

7503322-money-illusion-in-the-u-s-stock-market-and-economic-growth-a-revaluation-in-gold

SUMMARY: This research revalues U.S. stock market performance and economic growth in ounces of gold, a monetary unit used for five millennia but largely abandoned over the past 50 years. Systematic differences in stock market performance, economic growth, and risk compensation are found when these quantities are revalued in gold. The authors find that when stock market performance and economic growth are revalued in gold, both appear substantially weaker over the past 100 years. The differences are evident in the duration and severity of economic and market downturns, as well as in their subsequent recoveries. The authors use S&P 500 data from 1926 to 2024 to estimate latent expected returns and volatility using a particle filter and maximum likelihood, and measure risk compensation using the price of risk, defined as expected return per unit of risk. The results indicate that the decline in dollar-denominated risk compensation under nominal distortion is consistent with money illusion. The authors decompose inflation to examine the role of unanticipated inflation and find that M2 growth does not directly generate the money-illusion channel. Instead, incorporating money expansion reveals an asymmetric response of risk compensation to inflation surprises. The authors also find that revaluation in gold has implications for the risk-return relationship, with the positive gold-denominated risk-return relationship strengthening under nominal distortion.

METHOD: The authors revalue U.S. stock market performance and economic growth in ounces of gold, a monetary unit used for five millennia but largely abandoned over the past 50 years. They use S&P 500 data from 1926 to 2024 to estimate latent expected returns and volatility using a particle filter and maximum likelihood, and measure risk compensation using the price of risk, defined as expected return per unit of risk. The authors decompose inflation to examine the role of unanticipated inflation and find that M2 growth does not directly generate the money-illusion channel. Instead, incorporating money expansion reveals an asymmetric response of risk compensation to inflation surprises.

KEY FINDINGS:

- The revaluation of U.S. stock market performance and economic growth in gold reveals systematic differences in performance and growth.
- The price of risk, defined as expected return per unit of risk, declines under nominal distortion when measured in dollars.
- The price of risk remains fairly stable when inflation surprises increase when measured in gold.
- The positive gold-denominated risk-return relationship strengthens under nominal distortion.

IMPLICATION FOR TRADING: The findings suggest that the decline in dollar-denominated risk compensation under nominal distortion is consistent with money illusion. Practitioners can use this revaluation in gold to better understand the relationship between risk and return, particularly in environments of inflation and monetary expansion. This can help in making more informed investment decisions and potentially improve risk management strategies.

Money Illusion in the U.S. Stock Market and Economic Growth: A Revaluation in Gold*

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Abstract

We revalue U.S. stock market performance and economic growth in ounces of gold, a monetary unit used for five millennia but largely abandoned over the past 50 years. Substantially weaker gold-denominated stock market performance and economic growth show that strong dollar-denominated performance does not necessarily translate into purchasing-power gains. As inflation and M2 growth increase, dollar-denominated risk compensation falls because risk rises disproportionately relative to expected return. This disproportionate response is consistent with money illusion. Investors may interpret the corresponding nominal gains as real gains and fail to require sufficient expected return for the higher risk. Revaluation in gold considerably attenuates the decline in risk compensation and markedly strengthens the risk-return relationship.

Keywords: Gold Units, Alternative Numeraire, Nominal Distortion, Asset Pricing

JEL Classification: G12, E44, G17, C58

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1 Introduction

Although extensive research has examined asset pricing and economic growth dynamics, relatively little attention has been devoted to the role of the unit of account itself. Most economic and financial quantities are conventionally measured in dollars, implicitly treating the dollar as the natural unit of account. In this paper, we revalue these quantities in gold, a monetary unit used for five millennia but largely abandoned over the past 50 years. We find systematic differences in stock market performance, economic growth, and risk compensation when these quantities are revalued in gold.

These differences are economically important because nominal gains do not necessarily translate into gains in purchasing power. When stock market performance and economic growth are revalued in gold, both appear substantially weaker over the past 100 years. The differences are evident in the duration and severity of economic and market downturns, as well as in their subsequent recoveries. Both downturns last nearly three times as long and experience more than twice the cumulative decline when measured in gold.

Revaluation in gold also affects the forward-looking risk compensation. Using S&P 500 data from 1926 to 2024, we estimate latent expected returns and volatility using a particle filter and maximum likelihood. We measure risk compensation using the price of risk, defined as expected return per unit of risk. We refer to elevated inflation, monetary expansion, or their joint occurrence as nominal distortion. When measured in dollars, the price of risk falls under nominal distortion. Measuring returns in gold attenuates this decline by approximately 21% when M2 expansion coincides with elevated inflation and by 56% when it coincides with heightened uncertainty. Following positive inflation surprises, risk compensation declines when measured in dollars but increases modestly when measured in gold.

Our results indicate that the decline in dollar-denominated risk compensation under nominal distortion is consistent with money illusion. The decomposition analysis shows that this decline reflects a disproportionate increase in volatility relative to expected return under nominal distortion. In such environments, investors may misinterpret nominal gains as improvements in real performance. As a result, they may fail to require sufficient additional expected return when risk increases, leading to undercompensation. This suggests that money illusion is reflected in insufficient risk compensation rather than in lower expected returns. This imbalance is substantially less pronounced in gold terms. Revaluation in gold partially filters out nominal variation and reduces the disproportionate increase in volatility.

Existing studies of money illusion have focused primarily on inflation. We extend this perspective by considering M2 growth as an additional dimension of nominal distortion. In our analysis, inflation has a negative impact on the price of risk, whether measured in

dollars or gold. When inflation and M2 growth are considered jointly, however, the price of risk exhibits the divergent pattern across the two denominations documented above. This divergence cannot be captured when inflation is the only explanatory variable.

We further decompose inflation to examine the role of unanticipated inflation. With dollar-denominated returns, the price of risk falls following positive inflation surprises, while negative inflation surprises increase the price of risk only when they coincide with elevated M2 growth. M2 growth therefore does not directly generate the money-illusion channel. Instead, incorporating money expansion reveals an asymmetric response of risk compensation to inflation surprises that is not apparent when inflation surprises are considered alone. With gold-denominated returns, such asymmetry does not exist. Indeed, the price of risk remains fairly stable as inflation surprises increase.

Revaluation in gold also has implications for the risk-return relationship. In the baseline specification, both dollar-denominated and gold-denominated returns produce a positive risk-return relationship, consistent with standard asset-pricing theory. Under different inflationary and monetary conditions, however, the relationship observed with dollar-denominated returns differs from that with gold-denominated returns. Under nominal distortion, the positive gold-denominated risk-return relationship strengthens as the horizon increases. This delayed strengthening, documented in the risk-return literature ([Andrei et al. \(2019\)](#); [Lochstoer and Muir \(2022\)](#)), is much stronger and robust with gold-denominated returns than with dollar-denominated returns. Overall, the results indicate that denominating returns in gold rather than dollars stabilizes risk compensation across different inflationary and monetary conditions, and strengthens the positive risk-return relationship.

The motivation for revaluing economic and financial quantities in gold follows from the unprecedented fiscal and monetary expansion during the COVID-19 crisis. Such expansion produced a surge in the money supply and aggregate demand amid lockdown-driven supply constraints, contributing to a sharp rise in inflation. Since the breakdown of the Bretton Woods system, monetary policy has operated without a gold peg, allowing greater expansion of the money supply. The Quantity Theory of Money and subsequent empirical evidence document a near one-for-one relationship between monetary expansion and inflation ([Fisher \(1911\)](#); [Friedman and Schwartz \(1971\)](#); [Friedman \(1970\)](#); [Lucas \(1980\)](#); [Dwyer and Hafer \(1988\)](#); [Rolnick and Weber \(1994\)](#); [McCandless and Weber \(1995\)](#)). Sustained monetary expansion can therefore erode fiat-currency purchasing power, affecting the measurement and interpretation of real economic and financial performance.

Gold provides a natural benchmark for this purchasing-power dimension because its supply is limited and relatively inelastic. Although its inflation-hedging capability is mixed over short horizons, gold serves as a long-run inflation hedge and store of purchasing power ([Jas-](#)

tram and Leyland (2009); Aye et al. (2016); Conlon et al. (2018)). Our cointegration and rolling purchasing-power analyses confirm these properties. Gold prices track M2 growth over the long run and preserve purchasing power over most extended holding periods. These characteristics provide the economic foundation for revaluation in gold.

Our analyses connect to several strands of literature on liquidity, inflation, and nominal distortion in financial markets. Monetary and liquidity expansion generally support stock market performance and asset valuations. Stocks with greater liquidity sensitivity earn higher expected returns (Pástor and Stambaugh (2003)), while unexpected reductions in the federal funds rate are associated with increases in broad stock indexes (Bernanke and Kuttner (2005)). Inflation, by contrast, has a time-varying effect because inflation shocks can convey different information about future economic conditions (Boons et al. (2020)).

Beyond these information effects, inflation can generate nominal distortion in financial markets through money illusion. Investors can confuse nominal and real values, capitalizing equity earnings at nominal rather than real interest rates (Modigliani and Cohn (1979)), underpricing high-beta stocks more severely during high-inflation periods and producing a flatter Security Market Line than CAPM predicts (Cohen et al. (2005)), and exhibiting other forms of nominal price and inflation illusion (Brown et al. (2016); Birru and Wang (2016)). More broadly, money illusion affects economic decision-making through anchoring, mental accounting, and loss aversion (Shafir et al. (1997)). Taken together, this literature shows that money illusion can prevent changes in purchasing power from being fully incorporated into economic and financial decisions.

This literature, however, focuses primarily on inflation and gives comparatively less attention to money supply growth, despite its close relationship with inflation and its independent effects on liquidity and investor risk appetite (Bernanke and Mihov (1998); Bauer et al. (2023)). We address this gap from two perspectives. First, we evaluate stock market performance and economic growth from a purchasing-power perspective by revaluing conventional dollar-denominated measures in gold. Second, we examine the joint role of inflation and money supply growth in risk pricing rather than considering inflation in isolation.

Our analysis also contributes to the literature on the risk-return relationship. Existing evidence is mixed. Negative or insignificant relationships arise under alternative volatility-regime and GARCH specifications (Turner et al. (1989); Glosten et al. (1993)), while positive relationships emerge after accounting for additional state variables or the ICAPM hedge component (Scruggs (1998); Guo and Whitelaw (2006)). The risk-return relationship is also dynamic and asymmetric: conditional volatility can lead conditional expected returns rather than move contemporaneously with them (Whitelaw (1994)), while negative risk-return relationships can persist for several months before reverting (Brandt and Kang (2004)). Re-

searchers have therefore examined alternative econometric specifications, volatility measures, and macroeconomic controls. Yet these approaches retain the conventional dollar denomination. Our analysis shows that revaluation in gold provides another dimension through which the risk-return relationship can differ, particularly under nominal distortion.

The remainder of the paper is structured as follows. Section 2 introduces the transformation of dollar-denominated measures into gold-denominated measures and describes the empirical framework. Section 3 presents the foundational evidence, the main analysis of nominal distortion in risk pricing, and the risk-return relationship across the two measures. Sections 4 and 5 present the robustness analyses and concluding remarks.

2 Methodology

2.1 Data

U.S. economic output is proxied by real GDP, obtained at annual frequency from the Federal Reserve Bank of St. Louis (FRED)¹ from 1929-2024. We use real GDP rather than nominal GDP so that comparisons between dollar-denominated and gold-denominated measures reflect economic output rather than inflation. We revalue real GDP in ounces of gold to provide an alternative benchmark for economic performance. The U.S. stock market is proxied by the S&P 500 total return index. Monthly data on the S&P 500 index and its dividends are obtained from Amit Goyal’s website². Monthly gold prices in dollars are obtained from Datahub³, originally sourced from the World Gold Council. Both series span 1926-2024.

The analysis also uses three macroeconomic variables from August 1960 to December 2024. All three variables are transformed into rates of change: (i) inflation, computed from the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average⁴; (ii) M2 growth⁵, which captures monetary expansion⁶; and (iii) macroeconomic uncertainty, proxied by the percentage change in the JLN one-month-ahead uncertainty index⁷. We use the JLN measure rather than the VIX because it is available over a longer sample and more cleanly separates macroeconomic uncertainty from financial market risk (Jurado et al. (2015)). These variables are used to examine whether risk pricing differs between dollar-

¹<https://fred.stlouisfed.org/series/GDPA>

²<https://sites.google.com/view/agoyal145>

³<https://datahub.io/core/gold-prices>

⁴<https://fred.stlouisfed.org/series/CPIAUCSL>

⁵<https://fred.stlouisfed.org/series/M2SL>

⁶We use M2 rather than a narrower monetary aggregate such as M1 because M2 provides a broader quantity-based measure of liquidity (Eickmeier et al. (2013))

⁷<https://fred.stlouisfed.org/series/JLNUM1M>

denominated and gold-denominated measures under common macroeconomic conditions. Variables more directly related to gold price determination, including the trade-weighted dollar and real interest rates, are considered in the robustness analysis through the gold return decomposition.

2.2 Transformation into Gold Units

We revalue economic and financial quantities in ounces of gold by dividing their dollar-denominated values by the dollar price of gold. Let $\delta_t^\$$ denote U.S. real GDP measured in dollars at time t , G_t the dollar price of one ounce of gold, and δ_t real GDP measured in ounces of gold:

$$\delta_t = \frac{\delta_t^\$}{G_t}$$

Because GDP grows substantially in absolute terms, we work with $\log(\delta_t)$. This transformation facilitates comparisons between dollar-denominated and gold-denominated GDP. We identify downturns from cumulative declines in $\log$ GDP and use these declines to measure the severity and peak-to-trough duration of each episode. Following [Barro \(2006\)](#), [Barro \(2009\)](#), [Nakamura et al. \(2013\)](#), and [Muir \(2017\)](#), we classify a recession as a cumulative decline exceeding 1% but less than 5%, and a depression as a decline exceeding 5%.

GDP growth is measured as

$$g_t^{gold} = \log\left(\frac{\delta_{t+1}}{\delta_t}\right)$$

and

$$g_t^\$ = \log\left(\frac{\delta_{t+1}^\$}{\delta_t^\$}\right)$$

For each measure, we characterize real GDP dynamics using the mean, volatility, and the term structure of the variance ratio across horizons:

$$VR(k) = \frac{\text{Var}(y_t - y_{t-k})}{k \cdot \text{Var}(y_t - y_{t-1})}$$

where y_t denotes $\log$ GDP, k the time horizon, $y_t - y_{t-1}$ one-period $\log$ GDP growth, and $y_t - y_{t-k}$ cumulative $\log$ GDP growth over k periods. Values above one indicate persistence, values below one indicate mean reversion, and a value of one indicates a random walk.

We apply the same revaluation to the stock market. Let $S_t^\$$ denote the S&P 500 total return index measured in dollars at time t , with dividends reinvested:

$$S_{t+1}^{\$} = S_t^{\$} \left(\frac{P_{t+1}^{\$} + D_{t+1}^{\$}}{P_t^{\$}} \right)$$

where $P_{t+1}^{\$}$ denotes the dollar value of the S&P 500 at time $t + 1$ and $D_{t+1}^{\$}$ the dividend paid between t and $t + 1$. The corresponding gold-denominated index is

$$S_t = \frac{S_t^{\$}}{G_t}$$

We define a bear market as a peak-to-trough cumulative decline in $\log(S_t)$ exceeding 20%. Gold-denominated log market returns are

$$r_{t+1} = \log\left(\frac{S_{t+1}}{S_t}\right)$$

while dollar-denominated excess log market returns are

$$r_{t+1}^{\$} = \log\left(\frac{S_{t+1}^{\$}}{S_t^{\$}}\right) - r_{f,t}$$

where $r_{f,t} = \log(1 + R_{f,t})$ is the monthly log risk-free rate and $R_{f,t}$ is the three-month Treasury bill rate. Gold-denominated excess returns equal gold-denominated returns because holding gold earns no interest. Dollar-denominated excess returns subtract the risk-free rate because the opportunity cost of holding dollars is represented by the short-term Treasury bill rate. We examine the mean, volatility, and variance ratios of these returns using the same approach as for log GDP growth.

2.3 Particle-Filtering Maximum Likelihood

Average realized excess returns and volatility summarize stock market performance but provide limited information about its forward-looking and time-varying behavior. We therefore estimate latent expected return and volatility jointly using a particle-filtering maximum likelihood framework. This framework accommodates nonlinear and non-Gaussian dynamics in the latent states.

We model the latent expected return and volatility as AR(1) processes with long-run means, persistence parameters, and stochastic innovations:

$$r_{t+1} = \mu_t \Delta + \sigma_t \sqrt{\Delta} \epsilon_{r,t+1}$$

$$\mu_{t+1} = \alpha_{\mu} + \beta_{\mu} \mu_t + \sigma_{\mu} \epsilon_{\mu,t+1} \tag{1}$$

$$\sigma_{t+1} = \alpha_\sigma + \beta_\sigma \sigma_t + \sigma_\sigma \epsilon_{\sigma,t+1} \quad (2)$$

where $\Delta = 1/12$ is the monthly time interval. The variables μ_t and σ_t denote the time-varying conditional expected return and volatility, respectively. The innovations $\epsilon_{\mu,t}$ and $\epsilon_{\sigma,t}$ are standard normal, serially independent, and contemporaneously correlated, with $\text{Corr}(\epsilon_{\mu,t}, \epsilon_{\sigma,t}) = \rho_{\epsilon_\mu, \epsilon_\sigma}$. This correlation allows shocks to expected returns and volatility to move together. It differs from the leverage effect, which links volatility to realized rather than expected returns.

We use the filtered and smoothed latent states to examine forward-looking risk pricing for the dollar-denominated and gold-denominated measures. Details of the particle-filtering estimation, including the forward-filtering and backward-smoothing procedures, are provided in the Appendix.

2.4 Ex-ante Risk Pricing and Risk-Return Relationship

Using the latent expected return (μ_t) and volatility (σ_t) estimated from the particle-filtering procedure, we construct the ex-ante price of risk as

$$\lambda_t = \frac{\mu_t}{\sigma_{t-1}}$$

which measures expected return at t relative to the volatility observed at $t - 1$. Thus λ_t captures the risk compensation investors require at t conditional on the risk environment observed in the previous period. We relate λ_t to the lagged macroeconomic variables described in Section 2.1 to mitigate look-ahead bias and simultaneity. We also include lagged λ_{t-1} to capture persistence in the price of risk itself.

