing in κ , this cost parameter functions as an additional and largely overlooked policy lever: regulatory choices that raise or lower the operational cost of narrow-bank intermediation (e.g., reserve remuneration policy, licensing requirements, or technology standards for sCBDC issuers) directly affect the pace and magnitude of portfolio substitution toward sCBDC, both in steady

state and under stress. Unlike CBDC remuneration, which is a direct monetary policy choice, κ is more naturally a prudential and regulatory parameter, suggesting that the policy toolkit for managing sCBDC-driven disintermediation may sit closer to banking supervision than to monetary policy proper.

Hybrid architectures. Finally, the sharp asymmetry between the two regimes — full amplification under CBDC, full insulation under sCBDC, in our stylized framework — raises the question of whether hybrid arrangements could combine the payment efficiency benefits of retail CBDC with the structural insulation properties of synthetic architectures. One possibility, left for future research, is a regime in which only a fraction of CBDC issuance is direct (subject to tiered remuneration) while the remainder is routed through narrow-bank-like intermediaries; such a structure could in principle allow policymakers to calibrate the economy's position along the resilience-versus-credit-creation frontier identified in this paper, rather than being confined to one architecture's properties in full.

IX. Conclusion

This paper develops a unified DSGE framework with banking frictions to compare how central bank digital currency (CBDC) and synthetic CBDC (sCBDC) architectures shape the transmission of liquidity shocks, building on the classical fragility mechanism of Diamond and Dybvig (1983). Relative to a baseline banking economy in which deposits are the sole liquid asset, both digital architectures generate disintermediation in steady state — but through fundamentally different channels, and with starkly different implications for financial stability under stress.

Under the standard CBDC regime, disintermediation operates through a frictionless portfolio trade-off between deposits and a risk-free central bank liability. This same channel becomes a source of endogenous amplification once a liquidity preference shock hits the economy: households' flight to CBDC tightens bank funding, contracts credit, and propagates the shock to output, consumption, and employment. Tiered remuneration and holding ceilings mitigate this amplification but do not eliminate it, since the underlying substitutability between CBDC and deposits remains intact.

Under the synthetic CBDC regime, disintermediation is instead governed jointly by the structural participation of narrow banks λ and an intermediation cost friction (κ) that has no counterpart under direct CBDC issuance. This architecture yields a striking result: because narrow-bank reserve backing structurally separates payment liquidity from credit creation,

commercial bank lending becomes entirely insulated from the liquidity shock, even as households reallocate aggressively toward sCBDC during stress. The cost of this insulation is a permanently smaller banking sector and correspondingly lower steady-state credit, output, and employment as narrow-bank participation expands.

Taken together, these results identify a central trade-off in the design of digital monetary architectures: standard CBDC preserves greater steady-state credit intermediation but remains structurally exposed to liquidity-driven amplification, while synthetic CBDC offers resilience to such shocks at the cost of weaker long-run bank intermediation. Neither architecture dominates the other unconditionally; the appropriate choice depends on how policymakers weigh the variance-reducing benefits of structural insulation against the level cost of reduced credit creation.

Several extensions would enrich this framework. Introducing heterogeneous households would allow liquidity shocks to be modeled as genuine idiosyncratic withdrawal risk rather than an aggregate preference shift, closer to the original Diamond-Dybvig environment. Endogenizing the narrow-bank participation rate, λ allowing banks to convert in response to anticipated stress, rather than fixing λ , would test the robustness of the insulation result derived in Proposition 5 under a more realistic margin of adjustment. Finally, exploring hybrid CBDC-sCBDC arrangements, as discussed in Section 6, would help identify whether the resilience-versus-credit-creation frontier identified here can be improved upon through institutional design rather than navigated as a binary choice.

Appendix A

Assuming stationarity, all variables are constant over time (no trend growth). The key-state equilibrium satisfies:

- Interest rate in steady state:

$$R^D = \frac{1}{\beta} \quad (1A)$$

- Production and wages:

With capital normalized to one ($K = 1$), output is given by the Cobb-Douglas production function:

$$Y = AN^{1-\alpha} \quad (2A)$$

The firms maximize their profits :

$$\max_N AN^{1-\alpha} - W N \quad (3A)$$

FOC:

$$W = (1 - \alpha)AN^{-\alpha} = (1 - \alpha)Y \quad (4A)$$

$$WN = (1 - \alpha)AN^{1-\alpha} \quad (5A)$$

- The Labor-leisure trade-off

$$\chi N^\phi = \frac{W}{C} \quad (6A)$$

Soit

$$\chi N^{\phi+1} = \frac{W N}{AN^{1-\alpha}} = \frac{(1 - \alpha)AN^{1-\alpha}}{AN^{1-\alpha}} = (1 - \alpha) \quad (7A)$$

Hence the labour supply:

$$N = \left(\frac{1 - \alpha}{\chi} \right)^{\frac{1}{1+\phi}} \quad (8A)$$

- Output and consumption are therefore:

$$Y = C = A \left(\frac{1 - \alpha}{\chi} \right)^{\frac{1 - \alpha}{1 + \phi}} \quad (9A)$$

The equilibrium wage is:

$$W = (1 - \alpha)A \left(\frac{1 - \alpha}{\chi} \right)^{\frac{-\alpha}{1 + \phi}} \quad (10A)$$

The credit demand by the firms is then:

$$B = WN = (1 - \alpha)Y = (1 - \alpha)A \left(\frac{1 - \alpha}{\chi} \right)^{\frac{1 - \alpha}{1 + \phi}} \quad (11A)$$

- Bank equity is given by:

$$E = \frac{B}{\phi} \quad (12A)$$

Replacing (11A) in (12A), we obtain:

$$E = \frac{B}{\phi} = \frac{1 - \alpha}{\phi} A \left(\frac{1 - \alpha}{\chi} \right)^{\frac{1 - \alpha}{1 + \phi}} \quad (13A)$$

