

7505159-emotional-regret-expected-utility-and-a-formal-resolution-of-the-allais-paradox-

SUMMARY: This paper introduces Emotional Regret Expected Utility (EREU) within the framework of Relative Rationality Theory to explain the Allais paradox, a challenge to Expected Utility Theory. The Allais paradox involves individuals preferring a certain gain in the first experiment but choosing a risky lottery in the second, despite the Independence axiom requiring consistent preference rankings. The paper proposes that lottery outcomes are not treated as isolated monetary payoffs but as emotionally adjusted consequences, evaluated through Decision-EAW. Emotional weights are attached to these comparisons, capturing anticipated regret, relief, rejoicing, certainty protection, and life goals. Unlike Rank-Dependent Utility, EREU preserves linear probabilities and uses the alternative lottery as the counterfactual reference for emotionally weighted comparison. The paper suggests that the preference reversal can be explained by the strength of emotionally weighted certainty-zero comparisons relative to high-payoff comparisons.

METHOD: The study employs Relative Rationality Theory, which treats lottery outcomes as emotionally adjusted consequences (Decision-EAW) rather than isolated monetary payoffs. The model maintains linear probabilities and uses the alternative lottery as the counterfactual reference for emotionally weighted comparison. The analysis is based on the Allais paradox, where individuals prefer a certain gain in the first experiment but choose a risky lottery in the second, despite the Independence axiom requiring consistent preference rankings.

KEY FINDINGS:

- Lottery outcomes are evaluated as emotionally adjusted consequences.
- Emotional weights are attached to directional counterfactual comparisons.
- The preference reversal can be explained by the strength of emotionally weighted certainty-zero comparisons relative to high-payoff comparisons.
- EREU preserves linear probabilities and uses the alternative lottery as the counterfactual reference for emotionally weighted comparison.

IMPLICATION FOR TRADING: The implications of EREU for trading suggest that traders should consider the emotional impact of outcomes, rather than just the monetary value. This approach can help in understanding and predicting decision-making patterns, particularly in situations where certainty and risk are involved. By incorporating emotional weights into decision-making, traders can better account for the psychological factors that influence choices, leading to more accurate predictions and strategies. Additionally, understanding the role of emotional adjustment can aid in designing more effective risk management and hedging strategies.

Emotional Regret Expected Utility and a Formal Resolution of the Allais Paradox: A Relative Rationality Theory Approach

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Abstract

The Allais paradox remains one of the most important challenges to Expected Utility Theory, because many decision-makers prefer a certain gain in the first Allais experiment but choose the higher-payoff risky lottery when certainty is removed in the second. Under the standard monetary-consequence representation of the Allais problem, Expected Utility Theory has difficulty reconciling this pattern because identical monetary outcomes are represented as identical consequences across the two contexts, and the Independence axiom implies that the common consequences in the two Allais choices should cancel without reversing the preference ranking. This paper introduces Emotional Regret Expected Utility (EREU), within the framework of Relative Rationality Theory, as an alternative explanation of the Allais paradox. Lottery outcomes are not treated as isolated monetary payoffs; they are evaluated as emotionally adjusted consequences, represented through Decision-EAW, and compared with what the alternative lottery could have produced. Emotional weights are attached to these directional counterfactual comparisons, capturing anticipated regret, relief, rejoicing, certainty protection, and life goals. Unlike Rank-Dependent Utility, introduced by Quiggin (1982), which modifies decision weights according to the rank of outcomes within a lottery, EREU preserves linear probabilities and uses the alternative lottery as the counterfactual reference for emotionally weighted comparison. It differs from the canonical state-contingent representation of Regret Theory in its explicit decomposition of counterfactual evaluation: EREU separates the directional, comparison-specific emotional-intensity parameter w_{ij} from probability and from the utility difference, while evaluating that utility difference over Decision-EAW positions. Cross-outcome comparison itself is not claimed as novel, since Loomes and Sugden (1982) also considered statistically independent prospects. The EREU model shows that the preference $A_1 > B_1$ for the sure gain in the first Allais experiment can arise when the emotionally weighted certainty-zero comparison is sufficiently strong relative to the high-payoff comparison. Likewise in the second experiment, $B_2 > A_2$ can arise when, after certainty is removed, the emotionally weighted comparisons involving the €5 million outcome outweigh the emotionally weighted comparisons involving the €1 million outcome. The apparent reversal is therefore interpreted within EREU as a change in the dominant emotionally weighted counterfactual comparison rather than as a necessary consequence of nonlinear decision weighting. EREU thus provides a formal and psychologically motivated account of the Allais paradox while retaining linear probability and evaluating utility over Decision-EAW, the emotionally adjusted wealth position

generated by combining the decision-maker's pre-decision Emotional Adjusted Wealth (EAW) with the outcome-specific Emotional Adjusted Value (EAV).

Keywords: Allais paradox; Emotional Regret Expected Utility; Relative Rationality Theory; Expected Utility Theory; decision theory; regret; probability weighting; counterfactual comparison; Emotional Adjusted Wealth; uncertainty.

1. Introduction

The Allais paradox remains one of the most influential challenges to Expected Utility Theory (Allais, 1953). Standard Expected Utility Theory, grounded in the axiomatic tradition of von Neumann and Morgenstern (1944) and Savage (1954), has difficulty reconciling this pattern because the independence axiom requires a consistent ranking across the two comparisons. In the classical Allais choices, many individuals prefer a certain gain of €1 million to a risky lottery that offers a higher possible payoff. Yet, when certainty is removed in a second comparison, many of the same individuals prefer the lottery with the larger prize, even though the independence axiom of Expected Utility Theory requires a consistent ranking across the two problems. The paradox therefore reveals a central difficulty for standard decision theory: individuals appear to value certainty strongly in one context while accepting greater risk in another.

Traditional explanations of the Allais paradox have often relied on probability weighting (Kahneman & Tversky, 1979; Quiggin, 1982; Tversky & Kahneman, 1992). Prospect Theory and Rank-Dependent Utility explain the observed pattern by allowing probabilities to enter the decision function non-linearly. Small probabilities may receive disproportionate weight, while high but uncertain probabilities may be discounted relative to certainty. These models have been highly influential because they reproduce the empirical preference reversal. However, they leave open a further explanatory question: what psychological mechanism makes a given probability structure exert different decision influence across contexts.

This paper proposes Emotional Regret Expected Utility (EREU) as an alternative explanation within the framework of Relative Rationality Theory (Di Toro, 2026). The central claim is that the Allais reversal does not require probability distortion. Instead, it can be explained by recognising that lottery outcomes are not evaluated as isolated monetary payoffs. They are evaluated as emotionally adjusted consequences, and their meaning depends on what the alternative lottery could have produced.

In the present Allais application, the stated probabilities remain objective and enter the model without nonlinear transformation. The model does not transform the stated

probability of zero, the probability of a moderate gain, or the probability of a large payoff through a nonlinear probability-weighting function; instead, these probability terms remain analytically separate from the comparison-specific emotional-intensity parameter w_{ij} . What changes is the valuation layer. Each possible outcome generates a Decision-EAW consequence, and each such consequence is compared with the consequences that could have occurred under the alternative lottery. Emotional weights are then attached to these directional comparisons, representing the intensity of anticipated regret, relief, rejoicing, certainty protection, and the life-goal or future-opportunity significance associated with the specific counterfactual comparison.

This approach changes the interpretation of the Allais paradox. The zero payoff in the first Allais experiment is not emotionally equivalent to the zero payoff in the second experiment. In the first case, receiving zero under the risky lottery means receiving nothing after rejecting a guaranteed €1 million. In the second case, receiving zero occurs in a setting where both lotteries already involve a high probability of zero. The monetary outcome is the same, but the counterfactual meaning is different. EREU captures this difference directly through emotionally weighted comparisons.