We consider both regime-based and continuous-effect specifications to capture nonlinear macroeconomic effects. The regime-based specification is

$$\begin{aligned} \lambda_t = & \alpha + \beta_1 X_{1,c,dummy,t-1} + \beta_2 X_{2,c,t-1} + \beta_3 X_{3,c,t-1} + \beta_4 \lambda_{t-1} \\ & + \beta_5 (X_{2,c,t-1} * X_{1,c,dummy,t-1}) + \beta_6 (X_{3,c,t-1} * X_{1,c,dummy,t-1}) + \epsilon_t \end{aligned}$$

where λ_t denotes the price of risk measured in dollars or gold. Inflation, uncertainty, and M2 growth alternately define the regime variable $X_{1,c,dummy,t-1}$, while the remaining macroeconomic variables enter continuously. All continuous macroeconomic variables are demeaned before constructing interactions. This transformation mitigates multicollinearity and allows the coefficients to be interpreted at average macroeconomic conditions.

The continuous-effect specification is

$$\begin{aligned}\lambda_t = & \alpha + \beta_1 X_{A,c,t-1} + \beta_2 X_{B,c,t-1} + \beta_3 X_{3,c,t-1} \\ & + \beta_4 \lambda_{t-1} + \beta_5 (X_{A,c,t-1} * X_{B,c,t-1}) + \epsilon_t\end{aligned}$$

where the macroeconomic variables alternately serve as X_A and X_B , with the remaining variable entering as X_3 . We estimate all specifications using latent paths from both the 2,000- and 2,500-particle configurations to assess robustness. Because the price of risk combines expected returns and volatility, we further examine its underlying components, the decomposition of inflation into expected and surprise components, and the persistence of risk compensation across macroeconomic regimes.

The same latent estimates allow us to examine whether revaluation in gold helps explain the risk-return relationship. Here, expected return μ_t is the dependent variable and volatility σ_t is the primary explanatory variable. We use μ_t , rather than λ_t , because λ_t already embeds σ_{t-1} by construction. Using λ_t as the dependent variable would therefore mechanically relate the dependent variable to volatility, regardless of the underlying economic relationship. The risk-pricing and risk-return analyses thus address distinct but complementary questions. The former examines how nominal distortion affects risk compensation between dollar-denominated and gold-denominated measures. The latter examines whether revaluation in gold helps explain the risk-return relationship. Robustness analyses in the final section assess whether these findings reflect the informational content of revaluation in gold rather than mechanical features of commodity-based measurement.

3 Empirical Findings

3.1 Foundational Evidence

This section first examines gold’s long-run relationship with the money supply (M2) and its role as a store of purchasing power. These results provide the foundation for revaluation in gold. We then compare real GDP and S&P 500 performance using dollar-denominated and gold-denominated measures.

3.1.1 Long-Run Relationship between Gold Prices and M2

We begin by examining whether gold prices and M2 share a stable long-run relationship. Following the Engle-Granger procedure, we first test the stationarity of each series, estimate the long-run relationship between log gold prices and log M2, test the stationarity of the

residuals, and assess the speed at which deviations from equilibrium are corrected through an error correction model. To account for structural changes associated with the collapse of the Bretton Woods system, we conduct the analysis separately for the Bretton Woods period (1960-1971) and the post-Bretton Woods period (1972-2024).

The Bretton Woods subsample does not satisfy the stationarity precondition for the cointegration test because log M2 is stationary in levels. We therefore do not interpret the cointegration estimates for this period. In the post-Bretton Woods period, both log gold prices and log M2 are non-stationary, satisfying the precondition for the Engle-Granger test. The long-run regression yields a coefficient of 0.82 on log M2 ($t = 3.66$, $R^2 = 0.81$), indicating substantial long-run comovement between gold prices and M2. M2 explains 81% of the variation in gold prices.

The residual-based ADF test rejects the null of no cointegration at the 1% significance level, confirming a stable long-run relationship between gold prices and M2. Although gold prices and M2 may fluctuate and deviate temporarily, they revert toward a common long-run equilibrium rather than drifting apart indefinitely. The corresponding error-correction coefficient implies a monthly adjustment rate of 1.2%, with a half-life of approximately 4.8 years. Thus, deviations from the relationship between gold prices and M2 are persistent but gradually corrected over multi-year horizons.

As an additional comparison, we examine the long-run relationship between the CPI and M2 using the same Engle-Granger procedure. Unlike gold prices, CPI and M2 do not exhibit a stable long-run relationship over either the Bretton-Woods or post Bretton-Woods period. This contrast provides additional support for our gold revaluation framework, which is motivated by gold's stable long-run relationship with monetary expansion, an important dimension of nominal measurement that is not present for the CPI.

3.1.2 Rolling Purchasing-Power Preservation

Because the log CPI level is stationary over the post-Bretton Woods sample, we use a rolling purchasing-power analysis rather than a cointegration approach. We first construct the real gold price by dividing the nominal gold price by the CPI. On a rolling basis, for each starting month from January 1972 onward, we compute the ratio of the future to current real gold price over 5-, 10-, 20-, and 30-year horizons. A ratio above one indicates preservation of purchasing power. For each horizon, we report the mean, median, 25th and 75th percentiles, and the share of rolling windows with a ratio at or above one. Each rolling window requires at least 120 monthly observations.

Table 10.1 shows that gold preserves purchasing power in 53.3% of 5-year windows and 54.7% of 10-year windows. The share rises to 60.6% at 20 years and 83.0% at 30 years.

Median ratios exceed one at all horizons, with the highest median at 20 years. The 30-year horizon is the only period in which even the 25th percentile ratio exceeds one, indicating that the 83.0% preservation rate reflects a broad-based pattern rather than a pattern driven by a small number of favorable observations.

Taken together, gold provides only limited preservation of purchasing power over short and medium horizons but substantially stronger preservation over long horizons. Combined with the M2 cointegration evidence, these findings provide the economic foundation for using gold to revalue economic and financial quantities.

3.1.3 Real GDP in Dollars and Gold

Tables 10.2-10.3 show that recessions and depressions are generally more frequent, longer, more severe, and slower to recover when real GDP is measured in gold than when it is measured in dollars. Since 1970, gold-denominated real GDP has experienced several partial recoveries but has not surpassed its earlier peak, whereas dollar-denominated real GDP has continued to expand. Following the breakdown of the Bretton Woods system, gold prices became market-determined and rose substantially. Economic growth did not keep pace with the rise in gold prices, leaving gold-denominated real GDP persistently weaker than its dollar-denominated counterpart. Over 1929-2024, average real GDP growth is 3.12% in dollars but -2.05% in gold. Thus, while dollar-denominated output expanded over the sample, its value relative to gold declined.

Tables 10.5-10.6 further show that the two measures differ in their sensitivity to macroeconomic conditions. In dollar-denominated terms, inflation is significantly negatively associated with real GDP growth, largely independent of uncertainty and monetary conditions. In gold-denominated terms, this direct inflation effect is substantially weaker, while inflation interacts significantly with uncertainty. As inflation rises, the adverse effect of uncertainty on economic activity becomes more pronounced. This interaction is absent in dollar-denominated terms.

A similar divergence appears for monetary expansion. M2 growth is generally associated with stronger real GDP growth in dollar-denominated terms, consistent with the conventional view that monetary expansion supports economic activity. However, this effect is conditional on uncertainty. Uncertainty is insignificant under normal monetary conditions but becomes a significant drag on growth when M2 growth is elevated. The interaction is substantially stronger in gold-denominated terms. Rapid monetary expansion does not translate into comparable gains in purchasing power and instead amplifies the deterioration in gold-denominated real GDP during periods of elevated uncertainty. These interactions help explain the divergence between the two measures during the 1970s-1980s, when el-

evated inflation, monetary expansion, and uncertainty coincided. Gold-denominated real GDP declined substantially over this period, while dollar-denominated real GDP continued to grow.

Finally, real GDP exhibits greater persistence when measured in gold than when measured in dollars. M2 growth accounts for a substantial portion of movements in dollar-denominated real GDP, reducing its apparent persistence, whereas gold-denominated real GDP evolves more gradually and is less sensitive to short-term nominal fluctuations.

3.1.4 Stock Market Performance in Dollars and Gold

Compared with dollar-denominated terms, bear market episodes occur less frequently when the stock market is measured in gold but last longer, recover more slowly, and generate substantially larger cumulative losses. Since 2000, despite several recovery episodes, the gold-denominated stock market has yet to surpass its earlier peak, producing a prolonged bear episode, whereas the dollar-denominated stock market has experienced only intermittent bear episodes over the same period. Average excess returns are lower and volatility is higher when the stock market is measured in gold, indicating weaker risk-adjusted performance when returns are evaluated relative to gold.

As with real GDP, the key divergence arises from an interaction effect rather than a direct relationship. We introduce the ex-post price of risk, γ_t , defined as realized excess returns per unit of lagged volatility, to measure realized risk-adjusted performance. Under gold-denominated terms, monetary expansion is associated with deteriorating realized risk-adjusted performance when inflation is elevated. In dollar-denominated terms, however, monetary expansion offsets the adverse effect of heightened uncertainty. Together, these results suggest that monetary expansion does not necessarily translate into improved performance in purchasing-power terms when inflation is elevated, while strong dollar-denominated performance during heightened uncertainty may partly reflect nominal rather than purchasing-power gains. This pattern is consistent with the 1971–1980 divergence, when gold-denominated stock market performance deteriorated substantially while the dollar-denominated stock market experienced only a relatively short bear market between 1972 and 1974.

Only the gold-denominated specification produces a positive and significant coefficient on lagged γ_t , indicating meaningful persistence in realized risk-adjusted performance that is absent in the dollar-denominated specification. This persistence is consistent with the slower-moving dynamics introduced by gold prices. Stock market performance therefore evolves more gradually when viewed through a purchasing-power perspective.

Collectively, the analysis across the two measures documents systematic divergence in real

GDP and stock market performance. Relative to conventional dollar-denominated measures, gold-denominated measures reveal economic and financial dynamics that become particularly pronounced during periods of elevated inflation, monetary expansion, and uncertainty. This provides a complementary perspective on economic and financial performance.

3.2 Forward-Looking Risk Compensation

We now examine whether the divergence between dollar-denominated and gold-denominated measures established above extends to risk compensation. Specifically, we examine whether the compensation investors require for bearing risk differs when risk is measured in dollars and when it is measured in gold.

3.2.1 Reliability Check of the Particle-Filtering Estimation Procedure

Before presenting the risk-pricing results, we first assess the stability of the particle-filtering estimates reported in Table 3.2. This provides a validation of the latent expected return and volatility processes underlying the subsequent analysis. Overall, the maximum likelihood values and estimated parameters, with the exception of $\rho_{\epsilon_\mu, \epsilon_\sigma}$, remain relatively stable across particle configurations. The parameter $\rho_{\epsilon_\mu, \epsilon_\sigma}$ exhibits pronounced changes between 1,000 and 2,000 particles before stabilizing thereafter. We therefore adopt the 2,000-particle estimates as the baseline and the 2,500-particle estimates as a robustness benchmark.

Across the dollar-denominated and gold-denominated measures, the persistence parameters for expected returns (β_μ) and volatility (β_σ) are high, indicating slowly evolving but mean-reverting latent processes. The innovations to expected returns and volatility are positively correlated under both measures, suggesting that positive innovations in volatility tend to coincide with positive innovations in expected returns.

The remaining parameters are generally stable across particle configurations, although some variation is observed in α_σ , particularly for the dollar-denominated specification. The correlation between the smoothed paths generated by the 2,000- and 2,500-particle specifications are 0.9479 and 0.9938 for expected returns and volatility, respectively, for the gold-denominated specification. For the dollar-denominated specification, the corresponding correlations are lower, at 0.7992 and 0.9667, respectively. Although these correlations are lower for the dollar-denominated specification, they remain relatively high, suggesting that the two particle configurations capture broadly similar underlying dynamics. Overall, these results support the stability of the latent expected return and volatility processes across particle configurations and provide confidence in their use in the subsequent risk-pricing analysis.

Table 3.1: Estimated Parameters from Particle Filtering

Parameters	Dollars				Gold			
	1000	1500	2000	2500	1000	1500	2000	2500
α_μ	0.0054	0.0070	0.0065	0.0064	0.0056	0.0077	0.0051	0.0062
β_μ	0.9511	0.9335	0.9375	0.9496	0.9479	0.9304	0.9532	0.9444
σ_μ	0.0040	0.0066	0.0080	0.0086	0.0062	0.0026	0.0060	0.0051
α_σ	0.0086	0.0080	0.0076	0.0003	0.0062	0.0064	0.0059	0.0057
β_σ	0.9275	0.9306	0.9341	0.9678	0.9510	0.9511	0.9526	0.9559
σ_σ	0.0268	0.0260	0.0245	0.0248	0.0294	0.0290	0.0292	0.0254
$\rho_{\epsilon_\mu, \epsilon_\sigma}$	0.1123	0.2001	0.2050	0.1858	0.0766	0.0921	0.2276	0.2107
Maximum Likelihood	1959.3076	1959.8184	1959.6369	1967.0096	1679.0143	1679.7995	1678.9957	1681.2978

3.2.2 Nominal Distortion and Risk Compensation in Dollars and Gold

Across particle configurations, inflation exerts a significantly negative effect on the price of risk under both dollar-denominated and gold-denominated measures in the continuous-effect specifications. The magnitude of this effect is consistently larger under the dollar-denominated measure. A more nuanced pattern emerges when monetary growth is considered jointly with inflation. Under the dollar-denominated measure, the inflation $\times$ M2 interaction is significantly negative across particle configurations, and the regime-based results show the same state dependence. M2 growth is insignificant under low inflation but becomes significantly negative when inflation is elevated. In particular, higher M2 growth during periods of elevated inflation is associated with lower required risk compensation under the dollar-denominated measure, with the effect of M2 growth depending on prevailing inflation.

Two mechanisms could potentially account for this pattern. One possibility is genuine market risk reduction. During periods of elevated inflation, growth in M2 may ease financing conditions and expand aggregate risk-bearing capacity, thereby reducing the compensation investors require for bearing risk. Here, M2 growth reflects changes in the money supply and does not necessarily imply a deliberate expansionary monetary policy action. An alternative explanation is money illusion. The joint influence of inflation and M2 growth alters the perceived relationship between nominal returns and underlying real risk, leading to a reduction in dollar-denominated risk compensation. Distinguishing between these mechanisms is therefore important for interpreting the observed decline in the price of risk.

The interaction between M2 growth and uncertainty provides a further test of these competing explanations. If monetary expansion genuinely reduces market risk, higher M2 growth should weaken the sensitivity of risk compensation to uncertainty, implying a negative or insignificant M2 $\times$ JLN interaction. Instead, the interaction is significantly positive under

dollar-denominated measures across particle configurations. The evidence therefore provides little support for the interpretation that higher M2 growth uniformly mitigates the effects of heightened uncertainty. The negative inflation $\times$ M2 effect is consequently less consistent with a broad reduction in market risk.

The contrast with gold units is informative. The inflation $\times$ M2 interaction is attenuated and statistically insignificant under the gold-denominated measure, whereas the significantly positive M2 $\times$ JLN interaction remains, albeit smaller in magnitude. This cross-numeraire pattern suggests that revaluation in gold reduces the influence of nominal distortion on risk compensation while preserving the transmission of uncertainty into risk pricing. The contrast is therefore consistent with the interpretation that the decline in dollar-denominated risk compensation under elevated inflation and M2 growth contains a money-illusion component.

The interaction between inflation and uncertainty is insignificant under both measures. Together with the significant M2 $\times$ JLN interaction, this suggests that monetary conditions provide information about the risk-compensation response under heightened uncertainty beyond that captured by inflation alone. Incorporating M2 growth therefore reveals an additional dimension of the money-illusion channel.

The price of risk is highly persistent under both dollar- and gold-denominated measures, with its own lagged value accounting for most of the persistence relative to contemporaneous macroeconomic conditions. This persistence is systematically stronger under the gold-denominated measure, consistent with the GDP and stock market findings. Thus, beyond realized performance evaluation, revaluation in gold also shapes forward-looking risk pricing, extending the divergence documented above to the compensation investors require for bearing risk.

3.2.3 Decomposition of the Decline in Risk Compensation

To identify the mechanism underlying the decline in the price of risk, we decompose λ_t into its expected return and volatility components by separately modeling expected returns and volatility as dependent variables. We first estimate a conventional inflation-based specification as the baseline and then augment it with the interaction between inflation and M2 growth. This allows us to examine whether incorporating monetary growth reveals a risk-pricing pattern that is obscured when inflation is considered alone.

Baseline:

$$\mu_t = \alpha + \beta_1 inf_{t-1} + \beta_2 m2_{t-1} + \beta_3 jln_{t-1} + \beta_4 \mu_{t-1} + \epsilon_t$$

$$\sigma_t = \alpha + \beta_1 inf_{t-1} + \beta_2 m2_{t-1} + \beta_3 jln_{t-1} + \beta_4 \sigma_{t-1} + \epsilon_t$$

Table 3.2: Regime-based Results for the Forward-Looking Price of Risk

Regime Unit	Inflation			M2 Growth			Uncertainty (JLN)		
	Dollar		Gold	Dollar		Gold	Dollar		Gold
	2000	2500		2000	2500		2000	2500	
inf_{dm}	-0.0003 (0.0005)	-0.0004 (0.0007)	-0.0007** (0.0003)	-0.0006** (0.0003)					
$m2_{dm}$									
jln_{dm}									
inf_{t-1}									
$m2_{t-1}$									
jln_{t-1}									
$inf_{dm} \times jln_{t-1}$									
$inf_{dm} \times m2_{t-1}$									
$m2_{dm} \times inf_{t-1}$									
$m2_{dm} \times jln_{t-1}$									
$jln_{dm} \times inf_{t-1}$									
$jln_{dm} \times m2_{t-1}$									
λ_{t-1}	0.9823*** (0.0066)	0.9887*** (0.0062)	0.9886*** (0.0065)	0.9908*** (0.0056)	0.9870*** (0.0067)	0.9880*** (0.0068)	0.9795*** (0.0067)	0.9877*** (0.0063)	0.9902*** (0.0058)
R^2	0.9668	0.9805	0.9845	0.9894	0.9667	0.9845	0.9668	0.9805	0.9894
Adjusted R^2	0.9665	0.9803	0.9844	0.9893	0.9665	0.9844	0.9665	0.9804	0.9893

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Regime dummies are constructed using sample medians, with observations at or below the median assigned to the low regime (0) and observations above the median assigned the high regime (1). All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variable is the forward-looking price of risk (λ_t), measured in dollars or gold. All explanatory variables are lagged by one period to mitigate look-ahead bias, and lagged λ_{t-1} is included to capture persistence. $N=770$.