We can simply obtain the expression of deposits:

$$D = (\phi - 1)E = \left(1 - \frac{1}{\phi}\right) B = \left(1 - \frac{1}{\phi}\right) (1 - \alpha)A \left(\frac{1 - \alpha}{\chi} \right)^{\frac{1 - \alpha}{1 + \phi}} \quad (14A)$$

With all these equilibrium values of the variables let's verify that the household's period-by-period budget constraint is also verified, i.e. that:

$$C + D + E = W N + R^D D + \Pi^B$$

From (13) we have

$$\Pi^B = \left(\frac{1}{1 - \alpha} + \left(\frac{1}{\phi} - 1 \right) \frac{1}{\beta} \right) B$$

Rearranging, we obtain:

$$\frac{B}{1-\alpha} + \left(1 - \frac{1}{\beta}\right) \left(1 - \frac{1}{\varphi}\right) B + \frac{B}{\varphi} = B + \left(\frac{1}{1-\alpha} + \left(\frac{1}{\varphi} - 1\right) \frac{1}{\beta}\right) B \quad (15A)$$

which could be easily be shown that is verified.

Appendix B

The household maximizes utility subject to the budget constraint. The first-order conditions for deposits and CBDC are respectively :

$$\frac{1}{C} = \psi \cdot \frac{\partial L}{\partial D} + \frac{R^D}{\beta C} \quad (1B)$$

$$\frac{1}{C} = \psi \cdot \frac{\partial L}{\partial M} + \frac{R^M}{\beta C} \quad (2B)$$

From the CES liquidity aggregator:

$$L = [\omega \cdot D^{\frac{\sigma-1}{\sigma}} + (1-\omega) \cdot M^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad (3B)$$

the partial derivatives are:

$$\frac{\partial L}{\partial D} = \omega \cdot D^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} \quad (4B)$$

$$\frac{\partial L}{\partial M} = (1-\omega) \cdot M^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} \quad (5B)$$

Substituting (4B) and (5B) into (1B) and (2B):

$$\psi \omega \cdot D^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} = \frac{\beta - R^D}{\beta C} \quad (6B)$$

$$\psi (1-\omega) \cdot M^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} = \frac{\beta - R^M}{\beta C} \quad (7B)$$

Dividing (7B) by (6B) eliminates the common terms ψ , $L^{\frac{1}{\sigma}}$, and $\frac{1}{\beta C}$:

$$\frac{1-\omega}{\omega} \cdot \left(\frac{D^{CBDC}}{M} \right)^{\frac{1}{\sigma}} = \frac{\beta - R^M}{\beta - R^D}$$

Rearranging:

$$\begin{aligned} \left(\frac{D^{CBDC}}{M} \right)^{\frac{1}{\sigma}} &= \frac{\omega}{1-\omega} \cdot \frac{\beta - R^M}{\beta - R^D} \\ \frac{M}{D^{CBDC}} &= \left(\frac{1-\omega}{\omega} \right)^{\sigma} \left(\frac{\beta - R^D}{\beta - R^M} \right)^{\sigma} \end{aligned} \quad (8B)$$

This key equation determines the equilibrium ratio of CBDC to deposits as a function of relative returns and household preferences.

Case 1 — Coexistence ($R^M = R^D, M \leq \bar{M}$)

When $R^M = R^D$, we have $\beta - R^D = \beta - R^M$.

Equation (8B) becomes:

$$\frac{M}{D} = \left(\frac{1-\omega}{\omega} \right)^\sigma \equiv \Omega \quad (9B)$$

where $\Omega = \left(\frac{1-\omega}{\omega} \right)^\sigma$ captures the relative household preference for CBDC over deposits.

Using the identity

$$\begin{aligned} D^{CBDC} &= B^{CBDC} - E = B^{CBDC} - \frac{\bar{B}}{\varphi} \\ D^{CBDC} &= \bar{B} - M - \frac{\bar{B}}{\varphi} = \frac{\varphi-1}{\varphi} \bar{B} - M \\ M &= \Omega \left(\frac{\varphi-1}{\varphi} \bar{B} - M \right) \end{aligned} \quad (10B)$$

$$M(1 + \Omega) = \Omega \frac{\varphi-1}{\varphi} \bar{B} \quad (11B)$$

Solving for M and D^{CBDC} , and knowing that $\frac{\varphi-1}{\varphi} \bar{B} = \bar{D}$, we can write:

$$M = \frac{\Omega}{1 + \Omega} \frac{\varphi-1}{\varphi} \bar{B} = \frac{\Omega}{1 + \Omega} \bar{D} \quad (12B)$$

$$M = \frac{\Omega}{1+\Omega} \cdot \frac{\varphi-1}{\varphi} \cdot \bar{B} = \frac{\Omega}{1+\Omega} \cdot \bar{D} \quad (13B)$$

$$D^{cBDc} = \frac{1}{1 + \Omega} \cdot \bar{D} \quad (14B)$$

And the equilibrium credit supply:

$$B^{cBDc} = \bar{B} - M = \bar{B} \cdot \frac{\varphi + \Omega}{\varphi(1 + \Omega)} \quad (15B)$$

We can then derivate the other variables:

- Consumption and production

$$C^{cBDc} = \frac{B^{cBDc}}{1 - \alpha} \quad (16B)$$

$$C^{cBDc} = \frac{\bar{B}}{1 - \alpha} \cdot \frac{\varphi + \Omega}{\varphi(1 + \Omega)} \quad (17B)$$

$$Y^{cBDc} = C^{cBDc} = \bar{C} \cdot \frac{\varphi + \Omega}{\varphi(1 + \Omega)} \quad (18B)$$

- Labor:

$$B^{cBDc} = (1 - \alpha)A(N^{CBDC})^{1-\alpha} \quad (19B)$$

$$(N^{CBDC})^{1-\alpha} = \frac{B^{cBDc}}{(1 - \alpha)A} = N^{1-\alpha} \frac{\varphi + \Omega}{\varphi(1 + \Omega)} \quad (20B)$$

$$N^{CBDC} = \bar{N} \cdot \left(\frac{\varphi + \Omega}{\varphi(1 + \Omega)} \right)^{\frac{1}{1-\alpha}} \quad (21B)$$

