

Reading Tea Leaves: Benefits and Costs of Learning about Future Demand Shocks

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SUMMARY: This article explores the effects of learning about future demand shocks in financial markets within the framework of noisy rational expectations equilibrium (NREE). It examines how such information acquisition impacts investor behavior and market outcomes, particularly focusing on the front-running channel and a novel future uncertainty channel.

METHOD: The study proposes a model of a competitive CARA-normal economy with overlapping-generation investors. Investors in this model decide how to allocate precision to signals on current dividend innovations, current demand shocks, and future demand shocks, subject to a linear information capacity constraint.

KEY FINDINGS:

- Investors learn less about current dividend and demand shocks when they have access to information about future demand shocks.
- The front-running channel causes investors to front-run future demand shocks, while the future uncertainty channel makes future demand shocks relevant to current market behavior.
- Market-wide dividend information acquisition decreases, making dividends riskier and raising risk premiums.
- Individual investors benefit ex ante by reducing the conditional variance of future excess payoffs.
- Higher heterogeneity in private information increases price volatility and makes prices less informative contemporaneously but more informative in later periods.

IMPLICATION FOR TRADING: Traders should be aware that learning about future demand shocks can lead to increased market volatility and less informative prices in the short term. However, it can also provide better ex ante utility by reducing the uncertainty of future payoffs. Practitioners should consider the trade-offs between current and future information when making investment decisions.

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Abstract

In the noisy rational expectations equilibrium (NREE) literature, existing works have investigated information acquisition on current demand shocks, as well as current and future dividend innovations. This article fills an important gap by characterizing the equilibrium effects of learning about future demand shocks. It shows that with extra access to information about future demand shocks, investors learn less about current dividend and demand shocks, and allocate greater precision to signals about future demand shocks via the front-running channel and a novel *future uncertainty channel*. This future uncertainty channel operates on two levels and breaks the fallacy of composition: in aggregate, market-wide dividend information acquisition declines, rendering dividends riskier and raising risk premiums. At the individual level, information acquisition targeting future demand shocks reduces the conditional variance of future excess payoffs, improving ex ante utility. This article also demonstrates that higher heterogeneity in private information increases price volatility, and that learning about future demand shocks renders prices less informative contemporaneously yet more informative in later periods.

Keywords: NREE, future demand shocks, information acquisition

JEL: D82, D83, G11, G14

1. Introduction

Noisy rational expectations equilibrium (NREE) models have served as an established framework for information choice problems in the study of financial markets. These models usually introduce exogenous noise trades, alternatively termed as shocks to the demand/supply of the risky assets, to prevent their prices from fully revealing private information observed by investors (Grossman and Stiglitz, 1980; Hellwig, 1980; Verrecchia, 1982). These shocks are orthogonal to dividend innovations and do not affect the fundamental value, or liquidation value, of the risky assets. It is now well-understood that noise trades can be micro-founded by hedging needs (Liu and Wang, 2016; Medrano and Vives, 2004), liquidity needs (Han et al., 2016), or sentiment (Shleifer and Summers, 1990). This article abstracts from micro-foundations of noise trades and hereinafter refers to them as demand shocks.

Intuitively, learning about future demand shocks affects the financial markets via the front-running channel, that is if investors can learn about future shocks, then they have motives to buy (sell) shares now and sell (buy) them tomorrow if tomorrow's demand is predicted to be high (low). In this article, I propose a *future uncertainty channel*, through which future demand shocks affect the market regardless of whether the front-running motive is in place or not. If we shut down the front-running channel (e.g., in a case of a stream of dividend strips), at first glance, future demand shocks that cannot be learned about today, *should* be irrelevant to investor behavior and the current asset price. The future uncertainty channel, in fact, makes the future relevant to the present when there is information acquisition on future demand shocks. The future uncertainty channel works through two forces to increase

the investor's ex ante utility: on the aggregate level, market-wise dividend information acquisition decreases, which makes dividends riskier with higher returns; on the individual level, information acquisition on future demand shocks makes the conditional variance of future excess payoffs lower. Higher expected returns and lower conditional variance of future excess payoffs both increase investors' ex ante utility.

In the literature, investors are usually worse off if every one holds more private information. This is a manifestation of *fallacy of composition*, i.e. the error of assuming what is true of a member of a group is true for the group as a whole. In the context of financial markets, more private information is better for an atomic individual investor. On aggregate, however, when all investors acquire more private information, assets become less risky with lower returns, as discussed by Kurlat and Veldkamp (2015). Learning about current demand shocks is not sufficient to break the fallacy; but learning about *future* demand shocks does. It creates heterogeneity between generations, introduces risk-sharing across time and information spillover between generations, and reduces market-wise dividend information acquisition, thus making the asset riskier with a higher risk premium.