The contribution of the paper is therefore twofold. First, it provides a formal explanation of the Allais pattern without abandoning linear probabilities. Second, it integrates regret, relief, rejoicing, and psychological meaning into the evaluation of uncertain prospects through Decision-EAW and emotional comparison weights. The preference for the sure gain in the first experiment and the preference for the higher-payoff risky lottery in the second are shown to arise from the same underlying architecture.

The paper proceeds as follows. Section 2 presents the Allais paradox and Emotional Regret Expected Utility. Section 3 compares EREU with Prospect Theory, Rank-Dependent Utility, and classical regret theory. Sections 4 and 5 apply EREU to the first and second Allais experiments. Section 6 explains why the two choices become coherent under EREU. Section 7 discusses the theoretical implications, and Section 8 concludes.

2. The Allais Paradox and Emotional Regret Expected Utility

The Allais paradox remains one of the most important challenges to standard Expected Utility Theory (Allais, 1953). In the classical Allais structure, individuals commonly prefer a certain gain of €1 million to a risky lottery that offers a high probability of €1 million, a smaller probability of €5 million, and a small probability of receiving nothing. Yet, in a second choice problem in which certainty is removed, many of the same individuals prefer the lottery with the higher prize, even though the independence axiom of Expected

Utility Theory requires a consistent ranking across the two comparisons. The paradox therefore arises because the same decision-maker appears to value certainty strongly in one context but favours risk in another.

Emotional Regret Expected Utility (EREU) explains this pattern by incorporating anticipated regret, relief, and rejoicing into the RRT utility-comparison architecture used for the ex ante evaluation of lotteries. The central claim is not that probabilities are distorted, but that outcomes are emotionally interpreted through comparison with what the alternative lottery could have produced. A lottery is therefore not evaluated in isolation. Each possible outcome of the chosen lottery is compared with each possible outcome of the unchosen lottery, and the emotional force of that comparison enters the decision calculus.

Let lottery A contain outcomes A_i with probabilities $P_A(i)$, and let lottery B contain outcomes B_j with probabilities $P_B(j)$. The EREU comparison of A against B may be written as:

$$\text{EREU}(A|B) = \sum_i \sum_j P_A(i)P_B(j) \cdot w_{ij} \cdot [U(\text{Decision-EAW}_{A_i}) - U(\text{Decision-EAW}_{B_j})]$$

where $\text{Decision-EAW}_{A_i}$ is the Emotional Adjusted Wealth position generated by outcome A_i , $\text{Decision-EAW}_{B_j}$ is the Emotional Adjusted Wealth position generated by outcome B_j , $U(\cdot)$ is the utility function defined over Emotional Adjusted Wealth, and $w_{ij} \geq 0$ is the directional, comparison-specific emotional-intensity parameter attached to the ordered comparison between A_i and B_j . Under the statistically independent-lottery convention adopted here, the product term $P_A(i)P_B(j)$ represents the probability attached to the ex ante pairwise comparison between a possible outcome $A(i)$ and a possible counterfactual outcome $B(j)$. Both lotteries are evaluated before the choice is made; the product term does not imply that, in reality, the decision-maker receives an outcome from lottery (A) and an outcome from lottery (B) at the same time. The comparison is entirely ex ante and counterfactual: the outcome of the alternative lottery enters only as a psychologically relevant alternative against which the possible outcome of the reference lottery is compared. It is analytically distinct from probability and scales the emotional intensity of the comparison, while the sign of the utility difference determines the direction of the counterfactual effect (Di Toro, 2026).

The EREU comparison does not require the Allais lotteries to be classified primarily by the decision-maker's initial wealth zone. The relevant objects are the emotionally adjusted state consequences generated by each lottery outcome. Each outcome first produces a Decision-EAW position through its Emotional Adjusted Value, and utility is then applied to that emotionally adjusted consequence. The central mechanism remains the emotionally weighted comparison between what the chosen lottery delivers and what the alternative lottery could have delivered.

Because EREU is directional, both directional evaluations are calculated. Lottery A is preferred to lottery B when $EREU(A | B) > EREU(B | A)$; lottery B is preferred when $EREU(B | A) > EREU(A | B)$; and the decision-maker is indifferent when $EREU(A | B) = EREU(B | A)$.

In the purely monetary special case, Decision-EAW $_A$ may be approximated by EAW + A_i , where EAW is the pre decision Emotional Adjusted Wealth position. In the standard monetary representation of the Allais problem, the relevant consequences are expressed as monetary payoffs. In the full Relative Rationality Theory framework, however, the relevant consequence is not limited to the monetary payoff itself. Each possible outcome first generates an outcome-specific Emotional Adjusted Value, which modifies the decision-maker's pre-decision Emotional Adjusted Wealth and produces the corresponding Decision-EAW position. The object entering utility is therefore not the raw monetary payoff alone, but the emotionally adjusted wealth consequence generated by that outcome.

The emotional weight w_{ij} does not change the probability of an outcome. The probability term remains $(P_A(i)P_B(j))$. Instead, w_{ij} measures the emotional intensity of the counterfactual comparison. If choosing (A) and receiving $A(i)$ would be experienced as painful because $B(j)$ would have been better, the comparison carries regret. If choosing (A) and receiving $A(i)$ would be experienced positively because $B(j)$ would have been worse, the comparison carries relief or rejoicing. The sign of $U(\text{Decision-EAW}_A) - U(\text{Decision-EAW}_B)$ determines whether the comparison favours (A) or (B) , while w_{ij} determines how emotionally powerful that comparison is.

RRT uses objective or explicitly stated probabilities when they are available and permits subjective probabilities when such probabilities are unavailable. In the latter case, $(P_A(i))$ and $(P_B(j))$ may represent the decision-maker's subjective beliefs about the likelihood of the relevant future outcomes. This probabilistic component must, however, remain conceptually distinct from the emotional-intensity parameter w_{ij} .

The emotional-intensity parameter does not represent a further transformation of probability. Rather, it captures the intensity of the ex ante counterfactual emotional comparison between a possible future outcome of one option and a possible outcome under the alternative. It is therefore relational rather than attached exclusively to one lottery or to an outcome considered in isolation. In EREU, w_{ij} is associated with the comparison between the utility of (Decision-EAW_A) and the utility of (Decision-EAW_B) ; its magnitude may vary according to the emotional significance of that particular ordered counterfactual comparison.

The emotional-intensity parameter is specific to the ordered outcome comparison within the particular pair of lotteries being evaluated. Consequently, the emotional intensity attached to the same ordered monetary outcomes may differ across different lottery

comparisons. The monetary difference between (0) and (1) is identical across the two Allais experiments, but the emotional intensity of that comparison may differ because the counterfactual probability structure is different: in Experiment 1, (1) is a certain alternative, whereas in Experiment 2 it is only a possible outcome.

Accordingly, RRT separates three distinct elements: objective probability when it is known or explicitly stated, subjective probability when such probability is unavailable, and the comparison-specific emotional-intensity parameter w_{ij} , which captures the ex ante emotional intensity generated by the counterfactual relationship between a possible outcome of one lottery and a possible outcome of the alternative lottery.