- Wages :

$$W^{cBDc} = (1 - \alpha)A(N^{CBDC})^{-\alpha} \quad (22B)$$

$$W^{CBDC} = \bar{W} \cdot \left(\frac{\varphi + \Omega}{\varphi(1 + \Omega)} \right)^{\frac{-\alpha}{1-\alpha}} \quad (23B)$$

- Equity adjusts at the end of the period:

$$E^{CBDC} = \frac{B^{CBDC}}{\varphi} = \frac{\bar{B}}{\varphi} \cdot \frac{\varphi + \Omega}{\varphi(1 + \Omega)} \quad (24B)$$

$$E^{CBDC} = E \cdot \frac{\varphi + \Omega}{\varphi(1 + \Omega)} \quad (25B)$$

Case 2 — Penalization $(R^{2M} < R^D = \frac{1}{\beta}, M > \bar{M})$

When $R^{2M} < R^D$, equation (8B) becomes:

$$\frac{M}{D^{cBDc}} = \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma \quad (16B)$$

Since $R^{2M} < R^D$ we have $\beta - R^{2M} > \beta - R^D$, which implies:

$$\frac{\beta - R^D}{\beta - R^{2M}} < 1$$

Therefore :

$$\frac{M}{D^{cBDc}} < \Omega$$

If $\Omega < 1 \Rightarrow \frac{1-\omega}{\omega} < 1 \Leftrightarrow \omega > 1/2$

Households have a stronger structural preference for deposits than for CBDC. Deposits remain the dominant liquid asset in the household portfolio.

Then,

$$M = \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma \cdot D^{cBDc} \quad (17B)$$

If $\Omega = 1$, households are indifferent between the two assets and we have :

$$M = \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma \cdot D^{CBDC} \quad (26B)$$

And as we have $\beta - R^{2M} > \beta - R^D$, deposits will always dominate CBDC

We can conclude that in the case $\Omega \leq 1$, deposits will always dominate CBDC.

Using the identity $D^{CBDC} = \bar{B} - M - \frac{\bar{B}}{\varphi} = \frac{\varphi-1}{\varphi} \bar{B} - M$, (17B) becomes:

$$M = \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma \cdot \left[\frac{\varphi-1}{\varphi} \bar{B} - M \right] \quad (18B)$$

$$M \left[1 + \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma \right] = \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma \cdot \frac{\varphi-1}{\varphi} \bar{B} \quad (19B)$$

$$M = \frac{\Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma}{1 + \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma} \cdot \frac{\varphi-1}{\varphi} \bar{B} \quad (20B)$$

We can write $\Lambda = \Omega \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^\sigma$ which yields:

$$M = \frac{\Lambda}{1 + \Lambda} \cdot \frac{\varphi-1}{\varphi} \bar{B} \quad (21B)$$

$$D^{CBDC} = \frac{\varphi-1}{\varphi} \bar{B} - M = \frac{\varphi-1}{\varphi} \bar{B} - \frac{\Lambda}{1 + \Lambda} \cdot \frac{\varphi-1}{\varphi} \bar{B} \quad (22B)$$

$$D^{CBDC} = \left(1 - \frac{\Lambda}{1 + \Lambda} \right) \frac{\varphi-1}{\varphi} \bar{B} \quad (23B)$$

$$D^{CBDC} = \left(\frac{1}{1 + \Lambda} \right) \bar{D} \quad (24B)$$

We can simply verify from (22B) that if $M = 0$, $D^{CBDC} = \frac{\varphi-1}{\varphi} \bar{B} = \bar{D}$

- Credit supply

$$B^{CBDC} = \bar{B} - \left(\frac{\Lambda}{1 + \Lambda} \right) \bar{D} = \bar{B} - \left(\frac{\Lambda}{1 + \Lambda} \right) \frac{\varphi-1}{\varphi} \bar{B} \quad (25B)$$

$$B^{CBDC} = \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \cdot \bar{B} \quad (26B)$$

- consumption

$$C^{CBDC} = \frac{B^{CBDC}}{(1 - \alpha)} \quad (27B)$$

$$Y^{CBDC} = C^{CBDC} = \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \cdot \bar{C} \quad (28B)$$

- labor:

$$N^{CBDC} = \bar{N} \cdot \left(\frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \right)^{\frac{1}{1-\alpha}} \quad (29B)$$

- Wages :

$$W^{CBDC} = (1 - \alpha)A(N^{CBDC})^{-\alpha} \quad (30B)$$

$$W^{CBDC} = \bar{W} \cdot \left(\frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \right)^{\frac{-\alpha}{1-\alpha}} \quad (31B)$$

- Equity adjusts at the end of the period:

$$E^{CBDC} = \frac{B^{CBDC}}{\varphi} = \frac{\bar{B}}{\varphi} \cdot \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \quad (32B)$$

$$E^{CBDC} = E \cdot \frac{\varphi + \Lambda}{\varphi(1 + \Lambda)} \quad (33B)$$

If $\Omega > 1 \Rightarrow \frac{1-\omega}{\omega} > 1 \Leftrightarrow \omega < 1/2$

Households have a stronger structural preference for CBDC than deposits, but their holdings are limited through the threshold $\bar{M}$. In this case, the equilibrium is characterized by a corner solution:

$$M = \bar{M}$$

$$D^{CBDC} = \frac{\varphi - 1}{\varphi} \bar{B} - \bar{M} = \bar{D} - \bar{M} \quad (34B)$$

- Credit supply

$$B^{CBDC} = \bar{B} - \bar{M} \quad (35B)$$

- consumption

$$C^{CBDC} = \frac{\bar{B} - \bar{M}}{(1 - \alpha)} \quad (36B)$$

$$Y^{CBDC} = C^{CBDC} = \bar{C} \cdot \left(1 - \frac{\bar{M}}{\bar{B}}\right) \quad (37B)$$