Table 3.3: Continuous-Effect Results for the Forward-Looking Price of Risk

Unit	Dollar			Gold			Dollar			Gold		
	2000	2500		2000	2500		2000	2500		2000	2500	
N												
$\ln f_{t-1}$	-0.1308* (0.0741)	-0.1866* (0.0972)	-0.1230** (0.0603)	-0.1007* (0.0533)	-0.1461* (0.0784)	-0.1969** (0.0980)	-0.1209* (0.0649)	-0.0990* (0.0566)	-0.1350* (0.0732)	-0.1847* (0.0948)	-0.1170* (0.0606)	-0.0973* (0.0534)
$m2_{t-1}$	1.0771 (0.0596)	0.5101 (0.0765)	-6.4319 (0.0366)	-6.3764 (0.0339)	4.4426 (0.0563)	3.8508 (0.0758)	-1.9676 (0.0306)	-1.8545 (0.0299)	-0.0331 (0.0533)	-0.0927 (0.0681)	-0.0192 (0.0312)	-0.0201 (0.0299)
$j\ln_{t-1}$	-0.0150* (0.0087)	-0.0127 (0.0094)	-0.0011 (0.0044)	0.0006 (0.0040)	-0.0135 (0.0089)	-0.0110 (0.0094)	-0.0012 (0.0045)	0.0006 (0.0041)	-0.0168* (0.0099)	-0.0181* (0.0106)	-0.0029 (0.0050)	-0.0010 (0.0046)
$I \times M$	-11.2604*-18.5112** (6.5697)	-0.5387 (7.7327)	-2.7399 (3.7375)									
$I \times J$					-0.0990 (1.4108)	-1.4501 (2.0667)	-0.2882 (1.1428)	-0.5295 (0.9556)				
$M \times J$									1.0678* (0.5535)	2.4771*** (0.6925)	0.5787* (0.3413)	0.6032* (0.3165)
λ_{t-1}	0.9798*** (0.0067)	0.9865*** (0.0062)	0.9877*** (0.0068)	0.9903*** (0.0059)	0.9791*** (0.0068)	0.9863*** (0.0062)	0.9877*** (0.0068)	0.9902*** (0.0059)	0.9793*** (0.0067)	0.9862*** (0.0062)	0.9878*** (0.0068)	0.9903*** (0.0059)
R^2	0.9668	0.9804	0.9845	0.9893	0.9667	0.9804	0.9845	0.9893	0.9668	0.9805	0.9845	0.9894
Adjusted R^2	0.9666	0.9803	0.9844	0.9893	0.9665	0.9803	0.9844	0.9893	0.9666	0.9803	0.9844	0.9893

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. I denotes $\ln f_{t-1}$, M denotes $m2_{t-1}$, and J denotes $j\ln_{t-1}$. The dependent variable is the forward-looking price of risk (λ_t), measured in dollars or gold. All explanatory variables are lagged by one period to mitigate look-ahead bias, and lagged λ_{t-1} is included to capture persistence. $N=770$.

Augmented specification:

$$\mu_t = \alpha + \beta_1 inf_{t-1} + \beta_2 m2_{t-1} + \beta_3 jln_{t-1} + \beta_4 (inf_{t-1} \times m2_{t-1}) + \beta_5 \mu_{t-1} + \epsilon_t$$

$$\sigma_t = \alpha + \beta_1 inf_{t-1} + \beta_2 m2_{t-1} + \beta_3 jln_{t-1} + \beta_4 (inf_{t-1} \times m2_{t-1}) + \beta_5 \sigma_{t-1} + \epsilon_t$$

Under the baseline specification, the dollar-denominated model exhibits a positive inflation effect on expected returns, although the significance is less robust across particle configurations. The corresponding effect on volatility is consistently insignificant. By contrast, the augmented specification reveals that the inflation $\times$ M2 interaction is associated with increases in both expected returns and volatility. Expected returns therefore rise during periods of elevated inflation and M2 growth rather than decline. The increase in volatility, however, is substantially larger and more robust across particle configurations. The expected return effect also loses statistical significance under the 2,500-particle specification. Consequently, expected compensation does not rise proportionately with risk exposure, generating a decline in the price of risk. The decline in risk compensation therefore reflects a disproportionate increase in volatility relative to expected return under nominal distortion, rather than a genuine reduction in expected returns.

The baseline patterns under the gold-denominated measure are similar to those under the dollar-denominated measures. However, a different pattern emerges under the augmented specification. The interaction effect on expected returns is weaker and less consistent across particle configurations, while the corresponding effect on volatility is statistically insignificant in both specifications. The pronounced disproportionate increase in volatility relative to expected returns observed under the dollar-denominated measure is largely absent when returns are revalued in gold. The resulting decline in risk compensation is consequently attenuated. This difference suggests that some of the volatility associated with inflation and M2 growth in dollar-denominated measures reflects nominal variation. This variation is attenuated when returns are evaluated in purchasing-power terms. Revaluation in gold therefore leaves a smaller increase in volatility, pointing to greater emphasis on changes in underlying purchasing power rather than nominal variation.

This asymmetry accords with a money-illusion interpretation if investors interpret nominal gains accompanying inflation and M2 growth as improvements in real performance and consequently fail to require sufficient additional compensation for the associated increase in risk. M2 growth contributes primarily by making this undercompensation more visible. The low correlation between inflation and M2 growth in the sample ($r=-0.087$) and variance inflation factors ranging from 1.05 to 1.48 across the regressors further suggest that these findings are unlikely to be driven by material multicollinearity. Because the disproportion-

ate increase in volatility relative to expected returns weakens considerably when revalued in gold, the evidence is more consistent with money illusion in dollar-denominated risk pricing than with a genuine reduction in expected returns or a fundamental mispricing of risk.

3.2.4 Inflation Decomposition in Dollars and Gold

Since the nominal distortion documented above emerges from the joint occurrence of elevated inflation and M2 growth, an important question is whether such conditions can arise in practice. This combination may appear unusual because central banks typically respond to elevated inflation by tightening monetary policy. In theory, tighter monetary policy should constrain monetary expansion. Nevertheless, evidence on the relationship between monetary policy and money supply generated through bank lending is mixed (Bernanke and Blinder (1992); Borio and Gambacorta (2017)). We do not attempt to resolve this broader monetary-policy question. Instead, we investigate whether the observed nominal distortion can arise from unexpected nominal disturbances without requiring a deliberate policy response. We therefore decompose realized inflation into expected and unexpected components and examine their interactions with M2 growth in explaining risk compensation.

We proxy expected inflation with the 6-month-ahead CPI forecast from the Livingston Survey, obtained from the Federal Reserve Bank of Philadelphia⁸. We define unexpected inflation as the residual between realized and expected inflation. The data span from 1960, covering the major stagflation episodes central to the nominal distortion analysis. We interact expected and unexpected inflation separately with M2 growth under both dollar-denominated and gold-denominated measures, using the same regression framework as in the preceding analysis.

As reported in Table 3.6, expected inflation has no significant effect on the price of risk, either directly or through its interaction with M2 growth. These effects are consistent across all specifications and both measures. Anticipated changes in inflation therefore do not systematically alter risk compensation. This applies whether expected inflation is relatively higher or lower.

By contrast, unexpected inflation significantly reduces the price of risk in both measures. This negative association is further amplified by elevated M2 growth under the dollar-denominated measure. The decline in risk compensation is therefore concentrated in episodes in which realized inflation exceeds expectations, particularly when M2 growth is elevated. No comparable interaction is observed under the gold-denominated measure,

⁸<https://www.philadelphiafed.org/surveys-and-data/cpi-spf>. Because the Livingston Survey reports semi-annual forecasts, the same 6-month-ahead expected inflation rate is applied to each month within its corresponding forecast window.

Table 3.4: Decomposition of the Price of Risk: Expected Return and Volatility

σ_N Variable	Dollar Units				Gold Units			
	2000		2500		2000		2500	
	μ_t	σ_t	μ_t	σ_t	μ_t	σ_t	μ_t	σ_t
Baseline								
inf_{t-1}	0.0009*	0.0318	0.0006	0.0301	0.0002	0.0058	0.0002	0.0091
	(0.0005)	(0.0256)	(0.0005)	(0.0272)	(0.0005)	(0.0318)	(0.0004)	(0.0261)
jln_{t-1}	0.0000	0.0017	0.0001	0.0015	0.0001	0.0028	0.0000	0.0017
	(0.0001)	(0.0028)	(0.0001)	(0.0031)	(0.0000)	(0.0031)	(0.0000)	(0.0026)
$m2_{t-1}$	0.0001	-0.0023	0.0001	0.0114	-0.0000	-0.0046	0.0001	-0.0006
	(0.0003)	(0.0145)	(0.0003)	(0.0147)	(0.0003)	(0.0152)	(0.0002)	(0.0132)
μ_{t-1}	0.9795***		0.9921***		0.9944***		0.9940***	
	(0.0089)		(0.0069)		(0.0054)		(0.0065)	
σ_{t-1}		0.9854***		0.9917***		0.9943***		0.9949***
		(0.0091)		(0.0071)		(0.0052)		(0.0048)
R^2	0.9713	0.9770	0.9854	0.9880	0.9934	0.9921	0.9904	0.9942
Adjusted R^2	0.9711	0.9769	0.9853	0.9879	0.9933	0.9921	0.9904	0.9942
Augmented								
inf_{t-1}	0.0007	0.0258	0.0006	0.0233	0.0000	0.0015	0.0001	0.0056
	(0.0005)	(0.0258)	(0.0005)	(0.0273)	(0.0005)	(0.0321)	(0.0004)	(0.0266)
jln_{t-1}	0.0000	0.0023	0.0001	0.0022	0.0001	0.0032	0.0000	0.0021
	(0.0001)	(0.0029)	(0.0001)	(0.0031)	(0.0000)	(0.0030)	(0.0000)	(0.0025)
$m2_{t-1}$	0.0004	0.0095	0.0002	0.0254	0.0003	0.0051	0.0001	0.0075
	(0.0003)	(0.0170)	(0.0003)	(0.0174)	(0.0003)	(0.0195)	(0.0002)	(0.0165)
$inf_{t-1} \times m2_{t-1}$	0.0923**	4.0640*	0.0467	4.7895**	0.1043***	3.2743	0.0142	2.7360
	(0.0411)	(2.2853)	(0.0399)	(2.2934)	(0.0311)	(2.0675)	(0.0249)	(1.6930)
μ_{t-1}	0.9806***		0.9924***		0.9945***		0.9941***	
	(0.0090)		(0.0069)		(0.0053)		(0.0065)	
σ_{t-1}		0.9865***		0.9922***		0.9944***		0.9950***
		(0.0090)		(0.0071)		(0.0052)		(0.0048)
R^2	0.9714	0.9771	0.9854	0.9881	0.9934	0.9921	0.9904	0.9942
Adjusted R^2	0.9712	0.9769	0.9853	0.9880	0.9934	0.9921	0.9904	0.9942

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variables are expected returns (μ_t) and volatility (σ_t), measured in dollars or gold. All explanatory variables are lagged by one period to mitigate look-ahead bias. Lagged expected return (μ_{t-1}) and lagged volatility (σ_{t-1}) are included in their respective specifications to capture persistence. Inflation and M2 growth exhibit little linear correlation in the sample ($r = -0.087$), while variance inflation factors range from 1.05 to 1.48 across the regressors, indicating no material multicollinearity. $N = 770$

suggesting that unexpected inflation is an important source of nominal variation embedded in dollar-denominated risk compensation.

These results address the concern about the joint occurrence of elevated inflation and M2 growth. Such conditions can arise naturally when inflation unexpectedly exceeds expectations alongside elevated M2 growth, without requiring deliberate monetary accommodation.

Table 3.5: Inflation Decomposition in Dollars and Gold

Unit	Dollar		Gold	
	2000	2500	2000	2500
σ_N				
inf_{t-1}^E	-0.0710 (0.1233)	-0.0944 (0.1553)	-0.0748 (0.0901)	-0.0497 (0.0826)
inf_{t-1}^U	-0.1658** (0.0744)	-0.2321** (0.0947)	-0.1434** (0.0648)	-0.1215** (0.0572)
gln_{t-1}	-0.0148* (0.0087)	-0.0127 (0.0093)	-0.0014 (0.0043)	0.0004 (0.0039)
$m2_{t-1}$	-0.0565 (0.0602)	-0.1145 (0.0786)	-0.0124 (0.0372)	-0.0206 (0.0344)
$inf_{t-1}^E \times m2_{t-1}$	-9.7352 (22.5778)	-23.7107 (24.9026)	-11.2940 (13.8778)	-9.2946 (12.8304)
$inf_{t-1}^U \times m2_{t-1}$	-12.2429* (7.1714)	-18.2084** (8.7635)	1.4413 (5.0891)	-1.6971 (4.7865)
λ_{t-1}	0.9808*** (0.0069)	0.9879*** (0.0064)	0.9888*** (0.0069)	0.9916*** (0.0062)
R^2	0.9668	0.9805	0.9845	0.9894
Adjusted R^2	0.9665	0.9803	0.9844	0.9893

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variable is the forward-looking price of risk λ_t , measured in dollars or gold. Realized inflation is decomposed into expected (inf^E) and unexpected (inf^U) components, which are interacted separately with M2 growth. All explanatory variables are lagged by one period to mitigate look-ahead bias. Lagged λ_{t-1} is included to capture persistence. $N=770$.

3.2.5 Asymmetric Effects of Inflation Surprises

Decomposing unexpected inflation into positive and negative surprises reveals an asymmetric pattern under the dollar-denominated measures. Positive inflation surprises significantly

reduce the price of risk in the 2,500-particle specification. Risk compensation therefore responds negatively to positive inflation surprises even without conditioning on M2 growth. This negative effect is absent under the gold-denominated measure.

The difference between dollar- and gold-denominated risk compensation following positive inflation surprises is consistent with a money-illusion interpretation. When realized inflation exceeds expectations, nominal returns can increase without a corresponding improvement in real purchasing power. The accompanying decomposition of expected returns and volatility provides further support for this interpretation. Under the dollar-denominated measure, positive inflation surprises are associated with a relatively larger increase in volatility than in expected returns, consistent with insufficient compensation for the associated increase in risk. Under the gold-denominated measure, this disproportionate volatility response is less pronounced, and the overall effect on risk compensation is not statistically significant. Revaluation in gold therefore attenuates the nominal distortion associated with positive inflation surprises.

Negative inflation surprises, by contrast, are statistically insignificant on their own, but their interaction with M2 growth is significantly positive in both particle configurations under the dollar-denominated measure. When negative inflation surprises coincide with elevated M2 growth, risk compensation rises rather than falls. This interaction reveals that the risk-compensation response to negative inflation surprises is conditional on monetary conditions.

Repeating the analysis using a specification that excludes the M2 interaction confirms this conditioning role. Without the interaction, negative inflation surprises remain statistically insignificant. Negative inflation news therefore does not have a uniform effect on risk compensation. Its relevance depends on the prevailing monetary conditions, revealing an asymmetric response that is not captured by inflation surprises alone.

Under the gold-denominated measure, neither the direct nor interaction effect is significant. Risk pricing under gold revaluation therefore appears less sensitive to asymmetric inflation surprises and more closely tied to underlying purchasing-power dynamics.

3.2.6 Response to Nominal and Real Information

Gold revaluation does not exhibit the same disproportionate increase in volatility relative to expected return observed in dollar-denominated measures. This finding is consistent with the interpretation that gold revaluation attenuates nominal variation in risk compensation. To examine this interpretation, we compare the responses of dollar-denominated and gold-denominated volatility innovations to nominal and real information.