This structure is especially useful in the Allais paradox because the emotional meaning of zero is not constant across the two problems. In the standard monetary-consequence representation of the Allais problem, a zero monetary payoff is assigned the same utility wherever it appears. In EREU, by contrast, zero is interpreted relationally. Receiving zero after rejecting a guaranteed €1 million is not psychologically equivalent to receiving zero in a comparison where both lotteries already involve a high probability of zero. In the first case, zero is experienced as the painful loss of a secure opportunity. In the second case, zero is part of the common risk structure of both alternatives. The monetary amount is the same, but its emotional meaning is different.

3. Prospect Theory, Regret, and an EREU Resolution of the Allais Paradox

The Allais paradox is one of the most influential challenges to Expected Utility Theory. In the classical Allais choices, many individuals prefer a sure gain of €1 million to a risky lottery that offers a higher possible payoff. Yet, when certainty is removed in the second comparison, many of the same individuals prefer the lottery with the larger prize, even though the independence axiom of Expected Utility Theory requires a consistent ranking across the two problems. The paradox therefore reveals that decision-makers are highly sensitive to the presence or absence of certainty.

Prospect Theory explains this pattern through probability weighting (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). In that framework, individuals do not treat probabilities linearly. Small probabilities may receive disproportionately high decision weight, while high-probability outcomes may receive less weight than their objective probability would suggest. This allows Prospect Theory to reproduce the Allais pattern: in the first experiment, the certainty effect strengthens the appeal of the sure gain; in the second experiment, once certainty disappears, the small probability of the larger prize becomes more attractive.

From the standpoint of Relative Rationality Theory, this is an important descriptive insight, but it leaves open a deeper explanatory question. Why does a given probability become psychologically powerful in one context and less powerful in another? A small probability of receiving zero after rejecting a guaranteed gain is not emotionally equivalent to a small probability of receiving zero when both lotteries already involve a high probability of zero. Similarly, a small probability of a large prize may be experienced differently depending on whether it represents ordinary upside, the attainment of important life goals, the opening of significant future opportunities, or movement toward or beyond the Investment–Psychological threshold (W_2).

EREU shifts the explanation from probability distortion to emotionally weighted counterfactual comparison. Probabilities remain linear and objective. The terms $P_A(i)$ and $P_B(j)$ enter the model unchanged. What changes is the emotional intensity attached to the comparison between what the chosen lottery delivers and what the alternative lottery could have delivered. This emotional intensity is represented by the weight w_{ij} .

The difference between EREU and Rank-Dependent Utility is therefore not only that EREU keeps probabilities linear. Rank-Dependent Utility changes the decision weights attached to outcomes according to their rank within the same lottery, whereas EREU uses the alternative lottery as the counterfactual reference (Quiggin, 1982; Yaari, 1987). In EREU, each outcome of one lottery is compared with each possible outcome of the alternative lottery, and the emotional weight is attached to that directional comparison rather than to the rank of the outcome within a single lottery.

Classical regret theory, especially the approach associated with Loomes and Sugden, already recognises that outcomes may be evaluated relative to what could have occurred under an alternative choice (Bell, 1982; Loomes & Sugden, 1982). The canonical Loomes–Sugden formulation is commonly expressed in state-contingent form, but Loomes and Sugden (1982) also considered statistically independent prospects, for which cross-outcome comparisons arise through product probabilities. Later regret formulations broadened the informational basis of counterfactual evaluation. Bikhchandani and Segal (2014), for example, allow the regret or rejoicing associated with a particular outcome to depend on the entire alternative-lottery distribution. EREU adopts a different decomposition by retaining explicit pair-by-pair utility comparisons and assigning each ordered comparison its own directional emotional-intensity parameter w_{ij} . EREU's distinctive structure lies instead in the explicit separation of the directional, comparison-specific emotional-intensity parameter w_{ij} from probability and from the utility difference, while the outcomes entering that utility comparison are represented not as nominal monetary payoffs alone, but as Decision-EAW positions generated by the interaction between the decision-maker's pre-decision Emotional Adjusted Wealth (EAW) and the outcome-specific EAVs or, where RRT mapping is applied,

by the mapped EAVs of the emotional-economic consequences represented by those payoffs.

In EREU, each outcome of lottery A is compared with each outcome of lottery B. Each comparison is weighted by the product-probability term $P_A(i)P_B(j)$ and scaled by the directional, comparison-specific emotional-intensity parameter w_{ij} . The general emotional comparison term is:

$$w_{ij}[U(\text{Decision-EAW_}A_i) - U(\text{Decision-EAW_}B_j)]$$

This term is not a subjective probability. It measures how strongly the decision-maker reacts to receiving one emotionally adjusted consequence rather than the consequence that could have occurred under the alternative lottery. Thus, the emotional weight attached to “zero instead of a guaranteed million” may be much larger than the emotional weight attached to “zero instead of a merely possible million”. The monetary payoff is the same, but the counterfactual meaning is different.

This distinction is central to the EREU solution. In the first Allais experiment, the zero payoff of the risky lottery B_1 is emotionally severe because it is compared with the sure €1 million in A_1 . Choosing B_1 and ending with zero generates the counterfactual thought: “I could have had a guaranteed million.” This comparison receives a high emotional weight and strongly reduces the EREU of the risky lottery.

In the second experiment, by contrast, both lotteries assign a high probability to zero. If the decision-maker chooses B_2 and receives zero, the alternative A_2 also had a high probability of producing zero. The zero outcome is therefore no longer evaluated against a rejected certainty. Its emotional penalty is weaker. At the same time, the comparison between €5 million and €1 million becomes more salient, because the higher payoff may carry symbolic significance, enable important life goals or future opportunities, or generate a life-transforming emotionally adjusted outcome.

The crucial consequence is that zero does not have a single context-free decision meaning. The zero outcome in B_1 is not emotionally equivalent to the zero outcome in B_2 . In the standard monetary-consequence representation of the Allais problem, both zero outcomes are assigned the same utility, $(U(0))$, and under that consequence representation, the joint preference pattern conflicts with the Independence axiom of Expected Utility Theory. EREU instead evaluates each zero outcome through its counterfactual context. Zero after rejecting a guaranteed gain and zero in a lottery where both options are risky are different emotionally adjusted consequences.

EREU therefore differs from probability-weighting theories and from the canonical state-contingent representation of Regret Theory in specific respects. Prospect Theory retains stated probabilities as inputs but transforms their influence on valuation through decision weights, while rank-dependent models derive decision weights from

transformations of cumulative probabilities. EREU, by contrast, uses the stated probability terms directly in the probability component of the model and represents the additional psychological and emotional significance of the counterfactual comparison separately through the comparison-specific emotional-intensity parameter w_{ij} . Relative to canonical Regret Theory, its distinctive RRT structure lies not in originating cross-outcome comparison, but in explicitly separating directional emotional intensity w_{ij} from probability and from the utility difference and in evaluating utility differences between Decision-EAW positions.

The Allais paradox is therefore not necessarily a violation of rationality. It becomes coherent once the object of evaluation is changed. Decision-makers do not evaluate monetary outcomes alone. They evaluate emotionally adjusted consequences relative to what could have happened under the alternative lottery. The preference for A_1 in the first experiment reflects the emotional protection of certainty. The preference for (B_2) in the second experiment reflects the greater emotionally adjusted significance of the larger payoff once certainty disappears, particularly when that outcome enables important life goals or future opportunities, or moves the decision-maker toward or beyond the Investment–Psychological threshold (W_2). EREU explains both choices within a single architecture: linear probabilities, Decision-EAW consequences, and emotionally weighted counterfactual comparisons.