- labor:

$$N^{CBDC} = \bar{N} \cdot \left(1 - \frac{\bar{M}}{\bar{B}}\right)^{\frac{1}{1-\alpha}} \quad (38B)$$

- Wages :

$$W^{CBDC} = \bar{W} \cdot \left(1 - \frac{\bar{M}}{\bar{B}}\right)^{\frac{-\alpha}{1-\alpha}} \quad (39B)$$

- Equity adjusts at the end of the period:

$$E^{CBDC} = E \cdot \left(1 - \frac{\bar{M}}{\bar{B}}\right) \quad (40B)$$

Appendix C

$$D^{sCBDC} = B^{sCBDC} - E = (1 - \lambda)\bar{B} - \frac{\bar{B}}{\varphi} \quad (1C)$$

$$D^{sCBDC} = \bar{B} \left(1 - \lambda - \frac{1}{\varphi} \right) \quad (2C)$$

$$D^{sCBDC} = \bar{B} \left(\frac{\varphi - 1 - \lambda\varphi}{\varphi} \right) \quad (3C)$$

$$D^{sCBDC} = \bar{B} \left(\frac{\varphi - 1}{\varphi} \right) - \lambda\bar{B} \quad (4C)$$

$$D^{sCBDC} = \bar{D} - \lambda\bar{B} \quad (5C)$$

Expression of $\tilde{S}$:

The FOC give us:

$$\frac{S}{D^{sCBDC}} = \left(\frac{1 - \omega}{\omega} \right)^\sigma \cdot \left(\frac{\beta - R^D}{\beta - R^S} \right)^\sigma \quad (6C)$$

$$\frac{S}{D^{sCBDC}} = \Omega \cdot \left(\frac{\beta - R^D}{\beta - R^S} \right)^\sigma \quad (7C)$$

$$\tilde{S} = \Omega \cdot \left(\frac{\beta - R^D}{\beta - R^S} \right)^\sigma \cdot D^{sCBDC} \quad (8C)$$

Appendix D

The household maximizes utility subject to the budget constraint. Similarly to the CBDC regime, the first-order conditions for deposits and CBDC are respectively :

$$\frac{1}{C} = \psi \cdot \frac{\partial L}{\partial D} + \frac{R^D}{\beta C} \quad (1D)$$

$$\frac{1}{C} = \psi \cdot \frac{\partial L}{\partial S} + \frac{R^S}{\beta C} \quad (2D)$$

From the CES liquidity aggregator:

$$L = [\omega \cdot D^{\frac{\sigma-1}{\sigma}} + (1-\omega) \cdot S^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad (3D)$$

the partial derivatives are:

$$\frac{\partial L}{\partial D} = \omega \cdot D^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} \quad (4D)$$

$$\frac{\partial L}{\partial S} = (1-\omega) \cdot S^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} \quad (5D)$$

Substituting (4D) and (5D) into (1D) and (2D):

$$\psi \omega \cdot D^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} = \frac{\beta - R^D}{\beta C} \quad (6D)$$

$$\psi (1-\omega) \cdot S^{\frac{-1}{\sigma}} \cdot L^{\frac{1}{\sigma}} = \frac{\beta - R^S}{\beta C} \quad (7D)$$

Dividing (7D) by (6D) eliminates the common terms ψ , $L^{\frac{1}{\sigma}}$, and $\frac{1}{\beta C}$:

$$\frac{1-\omega}{\omega} \cdot \left(\frac{D^{SCBDC}}{S} \right)^{\frac{1}{\sigma}} = \frac{\beta - R^S}{\beta - R^D}$$

Rearranging:

$$\begin{aligned} \left(\frac{D^{SCBDC}}{S} \right)^{\frac{1}{\sigma}} &= \frac{\omega}{1-\omega} \cdot \frac{\beta - R^S}{\beta - R^D} \\ \frac{S}{D^{SCBDC}} &= \left(\frac{1-\omega}{\omega} \right)^{\sigma} \left(\frac{\beta - R^D}{\beta - R^S} \right)^{\sigma} \end{aligned} \quad (8D)$$

Replacing $R^S = R^D - \kappa$

$$\frac{S}{D^{SCBDC}} = \left(\frac{1-\omega}{\omega} \right)^{\sigma} \left(\frac{\beta - R^D}{\beta - R^D + \kappa} \right)^{\sigma} \quad (9D)$$

We note : $\Gamma = \left(\frac{1-\omega}{\omega}\right)^\sigma \left(\frac{\beta-R^D}{\beta-R^D+\kappa}\right)^\sigma$

$$\frac{S}{D^{SCBDC}} = \Gamma \quad (10D)$$

$$S = \Gamma \cdot D^{SCBDC} \quad (11D)$$

Knowing that : $D^{SCBDC} + S = \frac{\varphi(1-\lambda)-1}{\varphi} \bar{B}$ and substituting we obtain:

$$D^{SCBDC} + \Gamma \cdot D^{SCBDC} = \frac{\varphi(1-\lambda)-1}{\varphi} \bar{B} \quad (12D)$$

$$D^{SCBDC} (1 + \Gamma) = \frac{\varphi(1-\lambda)-1}{\varphi} \bar{B} \quad (13D)$$

$$D^{SCBDC} = \frac{1}{(1 + \Gamma)} \frac{\varphi(1-\lambda)-1}{\varphi} \bar{B} \quad (14D)$$

And from $S = \Gamma \cdot D^{SCBDC}$:

$$S = \frac{\Gamma}{(1 + \Gamma)} \frac{\varphi(1-\lambda)-1}{\varphi} \bar{B} \quad (14D)$$

Appendix E

Numerical illustration of the equilibrium under Regime 1

To illustrate the baseline equilibrium, we calibrate the model using standard parameter values. We then simulate the response of output, employment, wages, deposits and credit.