Table 3.6: Inflation Surprises in Dollars and Gold

Indicator	Dollar Unit		Gold Unit		Dollar Unit		Gold Unit	
	2000	2500	2000	2500	2000	2500	2000	2500
$inf_{E,t-1}$	-0.0719 (0.1163)	-0.1028 (0.1456)	-0.0849 (0.0880)	-0.0573 (0.0800)	-0.0888 (0.1155)	-0.1277 (0.1453)	-0.0879 (0.0867)	-0.0627 (0.0794)
$inf_{U,t-1}^+$	-0.2280 (0.1391)	-0.3996*** (0.1490)	-0.1363 (0.0964)	-0.1019 (0.0866)	-0.2306* (0.1391)	-0.4076*** (0.1479)	-0.1564 (0.0961)	-0.1208 (0.0869)
$inf_{U,t-1}^-$	0.0720 (0.1255)	0.0011 (0.1723)	0.1228 (0.1151)	0.1109 (0.1018)	0.1309 (0.1196)	0.1013 (0.1626)	0.1249 (0.1059)	0.1239 (0.0946)
jln_{t-1}	-0.0138 (0.0087)	-0.0103 (0.0092)	-0.0006 (0.0043)	0.0011 (0.0039)	-0.0129 (0.0085)	-0.0089 (0.0093)	-0.0009 (0.0042)	0.0010 (0.0039)
$m2_{t-1}$	-0.0931 (0.0786)	-0.1858 (0.1147)	-0.0427 (0.0519)	-0.0559 (0.0469)	-0.0182 (0.0567)	-0.0497 (0.0794)	-0.0113 (0.0301)	-0.0129 (0.0294)
$inf_{U,t-1}^+ \times m2_{t-1}$	15.0777 (28.0574)	33.2433 (37.9311)	25.5047 (20.4646)	26.4368 (17.4443)				
$inf_{U,t-1}^- \times m2_{t-1}$	21.5482** (10.5657)	37.8181** (15.9985)	5.3132 (6.9962)	8.9018 (6.2961)				
λ_{t-1}	0.9800*** (0.0068)	0.9865*** (0.0064)	0.9885*** (0.0069)	0.9914*** (0.0061)	0.9797*** (0.0069)	0.9867*** (0.0064)	0.9886*** (0.0069)	0.9915*** (0.0061)
R^2	0.9668	0.9805	0.9845	0.9894	0.9668	0.9804	0.9845	0.9894
Adjusted R^2	0.9665	0.9803	0.9844	0.9893	0.9665	0.9803	0.9844	0.9893

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variable is the forward-looking price of risk λ_t , measured in dollars or gold. Realized inflation is decomposed into expected (inf_E) and unexpected (inf_U) components, with unexpected inflation further decomposed into positive (inf_U^+) and negative (inf_U^-) inflation surprises, defined as demeaned inflation surprises above and below zero, respectively. One-period lagged positive and negative inflation surprises are each interacted with one-period lagged M2 growth. All explanatory variables are lagged by one period to mitigate look-ahead bias, and lagged λ_{t-1} is included to capture persistence. $N=770$.

$$\begin{aligned}
\sigma_t &= \alpha_\sigma + \beta_\sigma \sigma_{t-1} + \sigma_\sigma \epsilon_{\sigma,t}, \\
\hat{\epsilon}_{\sigma,t} &= \sigma_t - \left(\hat{\alpha}_\sigma + \hat{\beta}_\sigma \sigma_{t-1} \right), \\
\hat{\epsilon}_{\sigma,t} &= \alpha_0 + \gamma_1 X_{t-1}^{nominal} + \gamma_2 X_{t-1}^{real} + \varepsilon_t.
\end{aligned}$$

where $X^{nominal}$ denotes nominal distortion in the first specification, defined as a period in which both inflation and M2 growth are above their respective median values. In the second specification, $X^{nominal}$ denotes positive inflation surprises. X^{real} denotes the real interest rate, measured as the 10-year Treasury yield minus Livingston expected inflation. The real interest rate captures real interest-rate conditions and represents the opportunity cost of holding gold.

As reported in Table 3.8, volatility innovations in both dollar- and gold-denominated measures increase significantly under high nominal distortion. The gold-denominated measure also responds significantly to real interest-rate information, whereas the dollar-denominated measure does not. A clearer distinction emerges when positive inflation surprises are used to capture short-lived nominal variation. Dollar-denominated volatility innovations respond significantly to positive inflation surprises across specifications, whereas their response to real interest-rate conditions is significant only in the 2,500-particle specification. In contrast, gold-denominated volatility innovations respond significantly and robustly to real interest-rate conditions but not to positive inflation surprises.

The distinction between nominal and real interest-rate information provides further evidence that dollar-denominated risk measures are more sensitive to short-lived nominal variation. Gold revaluation attenuates short-lived nominal variation while retaining sensitivity to real interest-rate information and broader nominal distortion when elevated inflation and M2 growth occur jointly. Gold revaluation therefore attenuates rather than eliminates nominal distortion. This distinction between the response to nominal and real interest-rate information is consistent with the broader money-illusion literature, which emphasizes that investors may not fully distinguish between nominal changes and their real implications.

3.2.7 Quantifying the Attenuation of Nominal Distortion

Having established the presence of nominal distortion in risk pricing, we quantify its economic magnitude and the extent to which revaluation in gold attenuates it. We first compare the statistical difference between dollar- and gold-denominated prices of risk. We then report the corresponding attenuation ratios to quantify the economic magnitude of this difference across the macroeconomic regimes identified in the preceding analyses.

Under elevated inflation and M2 growth, both dollar- and gold-denominated measures

Table 3.7: Response to Nominal and Real Information in Dollars and Gold**(a)** Nominal Distortion

Unit	Dollar		Gold	
	2000	2500	2000	2500
N				
ND	0.0003** (0.0002)	0.0004*** (0.0002)	0.0006*** (0.0002)	0.0006*** (0.0002)
$Real$	0.0000 (0.0000)	0.0000 (0.0000)	0.0001* (0.0000)	0.0001** (0.0000)
R^2	0.0124	0.0257	0.0416	0.0582
Adjusted R^2	0.0098	0.0232	0.0391	0.0558

(b) Positive Inflation Surprise

Unit	Dollar		Gold	
	2000	2500	2000	2500
N				
Inf_U^+	0.0658** (0.0297)	0.0548*** (0.0263)	0.0521 (0.0403)	0.0510 (0.0350)
$Real$	0.0000 (0.0000)	0.0001** (0.0000)	0.0001** (0.0000)	0.0001*** (0.0000)
R^2	0.0154	0.0193	0.0305	0.0415
Adjusted R^2	0.0128	0.0168	0.0280	0.0390

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the volatility innovation, measured in dollars or gold. In panel A, ND denotes nominal distortion, defined as a period in which both inflation and M2 growth are above their respective median values. In panel B, Inf_U^+ denotes positive inflation surprises. $Real$ denotes the real interest rate, measured as the 10-year Treasury yield minus Livingston expected inflation. $N=771$.

exhibit a decline in the price of risk. The decline in the dollar-denominated price of risk is statistically stronger than the decline in the gold-denominated measure in the 2,500-particle specification ($\Delta = 0.0167$, one-sided $p = 0.0119$). The corresponding differential is not statistically significant in the 2,000-particle specification. In the 2,500-particle specification, revaluation in gold attenuates the decline in dollar-denominated risk compensation by approximately 40%. Averaging across the two particle configurations yields an attenuation of approximately 21%.

The attenuation is more pronounced when inflation is unexpected. The differences between the dollar- and gold-denominated measures are statistically significant in both particle specifications. Across positive and negative inflation-surprise regimes, the decline in the price of risk persists under the dollar-denominated measure but reverses under the gold-denominated measure. Gold-denominated risk compensation is higher in the positive-surprise regime than in the negative-surprise regime.

A further attenuation emerges under high-uncertainty and high-M2-growth regimes in the 2,500-particle specifications ($\Delta = 0.0120$, one-sided $p = 0.06$). Revaluation in gold attenuates the decline in dollar-denominated risk compensation by approximately 68% in this specification and by approximately 58% on average across the two particle configurations.

Overall, revaluation in gold reduces the short-lived nominal variation embedded in dollar-denominated risk compensation while preserving the underlying transmission of monetary and uncertainty conditions into risk pricing. The effects documented throughout Section 3.2 are economically meaningful.

4 Dynamics of Risk Compensation

Following the analysis of the level and composition of risk compensation, we next examine its dynamics. We first examine whether the negative effect of nominal distortion propagates over time. We then examine whether the persistence of risk compensation changes under nominal-distortion conditions and across different macroeconomic regimes.

4.1 Propagation of Risk Compensation over Horizons

We extend the analysis of nominal distortion on risk compensation beyond the one-period framework. We examine whether nominal distortion at time t is associated with risk compensation at subsequent horizons. The propagation of nominal distortion over time is estimated by

Table 3.8: Attenuation Magnitude Across Macroeconomic Regimes**(a)** Inflation and M2 Growth Regimes

Particles	Dollars	Gold	Dollars - Gold	Standard Errors	One-sided p Values
2000	0.0166	0.0215	-0.0048	0.0047	0.8452
2500	0.0419	0.0252	0.0167	0.0073	0.0116**
Average	0.0293	0.0233		Attenuation %	20.32%

(b) Positive and Negative Inflation Surprise Regimes

Particles	Dollars	Gold	Dollars - Gold	Standard Errors	One-sided p Values
2000	0.0022	-0.0046	0.0068	0.0027	0.0064***
2500	0.0004	-0.0051	0.0056	0.0041	0.0845*
Average	0.0013	-0.0049		Attenuation %	470.02%

(c) Uncertainty and M2 Growth Regimes

Particles	Dollars	Gold	Dollars - Gold	Standard Errors	One-sided p Values
2000	0.0082	0.0051	0.0031	0.0056	0.2912
2500	0.0177	0.0057	0.0120	0.0077	0.0601*
Average	0.0130	0.0054		Attenuation %	58.24%

Notes: In Panel A, the high regime is defined as periods in which both inflation and M2 growth simultaneously exceed their respective median values, while the low regime corresponds to periods in which both fall at or below their respective medians. In Panel B, the positive regime is defined as periods in which inflation surprises are positive, while the negative regime corresponds to periods in which inflation surprises are negative. In Panel C, the high regime is defined as periods in which both uncertainty and M2 growth simultaneously exceed their respective median values, while the low regime corresponds to periods in which both fall at or below their respective medians. The decline in risk compensation is computed as the difference between the average price of risk under low and high regimes. The reported average decline is calculated across particle configurations. The attenuation magnitude is computed as $1 - \frac{Compression_{gold}}{Compression_{dollar}}$.

$$\lambda_{t+h} = \alpha + \beta_1 \lambda_t + \beta_2 ND_t + \beta_3 inf_t + \beta_4 m2_t + \beta_5 jln_t + \epsilon_{t+h}$$

where ND denotes nominal distortion, defined as a period in which both inflation and M2 growth are above their respective median values. The other macroeconomic variables are demeaned as usual.

The propagation analysis shows that nominal distortion at time t is associated with lower future risk compensation across broadly longer horizons in both measures. The decline in future risk compensation tends to be larger in magnitude and persist over longer horizons under the dollar-denominated measure.

The dollar-gold difference in the propagation coefficients is statistically significant at the 6- and 9-month horizons in the 2,500-particle specification. This indicates that the attenuation associated with gold revaluation is most pronounced at medium horizons. Given the difference is significant only in the 2,500-particle specification, the attenuation is more consistently observed in the level of risk compensation than in its propagation across horizons.

4.2 Persistence Dynamics across Regimes

The gold-denominated measure exhibits consistently stronger persistence than its dollar-denominated counterpart. We therefore examine whether the persistence of risk compensation changes under nominal-distortion conditions and across different macroeconomic regimes. We estimate two specifications:

Nominal Distortion Regime:

$$\lambda_t = \alpha + \beta_1 inf_{t-1} + \beta_2 m2_{t-1} + \beta_3 jln_{t-1} + \beta_4 \lambda_{t-1} + \beta_5 (inf_{t-1} \times m2_{t-1}) + \beta_6 (\lambda_{t-1} \times inf_{t-1} \times m2_{t-1}) + \epsilon_t$$

Macroeconomic Regime:

$$\lambda_t = \alpha + \beta_1 R_{dummy,t-1} + \beta_2 X1_{t-1} + \beta_3 X2_{t-1} + \beta_4 \lambda_{t-1} + \beta_5 (\lambda_{t-1} \times R_{dummy,t-1}) + \epsilon_t$$

Persistence weakens under nominal-distortion regimes in the dollar-denominated measure, but this effect is not robust across particle configurations. In the gold-denominated measure, nominal distortion has no significant effect on persistence across specifications. Nominal distortion therefore affects the level and composition of risk compensation but does not robustly alter its persistence. The decline in dollar-denominated risk compensation primarily reflects the money-illusion mechanism operating through the effects of inflation and M2 growth on expected returns and volatility, rather than a systematic change in the persistence of risk compensation.

Table 4.1: Propagation of Risk Compensation over Horizons**(a)** Propagation of Nominal Distortion over Horizons

Unit	Dollar				Gold			
Horizon	2000		2500		2000		2500	
Variable	β_2	R^2	β_2	R^2	β_2	R^2	β_2	R^2
h=0	0.0073* (0.0040)	0.0877	-0.0001 (0.0059)	0.1251	0.0023 (0.0050)	0.1315	0.0005 (0.0053)	0.1530
h=1	-0.0013** (0.0006)	0.9667	-0.0019*** (0.0006)	0.9804	-0.0014*** (0.0005)	0.9846	-0.0012*** (0.0004)	0.9894
h=3	-0.0020* (0.0012)	0.9019	-0.0029** (0.0014)	0.9341	-0.0016* (0.0007)	0.9535	-0.0015** (0.0005)	0.9662
h=6	-0.0022 (0.0019)	0.7564	-0.0049** (0.0022)	0.8302	-0.0019 (0.0008)	0.8920	-0.0020* (0.0005)	0.9172
h=9	-0.0034 (0.0027)	0.5878	-0.0068** (0.0034)	0.6937	-0.0026 (0.0010)	0.8204	-0.0026 (0.0006)	0.8577
h=12	-0.0031 (0.0032)	0.4300	-0.0062 (0.0044)	0.5500	-0.0040* (0.0011)	0.7385	-0.0039* (0.0007)	0.7886

(b) Dollar–Gold Difference in Propagation over Horizons

Unit	2000		2500	
Variable	Dollar - Gold	Standard Error	Dollar - Gold	Standard Error
h=0	0.0050	0.0037	-0.0006	0.0047
h=1	0.0001	0.0005	-0.0006	0.0005
h=3	-0.0005	0.0010	-0.0014	0.0011
h=6	-0.0005	0.0014	-0.0029*	0.0017
h=9	-0.0014	0.0017	-0.0042*	0.0025
h=12	0.0000	0.0020	-0.0023	0.0030

Notes: Newey–West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. In Panel (a), the propagation regression is $\lambda_{t+h} = \alpha_h + \beta_1 \lambda_t + \beta_2 ND_t + \beta_3 inf_t + \beta_4 m2_t + \beta_5 jln_t + \epsilon_{t+h}$, where ND denotes nominal distortion, defined as a period in which both inflation and M2 growth are above their respective median values. β_2 measures the association between nominal distortion at time t and risk compensation at horizon h . Adjusted R^2 is reported below each R^2 value. In Panel (b), $D_t = \lambda_t^{dollar} - \lambda_t^{gold}$, and the regression is $D_{t+h} = \alpha_h + \beta_1 D_t + \beta_2 ND_t + \beta_3 inf_t + \beta_4 m2_t + \beta_5 jln_t + \epsilon_{t+h}$. A negative and significant β_2 indicates that nominal distortion is associated with a larger decline in dollar-denominated than gold-denominated risk compensation at horizon h .

In the macroeconomic regime specifications, the only robust positive interaction is between lagged price of risk and the uncertainty regime in the gold-denominated specification, with no comparable effect in the dollar-denominated specification. Elevated uncertainty therefore strengthens the persistence of the price of risk when revalued in gold, implying slower adjustment toward its long-run level. The absence of a comparable effect in dollars suggests that nominal variation in dollar-denominated risk compensation can obscure the dynamic response to underlying uncertainty. Once these short-lived nominal variations are attenuated through gold revaluation, the persistence of risk compensation under elevated uncertainty becomes more pronounced, providing a clearer view of investors' underlying required compensation for risk.

A marginally significant negative interaction between lagged dollar-denominated price of risk and the M2 regime emerges under the 2000-particle specification, suggesting faster adjustment during periods of elevated monetary growth. However, this effect is not robust across particle configurations and is therefore interpreted cautiously. There is little evidence that inflation conditions significantly alter persistence under either measure.

4.3 Risk-Return Relationship in Dollars and Gold

Up to this point, we have shown that the gold-denominated measure attenuates nominal distortion embedded in risk compensation and reveals risk-pricing dynamics that are partially obscured under the conventional dollar-denominated measure within a one-period framework. We now turn to our second question: whether revaluation in gold helps explain the risk-return puzzle. We examine the risk-return relationship while incorporating the role of money illusion and extending the analysis beyond the one-period framework to capture both contemporaneous and intertemporal dynamics.

4.3.1 Baseline Risk-Return Dynamics

We first establish the baseline relationship between expected returns and volatility across units of account, following the conventional risk-return specification

$$\mu_{t+h} = \alpha_h + \beta_1(\sigma_t)$$

estimated contemporaneously ($h = 0$) and over intertemporal horizons ($h = 1, 3, 6, 9, 12$). Volatility is consistently and positively associated with expected returns across horizons and units of account, consistent with the standard prediction that higher risk commands higher expected returns. The explanatory power of volatility is substantially higher when returns

are revalued in gold: σ_t alone accounts for 91-93% of the variation in contemporaneous expected returns under the gold-denominated specification, compared with 75-78% under the dollar-denominated specification. This gap persists and widens across longer horizons. The stronger explanatory power of volatility under the gold-denominated measure suggests that revaluing returns in purchasing-power terms yields a closer correspondence between measured risk and expected returns, consistent with the broader evidence that revaluation in gold attenuates nominal variation

Importantly, revaluation in gold does not mechanically generate the risk-return relationship. The persistence of the positive relationship across specifications and horizons establishes a meaningful baseline for the regime-augmented analysis that follows.

4.3.2 Regime-Augmented Risk-Return Dynamics

The baseline analysis establishes a positive risk-return relationship across horizons and units of account. We next examine whether this relationship varies with nominal conditions, motivated by the economic intuition that conditions observed at $t - 1$ may shape the subsequent risk-return relationship over horizon h . We consider two specifications: (A) the joint interaction between inflation and M2 growth, which captures the nominal distortion identified above, and (B) positive inflation surprises, which capture the direct effect of unexpected inflation.

Specification A:

$$\mu_{t+h} = \alpha + \beta_1 \sigma_t + \beta_2 \text{inf}_{t-1} + \beta_3 m2_{t-1} + \beta_4 (\sigma_t \times \text{inf}_{t-1}) + \beta_5 (\sigma_t \times m2_{t-1}) + \beta_6 (\sigma_t \times \text{inf}_{t-1} \times m2_{t-1}) + \epsilon_{t+h}$$

where β_6 captures whether the risk-return relationship varies with the joint inflation and M2 growth environment.

Specification B:

$$\mu_{t+h} = \alpha + \beta_1 \sigma_t + \beta_2 \text{inf}_{U,t-1}^+ + \beta_3 (\sigma_t \times \text{inf}_{U,t-1}^+) + \epsilon_{t+h}$$

where β_3 captures the change in the risk-return relationship following a positive inflation surprise. All explanatory variables are demeaned before constructing interaction terms. Both specifications are estimated over the same horizons as the baseline regression.