4. Emotional Regret Expected Utility and the First Allais Experiment

This section applies Emotional Regret Expected Utility (EREU) to the first Allais experiment. For simplicity, monetary payoffs are expressed in millions of euros. Thus, 0 denotes €0, 1 denotes €1 million, and 5 denotes €5 million. The two lotteries are:

A_1 : 1 with probability 1.00

B_1 : 1 with probability 0.89; 0 with probability 0.01; 5 with probability 0.10

The standard empirical pattern is:

$A_1 > B_1$

That is, many decision-makers prefer the certain €1 million to the risky lottery, even though B_1 contains a 10 per cent chance of receiving €5 million and only a 1 per cent chance of receiving nothing.

In the EREU framework, lotteries are not evaluated in isolation. Each outcome of one lottery is compared with each possible outcome of the alternative lottery. The general comparison between two lotteries A and B is:

$$\text{EREU}(A|B) = \sum_i \sum_j P_{A(i)} P_{B(j)} \cdot w_{ij} \cdot [U(\text{Decision-EAW}_{A_i}) - U(\text{Decision-EAW}_{B_j})]$$

where $P_{A(i)}$ is the probability of outcome i in lottery A , $P_{B(j)}$ is the probability of outcome j in lottery B , w_{ij} is the emotional weight attached to the directional comparison between outcome A_i and outcome B_j , and $U(\cdot)$ is the utility function defined over Decision-EAW.

In the present section, $U(0)$, $U(1)$, and $U(5)$ are used as shorthand for the utility of the Decision-EAW positions generated by the corresponding monetary outcomes. This shorthand does not imply that EREU is defined over monetary payoffs alone. In the full Relative Rationality Theory framework, each outcome first generates an emotionally adjusted state consequence, and utility is then evaluated over the resulting Decision-EAW.

The EREU of A_1 relative to B_1 is:

$$\text{EREU}(A_1|B_1) = 0.89w_{11}[U(1) - U(1)] + 0.01w_{10}[U(1) - U(0)] + 0.10w_{15}[U(1) - U(5)]$$

Since $U(1) - U(1) = 0$, the first term disappears. Therefore:

$$\text{EREU}(A_1|B_1) = 0.01w_{10}[U(1) - U(0)] + 0.10w_{15}[U(1) - U(5)]$$

Equivalently:

$$\text{EREU}(A_1|B_1) = 0.01w_{10}[U(1) - U(0)] - 0.10w_{15}[U(5) - U(1)]$$

The first remaining term captures the emotionally weighted comparison between receiving the certain €1 million under A_1 and the possibility of receiving €0 under B_1 . The second term captures the foregone-upside comparison between receiving €1 million under A_1 and the possibility of receiving €5 million under B_1 .

The EREU of B_1 relative to A_1 is:

$$\text{EREU}(B_1|A_1) = 0.89w_{11}[U(1) - U(1)] + 0.01w_{01}[U(0) - U(1)] + 0.10w_{51}[U(5) - U(1)]$$

Again, the first term disappears. Therefore:

$$\text{EREU}(B_1|A_1) = 0.01w_{01}[U(0) - U(1)] + 0.10w_{51}[U(5) - U(1)]$$

Equivalently:

$$\text{EREU}(B_1|A_1) = -0.01w_{01}[U(1) - U(0)] + 0.10w_{51}[U(5) - U(1)]$$

The first remaining term captures the emotionally weighted regret of receiving €0 under B_1 when A_1 would have guaranteed €1 million. The second term captures the emotionally weighted advantage of receiving €5 million under B_1 when A_1 would have produced only €1 million.

The observed preference $A_1 > B_1$ is represented in EREU as:

$$\text{EREU}(A_1|B_1) > \text{EREU}(B_1|A_1)$$

Substituting the two expressions gives:

$$0.01w_{10}[U(1) - U(0)] - 0.10w_{15}[U(5) - U(1)] > -0.01w_{01}[U(1) - U(0)] + 0.10w_{51}[U(5) - U(1)]$$

Rearranging terms yields the EREU condition for the first Allais preference:

$$0.01(w_{10} + w_{01})[U(1) - U(0)] > 0.10(w_{15} + w_{51})[U(5) - U(1)]$$

This inequality is the formal EREU condition under which the certain lottery A_1 is preferred to the risky lottery B_1 .

The left-hand side contains the two directional certainty–zero comparisons. The term w_{10} measures the emotional weight attached to receiving €1 million rather than €0. It captures the relief and security associated with having avoided the zero outcome. The term w_{01} measures the emotional weight attached to receiving €0 rather than the guaranteed €1 million. It captures the anticipated regret of ending with nothing after rejecting a certain gain. Both terms are attached to the same utility gap, $U(1) - U(0)$, but they need not have the same psychological intensity.

The right-hand side contains the two directional one–five million comparisons. The term w_{15} measures the emotional weight attached to receiving €1 million rather than €5 million. It captures the regret of missing the larger prize. The term w_{51} measures the emotional weight attached to receiving €5 million rather than €1 million. It captures the satisfaction, pride, or rejoicing associated with outperforming the safe option. Again, the two weights need not be equal.

The model therefore imposes no symmetry condition such as $w_{10} = w_{01}$ or $w_{15} = w_{51}$. This is important because regret, relief, and rejoicing are directional emotions. The emotional cost of receiving €0 when a certain €1 million was available may be much stronger than the relief of having avoided €0. Similarly, the regret of missing €5 million may differ from the rejoicing associated with obtaining it.

The inequality can be read as follows. The certain lottery A_1 is preferred when the total emotionally weighted force of the zero–one million contrast is greater than the total emotionally weighted force of the one–five million contrast. In other words, the emotional importance of securing €1 million and avoiding the possibility of ending with nothing outweighs the emotional attraction of the additional €4 million available in the risky lottery.

This result explains why the small 1 per cent probability of receiving €0 can dominate the 10 per cent probability of receiving €5 million. EREU does not require the decision-maker to overweight the 1 per cent probability. The probability remains 0.01. What matters is that this state carries a very large emotional comparison: zero is not merely zero, but zero after rejecting a guaranteed €1 million. The monetary outcome is small in probability but large in emotional significance.

The first Allais preference can therefore be explained without distorting probabilities. Probabilities remain linear, and the behavioural asymmetry is carried by utility differences over Decision-EAW and by the emotional weights attached to counterfactual comparisons. The preference for A_1 over B_1 is not a failure to understand expected value. It reflects the fact that the decision-maker evaluates the risky lottery through the anticipated emotional consequence of a possible failure state.

The EREU solution to Experiment 1 is therefore:

$$A_1 \succ B_1 \text{ if } 0.01(w_{10} + w_{01})[U(1) - U(0)] > 0.10(w_{15} + w_{51})[U(5) - U(1)]$$

This condition states that the sure €1 million is selected when the emotionally weighted utility of security exceeds the emotionally weighted utility of the additional upside. The Allais choice becomes coherent because the decision-maker is not comparing monetary outcomes alone. The decision-maker is comparing emotionally interpreted state consequences relative to what the alternative lottery could have produced.

4.1 Behavioural Interpretation: Why A_1 Is Preferred to B_1

The first Allais preference can now be interpreted behaviourally. In the Emotional Regret Expected Utility framework, the preference for the certain €1 million in A_1 over the risky lottery B_1 does not require any distortion of probabilities. The probabilities 0.89, 0.01, and 0.10 remain unchanged. What changes is the emotional significance of the counterfactual comparisons generated by the two lotteries.

From the previous derivation, the condition for $A_1 \succ B_1$ is:

$$0.01(w_{10} + w_{01})[U(1) - U(0)] > 0.10(w_{15} + w_{51})[U(5) - U(1)]$$

This inequality states that the sure lottery A_1 is preferred when the total emotionally weighted force of the zero–one million contrast is greater than the total emotionally weighted force of the one–five million contrast.