Table 1 Parameters' values chosen for the model calibration

Parameter	Description	Value	Admissible_range
β	Household subjective discount factor	0.7	(0,1)
α	Capital share in production	0.33	(0,1)
A	Total factor productivity	1	> 0
χ	Disutility of labor parameter	1	> 0
ϕ	Inverse Frisch elasticity of labor supply	2	> 0
φ	Maximum permissible leverage ratio	4	> 1

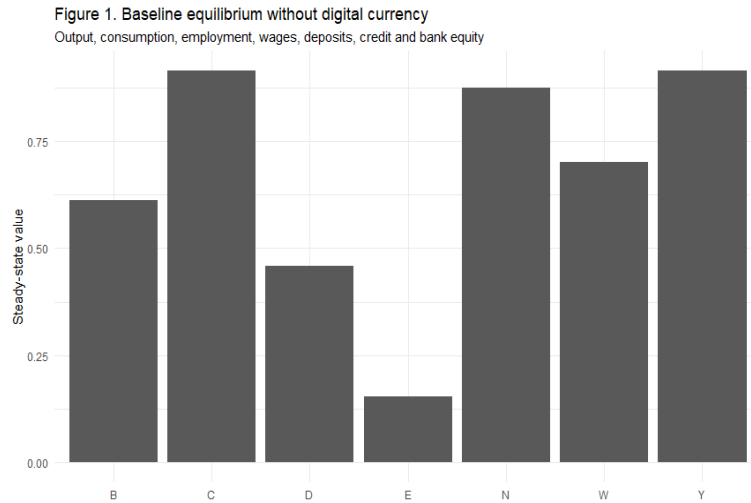


Figure 1 presents the calibrated steady-state equilibrium of the benchmark banking economy without digital currency, which is characterised in the following table.

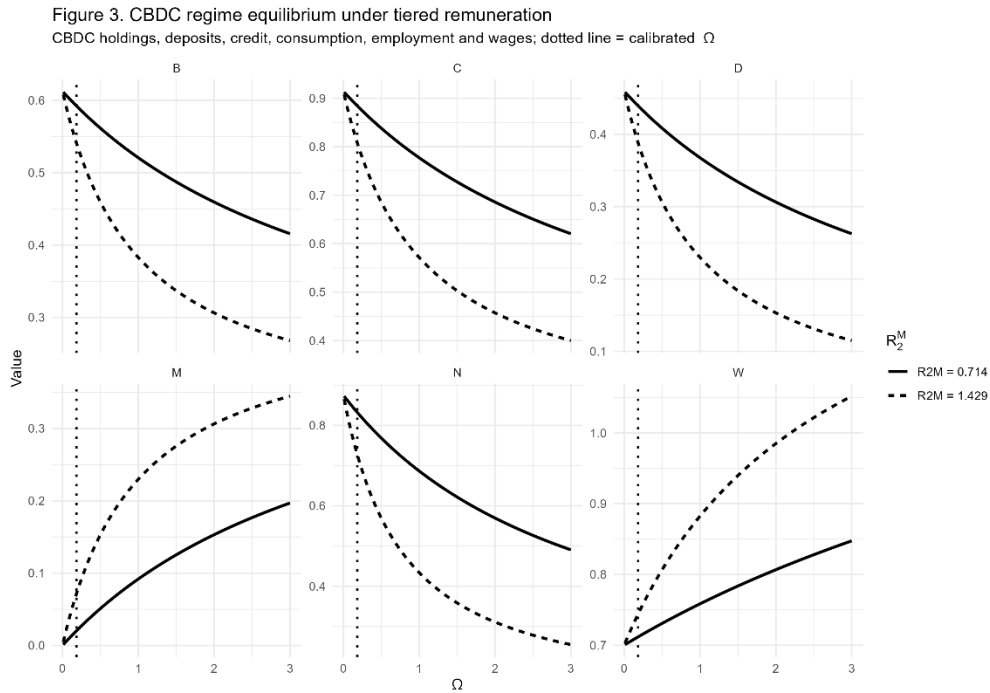
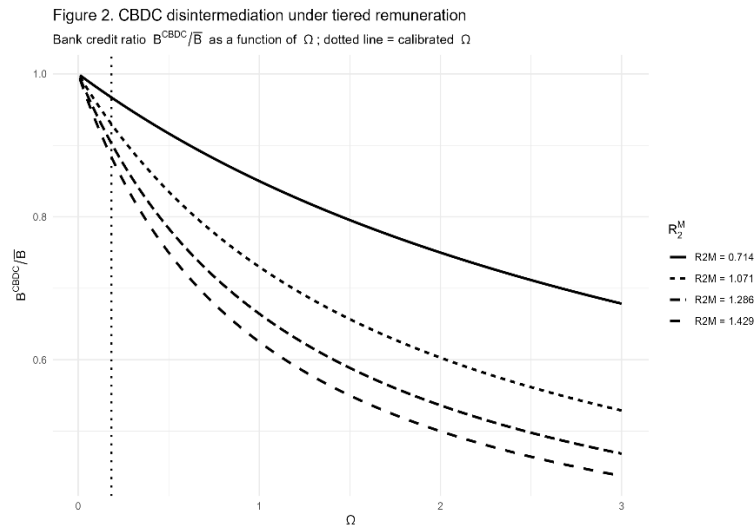
$\bar{N}$	$\bar{Y}$	$\bar{C}$	$\bar{W}$	$\bar{B}$	$\bar{D}$	$\bar{E}$	R^D	R^L	$\bar{\mu}$
0.88	0.91	0.91	0.70	0.61	0.46	0.15	1.43	1.49	0.06

Numerical illustration of the equilibrium under Regime 2

Table 2 Parameters' values chosen for the model calibration

Parameter	Description	Value	Admissible range
β	Household subjective discount factor	0.7	(0,1)

α	Capital share in production	0.33	(0,1)
A	Total factor productivity	1	> 0
χ	Disutility of labor parameter	1	> 0
ϕ	Inverse Frisch elasticity of labor supply	2	> 0
φ	Maximum permissible leverage ratio	4	> 1
ω	Relative preference weight for deposits in liquidity services (close to 1 implies stronger deposit preference)	0.7	(0,1)
σ	Elasticity of substitution between deposits and digital money (higher sigma implies stronger substitutability)	2	> 0
$\bar{M}$	CBDC holding ceiling under the corner solution (must remain below baseline deposits $\bar{D}$ in the corner solution).	0.15	≥ 0



Numerical Illustration of the Equilibrium under Regime 3 (sCBDC)

To illustrate the equilibrium dynamics under Regime 3, we calibrate the model using the same benchmark parameters retained under Regimes 1 and 2, in order to facilitate comparison across monetary architectures, while introducing the synthetic CBDC-specific parameters λ and κ .