For specification A, the σ_t coefficient remains significantly positive across horizons and particle configurations for both units of account. The explanatory power of volatility also remains higher under the gold-denominated specification. The triple interaction, however, is less robust across particle configurations. Under the dollar-denominated specification,

it is statistically insignificant in the 2,000-particle configuration and becomes significantly positive from $h=3$ onward in the 2,500-particle configuration. Under the gold-denominated specification, the interaction is positive and significant from $h=3$ onward in the 2,000-particle configuration, while significance emerges only at longer horizons in the 2,500-particle configuration. The positive risk-return relationship therefore strengthens at longer horizons under conditions of elevated inflation and M2 growth, particularly under the gold-denominated specification. This delayed strengthening is in line with evidence that the positive risk-return trade-off can emerge more clearly at longer horizons and depend on underlying economic conditions (Andrei et al. (2019); Lochstoer and Muir (2022)). Revaluation in gold makes this state-dependent variation in the risk-return relationship more visible.

For specification B, the positive σ_t coefficient remains significant throughout. The interaction with positive inflation surprises differs more sharply across configurations. Under the dollar-denominated specification, the interaction is positive and significant at $h = 1$ and $h = 3$ in the 2,000-particle configuration but is insignificant in the 2,500-particle configuration. Under the gold-denominated specification, the interaction is insignificant in the 2,000-particle configuration, while the 2,500-particle configuration produces a negative and increasingly significant effect from $h = 3$ onward. Thus, positive inflation surprises weaken the positive risk-return relationship when returns are revalued in gold at longer horizons in the finer particle configuration. Given its sensitivity to particle configuration, this evidence is suggestive rather than definitive.

Collectively, the baseline positive risk-return relationship persists across units of account, while its state dependence varies with the nominal environment. These findings do not resolve the risk-return puzzle, but suggest that revaluation in gold is a relevant consideration in its empirical analysis.

5 Robustness Checks

We conduct a series of robustness checks to assess whether the findings reflect information content embedded in the gold-denominated measure rather than model-specific effects.

5.1 Mechanical or Informational Divergence

A key concern is that the documented divergence between the dollar- and gold-denominated measures arise mechanically from the transformation into gold units rather than from economically meaningful variation in the gold-denominated measures. To address this concern, we conduct subsample analyses based on gold price volatility and major monetary regimes.

Table 4.2: Baseline Risk-Return Dynamics

Unit	Dollar				Gold			
Horizon	2000		2500		2000		2500	
Variable	σ_t	R^2	σ_t	R^2	σ_t	R^2	σ_t	R^2
h=0	0.0171***	0.7474	0.0171***	0.7816	0.0164***	0.9343	0.0108***	0.9057
	(0.0014)	0.7471	(0.0011)	0.7813	(0.0005)	0.9342	(0.0004)	0.9056
h=1	0.0169***	0.7317	0.0170***	0.7697	0.0163***	0.9240	0.0108***	0.8996
	(0.0014)	0.7314	(0.0011)	0.7694	(0.0006)	0.9239	(0.0004)	0.8894
h=3	0.0163***	0.6777	0.0166***	0.7295	0.0160***	0.8921	0.0106***	0.8751
	(0.0014)	0.6773	(0.0011)	0.7292	(0.0007)	0.8919	(0.0005)	0.8750
h=6	0.0146***	0.5481	0.0155***	0.6345	0.0153***	0.8243	0.0102***	0.8154
	(0.0015)	0.5475	(0.0013)	0.6341	(0.0008)	0.8241	(0.0005)	0.8152
h=9	0.0124***	0.4005	0.0140***	0.5176	0.0145***	0.7429	0.0097***	0.7417
	(0.0017)	0.3997	(0.0014)	0.5170	(0.0010)	0.7425	(0.0006)	0.7414
h=12	0.0102***	0.2696	0.0124***	0.3996	0.0135***	0.6519	0.0091***	0.6618
	(0.0017)	0.2687	(0.0015)	0.3988	(0.0011)	0.6515	(0.0007)	0.6614

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Adjusted R^2 is reported below each R^2 value. The dynamic horizon regression is specified as $\mu_{t+h} = \alpha_h + \beta_1 \sigma_t$. The horizon $h = 0$ captures the contemporaneous relationship between expected returns μ and volatility σ , while $h = 1, 3, 6, 9, 12$ captures the effect of current volatility on expected return h months ahead.

Table 4.3: Regime-Augmented Risk-Return Relationship (Inflation $\times$ M2 Growth)

Horizon	0		1		3		6		9		12	
	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500
Dollars												
σ_t	0.0161***	0.0178***	0.0158***	0.0177***	0.0152***	0.0172***	0.0133***	0.0160***	0.0110***	0.0142***	0.0086***	0.0123***
	(0.0013)	(0.0010)	(0.0013)	(0.0011)	(0.0013)	(0.0012)	(0.0014)	(0.0014)	(0.0015)	(0.0015)	(0.0016)	(0.0016)
inf_{t-1}	0.0115***	-0.0087***	0.0119**	-0.0085***	0.0120***	-0.0080**	0.0130***	-0.0061	0.0147***	-0.0032	0.0143***	-0.0011
	(0.0023)	(0.0028)	(0.0023)	(0.0029)	(0.0026)	(0.0032)	(0.0030)	(0.0037)	(0.0036)	(0.0044)	(0.0041)	(0.0050)
$m2_{t-1}$	0.0032*	-0.0005	0.0032*	-0.0004	0.0039**	0.0005	0.0050**	0.0018	0.0062**	0.0040	0.0079***	0.0061*
	(0.0018)	(0.0020)	(0.0018)	(0.0020)	(0.0019)	(0.0020)	(0.0021)	(0.0023)	(0.0024)	(0.0028)	(0.0028)	(0.0032)
$jl n_{t-1}$	-0.0003	-0.0004	-0.0002	-0.0003	-0.0001	-0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0001
	(0.0002)	(0.0003)	(0.0002)	(0.0003)	(0.0002)	(0.0003)	(0.0002)	(0.0004)	(0.0003)	(0.0005)	(0.0003)	(0.0005)
$\sigma_t \cdot inf_{t-1}$	-0.1974	-0.2982	-0.1971	-0.2874	-0.2079	-0.2589	-0.3073	-0.2880	-0.4773*	-0.4104	-0.5990**	-0.4777*
	(0.2477)	(0.2222)	(0.2378)	(0.2279)	(0.2371)	(0.2445)	(0.2455)	(0.2600)	(0.2599)	(0.2688)	(0.2659)	(0.2772)
$\sigma_t \cdot m2_{t-1}$	-0.0368	-0.0463	-0.0726	-0.0331	-0.0692	0.0144	-0.1511	0.0273	-0.1480	0.0769	-0.2438	0.0621
	(0.1863)	(0.1513)	(0.1833)	(0.1544)	(0.1886)	(0.1657)	(0.2171)	(0.1983)	(0.2411)	(0.2305)	(0.2649)	(0.2497)
$\sigma_t \cdot inf_{t-1} \cdot m2_{t-1}$	-12.2833	17.9049	-14.4367	24.5415	-0.4397	46.0067*	14.6589	70.9854**	23.1909	87.5290***	23.4254	93.6485**
	(19.2414)	(23.4107)	(17.7725)	(24.9415)	(16.4192)	(25.6384)	(17.2387)	(28.3605)	(20.0675)	(32.6193)	(19.9787)	(35.5961)
R^2	0.7771	0.8006	0.7632	0.7870	0.7107	0.7443	0.5908	0.6473	0.4563	0.5327	0.3329	0.4197
Adjusted R^2	0.7751	0.7988	0.7611	0.7850	0.7080	0.7419	0.5870	0.6440	0.4513	0.5284	0.3266	0.4143

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dynamic horizon regression is defined as $\mu_{t+h} = \alpha + \beta_1 \sigma_t + \beta_2 inf_{t-1} + \beta_3 m2_{t-1} + \beta_4 (\sigma_t \times inf_{t-1}) + \beta_5 (\sigma_t \times m2_{t-1}) + \beta_6 (\sigma_t \times inf_{t-1} \times m2_{t-1}) + \epsilon_{t+h}$. The horizon $h = 0$ captures the contemporaneous relationship between expected returns μ and volatility σ , while $h = 1, 3, 6, 9, 12$ examines the relationship between current volatility and expected returns at future horizons.

Table 4.4: Regime-Augmented Risk-Return Relationship (Inflation $\times$ M2 Growth)

Horizon	0		1		3		6		9		12	
	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500
Gold												
σ_t	0.0159***	0.0112***	0.0158***	0.0111***	0.0154***	0.0109***	0.0146***	0.0105***	0.0137***	0.0099***	0.0127***	0.0093***
	(0.0004)	(0.0004)	(0.0005)	(0.0004)	(0.0006)	(0.0005)	(0.0007)	(0.0005)	(0.0009)	(0.0006)	(0.0010)	(0.0007)
inf_{t-1}	0.0043**	-0.0021	0.0043**	-0.0020	0.0055**	-0.0008	0.0079***	0.0003	0.0097***	0.0012	0.0115***	0.0022
	(0.0020)	(0.0014)	(0.0021)	(0.0015)	(0.0023)	.0017	(0.0026)	(0.0019)	(0.0031)	(0.0021)	(0.0039)	(0.0025)
$m2_{t-1}$	0.0023*	-0.0018*	0.0023*	-0.0019*	0.0031**	-0.0014	0.0043**	-0.0005	0.0050**	0.0004	0.0055**	0.0011
	(0.0012)	(0.0010)	(0.0012)	(0.0010)	(0.0014)	(0.0010)	(0.0018)	(0.0011)	(0.0023)	(0.0012)	(0.0026)	(0.0014)
$jl n_{t-1}$	-0.0000	0.0002	0.0000	0.0002	0.0002	0.0003	0.0003	0.0005**	0.0003	0.0005*	0.0001	0.0003
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0003)	(0.0003)
$\sigma_t \cdot inf_{t-1}$	0.1495	-0.1798**	0.1193	-0.1986***	0.0471	-0.2589***	-0.0753	-0.3257***	-0.2093	-0.4074***	-0.3459*	-0.5013***
	(0.1192)	(0.0710)	(0.1225)	(0.0719)	(0.1274)	(0.0777)	(0.1460)	(0.0959)	(0.1667)	(0.1157)	(0.1846)	(0.1311)
$\sigma_t \cdot m2_{t-1}$	0.2956***	-0.1393*	0.2935***	-0.1310*	0.2981**	-0.1254	0.2595*	-0.1283	0.2949*	-0.0780	0.3394*	-0.0315
	(0.0946)	(0.0824)	(0.1003)	(0.0788)	(0.1173)	(0.0786)	(0.1430)	(0.0837)	(0.1650)	(0.1001)	(0.1907)	(0.1188)
$\sigma_t \cdot R_{t-1}$	5.6752	-20.5228*	12.9723	-15.4438	33.2281***	-0.0992	50.6155***	15.2522	62.4177***	26.8481**	68.7025***	33.6279**
	(9.3652)	(12.3981)	(9.3638)	(12.5698)	(10.8650)	(13.2703)	(14.6959)	(13.1912)	(17.4689)	(12.7968)	(22.8870)	(14.7142)
R^2	0.9418	0.9160	0.9311	0.9101	0.9010	0.8859	0.8365	0.8285	0.7590	0.7589	0.6729	0.6858
Adjusted R^2	0.9413	0.9152	0.9305	0.9093	0.9001	0.8848	0.8350	0.8269	0.7568	0.7566	0.6698	0.6828

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dynamic horizon regression is defined as $\mu_{t+h} = \alpha + \beta_1 \sigma_t + \beta_2 inf_{t-1} + \beta_3 m2_{t-1} + \beta_4 (\sigma_t \times inf_{t-1}) + \beta_5 (\sigma_t \times m2_{t-1}) + \beta_6 (\sigma_t \times R_{t-1}) + \epsilon_{t+h}$, where R_{t-1} represents $inf_{t-1} \times m2_{t-1}$. The horizon $h = 0$ captures the contemporaneous relationship between expected returns μ and volatility σ , while $h = 1, 3, 6, 9, 12$ examines the relationship between current volatility and expected returns at future horizons.

Table 4.5: Regime-Augmented Risk-Return Relationship (Inflation Suprises)

Horizon	0		1		3		6		9		12	
	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500
Dollars												
σ_t	0.0164*** (0.0014)	0.0176*** (0.0010)	0.0161*** (0.0014)	0.0174*** (0.0011)	0.0154*** (0.0014)	0.0169*** (0.0011)	0.0137*** (0.0016)	0.0158*** (0.0013)	0.0117*** (0.0018)	0.0145*** (0.0015)	0.0096*** (0.0019)	0.0130*** (0.0016)
$inf_{U,t-1}^+$	0.0059* (0.0032)	-0.0132*** (0.0041)	0.0066** (0.0033)	-0.0130*** (0.0042)	0.0072* (0.0039)	-0.0119** (0.0048)	0.0085* (0.0048)	-0.0101* (0.0053)	0.0082 (0.0052)	-0.0103* (0.0061)	0.0053 (0.0057)	-0.0119* (0.0069)
$\sigma_t \cdot inf_{U,t-1}^+$	0.5042 (0.3394)	-0.2687 (0.2113)	0.6115* (0.3236)	-0.1838 (0.2205)	0.6687* (0.3600)	-0.1307 (0.2500)	0.6684 (0.4153)	-0.1555 (0.2959)	0.5326 (0.4495)	-0.3526 (0.3514)	0.4097 (0.4528)	-0.4845 (0.3773)
R^2	0.7535	0.7894	0.7401	0.7765	0.6875	0.7350	0.5600	0.6388	0.4096	0.5233	0.2739	0.4082
Adjusted R^2	0.7525	0.7765	0.7391	0.7756	0.6862	0.7340	0.5582	0.6373	0.4073	0.5214	0.2710	0.4059
Gold												
σ_t	0.0164*** (0.0006)	0.0110*** (0.0004)	0.0162*** (0.0006)	0.0109*** (0.0004)	0.0159*** (0.0007)	0.0108*** (0.0005)	0.0153*** (0.0009)	0.0105*** (0.0006)	0.0145*** (0.0011)	0.0101*** (0.0007)	0.0137*** (0.0013)	0.0097*** (0.0008)
$inf_{U,t-1}^+$	0.0006 (0.0035)	0.0037* (0.0020)	0.0002 (0.0038)	0.0037* (0.0021)	0.0018 (0.0040)	0.0053** (0.0022)	0.0024 (0.0047)	0.0050* (0.0029)	0.0006 (0.0052)	0.0037 (0.0033)	-0.0019 (0.0060)	0.0015 (0.0040)
$\sigma_t \cdot inf_{U,t-1}^+$	0.0635 (0.1679)	-0.2079 (0.1569)	0.0742 (0.1774)	-0.2086 (0.1549)	0.0222 (0.1960)	-0.2691* (0.1533)	0.0038 (0.2330)	-0.3045* (0.1630)	-0.0997 (0.2493)	-0.3914** (0.1552)	-0.2194 (0.2860)	-0.5225*** (0.1845)
R^2	0.9342	0.9066	0.9239	0.9004	0.8920	0.8767	0.8241	0.8170	0.7424	0.7439	0.6519	0.6666
Adjusted R^2	0.9340	0.9063	0.9236	0.9000	0.8915	0.8763	0.8234	0.8162	0.7414	0.7429	0.6506	0.6652

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dynamic horizon regression is defined as $\mu_{t+h} = \alpha + \beta_1 \sigma_t + \beta_2 inf_{U,t-1}^+ + \beta_3 (\sigma_t \times inf_{U,t-1}^+) + \epsilon_{t+h}$. The horizon $h = 0$ captures the contemporaneous relationship between expected returns μ and volatility σ , while $h = 1, 3, 6, 9, 12$ examines the relationship between current volatility and expected returns at future horizons.

Appendix Tables 10.10 and 10.11 show that, under low gold price volatility and institutionally anchored gold prices, the divergence in real GDP growth between the two measures is small. Similar patterns are observed for the price of risk across both the 2,000- and 2,500-particle specifications. When gold prices are relatively stable or unable to adjust freely to prevailing macroeconomic conditions, the dollar- and gold-denominated measures produce broadly similar results, providing little evidence that the transformation into gold units mechanically generates systematic divergence.

A markedly different pattern emerges once gold prices become market-determined following the breakdown of the Bretton Woods system. The divergence in both real GDP growth and the price of risk increases with gold price volatility and is substantially larger under the fiat regime, when gold prices become more responsive to market conditions. This stronger divergence between dollar- and gold-denominated measures reflects economically meaningful variation rather than the arithmetic transformation alone.

5.2 Fundamental or Speculative Gold Prices

We next examine whether the divergence documented in Section 4.1 is attributable to macroeconomic fundamentals embedded in gold prices or non-fundamental price movements.

We decompose monthly log gold returns into a macro-fundamental component and a residual by regressing gold returns on contemporaneous inflation, M2 growth, changes in JLN uncertainty, changes in the dollar index⁹, and changes in real interest rate¹⁰. The estimated coefficients have the expected signs documented in the literature.¹¹ These variables jointly explain approximately 17% of the variations in monthly log gold returns. We use the fitted returns to construct a gold-prices series that retains only the macro-fundamental component of gold price movements, and re-estimate the gold price volatility analysis using this series.

As reported in Table 10.12, the key divergence patterns remain after replacing actual gold prices with the fitted series. The divergence in real GDP growth remains monotonic across volatility regimes. For the price of risk, the pattern is monotonic under the 2,500-particle specification but flatter under the 2,000-particle specification. Thus, the main divergence

⁹The dollar index is the monthly nominal Major Currencies U.S. Dollar Index (Goods Only), used to compute month-on-month changes as a proxy for dollar strength. <https://fred.stlouisfed.org/series/TWEXMMTH>.

¹⁰The real interest rate is computed as the 10-year U.S. Treasury constant maturity yield minus expected inflation from the Livingston Survey. <https://fred.stlouisfed.org/series/GS10>.