Acquiring a signal about future demand shocks also echoes the general idea of news-driven business cycles in the macroeconomic literature. The news shocks are mainly shocks on future total factor productivity, ignoring an important aspect of price formation: demand shocks can drive the prices of goods away from their fundamental values. Learning about future demand shocks is even more important when we turn our attention to the financial market, which is the focus of this article.

This article studies how learning about future demand shocks affects investor behavior, including choices of information and investment, and market outcomes, including price informativeness and volatility. Specifically, I propose a model of a competitive CARA-normal economy with overlapping-generation investors who live for two periods. In each period, investors first decide, subject to a linear information capacity constraint, how to allocate precision to signals on current dividend innovations, current demand shocks and future demand shocks. Then they make subsequent investment decisions after observing signal realizations and the equilibrium asset price. To disentangle the front-running channel and the future uncertainty channel, I study two versions of the model, one with a long-lived asset (“dynamic model”) and one with dividend strips (“static model”).

The model features risk sharing across time and information spillover between generations. On the one hand, with signals of future demand shocks, future risks are incorporated into today’s price. As a result, tomorrow’s market risks are partially shared by today’s investors. On the other hand, only the young generation optimally choose to learn about future demand shocks. They will be able to extract *more* dividend innovation from the equilibrium price in their second period, but *less* in the current period. The old generation know more about the current demand shocks from the previous period, and their private information is aggregated by and then spill over to the young generation through the equilibrium price.

This model also features two margins of information choices. On the intensive margin, learning about tomorrow’s demand shocks today and tomorrow are imperfect substitutes, with the former also affecting the marginal

value of learning tomorrow. Assuming learning about future demand shocks is equally costly today and tomorrow, learning about tomorrow's demand shock today can fully substitute for learning about it tomorrow and offer an extra benefit from a higher marginal value of learning. On the extensive margin, we are weighing the cost-adjusted marginal benefits of one unit of precision allocated to signals about dividend innovations and demand shocks. The extra benefit brought in by learning about future demand shocks makes learning about them more beneficial than learning about current dividend innovations. Therefore, with information acquisition equally costly (both across signals and for each unit of precision of the same signal), investors specialize in information acquisition and optimally choose to solely acquire signals about future demand shocks in the first period and signals about dividend innovations in the second period. On the aggregate level, less dividend information is learned and thus price informativeness decreases, while price volatility increases due to more heterogeneous private information.

To put the question into empirical contexts, I discuss a few examples where learning about future demand shocks plays an important role. One notable example is index recomposition. As Han et al. (2016) points out, institutional traders, e.g. index funds or ETFs, need to rebalance portfolios for non-informational reasons around index recomposition. The existence of index funds and the requirement that they replicate indices create demand shocks that are non-fundamental to the underlying securities. On the announcement date of index recomposition, investors receive a signal of effective-date demand shocks. Sophisticated investors can even adjust portfolios before the index recomposition announcement by making informed predictions about how

index funds will have to rebalance. Another set of examples are in a similar vein as the news shock, except that here the news that investors receive is not about future total factor productivity, but about future demand shocks. One case with far-reaching impact is the BlackRock-BGI acquisition. BlackRock, Inc. announced on June 11, 2009 that it would acquire Barclays Global Investors (BGI), including iShares, from Barclays Bank PLC. The deal closed on December 1, 2009. Consequently, BlackRock became the world's largest investment management company. Observing the news of the BGI transaction on June 11, sophisticated investors would expect the iShares asset under management (AUM) to expand in the following years. For the component stocks of iShares ETFs, these are future demand shocks that are not related to their firm fundamentals. Some other examples may include the future liquidity crisis, sentiment, or behavioral bias. In the financial markets, it is usually the high-frequency traders (HFTs) who are learning about future demand shocks.