Figure 4. sCBDC disintermediation by narrow-bank fraction

Bank credit ratio $B^{\text{sCBDC}}/\bar{B}$ as a function of λ

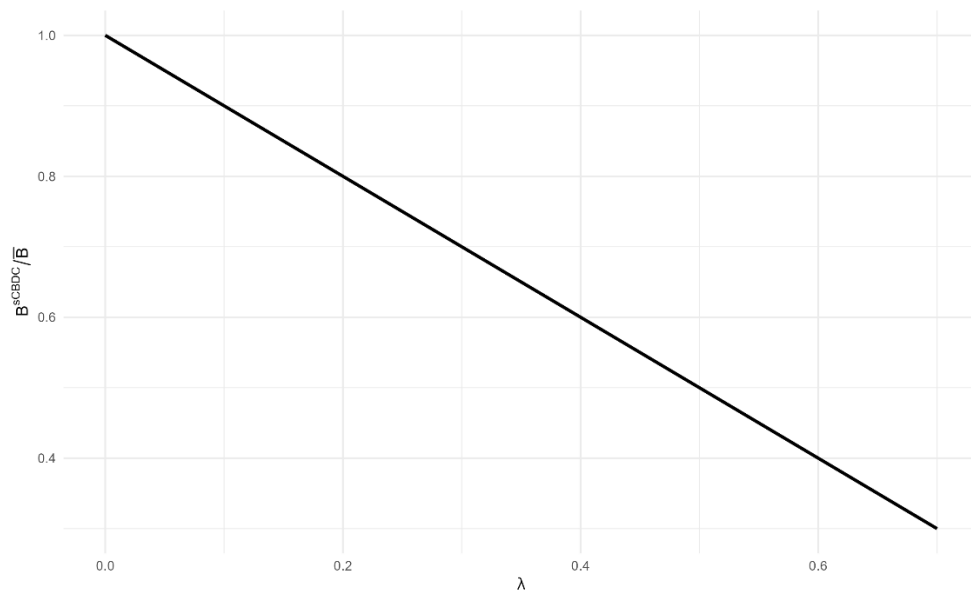


Figure 5. sCBDC regime: credit ratio over lambda and kappa

$B^{\text{sCBDC}}/\bar{B}$

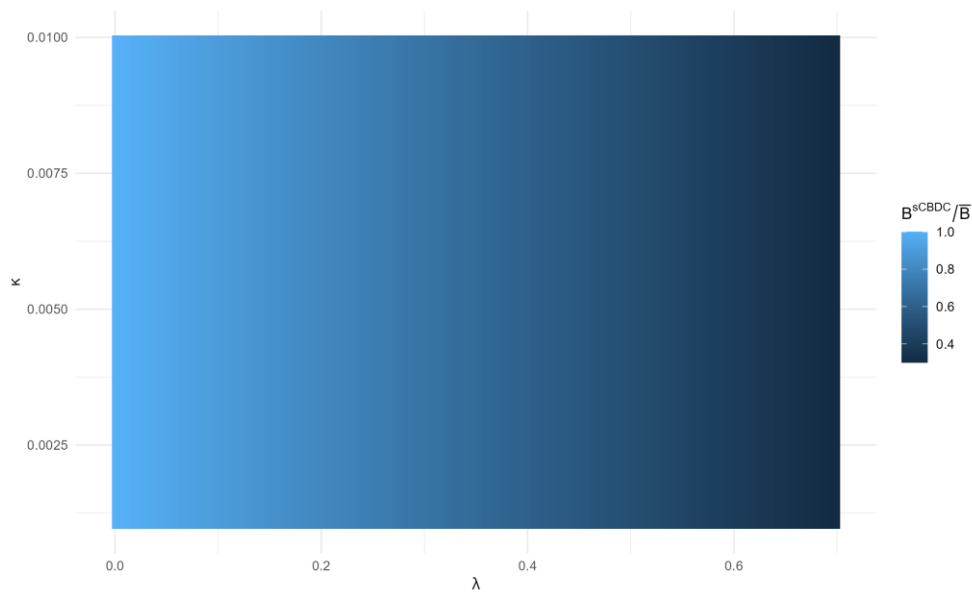


Figure 6. sCBDC regime: deposits and sCBDC holdings
Traditional deposits decline while sCBDC holdings increase with lambda