The left-hand side contains the certainty–zero comparison. This is the emotionally decisive contrast in the first Allais experiment. If the decision-maker chooses B_1 and receives €0, the outcome is not experienced merely as receiving nothing. It is experienced as receiving nothing after rejecting a guaranteed €1 million. The comparison therefore carries a strong anticipated regret: “I could have had a certain million, but I chose the risky lottery and ended with nothing”. In the EREU model, this emotional intensity is represented especially by w_{01} , the weight attached to receiving 0 when the alternative would have produced 1. The corresponding utility gap, $U(1) - U(0)$, is also large because the movement from nothing to €1 million represents a major improvement in Emotional Adjusted Wealth.

The right-hand side contains the one–five million comparison. This captures the emotional attraction of the higher payoff in B_1 . If the decision-maker chooses B_1 and receives €5 million, the outcome is experienced as superior to the certain €1 million in A_1 . This comparison is represented especially by w_{51} and by the utility gap $U(5) - U(1)$. However, the force of this upside comparison depends not only on the monetary difference between €1 million and €5 million, but also on the emotional weight attached to the relevant counterfactual comparison. In the first Allais experiment, the emotional attraction of the larger payoff may be outweighed by the anticipated regret attached to the zero branch.

The preference for A_1 therefore arises when the emotional cost of possible failure dominates the emotional attraction of the larger prize. The small probability of zero remains only 0.01, but the emotional comparison attached to that state is extremely powerful. The state “0 instead of 1” is not equivalent to an isolated zero payoff. It is zero relative to a foregone certainty. Within EREU, the observed preference is represented when the anticipated emotional intensity of the certainty–zero comparison is sufficiently strong to outweigh the emotionally weighted attraction of the 10 per cent chance of the €5 million outcome.

Nevertheless, in the standard first Allais experiment, many decision-makers continue to prefer A_1 because the certainty–zero comparison remains psychologically dominant. The emotional weight attached to the thought of ending with nothing after refusing a guaranteed €1 million is sufficiently high to offset the attraction of the larger prize. In this case, the decision-maker does not reject B_1 because the €5 million outcome is ignored. The decision-maker rejects B_1 because the possible regret associated with the zero branch is too emotionally costly.

The EREU explanation can therefore be summarised as follows. The first Allais preference is generated by a trade-off between security and the potential life-goal significance of the larger payoff. The risky lottery offers a higher possible gain, but it also exposes the decision-maker to a uniquely painful counterfactual comparison: receiving zero when a certain million was available. When the emotionally weighted certainty–zero contrast dominates the emotionally weighted one–five million contrast, A_1 is preferred to B_1 .

Thus, $A_1 > B_1$ is not treated as irrational. Within Relative Rationality Theory, the choice is relatively rational in relation to the decision-maker’s emotionally adjusted evaluation of the alternatives, while EREU provides the formal structure through which that choice becomes internally coherent. The model preserves linear probabilities and explains the certainty effect primarily through emotionally weighted counterfactual comparison. The Allais preference pattern can be reconciled within EREU because the model evaluates not only monetary payoffs, but emotionally interpreted consequences relative to what could have happened under the alternative lottery.

5. Emotional Regret Expected Utility and the Second Allais Experiment

This section applies Emotional Regret Expected Utility (EREU) to the second Allais experiment. As before, monetary payoffs are expressed in millions of euros. Thus, 0 denotes €0, 1 denotes €1 million, and 5 denotes €5 million. The two lotteries are:

A_2 : 1 with probability 0.11; 0 with probability 0.89

B_2 : 5 with probability 0.10; 0 with probability 0.90

The standard empirical pattern is:

$$B_2 > A_2$$

That is, many decision-makers prefer the lottery with a 10 per cent chance of €5 million to the lottery with an 11 per cent chance of €1 million, even though B_2 has a slightly lower probability of yielding any positive payoff.

The key difference from the first Allais experiment is that certainty has disappeared. In Experiment 1, the decision-maker compared a guaranteed €1 million with a risky alternative that included a small probability of receiving nothing. In Experiment 2, both lotteries are risky, and both assign a high probability to receiving zero. The emotional structure of the choice is therefore no longer dominated by the fear of rejecting a certain gain. Instead, the comparison shifts towards the contrast between a moderate gain and a potentially transformative gain.

The general EREU comparison between two lotteries A and B is:

$$\text{EREU}(A|B) = \sum_i \sum_j P_A(i)P_B(j) \cdot w_{ij} \cdot [U(\text{Decision-EAW_}A_i) - U(\text{Decision-EAW_}B_j)]$$

where $P_A(i)$ and $P_B(j)$ are the probabilities of the relevant outcomes, represented by objective or explicitly stated probabilities when available and by subjective probabilities when objective or stated probabilities are unavailable, w_{ij} is the emotional weight attached to the directional comparison between outcome A_i and outcome B_j , and $U(\cdot)$ is the utility function defined over Decision-EAW.

In the present section, $U(0)$, $U(1)$, and $U(5)$ are used as shorthand for the utility of the Decision-EAW positions generated by the corresponding monetary outcomes. This shorthand is used only for simplicity. In the full Relative Rationality Theory framework, each monetary outcome first generates an emotionally adjusted state consequence, and utility is then evaluated over the resulting Decision-EAW.

The EREU of A_2 relative to B_2 is:

$$\text{EREU}(A_2|B_2) = 0.89 \cdot 0.90w_{00}[U(0) - U(0)] + 0.89 \cdot 0.10w_{05}[U(0) - U(5)] + 0.11 \cdot 0.90w_{10}[U(1) - U(0)] + 0.11 \cdot 0.10w_{15}[U(1) - U(5)]$$

Since $U(0) - U(0) = 0$, the first term disappears. Therefore:

$$\text{EREU}(A_2|B_2) = 0.089w_{05}[U(0) - U(5)] + 0.099w_{10}[U(1) - U(0)] + 0.011w_{15}[U(1) - U(5)]$$

Equivalently:

$$\text{EREU}(A_2|B_2) = -0.089w_{05}[U(5) - U(0)] + 0.099w_{10}[U(1) - U(0)] - 0.011w_{15}[U(5) - U(1)]$$

The first term captures the regret of receiving €0 under A_2 when B_2 would have produced €5 million. The second term captures the relief or satisfaction of receiving €1 million under A_2 when B_2 would have produced €0. The third term captures the regret of receiving €1 million under A_2 when B_2 would have produced €5 million.

The EREU of B_2 relative to A_2 is:

$$\text{EREU}(B_2|A_2) = 0.90 \cdot 0.89w_{00}[U(0) - U(0)] + 0.90 \cdot 0.11w_{01}[U(0) - U(1)] + 0.10 \cdot 0.89w_{50}[U(5) - U(0)] + 0.10 \cdot 0.11w_{51}[U(5) - U(1)]$$

Again, the first term disappears. Therefore:

$$\text{EREU}(B_2|A_2) = 0.099w_{01}[U(0) - U(1)] + 0.089w_{50}[U(5) - U(0)] + 0.011w_{51}[U(5) - U(1)]$$

Equivalently:

$$\text{EREU}(B_2|A_2) = -0.099w_{01}[U(1) - U(0)] + 0.089w_{50}[U(5) - U(0)] + 0.011w_{51}[U(5) - U(1)]$$

The first term captures the regret of receiving €0 under B_2 when A_2 would have produced €1 million. The second term captures the rejoicing or satisfaction of receiving €5 million under B_2 when A_2 would have produced €0. The third term captures the satisfaction of receiving €5 million under B_2 when A_2 would have produced only €1 million. No symmetry is imposed among these weights.