¹¹Dollar index changes and real interest rate changes are included as additional explanatory variables because they are key drivers of gold returns and help mitigate omitted-variable concerns. Inflation, M2 growth, and changes in JLN uncertainty enter positively, while changes in the dollar index and real interest rate enter negatively. The fitted gold-return regression is given by $\widehat{gold_ret}_t = 0.0028 + 2.4272 \cdot inf_t + 0.8692 \cdot m2_t + 0.0629 \cdot jln_t - 1.0337 \cdot dollar_t - 0.1700 \cdot real_rate_t$. The fitted gold price series is constructed as $G_{fitted,t} = G_0 \times \exp\left(\sum_{s=1}^t \widehat{gold_ret}_s\right)$, where G_0 is the actual gold price at the start of the sample.

documented in Section 4.1 persists when gold price movements are restricted to their macro-fundamental component. The persistence indicates that the divergence is not driven solely by non-fundamental gold price movements and is consistent with macroeconomic information embedded in gold prices contributing to the differences between the two measures.

5.3 Business-Cycle Control

To assess whether our main findings are robust to business-cycle conditions, we explicitly control for real GDP growth. This addresses the concern that the observed decline in risk compensation could be confounded by business-cycle fluctuations. The VIF for real GDP growth ranges from 1.26 to 1.64 across specifications involving interactions with other macroeconomic variables, indicating no material multicollinearity concern.

We re-estimate the tests of nominal distortion in risk compensation, asymmetric responses to inflation surprises, the disproportionate response of volatility relative to expected returns, persistence across macroeconomic regimes, and risk-return dynamics.

Real GDP growth is statistically significant in most specifications, indicating that business-cycle conditions contain additional information about risk compensation. Importantly, controlling for real GDP growth does not materially alter the main findings reported in Section 3. The disproportionate increase in volatility relative to expected returns under nominal distortion remains evident in the dollar-denominated measure. The asymmetric responses to positive and negative inflation surprises are also preserved. In addition, elevated uncertainty continues to strengthen persistence in the gold-denominated measure, while the risk-return relationship continues to strengthen at long horizons under gold revaluation.

The inflation $\times$ M2 interaction in the dollar-denominated measure becomes insignificant in the 2,000-particle continuous-effect specification, but remains significantly negative in the 2,500-particle and regime-based specification. The decline in dollar-denominated risk compensation and the stronger negative effect of M2 growth under elevated inflation are therefore preserved across the broader set of specifications after accounting for business-cycle conditions.

Overall, the main differences between the dollar- and gold-denominated measures documented in Section 3 are consistent with a nominal-distortion interpretation rather than being solely attributable to business-cycle fluctuations.

5.4 Cross-Country Analysis

Canada, Japan, Switzerland and Turkey are selected to represent distinct monetary environments. Canada shares broadly similar monetary characteristics with the U.S.; Japan

represents low inflation and accommodative monetary conditions; Switzerland represents low inflation and a comparatively disciplined monetary regime; and Turkey represents an inflationary environment characterized by monetary instability. Each sample contains at least 40 years of monthly observations.

The cointegration results reveal systematic cross-country variation in the strength of the long-run gold-money relationship and the preservation of purchasing power. The relationship is strongest for Turkey, while purchasing-power preservation is strongest for Canada; Japan exhibits the weakest overall relationship, with Switzerland falling between these cases.

The Turkey results are particularly informative. The relatively high long-run coefficient and short half-life of reversion indicate a strong and rapidly adjusting long-run relationship between domestic currency and gold prices. The rolling purchasing-power results provide complementary evidence. The share of periods in which gold preserves purchasing power rises steadily with the holding period, from 62.4% at 5 years to 83.8% at 30 years.

Japan provides a useful contrast. Its lower long-run coefficient, lower purchasing-power preservation, and slower speed of reversion correspond to a weaker and more persistent gold-money relationship. Rolling purchasing-power preservation is also consistently lower than in Turkey across all horizons. This contrast is consistent with gold becoming a more relevant reference for purchasing power when domestic inflation and monetary conditions place greater pressure on the currency value.

Despite these cross-country differences, the real GDP divergence pattern is remarkably consistent. Across all four countries, the divergence between dollar- and gold-denominated real GDP widens monotonically with increasing gold price volatility. The divergence is therefore not specific to a particular monetary context and does not appear to arise mechanically from the transformation into gold units.

The cross-country evidence supports the robustness of the gold revaluation framework across diverse monetary environments. Its value is particularly pronounced under high inflation and monetary instability, when conventional nominal measures are most susceptible to nominal distortion.

5.5 Gold-Specificity of the Findings

A final question is whether the findings documented throughout the paper are specific to gold or simply reflect the use of any real asset as an alternative unit of measurement. We replace gold prices with West Texas Intermediate (WTI)¹² oil prices and replicate the same empirical framework using oil units as a comparative case.

¹²<https://fred.stlouisfed.org/series/WTISPLC>

The results reported in Table 10.14 reveal several important differences between gold and oil units. First, the estimated long-run coefficient between oil prices and M2 is substantially smaller than that for gold, at approximately 0.68, with an R^2 of 66%, indicating a weaker long-run association between oil prices and M2. Under the interaction between high inflation and high M2 growth, the price of risk measured in oil units exhibits largely insignificant coefficients across the macroeconomic specifications. Similar insignificance is observed for positive inflation surprises and for the interactions between negative inflation surprises and M2 growth.

In terms of quantitative attenuation, oil units produce a sign reversal under both inflationary-monetary expansion and uncertainty-monetary expansion regimes, with the mean price of risk expanding rather than compressing. This expansion does not necessarily represent a restoration of risk compensation. Oil is a production input whose price contributes directly to inflationary pressures and cyclical economic conditions. Expressing returns relative to oil may therefore introduce additional variation associated with these conditions rather than filter the nominal component identified under gold revaluation.

The divergence between real GDP measured in dollar units and oil units increases from low to medium oil-price-volatility regimes before plateauing at high-volatility, in contrast to the pronounced monotonic divergence observed when real GDP is revalued in gold. The weaker and less systematic divergence further limits the usefulness of oil as a comparative unit of measurement.

The oil results indicate that the findings documented throughout the paper are not a generic consequence of expressing economic and financial variables in real-asset units. The distinct patterns observed under gold revaluation, including a more stable long-run relationship with money supply and higher explanatory power, are not replicated by oil.

6 Conclusion

Real GDP and stock market performance differ systematically when measured in dollar-denominated and gold-denominated terms from a purchasing-power perspective. This systematic divergence extends to forward-looking asset pricing.

Our paper makes two main contributions to the literature. First, gold revaluation attenuates the decline in dollar-denominated risk compensation by approximately 20% when M2 expansion coincides with elevated inflation and by 58% when M2 expansion coincides with heightened uncertainty. Gold-denominated measures also eliminate the decline associated with positive inflation surprises. This attenuation reflects weaker sensitivity to short-lived nominal variation and greater sensitivity to real interest-rate information. This reduces

the disproportionate increase in volatility relative to expected return observed in dollar-denominated measures. Gold revaluation also reveals stronger persistence in risk compensation under elevated uncertainty and stronger long-run risk-return relationship.

Second, by incorporating M2 growth, we reveal an additional dimension of nominal distortion beyond inflation alone. M2 growth makes the decline in dollar-denominated risk compensation more visible when it coincides with elevated inflation, particularly when inflation exceeds expectations. M2 growth also reveals asymmetric responses of risk compensation to inflation surprises.

Gold revaluation therefore provides a different perspective on purchasing power, risk compensation and the risk-return relationship than conventional dollar-denominated measures. These findings are consistent with money illusion in the stock market and economic growth. Future research may extend the gold-denominated framework to cross-asset valuation and return predictability, including whether its persistence dynamics under elevated uncertainty help explain broader asset-pricing anomalies.

7 References

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8 Appendix

Appendix 1 Initialization of Parameter Values

We specify initial parameter values subject to economically meaningful constraints. The parameter vector is defined as:

$$\Theta = (\alpha_\mu, \beta_\mu, \sigma_\mu, \alpha_\sigma, \beta_\sigma, \sigma_\sigma, \rho_{\epsilon_\mu, \epsilon_\sigma})$$

We initialize the long-run means of expected returns and volatility, $\frac{\alpha_\mu}{1-\beta_\mu}$ and $\frac{\alpha_\sigma}{1-\beta_\sigma}$, using the sample mean and volatility of realized excess returns, from which α_μ and α_σ are derived. We set the persistence parameters β_μ and β_σ to 0.90 and constrain $0.5 \leq \beta \leq 0.9999$ to ensure stationarity and avoid near-explosive persistence. The innovation volatilities σ_μ and σ_σ are initialized at 10% of the corresponding long-run mean and volatility.

The correlation parameter $\rho_{\epsilon_\mu, \epsilon_\sigma}$ captures the co-movement between shocks to expected returns and volatility. In the absence of a universally accepted benchmark, we initialize it at 0.20. The derivation of these correlated shocks is presented in Appendix 2. During estimation, $\rho_{\epsilon_\mu, \epsilon_\sigma}$ is constrained to (-1,1) and subject to a soft Gaussian penalty to discourage boundary solutions:

$$-\frac{1}{2} \left(\frac{\rho - \mu_\rho}{\sigma_\rho} \right)^2$$

where μ_ρ and σ_ρ denote the prior mean and standard deviation. Similar penalization terms are applied to the μ -state parameters (β_μ, σ_μ) to stabilize estimation given their weaker identification relative to the σ -related parameters. Details of the soft-penalty procedure are provided in Appendix 5.

Appendix 2 Derivation of Correlated Shocks

After parameter initialization, we specify the state dynamics using simulated innovations. We generate two independent standard normal random variables,

$$z_1, z_2 \sim \mathcal{N}(0, 1), \quad \text{i.i.d.}$$

and define the expected return shock as $\epsilon_{\mu,t} = z_1$. The volatility shock is constructed as a linear combination of the two independent innovations:

$$\epsilon_{\sigma,t} = az_1 + bz_2$$

Because $\epsilon_{\mu,t} = z_1$,

$$\begin{aligned}\text{Cov}(\epsilon_{\mu,t}, \epsilon_{\sigma,t}) &= \text{Cov}(z_1, az_1 + bz_2) \\ &= a \underbrace{\text{Cov}(z_1, z_1)}_{=1} + b \underbrace{\text{Cov}(z_1, z_2)}_{=0} \\ &= a.\end{aligned}$$

Since both shocks are standardized to have unit variance.

$$\rho_{\epsilon_{\mu}, \epsilon_{\sigma}} = \text{Cov}(\epsilon_{\mu,t}, \epsilon_{\sigma,t}) = a.$$

To ensure $\text{Var}(\epsilon_{\sigma,t}) = 1$

$$\begin{aligned}\text{Var}(\epsilon_{\sigma,t}) &= \text{Var}(az_1 + bz_2) = 1 \\ &= a^2(\text{Var}(z_1)) + b^2(\text{Var}(z_2)) \\ 1 &= a^2 + b^2\end{aligned}$$

Setting $a = \rho_{\epsilon_{\mu}, \epsilon_{\sigma}}$, therefore gives

$$b = \sqrt{1 - \rho_{\epsilon_{\mu}, \epsilon_{\sigma}}^2},$$

and hence

$$\epsilon_{\sigma,t} = \underbrace{\rho_{\epsilon_{\mu}, \epsilon_{\sigma}} z_1}_{\text{common component}} + \underbrace{\sqrt{1 - \rho_{\epsilon_{\mu}, \epsilon_{\sigma}}^2} z_2}_{\text{idiosyncratic component}}.$$

Thus, the first component induces correlation between expected return and volatility shocks, while the second component is specific to the volatility shock. These innovations are then incorporated into the state equations in Equations (1) and (2).

Appendix 3 Systematic Resampling

In the forward-filtering step, we employ systematic resampling rather than simple random resampling (Douc et al. (2005)). Each particle, characterized by its associated expected return and volatility states, is assigned a weight reflecting its relative importance, and resampling is performed in proportion to these weights.

Systematic resampling generates N equally spaced sampling positions from a single draw from a uniform distribution:

$$u_i = u_1 + \frac{i-1}{N}, \quad i = 1, \dots, N$$

where $u_1 \sim U(0, 1/N)$. Each u_i is then mapped to the cumulative weight intervals, selecting the particle whose interval contains the sampling position.

This procedure selects particles in proportion to their weights while reducing sampling variability relative to independent random draws. Particles with higher weights may therefore be selected multiple times, whereas particles with relatively low weights may be discarded during resampling.

Appendix 4 Log-Likelihood Evaluation via Particle Filtering

This appendix describes how the likelihood function is evaluated within the particle-filtering framework and how particle weights are constructed.

Conditional on the latent states (μ_t, σ_t) , the observed return r_{t+1} follows

$$r_{t+1} \mid \mu_t, \sigma_t \sim \mathcal{N}(\mu_t \Delta, \sigma_t^2 \Delta)$$

For particle i , the conditional density is

$$p(r_{t+1} \mid \mu_t^{(i)}, \sigma_t^{(i)}) = \frac{1}{\sqrt{2\pi}\sigma_t^{(i)}\sqrt{\Delta}} \exp \left[-\frac{(r_{t+1} - \mu_t^{(i)} \Delta)^2}{2(\sigma_t^{(i)})^2 \Delta} \right]$$

Let $LL_t^{(i)} = p(r_{t+1} \mid \mu_t^{(i)}, \sigma_t^{(i)})$ denote the likelihood, and $\log(LL_t^{(i)})$ the corresponding log-likelihood, for each particle:

$$\log(LL_t^{(i)}) = \log \left(\frac{1}{\sqrt{2\pi}\sigma_t^{(i)}\sqrt{\Delta}} \times \exp \left[-\frac{(r_{t+1} - \mu_t^{(i)} \Delta)^2}{2(\sigma_t^{(i)})^2 \Delta} \right] \right)$$

Each particle is assigned a weight proportional to its conditional likelihood,

$$w_t^{(i)} \propto p(r_{t+1} \mid \mu_t^{(i)}, \sigma_t^{(i)}) \propto \exp^{\log(LL_t^{(i)})},$$

and the weight are normalized as:

$$\tilde{w}_t^{(i)} = \frac{w_t^{(i)}}{\sum_{j=1}^N w_t^{(j)}}.$$

To avoid numerical underflow when exponentiating log-likelihoods, we apply a standard

log-sum-exp transformation.

Let

$$m_t = \max_{1 \leq i \leq N} \log(LL_t^{(i)}).$$

The likelihood contribution at time t is then evaluated as

$$\begin{aligned} \log L_t(\theta) &= \log \left[\frac{1}{N} \sum_{i=1}^N \exp \left(\log(LL_t^{(i)}) \right) \right] \\ &= m_t + \log \left[\frac{1}{N} \sum_{i=1}^N \exp \left(\log(LL_t^{(i)}) - m_t \right) \right]. \end{aligned}$$

where $\log L_t(\theta)$ denotes the likelihood contribution from the observed return at time $t + 1$, computed by aggregating the particle likelihoods. Summing these contributions over the sample gives the log-likelihood for each simulation run, denoted by $\log L^{(k)}(\theta)$.

We conduct $K = 30$ independent simulation runs, using different random draws in each run. The final objective function is obtained by averaging the resulting log-likelihoods:

$$\log L^*(\theta) = \frac{1}{K} \sum_{k=1}^K \log L^{(k)}(\theta), \quad K = 30.$$

Thus, the reported log-likelihood is the average across the 30 independent simulation runs, reducing the influence of simulation-specific random variation.

Appendix 5 Soft Penalization and Particle Degeneracy Mitigation

Because the μ_t s process is relatively weakly identified, we impose soft penalization on its persistence and innovation-volatility parameters, β_μ and σ_μ , as well as on the shock correction parameter ρ . The σ_t -related parameters are left unpenalized because they are more directly identified by realized return volatility.

$$\log L^*(\theta) = \log L^*(\theta) - \frac{1}{2} \left(\frac{\rho - \mu_\rho}{\sigma_\rho} \right)^2 - \frac{1}{2} \left(\frac{\sigma_\mu - \mu_{\sigma_\mu}}{\sigma_{\sigma_\mu}} \right)^2 - \frac{1}{2} \left(\frac{\beta_\mu - 0.9}{0.05} \right)^2$$

The prior mean and standard deviation for ρ are set to 0.20 and 0.20, respectively. For σ_μ , both the prior mean and standard deviation are set to 10% of the absolute long-run mean of realized returns. For β_μ , the penalty is centered at 0.9 with a standard deviation of 0.05, reflecting a conventional persistence value in the empirical literature.

To mitigate particle degeneracy, we compute the effective sample size (ESS) at each time

step Martino et al. (2017):

$$ESS_t = \frac{1}{\sum_{i=1}^N (\tilde{w}_t^{(i)})^2}.$$

Particles are resampled whenever ESS_t falls below half the number of particles.

Appendix 6 Parameters Estimation and Optimization

Given that the likelihood is evaluated using particle filtering, the resulting objective function is subject to simulation noise. We therefore employ the derivative-free Nelder-Mead method, which is well suited to optimization problems with noisy objective functions and does not require gradient evaluation (Nelder and Mead (1965); Larson et al. (2019)). We allow a maximum of 1,000 iterations and terminate the procedure when changes in both the parameters and the objective function fall below a tolerance value of 0.0001.

We repeat the optimization procedure across three independent runs and select the parameter set yielding the highest log-likelihood. Small perturbations to the initial parameter values across runs are used to reduce sensitivity to starting values and improve robustness to local optima.

Appendix 7 Backward Smoothing

Forward filtering yields weighted particles at each point in time based only on information available up to that point. To incorporate information from the full sample and obtain smoothed trajectories for the subsequent regression analysis, we apply a backward smoothing procedure (Kitagawa (2012); Godsill et al. (2012)). The procedure samples particles at each preceding time point according to both their filtering weights and the transition density linking them to the selected future state.