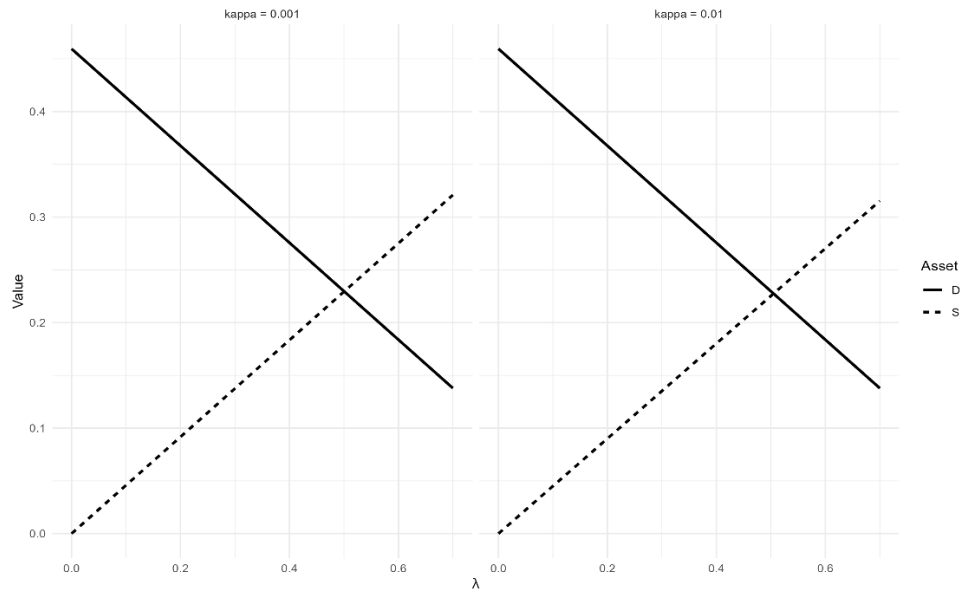
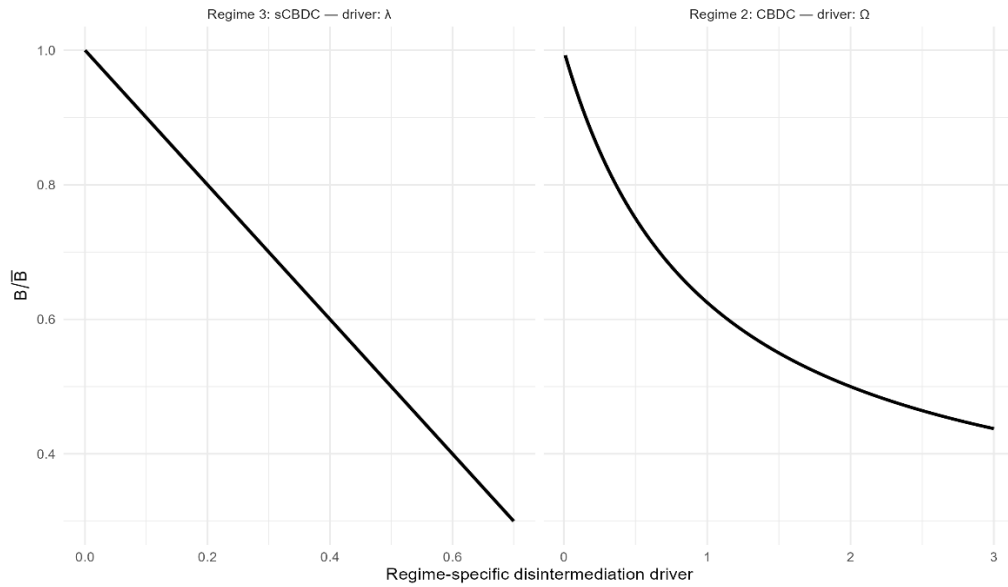


Figure 8. Comparison of disintermediation mechanisms under CBDC and sCBDC
CBDC is driven by liquidity preference; sCBDC by narrow-bank participation



Appendix F

We consider a liquidity preference shock $\theta_t \in [0,1]$ that increases households' demand for liquid assets.

In this regime, the liquidity aggregator is given by

$$\mathcal{L}(D, M) = \left[(1 - \theta_t) \omega D^{\frac{\sigma-1}{\sigma}} + (1 - (1 - \theta_t) \omega) M^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (\text{F1})$$

The household optimality conditions give:

$$\frac{\partial \mathcal{L}}{\partial D} = (1 - \theta_t) \omega \cdot D^{-\frac{1}{\sigma}} \cdot \mathcal{L}^{\frac{1}{\sigma}} \quad (\text{F2})$$

$$\frac{\partial \mathcal{L}}{\partial M} = (1 - (1 - \theta_t) \omega) \cdot M^{-\frac{1}{\sigma}} \cdot \mathcal{L}^{\frac{1}{\sigma}} \quad (\text{F3})$$

Dividing the CBDC Euler by the deposit Euler and using $R^D = \frac{1}{\beta}$

$$\frac{M^*}{D^{CBDC*}} = \left(\frac{1 - (1 - \theta_t) \omega}{(1 - \theta_t) \omega} \right)^{\sigma} \cdot \left(\frac{\beta - R^D}{\beta - R^{2M}} \right)^{\sigma} \equiv \Lambda(\theta_t) \quad (\text{F4})$$

Where $\Omega(\theta_t) = \left(\frac{1 - (1 - \theta_t) \omega}{(1 - \theta_t) \omega} \right)^{\sigma}$ is strictly increasing in θ_t since:

$$\frac{\partial \Omega(\theta_t)}{\partial \theta_t} = \sigma \left(\frac{1 - (1 - \theta_t) \omega}{(1 - \theta_t) \omega} \right)^{\sigma-1} \cdot \frac{\omega}{[(1 - \theta_t) \omega]^2} > 0 \quad (\text{F5})$$

When $\theta_t = 0$: $\Omega(0) = \left(\frac{1 - \omega}{\omega} \right)^{\sigma} = \Omega$, recovering the steady state conditions.

Using the balance sheet identity $D^{CBDC*} = \frac{\varphi-1}{\varphi} \bar{B} - M^*$ and substituting in (C4):

$$M^* = \Lambda(\theta_t) \left(\frac{\varphi-1}{\varphi} \bar{B} - M^* \right) \quad (\text{F6})$$

$$M^* (1 + \Lambda(\theta_t)) = \Lambda(\theta_t) \frac{\varphi-1}{\varphi} \bar{B} \quad (\text{F7})$$

$$M^* = \frac{\Lambda(\theta_t)}{(1 + \Lambda(\theta_t))} \bar{B} \quad (\text{F8})$$

Since $\frac{\Lambda}{(1+\Lambda)}$ is strictly increasing in Λ , and $\Lambda(\theta_t)$ is strictly increasing in θ_t , we obtain:

$$\frac{\partial M^*}{\partial \theta_t} > 0 \quad (\text{F9})$$

From (C8) and $D^{CBDC*} = \bar{D} - M^*$:

$$D^{CBDC*} = \frac{1}{(1 + \Lambda(\theta_t))} \bar{D} \quad (\text{F10})$$

Since $\frac{1}{(1 + \Lambda(\theta_t))}$ is strictly decreasing:

$$\frac{\partial D^{CBDC*}}{\partial \theta_t} = -\frac{1}{(1 + \Lambda(\theta_t))^2} \cdot \frac{\partial \Lambda}{\partial \theta_t} \cdot \bar{D} < 0 \quad (\text{F11})$$

A positive liquidity shock reduces equilibrium bank deposits.