The observed preference $B_2 > A_2$ is represented in EREU as:

$$\text{EREU}(B_2|A_2) > \text{EREU}(A_2|B_2)$$

Substituting the two expressions gives:

$$-0.099w_{01}[U(1) - U(0)] + 0.089w_{50}[U(5) - U(0)] + 0.011w_{51}[U(5) - U(1)] > -0.089w_{05}[U(5) - U(0)] + 0.099w_{10}[U(1) - U(0)] - 0.011w_{15}[U(5) - U(1)]$$

Rearranging terms yields the EREU condition for the second Allais preference:

$$0.089(w_{50} + w_{05})[U(5) - U(0)] + 0.011(w_{51} + w_{15})[U(5) - U(1)] > 0.099(w_{01} + w_{10})[U(1) - U(0)]$$

This inequality is the formal EREU condition under which B_2 is preferred to A_2 .

The behavioural meaning of this condition is straightforward. In Experiment 2, certainty has disappeared. The decision-maker is no longer comparing a risky lottery with a guaranteed €1 million. Instead, both options involve a high probability of receiving nothing. This changes the emotional structure of the comparison. The thought “I rejected

a sure million and ended with zero”, which dominated Experiment 1, is no longer available. The relevant comparison now shifts towards the possibility of obtaining a much larger payoff.

In EREU terms, the emotionally decisive contrasts are no longer centred on certainty versus zero, but on the high-payoff comparisons involving €5 million. The terms involving $U(5) - U(0)$ and $U(5) - U(1)$ capture the emotional and utility force of receiving €5 million rather than nothing or rather than €1 million. These terms may become especially powerful when the €5 million outcome is experienced as transformational, because it may represent not only additional consumption but also strategic freedom, identity expansion, symbolic achievement, entrepreneurial possibility, or long-term life change.

The regret terms associated with receiving zero when €1 million was possible remain relevant, but they no longer refer to a rejected certainty. In Experiment 2, €1 million was only an 11 per cent possibility. Therefore, within EREU, the emotional weight attached to “zero instead of one million” may be weaker than in Experiment 1, where zero meant failure after rejecting a guaranteed gain. Once the certainty anchor disappears, the emotional system can give greater importance to the upper-tail outcome.

Thus, B_2 is preferred when the emotionally weighted pull of €5 million dominates the emotionally weighted comfort of the more frequent €1 million possibility. The decision-maker accepts a slightly lower chance of receiving a positive payoff because the possible positive payoff is much more emotionally and psychologically significant. EREU therefore explains $B_2 > A_2$ without distorting probabilities: the probabilities remain 0.89, 0.90, 0.11, and 0.10, while the behavioural asymmetry is generated by utility differences over Decision-EAW and by the emotional weights attached to each counterfactual comparison.

The left-hand side contains the emotionally weighted contrasts involving the high payoff of €5 million. The first component, $0.089(w_{50} + w_{05})[U(5) - U(0)]$, captures the contrast between receiving €5 million and receiving nothing. This comparison may be especially powerful when the €5 million outcome is experienced as transformational. The second component, $0.011(w_{51} + w_{15})[U(5) - U(1)]$, captures the contrast between receiving €5 million and receiving €1 million. This comparison expresses the additional emotional and utility force of moving from moderate wealth to a much higher wealth position.

The right-hand side contains the emotionally weighted contrast between €1 million and €0. This is represented by $0.099(w_{01} + w_{10})[U(1) - U(0)]$. The term w_{01} captures the regret of receiving €0 under B_2 when A_2 would have produced €1 million. The term w_{10} captures the satisfaction of receiving €1 million under A_2 when B_2 would have produced €0. These emotions remain relevant, but they no longer refer to a rejected certainty. In Experiment 2, €1 million was never guaranteed. It was only an 11 per cent possibility.

This distinction is central. In Experiment 1, receiving zero under the risky lottery meant receiving nothing after rejecting a certain €1 million. In Experiment 2, receiving zero means receiving nothing in a situation where both lotteries already involved a high probability of nothing. The emotional meaning of zero is therefore weaker in Experiment 2 than in Experiment 1. The zero branch is no longer a unique failure relative to a secure benchmark; it is part of the common risk structure of both options.

Within EREU, the observed preference B_2 arises when the combined emotionally weighted comparisons between €5 million and €0 and between €5 million and €1 million exceed the emotionally weighted comparison between €1 million and €0. The decision-maker accepts a slightly lower probability of receiving a positive outcome because the possible positive outcome is emotionally and psychologically much more powerful. In this case, the attraction of entering a qualitatively different wealth regime dominates the regret associated with missing the smaller €1 million gain.

The model does not impose symmetry among the weights. The regret of receiving €0 when €1 million was possible need not equal the satisfaction of receiving €1 million when €0 was possible. Similarly, the regret of missing €5 million need not equal the satisfaction of obtaining €5 million. The six directional weights w_{05} , w_{10} , w_{15} , w_{01} , w_{50} , and w_{51} remain distinct psychological parameters. This is important because regret, relief, and rejoicing are directional emotions, and their intensities may differ across individuals and contexts.

The EREU explanation of Experiment 2 can therefore be summarised as follows. In the absence of certainty, the comparison “zero instead of a guaranteed million” disappears. The emotional dominance of the certainty–zero contrast is replaced by the stronger emotionally adjusted significance of the €5 million payoff, particularly when that outcome enables important life goals or future opportunities, or moves the decision-maker toward or beyond the Investment–Psychological threshold (W_2). When the emotionally weighted utility of the €5 million contrasts dominates the emotionally weighted utility of the €1 million contrasts, B_2 is preferred to A_2 .

Thus, $B_2 > A_2$ is not treated as an irrational reversal. Within Relative Rationality Theory, the choice is relatively rational in relation to the decision-maker’s emotionally adjusted evaluation of the lotteries, while EREU provides the formal structure through which that choice becomes internally coherent. The model preserves linear probabilities: 0.89, 0.90, 0.11, and 0.10 enter the formula without distortion. What changes is the emotional weighting of the outcome contrasts in Decision-EAW space. In Experiment 1, the largest emotional force is attached to the possibility of ending with zero after rejecting certainty, which favours A_1 . In Experiment 2, once certainty is removed, the largest emotional force may shift towards the high-payoff contrasts, which favours B_2 .

The second Allais preference is therefore coherent within EREU:

$$B_2 \succ A_2 \text{ if } 0.089(w_{50} + w_{05})[U(5) - U(0)] + 0.011(w_{51} + w_{15})[U(5) - U(1)] > 0.099(w_{01} + w_{10})[U(1) - U(0)]$$

The Allais reversal is explained by a change in the dominant emotional comparison. The first experiment is governed by security and the fear of catastrophic regret relative to certainty. The second experiment is governed by psychological life change and the possibility of a transformational gain. The Allais preference pattern can be reconciled within EREU because the model evaluates lotteries not as isolated monetary distributions, but as emotionally weighted counterfactual comparisons over Decision-EAW.

6. An EREU Resolution of the Allais Paradox

Traditional Expected Utility Theory cannot easily reconcile the joint choice of A_1 in Experiment 1 and B_2 in Experiment 2. With a single utility function and linear expectations, the independence axiom requires common consequences to cancel out, preserving the ranking across the two Allais comparisons. Yet the standard Allais pattern is precisely the opposite: many decision-makers prefer the certain €1 million in the first experiment and the higher-payoff risky lottery in the second.