This article contributes to the literature on rational expectations models with asymmetric information. Our understanding of endogenous information acquisition started from static descriptions about financial markets where signals are only on current fundamentals (Grossman and Stiglitz, 1980; Hellwig, 1980; Verrecchia, 1982): there is strategic substitution in information acquisition of informed investors. On the one hand, they can gather private information about the liquidation value or dividend to make better portfolio choices, collectively making price more informative. On the other hand, when facing a more informative price, investors would rely less on private signals and more on price, which reduces their incentive to acquire private

information. There are more mechanisms at force when we consider a more complicated information structure. For example, when different investors are informed of different fundamentals of the asset payoff, trading aggressively on information about one fundamental reduces uncertainty of other fundamentals other investors face, encouraging those investors to learn more about those fundamentals (Goldstein and Yang, 2015). Goldstein and Yang (2019) further specify dimensions of fundamental uncertainty to be one that is related to the aggregate economy and one that is firm-specific. Disclosing firm-specific information that the real decision maker already knows improves price informativeness and real efficiency. Learning about current demand shocks plays a slightly different role. In Medrano and Vives (2004), Ganguli and Yang (2009) and Manzano and Vives (2011), an investor receives an endowment shock, which can be viewed as her private information about the aggregate endowment shock and helps her make more precise inference about fundamentals. Marmora and Rytchkov (2018) show that learning about noise increases price informativeness, and the price can be the most informative when the majority of investors acquire non-fundamental information.

In dynamic settings, interpretations about fundamental and non-fundamental information can be different, and it becomes meaningful to distinguish current and future shocks. However, learning is still limited to future dividend innovations, e.g. Cai (2017) and Avdis (2016). In Cai (2017), learning about the future is achieved through learning about the persistent part of current and past dividends, which can then be thought of as a signal for future dividends. And in Avdis (2016), the equilibrium price contains both dividend and discount rates information, where discount rates shocks

are modeled as demand shocks. Thus dividend information acquisition offers an extra benefit of revealing non-dividend information about conditional expected returns. This article approaches dynamic information acquisition in a different way where signals of i.i.d. future demand shocks are acquired today. I emphasize the future uncertainty channel where the asset becomes riskier because less dividend information is learned on the aggregate level and an atomic investor is more certain about asset payoff in the second period thanks to the signal about second-period demand shocks they acquire in the first period.

This article also connects with the macroeconomic literature on news-driven business cycles. As discussed by Beaudry and Portier (2014), “if an agent can properly anticipate a future need, he can gain by trying to preempt the market and invest early as to make good readily available when the predicted needs eventually appear”. The front-running channel in this article echoes with this general idea. However, the macroeconomic literature usually treats news shock as signals about future fundamentals. For example, Jaimovich and Rebelo (2009) shows how news about future total factor productivity affects current output, consumption and investment. This article complements this literature by bringing to the spotlight the importance of learning about future demand shocks.

This article also contributes to the literature on HFT¹. In reality, HFTs actively manage their limit orders and reveal information through quote updates (Hasbrouck and Saar, 2013). Empirical evidence also shows that

¹Please refer to Menkveld (2016) for a more thorough literature review on HFT.

HFTs predict order flows (Hirschey, 2018) or price changes (Harris and Saad, 2014), which can be mapped into acquiring a signal for future demand shocks in this article. This article also finds that with access to information about future demand shocks, investors are better off, which is consistent with Jovanovic and Menkveld (2017) who also find a moderately positive welfare effect.

The rest of this article is organized as follows. Section 2 sets up the model and present model solutions, Section 3 presents the analytical and numerical results from the static model, Section 4 discusses the empirical applications of the dynamic model, and Section 5 concludes.

2. The Model

2.1. Setup

2.1.1. Assets

There is a single tradable, risky asset, with supply $\bar{x}_t$. To disentangle the future uncertainty and the front-running channels, I introduce a parameter $\alpha \in \{0, 1\}$ as in Farboodi and Veldkamp (2020) and model the payoff of the risky asset as $\alpha p_{t+1} + \tilde{d}_t$, the sum of the asset price and the dividend paid out at the end of period t .

When $\alpha = 1$, the risky asset pays $p_{t+1} + \tilde{d}_t$, the future price of the long-lived asset plus its dividend. The dividend is modeled as $\tilde{d}_t = \mu + g d_{t-1} + \tilde{y}_t$, where μ and g are known parameters, and the dividend shock $\tilde{y}_t \sim i.i.d. N(0, \tau_y^{-1})$. In this case, future demand shocks will affect future equilibrium prices, and thus affect reselling profits and current choices. I call this the “dynamic” model.