The procedure begins at time T , where a reference particle is sampled according to the filtering weights. For each preceding time t , the probability of selecting particle i is determined by its filtering weight and the transition density from $x_t^{(i)}$ to the selected state x_{t+1} , where

$$x_t = (\mu_t, \sigma_t)'$$

Given the state equations, the transition density is

$$p(x_{t+1} \mid x_t) \sim \mathcal{N}(m_t, \Sigma),$$

where

$$m_t = \begin{pmatrix} \alpha_\mu + \beta_\mu \mu_t \\ \alpha_\sigma + \beta_\sigma \sigma_t \end{pmatrix},$$

and the covariance matrix of the correlated state innovations is

$$\Sigma = \begin{pmatrix} \sigma_\mu^2 & \rho \sigma_\mu \sigma_\sigma \\ \rho \sigma_\mu \sigma_\sigma & \sigma_\sigma^2 \end{pmatrix}.$$

Its determinant and inverse are

$$|\Sigma| = \sigma_\mu^2 \sigma_\sigma^2 - (\rho \sigma_\mu \sigma_\sigma)^2,$$

and

$$\Sigma^{-1} = \frac{1}{|\Sigma|} \begin{pmatrix} \sigma_\sigma^2 & -\rho \sigma_\mu \sigma_\sigma \\ -\rho \sigma_\mu \sigma_\sigma & \sigma_\mu^2 \end{pmatrix}, \quad |\Sigma| > 0.$$

For the two-dimensional state vector, the transition density is therefore

$$p(x_{t+1} | x_t) = \frac{1}{(2\pi) |\Sigma|^{\frac{1}{2}}} \exp \left[-\frac{1}{2} (x_{t+1} - m_t)' \Sigma^{-1} (x_{t+1} - m_t) \right].$$

The corresponding log-transition density is

$$\log p(x_{t+1} | x_t) = \underbrace{-\log(2\pi) - \frac{1}{2} \log |\Sigma|}_{\text{constant part}} \underbrace{- \frac{1}{2} (x_{t+1} - m_t)' \Sigma^{-1} (x_{t+1} - m_t)}_{\text{variable part}}.$$

The constant part depends only on Σ and is identical across particles at a given time. The variable part, by contrast, depends on the deviation of x_{t+1} from the conditional mean m_t , accounting for correlation and scale, and therefore varies across particles.

The conditional expectations of expected returns and volatility for each particle i , given information at time t are

$$E[\mu_{t+1}^{(i)} | \mu_t^{(i)}] = \alpha_\mu + \beta_\mu \mu_t^{(i)} \quad E[\sigma_{t+1}^{(i)} | \sigma_t^{(i)}] = \alpha_\sigma + \beta_\sigma \sigma_t^{(i)}$$

with corresponding prediction errors

$$\Delta\mu^{(i)} = \mu_{t+1} - E[\mu_{t+1}^{(i)} | \mu_t^{(i)}] \quad \Delta\sigma^{(i)} = \sigma_{t+1} - E[\sigma_{t+1}^{(i)} | \sigma_t^{(i)}],$$

where μ_{t+1} and σ_{t+1} are the state values of the selected reference particle at time $t + 1$.

The variable component for each particle can then be written as

$$\begin{aligned}
(x_{t+1} - m_t)^\top \Sigma^{-1} (x_{t+1} - m_t) &= \begin{pmatrix} \Delta\mu^{(i)} & \Delta\sigma^{(i)} \end{pmatrix} \frac{1}{|\Sigma|} \begin{pmatrix} \sigma_\sigma^2 & -\rho \sigma_\mu \sigma_\sigma \\ -\rho \sigma_\mu \sigma_\sigma & \sigma_\mu^2 \end{pmatrix} \begin{pmatrix} \Delta\mu^{(i)} \\ \Delta\sigma^{(i)} \end{pmatrix} \\
&= \frac{1}{|\Sigma|} [\sigma_\sigma^2 (\Delta\mu^{(i)})^2 - 2\rho \sigma_\mu \sigma_\sigma (\Delta\mu^{(i)})(\Delta\sigma^{(i)}) + \sigma_\mu^2 (\Delta\sigma^{(i)})^2].
\end{aligned}$$

Thus, particles whose predicted states are closer to the selected future state receive a higher transition density and are more likely to be selected in the backward sampling step.

The backward sampling weight for particle i is given by

$$b_t^{(i)} \propto \tilde{w}_t^{(i)} \times p(x_{t+1} | x_t^{(i)}),$$

where $\tilde{w}_t^{(i)}$ is the normalized filtering weight. To improve numerical stability, the transition log-density is centered by its maximum across particles before exponentiation:

$$\widetilde{\log p}^{(i)} = \log p(x_{t+1} | x_t^{(i)}) - \max_j \left\{ \log p(x_{t+1} | x_t^{(j)}) \right\}.$$

Because different backward draws may produce different state trajectories, we conduct multiple backward smoothing runs. Specifically, we generate $M = 2,000$ or $2,500$ smoothing paths, matching the number of particles used in the corresponding forward-filtering specification. The smoothed expected return ($\tilde{\mu}_t$) and volatility ($\tilde{\sigma}_t$) are obtained by averaging across the simulated paths:

$$\tilde{\mu}_t = \frac{1}{M} \sum_{m=1}^M \mu_t^{(m)} \quad \tilde{\sigma}_t = \frac{1}{M} \sum_{m=1}^M \sigma_t^{(m)}$$

These smoothed latent paths are then used in the ex-ante analysis.

10 Tables and Figures

Table 10.1: Long-Run Relationships between Gold Prices, CPI, and M2

(a) Long-Run Relationship between Gold Prices and M2 Supply

Sample	Long-Run β_1	R^2	Residual ADF p-value	Result	Reversion β_1^R	Half Life
Bretton Woods ^a	0.1821***	0.4197	0.0081	X	-0.0358	1.6 years
Post-Bretton Woods	0.8193***	0.8079	0.0016	Yes (1%)	-0.0120	4.8 years

(b) Long-Run Relationship between CPI and M2 Supply

Sample	Long-Run β_1	R^2	Residual ADF p-value	Result	Reversion β_1^R	Half Life
Bretton Woods ^b	0.4081**	0.9009	0.2452	X	X	X
Post-Bretton Woods ^c	0.5559**	0.9500	0.1209	X	-0.0036	15.8 years

Notes: ^a log M2 series; ^b log M2 and log CPI series; ^c log CPI series do not satisfy the non-stationarity precondition required for the cointegration test during the Bretton Woods subsample. The reported statistics for this period are therefore included for illustrative purposes only. For the Post-Bretton Woods subsample in Panel (a), Newey-West standard errors with twelve lags are used to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The long-run coefficient (β_1) is estimated from $\log(\text{Gold_Price}_t) = \alpha + \beta_1 \log(\text{M2}_t) + \epsilon_t$, capturing the long-run association between gold prices and M2. Residuals are computed as the difference between realized and fitted log gold prices and tested for stationarity using the Augmented Dickey-Fuller (ADF) test. Cointegration is inferred when the residual series is stationary. The reversion coefficient β_1^R is estimated from the error-correction specification $\Delta \epsilon_t = \alpha + \beta_1^R \epsilon_{t-1} + u_t$, where ϵ_t denotes the residual from the long-run regression. A negative β_1^R indicates gradual correction of deviations from the long-run equilibrium. The half-life is calculated as $\frac{\ln(2)}{|\ln(1 + \beta_1^R)|}$, measuring the time required for a deviation from the long-run equilibrium to decline by one-half.

Table 10.2: Rolling Purchasing-Power Preservation of Gold

Rolling Window (Years)	5	10	20	30
Share of Windows Preserving Purchasing Power	53.3%	54.7%	60.6%	83.0%
Mean Purchasing Power Ratio	1.236	1.432	1.769	1.584
25th Percentile Purchasing Power Ratio	0.813	0.720	0.764	1.198
Median Purchasing Power Ratio	1.074	1.107	1.494	1.448
75th Percentile Purchasing Power Ratio	1.530	1.754	2.763	1.877
Number of Observations	576	516	396	276

Notes: The real gold price is calculated as the nominal gold price divided by the CPI level. The purchasing-power ratio is the future real gold price relative to the current real gold price over 5-, 10-, 20-, and 30-year horizons. A ratio above one indicates an increase in purchasing power, while a ratio below one indicates a decrease. The share of windows preserving purchasing power is the proportion of rolling windows with a ratio at or above one. Each rolling window requires at least 120 monthly observations to ensure that the reported shares are not driven by an unusually small number of observations.

Table 10.3: Real GDP and Stock Market Performance in Dollars and Gold

Indicator	Real GDP		Stock Market	
	Dollars	Gold	Dollars	Gold
Number of Recessions	4.0	2.0		
Number of Depressions	2.0	5.0		
Number of Bear Market Events			10.0	4.0
Average Duration (year)	1.8	9.4	1.2	3.4
Average Cumulative Decline (log)	0.0916	0.5758	0.540	1.186
Average Recovery Time (year)	1.8	2.8	2.9	7.9
Mean	0.0312	-0.0205	0.0055	0.0041
Volatility	0.0461	0.1676	0.0538	0.0662
Variance Ratio (Horizon)				
2	1.4970	1.4315	1.0724	1.1305
3	1.7402	1.5925	1.0834	1.1500
5	1.4523	1.6642	1.0392	1.1589
10	1.2285	2.0064	1.1346	1.2625

Notes: A recession is defined as a peak-to-trough cumulative decline in log real GDP exceeding 1% but less than 5%; a depression as a decline exceeding 5%. A bear market is defined as a peak-to-trough cumulative decline in log stock market value exceeding 20%. Real GDP statistics are computed over 1929–2024, while stock market statistics are computed over 1926–2024. Average duration, cumulative decline, and recovery time are computed across all identified events. Mean represents average real GDP growth and average stock market excess return for the dollar-denominated and gold-denominated measures; volatility represents the standard deviation of real GDP growth and stock market excess returns for the dollar-denominated and gold-denominated measures. The variance ratio captures the propagation of shocks across horizons: $VR > 1$ indicates persistence, $VR < 1$ indicates mean reversion, and $VR = 1$ indicates a random walk.

Table 10.4: Frequency and Severity of Recession and Depression Episodes**(a)** Recession and Depression Episodes (Dollars)

Peak Date	Trough Date	Cumulative Decline	Type	Recovery Date	Duration(year)	Recovery(year)
1929-01-01	1933-01-01	0.3057	Depression	1936-01-01	4.0	3.0
1937-01-01	1938-01-01	0.0337	Recession	1939-01-01	1.0	1.0
1944-01-01	1947-01-01	0.1447	Depression	1951-01-01	3.0	4.0
1981-01-01	1982-01-01	0.0182	Recession	1983-01-01	1.0	1.0
2008-01-01	2009-01-01	0.0261	Recession	2010-01-01	1.0	1.0
2019-01-01	2020-01-01	0.0210	Recession	2021-01-01	1.0	1.0

(b) Recession and Depression Episodes (Gold)

Peak Date	Trough Date	Cumulative Decline	Type	Recovery Date	Duration(year)	Recovery(year)
1929-01-01	1930-01-01	0.0889	Depression	1931-01-01	1.0	1.0
1931-01-01	1934-01-01	0.7576	Depression	1943-01-01	3.0	9.0
1944-01-01	1947-01-01	0.1698	Depression	1951-01-01	3.0	4.0
1953-01-01	1954-01-01	0.0115	Recession	1955-01-01	1.0	1.0
1957-01-01	1958-01-01	0.0117	Recession	1959-01-01	1.0	1.0
1967-01-01	1969-01-01	0.0830	Depression	1970-01-01	2.0	1.0
1970-01-01	Present	2.9070	Depression	Ongoing		

Notes: A recession is defined as a peak-to-trough cumulative decline in log real GDP exceeding 1% but less than 5%; a depression as a decline exceeding 5%. Recovery date is defined as the date on which log real GDP surpasses its previous peak following the peak-to-trough decline. Duration is measured as the number of years from the peak date to the trough date, and recovery time as the number of years from the trough date to the recovery date. "Present" denotes the end of the sample period (December 2024).

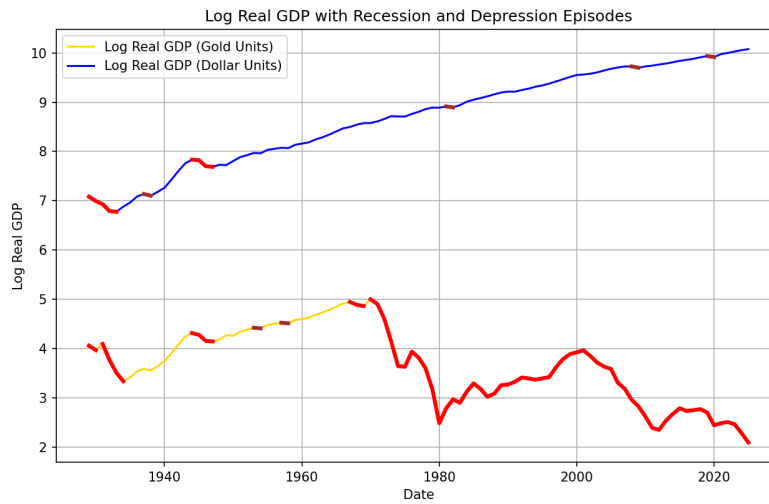
**Figure 1:** Recession and Depression Episodes In Dollars and Gold

Table 10.5: Frequency and Severity of Bear Market Episodes**(a)** S&P 500 Log Stock Market Value (Dollars)

Peak Date	Trough Date	Cumulative Decline	Recovery Date	Duration (year)	Recovery (year)
1929-08-01	1932-05-01	1.8648	1945-04-01	2.8	12.9
1946-05-01	1946-11-01	0.2544	1949-11-01	0.5	3.0
1961-12-01	1962-06-01	0.2526	1963-04-01	0.5	0.8
1968-11-01	1970-06-01	0.3459	1971-03-01	1.6	0.7
1972-12-01	1974-09-01	0.5574	1976-06-01	1.8	1.8
1987-08-01	1987-11-01	0.3507	1989-05-01	0.3	1.5
2000-08-01	2002-09-01	0.5875	2006-10-01	4.1	2.1
2007-10-01	2009-02-01	0.6975	2012-03-01	1.3	3.1
2020-01-01	2020-03-01	0.2156	2020-07-01	0.2	0.3
2021-12-01	2022-09-01	0.2701	2023-12-01	0.8	1.2
2024-11-01	Present	0.0237	Ongoing	Ongoing (No Bear)	NA

(b) S&P 500 Log Stock Market Value (Gold)

Peak Date	Trough Date	Cumulative Decline	Recovery Date	Duration (year)	Recovery (year)
1929-08-01	1932-05-01	1.8677	1950-09-01	2.8	18.3
1961-12-01	1962-06-01	0.2520	1963-04-01	0.5	0.8
1968-11-01	1970-06-01	0.2298	1971-01-01	1.6	0.6
1971-04-01	1980-01-01	2.3943	1991-12-01	8.8	11.9
2000-08-01	Present	0.4224	Ongoing	Ongoing (Bear)	NA

Notes: A bear market is defined as a peak-to-trough cumulative decline in log stock market value exceeding 20%. Recovery date is defined as the date on which log stock market value surpasses its previous peak following a peak-to-trough decline. Duration is measured as the number of years from the peak date to the trough date, and recovery time as the number of years from the trough date to the recovery date. “Present” denotes the end of the sample period, corresponding to December 2024. “Ongoing” indicates that the episode has not recovered by the end of the sample period; for ongoing episodes, the cumulative decline is measured through December 2024.

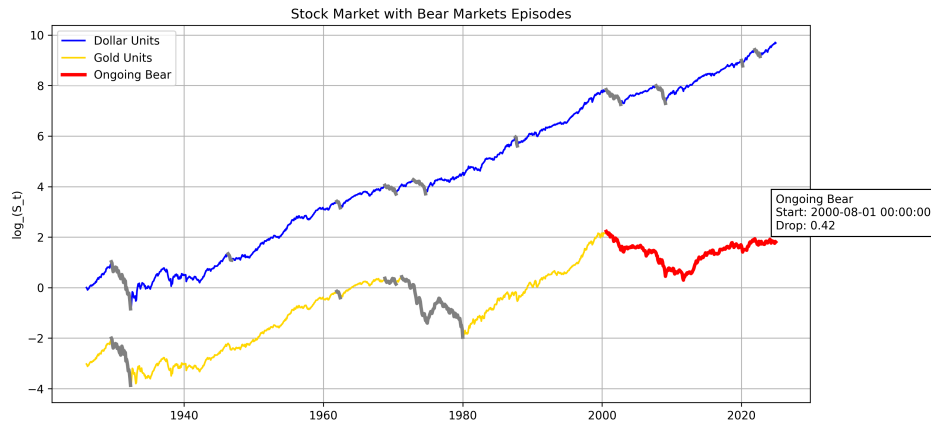
**Figure 2:** Stock Market Bear Market Episodes In Dollars and Gold

Table 10.6: Regime-based Results for Real GDP Growth

Regime Unit	Inflation		M2 Growth		Uncertainty (JLN)	
	Dollar	Gold	Dollar	Gold	Dollar	Gold
inf_{dm}	-0.0057 (0.0035)	-0.0365 (0.0365)				
$m2_{dm}$			0.0098*** (0.0029)	-0.0250 (0.0375)		
jln_{dm}					-0.0150*** (0.0038)	-0.0385 (0.0327)
inf_{t-1}			-2.6090*** (0.9844)	12.1919 (11.4418)	-0.8337 (1.5511)	-20.6718 (20.9159)
$m2_{t-1}$	2.9834*** (1.0159)	8.4735 (5.2074)			1.7256** (0.6854)	5.0415 (8.6319)
jln_{t-1}	-0.8970*** (0.2766)	-0.6192 (1.6772)	-0.3157 (0.2003)	1.5593 (2.6224)		
$inf_{dm} \times jln_{t-1}$	-0.4316 (0.3621)	-7.2325 (4.4425)				
$inf_{dm} \times m2_{t-1}$	-1.5197 (1.4274)	-17.5495 (17.4716)				
$m2_{dm} \times inf_{t-1}$			-0.3057 (1.3587)	-24.8695 (21.2253)		
$m2_{dm} \times jln_{t-1}$			-0.9438*** (0.2953)	-7.1378* (4.1235)		
$jln_{dm} \times inf_{t-1}$					-2.0180 (1.9371)	29.0089 (18.4587)
$jln_{dm} \times m2_{t-1}$					0.7917 (1.1881)	-0.4763 (11.6570)
log_rgdp_{t-1}	0.3234*** (0.1010)	0.3351* (0.1788)	0.1536 (0.1277)	0.2923* (0.1750)	0.2147* (0.1225)	0.4450*** (0.1465)
R-squared	0.4566	0.2529	0.4630	0.2489	0.3705	0.1965
R-squared Adj	0.3994	0.1742	0.4065	0.1698	0.3042	0.1119

Notes: Newey-West standard errors with two lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Regime dummies are constructed using sample medians, with observations at or below the median assigning to the low regime (0) and observations above the median assigned the high regime (1). All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variable is log real GDP growth measured in dollars or gold. All explanatory variables are lagged by one period to mitigate look-ahead bias, and lagged log real GDP growth is included to capture persistence. $N=64$.