Emotional Regret Expected Utility provides a formal resolution of this pattern within a single decision architecture. Probabilities remain linear and objective. The model does not transform 0.01, 0.10, 0.11, 0.89, or 0.90 into nonlinear decision weights. Instead, EREU changes the object of comparison. Lotteries are not evaluated as isolated monetary distributions. Each outcome first generates an emotionally adjusted state consequence, represented as a Decision-EAW position, and each such consequence is compared with what the alternative lottery could have produced.

For each directional comparison, EREU assigns an emotional parameter w_{ij} to the utility difference between the two Decision-EAW consequences:

$$w_{ij}[U(\text{Decision-EAW}_i) - U(\text{Decision-EAW}_j)]$$

The parameter w_{ij} captures the emotional intensity of the comparison. It may represent anticipated regret, relief, rejoicing, certainty protection. The key point is that the same monetary payoff can have different emotional meaning depending on the alternative that was available. A zero payoff after rejecting a guaranteed €1 million is not emotionally equivalent to a zero payoff in a choice where both lotteries already involve a high probability of zero. Thus, probability and emotional intensity remain analytically distinct, even though the probability structure can affect the magnitude of w_{ij} .

In the first Allais experiment, the dominant comparison is the possibility of receiving €0 under B_1 when A_1 would have guaranteed €1 million. This is not merely a low monetary outcome. It is the counterfactual experience of ending with nothing after refusing

certainty. In EREU, this comparison receives a high emotional weight. The regret attached to “0 instead of 1” is strong enough to dominate the attraction of the higher prize in B_1 . Therefore, A_1 is preferred to B_1 .

In the second Allais experiment, the emotional structure changes. There is no certain €1 million to reject. Both lotteries involve a high probability of receiving zero. As a result, the zero outcome no longer carries the same counterfactual meaning as in Experiment 1. The thought “I rejected a guaranteed million and received nothing” is absent. The comparison now shifts toward the larger payoff in B_2 . The contrasts “5 instead of 0” and “5 instead of 1” may receive strong emotional weights because the €5 million outcome may carry strong life-goal significance, symbolic meaning, or life-changing implications. Therefore, B_2 can be preferred to A_2 .

The Allais reversal is therefore explained by a change in the dominant emotional comparison. In Experiment 1, the decisive comparison is security against catastrophic regret: receiving zero after rejecting certainty. In Experiment 2, the decisive comparison may shift toward the possibility of receiving €5 million when no certain alternative exists, particularly when that outcome carries strong life-goal significance, enables important future opportunities, or is capable of moving the decision-maker toward or beyond the Investment–Psychological threshold W_2 , where additional wealth can expand the set of attainable life goals, projects, opportunities, and future possibilities. The monetary outcomes are formally similar across the two experiments, but their emotionally adjusted meanings and resulting Decision-EAW implications are not the same.

This is why EREU does not require probability distortion. The probability of the zero branch in B_1 remains 0.01. The probability of the €5 million branch in B_2 remains 0.10. What changes is the emotional weight attached to the comparison generated by each outcome. Probability remains probability; emotional salience enters through w_{ij} .

Under the standard monetary-consequence representation of the Allais problem, identical monetary consequences are assigned the same utility wherever they occur, and the joint preference pattern conflicts with the Independence axiom. EREU instead represents those monetary outcomes as Decision-EAW consequences embedded in alternative-dependent counterfactual comparisons. The zero in B_1 and the zero in B_2 are not the same decision object, because they are compared with different alternatives. Once this distinction is recognised, the pair of choices $A_1 \succ B_1$ and $B_2 \succ A_2$ becomes internally coherent.

EREU therefore provides a formal resolution of the Allais preference pattern by preserving linear probability while replacing isolated payoff evaluation with emotionally weighted counterfactual comparison over Decision-EAW. The model explains the certainty effect in Experiment 1 and the preference for the higher-payoff risky lottery in Experiment 2

using the same mechanism: emotionally weighted regret, relief, and psychological meaning attached to counterfactual outcome comparisons.

7. Discussion

The analysis developed in this paper suggests that the Allais paradox does not necessarily reveal irrationality. Rather, it highlights a limitation of the standard monetary-consequence representation of the Allais problem. In the standard monetary-consequence representation of the Allais problem, lotteries are evaluated through monetary outcomes, their associated probabilities, and a utility function. Identical monetary outcomes are therefore assigned the same utility wherever they appear within that representation. From this perspective, the zero payoff in the first Allais experiment and the zero payoff in the second Allais experiment are formally the same object, $U(0)$. Once common consequences are cancelled through the independence axiom, the observed preference pattern $A_1 > B_1$ and $B_2 > A_2$ appears inconsistent.

Emotional Regret Expected Utility changes this interpretation. The central claim is that lottery outcomes are not evaluated as isolated monetary payoffs. They are evaluated as emotionally adjusted consequences, and their meaning depends on the counterfactual alternative against which they are compared. A zero payoff after rejecting a guaranteed €1 million is not psychologically equivalent to a zero payoff in a choice where both lotteries already involve a high probability of zero. The monetary amount is the same, but the emotionally adjusted decision consequence is different. This is the point at which the Allais paradox begins to dissolve.

The contribution of EREU is therefore not to abandon expectation, nor to replace probability with emotion. In the present Allais application, probabilities remain linear and objective. The terms 0.01, 0.10, 0.11, 0.89, and 0.90 enter the analysis exactly as they are. What changes is the valuation layer. Each lottery outcome first generates a Decision-EAW consequence, and each consequence is then compared with the consequences that could have occurred under the alternative lottery. The emotional weight w_{ij} captures the intensity of that directional comparison. Regret, relief, rejoicing, certainty protection, and psychological meaning enter the model through these emotional parameters rather than through a transformation of the stated probability terms.

This distinction is important because it separates EREU from nonlinear decision-weighting explanations of the Allais paradox. Prospect Theory and Rank-Dependent Utility explain the observed pattern by applying nonlinear transformations to stated probabilities to generate decision weights (Kahneman & Tversky, 1979; Quiggin, 1982; Tversky & Kahneman, 1992). These decision weights capture the nonlinear impact of stated probabilities on prospect valuation without decomposing that impact into explicit comparison-specific emotional components. EREU preserves the stated probability

terms and represents the additional comparison-specific emotional intensity separately through w_{ij} . The small probability of receiving zero in B_1 may become decisive because the counterfactual comparison “zero instead of a guaranteed million” carries exceptional emotional salience.

Classical regret theory recognises that outcomes are evaluated relative to what might have happened under an alternative choice (Bell, 1982; Loomes & Sugden, 1982). EREU’s distinctive contribution is not the introduction of cross-outcome comparison itself, but the RRT-specific decomposition in which probability, directional comparison-specific emotional intensity w_{ij} , and utility differences over Decision-EAW remain analytically distinct.

The Allais pattern is then explained by a shift in the dominant emotional comparison across the two experiments. In Experiment 1, the dominant comparison is security against catastrophic regret. Choosing B_1 and receiving zero is evaluated against the rejected certainty of A_1 . The emotional weight attached to “0 instead of 1” is therefore large, and the sure option A_1 becomes attractive even though B_1 contains a higher possible prize. In Experiment 2, certainty disappears. Both lotteries contain a high probability of zero, so the zero outcome no longer means failure after rejecting a guaranteed gain. The emotional system can shift towards the high-payoff comparisons involving €5 million. The result is that B_2 can be preferred to A_2 without any change in the probability structure.