The bank balance sheet with endogenous equity $E^{CBDC*} = \frac{B^{CBDC*}}{\varphi}$ gives:

$$B^{CBDC*} = D^{CBDC*} + E^{CBDC*} = D^{CBDC*} + \frac{B^{CBDC*}}{\varphi} \quad (\text{F12})$$

$$B^{CBDC*} \left(1 - \frac{1}{\varphi}\right) = D^{CBDC*} \quad (\text{F13})$$

$$B^{CBDC*} = \bar{B} \cdot \left(\frac{\varphi + \Lambda(\theta_t)}{\varphi(1 + \Lambda(\theta_t))} \right) \quad (\text{F14})$$

Differentiating with respect to θ_t :

$$\frac{\partial B^{CBDC*}}{\partial \theta_t} = \left(\frac{\varphi}{\varphi - 1} \right) \cdot \frac{\partial D^{CBDC*}}{\partial \theta_t} < 0 \quad (\text{F15})$$

An increase in θ_t reduces the bank's lending capacity proportionally, via the contraction of the deposit base.

From $B^{CBDC*} = (1 - \alpha)A(N^{CBDC*})^{-\alpha}$:

$$\frac{\partial W^{CBDC*}}{\partial \theta_t} = -\alpha(1 - \alpha)(1 - \alpha)A(N^{CBDC*})^{-\alpha-1} \frac{\partial N^{CBDC*}}{\partial \theta_t} > 0 \quad (\text{F16})$$

Appendix G

Consider a positive liquidity shock $\theta_t \in [0,1]$ that increases households' relative preference for synthetic CBDC holdings. The shock affects the optimal portfolio allocation between deposits and sCBDC through the liquidity services function.

Let the optimal portfolio ratio under the liquidity shock be defined as:

$$\frac{S_t}{D_t^{sCBDC}} = \Gamma(\theta_t) \quad (G1)$$

where

$$\Gamma(\theta_t) > 0$$

And

$$\Gamma'(\theta_t) > 0$$

The latter condition reflects the fact that a larger liquidity shock increases the attractiveness of sCBDC relative to traditional bank deposits.

From the banking sector balance-sheet identity, total private liquid assets held by households are equal to deposits plus sCBDC holdings:

$$D_t^{sCBDC} + S_t = B_t^{sCBDC} - E_t^{sCBDC} \quad (G2)$$

In the synthetic CBDC regime, bank credit is constrained by

$$B_t^{sCBDC} = (1 - \lambda)\bar{B} \quad (G3)$$

while bank equity satisfies

$$E_t^{sCBDC} = \frac{B_t^{sCBDC}}{\varphi} \quad (G4)$$

Substituting equation (G3) into equation (G4) yields

$$E_t^{sCBDC} = \frac{(1 - \lambda)\bar{B}}{\varphi} \quad (G5)$$

Replacing equations (G3) and (G5) into equation (G2), we obtain

$$D_t^{sCBDC} + S_t = (1 - \lambda)\bar{B} - \frac{(1 - \lambda)\bar{B}}{\varphi} \quad (G6)$$

which simplifies to

$$D_t^{sCBDC} + S_t = \frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi} \quad (G7)$$

Using the optimal portfolio condition (G1),

$$S_t = \Gamma(\theta_t)D_t^{sCBDC} \quad (G8)$$

equation (G6) becomes

$$D_t^{sCBDC} + \Gamma(\theta_t)D_t^{sCBDC} = \frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi} \quad (G9)$$

or equivalently,

$$(1 + \Gamma(\theta_t))D_t^{sCBDC} = \frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi} \quad (G10)$$

Hence, equilibrium deposits are given by

$$D_t^{sCBDC} = \frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi(1 + \Gamma(\theta_t))} \quad (G11)$$

Substituting equation (G11) into equation (G8) yields equilibrium sCBDC holdings:

$$S_t^* = \Gamma(\theta_t) \left[\frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi(1 + \Gamma(\theta_t))} \right] \quad (G12)$$

$$S_t^* = \frac{\Gamma(\theta_t)}{(1 + \Gamma(\theta_t))} \left[\frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi} \right] \quad (G13)$$

To establish the comparative statics, differentiate equation (G13) with respect to θ_t :

$$\frac{\partial S_t^*}{\partial \theta_t} = \frac{\Gamma'(\theta_t)}{(1 + \Gamma(\theta_t))^2} \left[\frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi} \right] \quad (G14)$$

Since

$$\Gamma'(\theta_t) > 0$$

it follows that

$$\frac{\partial S_t^*}{\partial \theta_t} > 0 \quad (\text{G15})$$

Similarly, differentiating equation (G11) gives

$$\frac{\partial D_t^{SCBDC}}{\partial \theta_t} = -\frac{\Gamma'(\theta_t)}{(1 + \Gamma(\theta_t))^2} \left[\frac{(\varphi - 1)(1 - \lambda)\bar{B}}{\varphi} \right] \quad (\text{G16})$$

This implies:

$$\frac{\partial D_t^{SCBDC}}{\partial \theta_t} > 0$$

Finally, because the volume of bank credit is solely determined by the fraction of remaining traditional banks,

$$B_t^{SCBDC} = (1 - \lambda)\bar{B}$$

bank lending does not depend on the liquidity shock θ_t . Therefore,

$$\frac{\partial B_t^{SCBDC}}{\partial \theta_t} = 0$$

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