Table 10.7: Continuous-Effect Results for Real GDP Growth

Unit	Dollar	Gold	Dollar	Gold	Dollar	Gold
inf_{t-1}	-1.8044** (0.8617)	1.2920 (11.9489)	-2.5626** (1.0363)	11.2408 (14.7450)	-2.2903*** (0.8097)	-3.9684 (13.9847)
$m2_{t-1}$	2.1510*** (0.4335)	2.3095 (7.5754)	2.4542*** (0.5591)	-1.1563 (8.4304)	2.3854*** (0.5441)	2.4836 (8.2735)
jln_{t-1}	-1.0646*** (0.2023)	-2.9827 (2.1673)	-0.9413*** (0.1638)	-3.4686*** (1.4938))	-0.9545*** (0.1831)	-2.4096 (1.8530)
$inf_{t-1} \times M2_{t-1}$	-705.0363 (443.4843)	-2504.4796 (4569.0661)				
$inf_{t-1} \times jln_{t-1}$			37.6141 (76.3599)	-2082.9405** (871.6476)		
$m2_{t-1} \times jln_{t-1}$					1.0898 (61.1831)	-519.6135 (833.2067)
log_rgdp_{t-1}	0.3098*** (0.1077)	0.3941* (0.1763)	0.2631** (0.1103)	0.3047* (0.1758)	0.2665** (0.1050)	0.3454** (0.1726)
R-squared	0.5001	0.1902	0.4790	0.2515	0.4771	0.1940
R-squared Adj	0.4570	0.1204	0.4340	0.1870	0.4320	0.1245

Notes: Newey-West standard errors with two lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variable is log real GDP growth measured in dollars or gold. All explanatory variables are lagged by one period to mitigate look-ahead bias, and lagged log real GDP growth is included to capture persistence. $N=64$.

Table 10.9: Continuous-Effects Results for Realized Risk-Adjusted Stock Market Performance

Unit	Dollar			Gold			Dollar			Gold		
	2000	2500		2000	2500		2000	2500		2000	2500	
σ_N												
inf_{t-1}	-24.1853** (10.8374)	-24.3650** (10.2005)	-18.5112 (11.2973)	-18.8537 (11.5688)	-24.9814** (11.1383)	-25.0879** (10.4145)	-20.3003 (12.7125)	-20.7644 (13.0335)	-21.7487* (11.2580)	-21.8282** (10.5716)	-18.3599 (11.8626)	-18.6115 (12.1489)
$m2_{t-1}$	1.0771 (7.6536)	0.5101 (7.2686)	-6.4319 (6.9884)	-6.3764 (7.1016)	4.4426 (6.9970)	3.8508 (6.8417)	-1.9676 (6.1798)	-1.8545 (6.3165)	-0.7874 (6.4943)	-1.4595 (6.0983)	-4.7286 (6.3039)	-4.7907 (6.4269)
jln_{t-1}	-4.0553*** (1.3271)	-3.6514*** (1.2171)	-2.5026** (1.1921)	-2.5429** (1.2176)	-3.9266*** (1.3076)	-3.5254*** (1.2043)	-2.2947* (1.2477)	-2.3271* (1.2739)	-4.9949*** (1.6287)	-4.6098*** (1.5328)	-2.8619* (1.4904)	-2.9270* (1.5278)
$I \times M$	-1245.3808 (1015.9665)	-1247.2745 (954.5641)	-1525.0871 (939.0419)	-1527.3303 (952.7827)								
$I \times J$					-79.2161 (346.6022)	-86.0034 (334.5511)	-16.2726 (317.1441)	-4.3285 (324.6439)				
$M \times J$									354.7683*** (126.0967)	361.6359*** (119.0781)	182.3211 (137.5100)	190.3489 (139.9205)
γ_{t-1}	-0.0295 (0.0366)	-0.0301 (0.0374)	0.0856** (0.0361)	0.0876** (0.0366)	-0.0294 (0.0364)	-0.0297 (0.0370)	0.0852** (0.0362)	0.0872** (0.0367)	-0.0243 (0.0367)	-0.0238 (0.0373)	0.0883** (0.0369)	0.0904** (0.0373)
R^2	0.0244	0.0253	0.0240	0.0243	0.0234	0.0241	0.0223	0.0226	0.0318	0.0342	0.0246	0.0250
Adjusted R^2	0.0181	0.0189	0.0177	0.0179	0.0170	0.0177	0.0159	0.0162	0.0255	0.0278	0.0182	0.0186

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. I denotes inf_{t-1} , M denotes $m2_{t-1}$, and J denotes jln_{t-1} . The dependent variable is the ex-post price of risk (γ_t) measured in dollars or gold. All explanatory variables are lagged by one period to mitigate look-ahead bias, and lagged γ_{t-1} is included to capture persistence. $N=770$.

Table 10.10: Persistence of the Price of Risk Across Macroeconomic Regimes

Regime Unit	Inflation				M2 Growth				Uncertainty (JLN)			
	Dollar		Gold		Dollar		Gold		Dollar		Gold	
	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500	2000	2500
σ_N												
inf_{dm}	-0.0029 (0.0023)	-0.0052* (0.0030)	0.0003 (0.0013)	0.0004 (0.0013)								
$m2_{dm}$					0.0042** (0.0021)	0.0003 (0.0022)	-0.0005 (0.0013)	-0.0005 (0.0012)				
jln_{dm}									-0.0017 (0.0023)	-0.0011 (0.0021)	-0.0029** (0.0014)	-0.0029** (0.0012)
inf_{t-1}					-0.1333* (0.0707)	-0.2013** (0.0929)	-0.1225** (0.0590)	-0.1028* (0.0519)	-0.1439** (0.0729)	-0.2064** (0.0957)	-0.1198** (0.0592)	-0.0997* (0.0525)
$m2_{t-1}$	-0.0115 (0.0591)	-0.0424 (0.0798)	-0.0045 (0.0308)	-0.0063 (0.0304)					-0.0200 (0.0555)	-0.0553 (0.0779)	-0.0083 (0.0303)	-0.0078 (0.0299)
jln_{t-1}	-0.0141* (0.0083)	-0.0111 (0.0093)	-0.0018 (0.0044)	0.0003 (0.0040)	-0.0146* (0.0081)	-0.0115 (0.0095)	-0.0011 (0.0043)	0.0008 (0.0039)				
λ_{t-1}	0.9754*** (0.0080)	0.9797*** (0.0087)	0.9917*** (0.0084)	0.9940*** (0.0076)								
$\lambda_{t-1} \times inf_{dm}$	0.0137 (0.0126)	0.0222 (0.0153)	-0.0069 (0.0100)	-0.0068 (0.0098)								
λ_{t-1}									0.9767*** (0.0091)	0.9853*** (0.0083)	0.9776*** (0.0063)	0.9803*** (0.0056)
$\lambda_{t-1} \times jln_{dm}$									0.0056 (0.0119)	0.0018 (0.0103)	0.0210** (0.0106)	0.0209** (0.0091)
λ_{t-1}					0.9897*** (0.0086)	0.9877*** (0.0069)	0.9852*** (0.0079)	0.9882*** (0.0066)				
$\lambda_{t-1} \times m2_{dm}$					-0.0231* (0.0118)	-0.0022 (0.0115)	0.0047 (0.0109)	0.0037 (0.0096)				
R^2	0.9666	0.9804	0.9845	0.9894	0.9669	0.9804	0.9845	0.9893	0.9667	0.9804	0.9846	0.9895
Adjusted R^2	0.9664	0.9803	0.9844	0.9893	0.9666	0.9802	0.9844	0.9893	0.9665	0.9803	0.9845	0.9894

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All explanatory variables are demeaned before constructing interaction terms to facilitate interpretation and reduce multicollinearity. The dependent variable is the forward-looking price of risk λ_t , measured in dollars or gold. Persistence across macroeconomic regimes is captured by the interaction between λ_{t-1} and each macroeconomic regime dummy. Regime dummies are constructed using sample medians, with observations at or below the median assigning to the low regime (0) and observations above the median assigned the high regime (1). All explanatory variables are lagged by one period to mitigate look-ahead bias. $N=770$.

Table 10.11: Real GDP Growth and Price of Risk Across Gold Price Volatility Regimes**(a) Real GDP**

Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
Low	0.0479	0.0478	0.0001	0.0589	0.0609
Medium	0.0282	-0.0202	0.0484	0.0429	0.1713
High	0.0212	-0.0869	0.1081	0.0198	0.2109

Notes: Rolling gold price volatility is computed using a 3-year window, matching the annual frequency of real GDP data. Gold price volatility regimes are constructed by partitioning rolling volatility into equal-frequency low, medium, and high quantiles. Mean annual real GDP growth and its volatility measured in dollars or gold are reported for each regime.

(b) Price of Risk: λ (N = 2000)

Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
Low	0.1903	0.1715	0.0187	0.0423	0.0472
Medium	0.1785	0.1332	0.0453	0.0449	0.0342
High	0.1844	0.1317	0.0527	0.0353	0.0275

Notes: Rolling gold price volatility is computed using a 12-month window, matching the monthly frequency of the price of risk series. Gold price volatility regimes are constructed by partitioning rolling volatility into equal-frequency low, medium, and high quantiles. Mean monthly price of risk and its volatility measured in dollars or gold are reported for each regime.

(c) Price of Risk: λ (N = 2500)

Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
Low	0.2098	0.1789	0.0310	0.0653	0.0496
Medium	0.2007	0.1372	0.0635	0.0670	0.0355
High	0.2108	0.1362	0.0746	0.0535	0.0292

Notes: Rolling gold price volatility is computed using a 12-month window, matching the monthly frequency of the price of risk series. Gold price volatility regimes are constructed by partitioning rolling volatility into equal-frequency low, medium, and high quantiles. Mean monthly price of risk and its volatility measured in dollars or gold are reported for each regime.

Table 10.12: Real GDP Growth and Price of Risk Across Monetary Regimes**(a)** Real GDP

Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
1929-1971 (Anchored Regime)	0.0364	0.0201	0.0162	0.0661	0.1032
1972-2024 (Fiat Currency)	0.0273	-0.0495	0.0768	0.0200	0.2004

Notes: Two regimes are defined: the Anchored Regime (1929-1971), during which gold prices were institutionally anchored and could not adjust freely, and the Fiat Currency Regime (1972-2024), in which currencies are no longer pegged to gold and gold prices are market-determined. The mean and volatility of annual real GDP measured in dollars or gold are reported for each regime.

(b) Price of Risk: λ (N = 2000)

Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
1929-1971 (Anchored Regime)	0.1823	0.1621	0.0202	0.0473	0.0493
1972-2024 (Fiat Currency)	0.1870	0.1330	0.0541	0.0361	0.0284

Notes: Two regimes are defined: the Anchored Regime (1929-1971), during which gold prices were institutionally anchored and could not adjust freely, and the Fiat Currency Regime (1972-2024), in which currencies are no longer pegged to gold and gold prices are market-determined. The mean and volatility of the monthly price of risk measured in dollars or gold are reported for each regime.

(c) Price of Risk: λ (N = 2500)

Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
1929-1971 (Anchored Regime)	0.2007	0.1685	0.0322	0.0675	0.0522
1972-2024 (Fiat Currency)	0.2143	0.1374	0.0769	0.0576	0.0296

Notes: Two regimes are defined: the Anchored Regime (1929-1971), during which gold prices were institutionally anchored and could not adjust freely, and the Fiat Currency Regime (1972-2024), in which currencies are no longer pegged to gold and gold prices are market-determined. The mean and volatility of the monthly price of risk measured in dollars or gold are reported for each regime.

Table 10.13: Real GDP and Price of Risk Across Fitted Gold Price Volatility Regimes

(a) Real GDP					
Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
Low	0.0362	0.0889	0.0526	0.0163	0.1099
Medium	0.0260	-0.0584	0.0844	0.0173	0.1819
High	0.0185	-0.1139	0.1325	0.0208	0.1885

(b) Price of Risk: λ (N = 2000)					
Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
Low	0.1863	0.1349	0.0514	0.0350	0.0296
Medium	0.1851	0.1277	0.0574	0.0389	0.0282
High	0.1899	0.1400	0.0498	0.0349	0.0257

(c) Price of Risk: λ (N = 2500)					
Regimes	Mean			Volatility	
	Dollar	Gold	Divergence	Dollar	Gold
Low	0.2070	0.1389	0.0681	0.0591	0.0309
Medium	0.2097	0.1320	0.0778	0.0616	0.0292
High	0.2264	0.1453	0.0811	0.0518	0.0268

Notes: Rolling volatility of fitted gold prices is computed using a 3-year window for real GDP and a 12-month window for the price of risk, matching the frequency of each series. Fitted gold price volatility regimes are constructed by partitioning rolling volatility into equal-frequency quantiles: low, medium, and high. The fitted gold-price series is constructed from gold returns regressed on contemporaneous inflation, M2 growth, changes in JLN uncertainty, changes in the dollar index, and changes in the real interest rate. The mean and volatility of real GDP growth and the price of risk measured in dollars or gold are reported for each regime.

Table 10.14: Cross-Country Analysis Summary

Sample		Canada	Japan	Switzerland	Turkey
Cointegration Test					
Long-Run β_1		0.7141***	0.2779***	0.5712***	0.7989***
R^2		0.8186	0.1372	0.5454	0.9941
Engle–Granger p-value		0.0276**	0.0561*	0.0402**	0.0032***
Reversion β_1^R		-0.0079	-0.0072	-0.0096	-0.0301
Half Life (year)		7.3	8.0	6.0	1.9
Rolling Purchasing Power Preserved					
5 years		64.4%	51.0%	51.1%	62.4%
10 years		64.2%	64.9%	65.9%	62.8%
20 years		76.9%	66.1%	63.8%	75.7%
30 years		90.8%	42.1%	63.2%	83.8%
Low Regime					
Dollar	Mean	0.0438	0.0745	0.0388	0.0504
	Volatility	0.0248	0.0358	0.0245	0.0606
Gold	Mean	0.0359	0.0667	0.0310	0.0426
	Volatility	0.0511	0.0610	0.0559	0.0731
Medium Regime					
Dollar	Mean	0.0319	0.0298	0.0136	0.0439
	Volatility	0.0194	0.0246	0.0244	0.0376
Gold	Mean	-0.0109	-0.0130	-0.0293	0.0010
	Volatility	0.1714	0.1636	0.1732	0.1799
High Regime					
Dollar	Mean	0.0185	0.0137	0.0188	0.0466
	Volatility	0.0246	0.0269	0.0189	0.0398
Gold	Mean	-0.0985	-0.1033	-0.0982	-0.0704
	Volatility	0.2282	0.2339	0.2251	0.2429

Notes: Newey–West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Country-specific gold prices are computed by converting U.S. dollar gold prices into local currency using monthly average exchange rates. Money supply and real GDP are expressed in local currency. The cointegration test, reversion coefficient, half-life, and rolling purchasing power calculations follow the same procedure as the U.S. analysis. Rolling volatility of gold prices is computed using a 3-year window. Gold price volatility regimes are constructed by partitioning rolling volatility into equal-frequency quantiles: low, medium and high. The mean and volatility of annual real GDP growth measured in dollars or gold are reported for each regime.

Table 10.15: Oil Units Analysis Summary

Cointegration Test	Long-Run β_1	R^2	Engle-Granger p-value	Reversion β_1^R	Half Life
Post-Bretton Woods	0.6849***	0.6598	0.0110***	-0.0236	2.4
Inflation	2000	2500	Inflation Surprises	2000	2500
inf_{dummy}	-0.0008	-0.0008	$inf_{E,t-1}$	-0.0724	-0.0772
jln_{t-1}	-0.0069	-0.0074	$inf_{U,t-1}^+$	-0.1392	-0.0982
$m2_{t-1}$	0.0516	0.0581	$inf_{U,t-1}^-$	0.0364	0.0818
$inf_{dummy} \times m2_{t-1}$	-0.0051	-0.0016	jln_{t-1}	-0.0096	-0.0085
$inf_{dummy} \times jln_{t-1}$	-0.0887	-0.1001	$m2_{t-1}$	0.0715	0.0579
λ_{t-1}	0.9733***	0.9726***	$m2_{t-1} \times inf_{U,t-1}^+$	-50.5961	-40.3528
			$m2_{t-1} \times inf_{U,t-1}^-$	-5.0265	-2.9913
			λ_{t-1}	0.9723***	0.9722***
Attenuation Magnitude	Particles	λ_{low}	λ_{high}	Compression	Average
$inf \times m2$	2000	0.0997	0.1152	-0.0155	
	2500	0.0999	0.1141	-0.0143	-0.0149
$jln \times m2$	2000	0.0968	0.1124	-0.0156	
	2500	0.0968	0.1114	-0.0146	-0.0151
GDP Divergence	$Mean_{Dollar}$	$Mean_{Oil}$	Divergence	σ_{Dollar}	σ_{Oil}
Low	0.0395	0.0109	0.0286	0.0256	0.0758
Medium	0.0304	-0.0241	0.0545	0.0202	0.2457
High	0.0220	-0.0322	0.0543	0.0181	0.3282

Notes: Newey-West standard errors with twelve lags are reported in parentheses to account for autocorrelation and heteroscedasticity. Statistical significance is indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The cointegration test, reversion coefficient, half-life, nominal distortion regressions, and attenuation magnitude calculations follow the same procedure as the gold revaluation analysis. Rolling volatility of oil prices is computed using a 3-year window. Oil price volatility regimes are constructed by partitioning rolling volatility into equal-frequency quantiles: low, medium and high. The mean and volatility of annual real GDP growth measured in dollars or gold are reported for each regime.