This account also clarifies why the Independence axiom fails descriptively in the Allais setting. The Independence axiom implies that mixing the same third lottery with two alternatives should not reverse their ranking; in the standard monetary-consequence representation of the Allais problem, this yields cancellation of common consequences. EREU departs from this separability by allowing the decision value of an outcome to depend on the counterfactual alternative against which it is evaluated. A common monetary consequence is therefore not necessarily a common emotionally adjusted consequence. Its decision meaning may depend on the alternative that was available, the regret or relief associated with that comparison, the psychological meaning of the outcome, and the emotional intensity assigned to the relevant contrast. The descriptive failure of Independence in the Allais setting is therefore not arbitrary within EREU; it reflects the fact that decision-makers may experience outcomes in relation to the psychological and counterfactual structure in which they occur.

The broader implication is that rationality under uncertainty cannot be fully understood if outcomes are stripped of their emotional and counterfactual meaning. EREU preserves formal discipline: it uses explicit probabilities, utility differences, and comparison weights. At the same time, it recognises that real decisions may involve anticipated regret, relief, and rejoicing, whose intensity can vary across counterfactual comparisons. The framework therefore offers a middle path between purely monetary expected utility

and purely descriptive behavioural correction. It does not merely fit the Allais choices; it provides a psychological reason why the same monetary payoff can rationally carry different decision value in different contexts.

Different individuals may attach different emotional weights to certainty, regret, and the life-goal significance of possible outcomes. Some may be highly sensitive to the possibility of ending with zero after rejecting certainty; others may attach stronger emotional force to the possibility of a transformative high payoff. This variability allows the model to represent heterogeneous emotional decision profiles through comparison-specific emotional-intensity parameters, providing a distinct source of behavioural heterogeneity from differences in how probabilities influence decision valuation.

The purpose of the present paper is theoretical. It establishes the mathematical conditions under which the interaction of stated probabilities, utility differences over Decision-EAW positions generated through EAV, and comparison-specific emotional-intensity parameters w_{ij} allows EREU to reproduce the observed Allais preference pattern.

The theoretical contribution of EREU is therefore twofold. First, it provides an RRT-specific formal explanation of the Allais preference pattern that does not rely on nonlinear decision weighting of stated probabilities. Second, it integrates counterfactual regret, relief, and rejoicing into the RRT decision architecture by connecting lottery outcomes to Decision-EAW consequences and explicit comparison-specific emotional-intensity parameters within counterfactual utility comparisons. In this framework, the Allais preference pattern is no longer interpreted as a contradiction between rationality and observed behaviour. Instead, it suggests that the standard monetary-consequence representation of the Allais problem may omit emotionally and counterfactually relevant features of the outcomes that EREU represents explicitly through Decision-EAW and comparison-specific emotional intensity.

The discussion provides an EREU account of the Allais pattern by showing that people do not choose between abstract probability distributions alone; they choose between emotionally interpreted consequences, each evaluated in relation to what the alternative could have produced. Once this is recognised, the preference for A_1 in the first experiment and B_2 in the second becomes coherent within a single architecture of linear probability, Decision-EAW valuation, and emotionally weighted regret comparison.

8. Conclusion

This paper has proposed Emotional Regret Expected Utility (EREU) as an RRT-specific formal resolution of the Allais preference pattern. EREU retains utility-based evaluation and explicit probability terms while changing the object and structure of counterfactual comparison.

The central claim is that, under the standard monetary-consequence representation of the Allais problem, the Independence axiom does not permit the context-dependent valuation required to reproduce the observed preference reversal, whereas EREU represents outcomes as Decision-EAW consequences embedded in counterfactual comparison. A monetary outcome does not have the same decision meaning in every context. Receiving zero after rejecting a guaranteed €1 million is not equivalent to receiving zero in a choice where both lotteries already involve a high probability of zero.

EREU retains utility-based evaluation and explicit probability terms while changing both the object of evaluation and the structure through which outcomes are compared counterfactually. Instead, each outcome generates a Decision-EAW consequence, which is then compared with what the alternative lottery could have produced. Emotional weights w_{ij} are then attached to these directional comparisons, capturing the intensity of anticipated regret, relief, rejoicing, certainty protection, and the life-goal significance associated with the specific counterfactual comparison.

The Allais pattern becomes coherent once this relational structure is introduced. In the first experiment, the sure option A_1 is preferred because the risky lottery B_1 exposes the decision-maker to a small but emotionally powerful possibility of receiving zero after rejecting certainty. The comparison “0 instead of 1” carries exceptional regret weight. In the second experiment, certainty disappears. Both lotteries involve a high probability of zero, and the emotional dominance of the certainty–zero comparison weakens. The comparison shifts towards the larger payoff in B_2 , where “5 instead of 0” and “5 instead of 1” may carry strong rejoicing weights and greater life-goal or future-opportunity significance, particularly when the larger payoff moves the decision-maker toward or beyond the Investment–Psychological threshold (W_2). The same architecture therefore explains both $A_1 > B_1$ and $B_2 > A_2$.

The contribution of EREU is not merely that it reproduces the observed Allais choices. Its deeper contribution is explanatory. Probability-weighting models represent the behaviour through nonlinear decision weights derived from stated probabilities, whereas EREU preserves the stated probability terms and represents the additional comparison-specific emotional intensity separately through w_{ij} . EREU identifies an emotional mechanism that may generate this behaviour: some counterfactual comparisons may carry substantially greater emotional intensity than others. What appears behaviourally similar to nonlinear decision weighting may therefore arise, within EREU, from emotionally weighted counterfactual comparison over Decision-EAW. EREU does not transform the stated probability terms themselves. However, the comparison-specific emotional-intensity parameter w_{ij} may vary because the probability structure of the comparison can affect its emotional force. In particular, even a low-probability adverse outcome under the evaluated lottery may generate strong anticipated regret when it is compared with an alternative outcome that would have been certain. Thus, in the first

Allais experiment, the emotional intensity attached to receiving zero may be high despite its probability being only 1 per cent, because that outcome is evaluated against a foregone guaranteed payoff of €1 million. The emotional force therefore arises from the counterfactual comparison with certainty rather than from the probability magnitude of the zero outcome itself. Probability and emotional intensity remain analytically distinct, even though the certainty or likelihood of the counterfactual alternative may influence the emotional significance of the comparison.

This approach also clarifies why the Independence axiom fails descriptively in the Allais setting. The Independence axiom implies that mixing the same third lottery with two alternatives should not reverse their ranking; under the standard monetary-consequence representation of the Allais problem, this yields cancellation of common consequences. EREU shows why this assumption may be psychologically too narrow. A common monetary consequence is not necessarily a common emotionally adjusted consequence. Its decision meaning may depend on the alternative that was available and on the regret, relief, or broader psychological meaning associated with that comparison. EREU therefore departs from the Independence-based assumption that a common consequence has the same decision value across alternative contexts, by allowing the decision value of an outcome to depend both on its outcome-specific EAV, which contributes to the resulting Decision-EAW position, and on the counterfactual alternative against which that Decision-EAW position is evaluated.

The Allais preference pattern therefore need not be interpreted as evidence of simple irrationality. Instead, it suggests that the standard monetary-consequence representation of the Allais problem may omit emotionally and counterfactually relevant features of outcomes that EREU represents explicitly through Decision-EAW and comparison-specific emotional intensity. Within EREU, decision-makers are modelled as evaluating these enriched consequences in relation to the outcomes that could have occurred under the alternative lottery. Their evaluation therefore extends beyond monetary payoffs alone to emotionally interpreted consequences, including what they might regret, what they might be relieved to avoid, and the life-goal or future-opportunity significance associated with possible outcomes. By formally incorporating these elements through Decision-EAW and the comparison-specific emotional-intensity parameters w_{ij} , EREU provides an RRT-specific formal explanation of the Allais preference pattern without abandoning linear probability.

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