When $\alpha = 0$, the asset is not long-lived, which can be reinterpreted it as a series of dividend strips. That is, in each period, there is a *new* one-period risky asset that can be traded. I call this the “static” model. Studying the static model is important. Firstly, if demand shocks can be insured or hedged, then the static model and the dynamic model are equivalent, only differ in some constant and multiplier.² Thus the static model can be interpreted as equivalent to a model with a long-lived risky asset where demand shocks can be fully-insured or fully-hedged. Secondly, using dividend strips allows isolation of the new future uncertainty channel. In the static model, the front-running channel is shut down, signals about future demand shocks can still transmit via the future uncertainty channel. Thirdly, studying dividend strips per se is also important since they provide us information about how stock prices are formed³.

2.1.2. Signals

Each investor receives an unbiased signal $\tilde{\eta}_{it}^y = \tilde{y}_t + \tilde{\epsilon}_{it}^y$ about current dividend innovation $\tilde{y}_t$, where $\tilde{\epsilon}_{it}^y \sim N(0, \tau_{eyt}^{-1})$, and an unbiased signal $\tilde{\eta}_{it}^x = \tilde{x}_t + \tilde{\epsilon}_{it}^x$ about current demand shocks $\tilde{x}_t$, where $\tilde{x}_t \sim iid. N(0, \tau_x^{-1})$ and

²Consider an insurance company. $\mathcal{G}_t$ -investor i pays an insurance fee at the beginning of their life, and receives $f_t \mid \mathcal{I}_t^+$ (measurable at the end of period t) at the end of period t and $f_{t+1} \mid \mathcal{I}_{t+1}^+$ at the end of period $t+1$, where $\tilde{f}_t$ is the fundamental value of this long-living asset, and $\tilde{f}_t \mid \mathcal{I}_t^+ = r^2\mu/[(r-1)(r-g)] + rgd_{t-1}/(r-g) + ry_t/(r-g)$. Thus the budget constraint is $\tilde{W}_{i,t+1} = r[rW_{it-1} + q'_{it}(\tilde{f}_t - p_t)] + q'_{it+1}(\tilde{f}_{t+1} - p_{t+1}r)$. For investor i , current and future demand shocks are insured. The aforementioned model is different to the static model only by some constants and multipliers.

³For example, van Binsbergen et al. (2012) empirically measure dividend strip prices to study the term structure of the equity premium.

$\tilde{\epsilon}_{it}^x \sim N(0, \tau_{\epsilon xt}^{-1})$. Investors can also predict demand shocks in the next period $\tilde{x}_{t+1}$ by an unbiased signal $\tilde{\xi}_{it}^x = \tilde{x}_{t+1} + \tilde{\nu}_{it}^x$, where $\tilde{\nu}_{it}^x \sim i.i.d. N(0, \tau_{\nu xt}^{-1})$. All shocks are orthogonal. Signal precisions⁴ $\tau_{\epsilon xt}$, $\tau_{\epsilon yt}$, and $\tau_{\nu xt}$ are choice variables and investors optimally allocate them with a finite information capacity constraint K .

2.1.3. Investors

At the start of each period t , a continuum of measure- $\frac{1}{2}$ investors, denoted by $\mathcal{G}_t$, with initial investable wealth $W_{it-1+} = W_0$ are born. They live for two periods⁵, making both information and portfolio choices in each period. Investor i only consumes at the terminal date, and the utility function over the terminal wealth is $-\exp(-\rho W_{it+1+})$, where ρ is the risk aversion coefficient.

At the portfolio choice stage of period $t+1$, after receiving private signals and observing the equilibrium price, $\mathcal{G}_t$ -investor i maximizes the expected utility conditional on information they have by choosing how much to invest in the risky asset, that is investor i chooses the optimal portfolio $\hat{q}_{it+1}$ to maximize her interim expected utility $\mathbb{E}[-\exp(-\rho \tilde{W}_{it+1+}) \mid \hat{\mathcal{I}}_{it+1}]$ subject to the budget constraint⁶ :

$$\tilde{W}_{it+1+} = rW_{i,t+} + \hat{q}_{it+1}(\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1}r). \quad (1)$$

At the beginning of period $t+1$, investor i chooses signal precision $\hat{\tau}_{\epsilon yit+1}$, $\hat{\tau}_{\epsilon xit+1}$, and $\hat{\tau}_{\nu xit+1}$ to maximize expected utility $\mathbb{E}[-\exp(-\rho \tilde{W}_{it+1+}) \mid$

⁴The subscript i is dropped because investors are ex ante identical.

⁵Several papers also model two trading periods, e.g. Holden and Subrahmanyam (2002), Chordia and Subrahmanyam (2004) and Avdis (2016), among others.

⁶Throughout this article, the notation $\hat{\cdot}$ is used on variables that are related to the old generation.

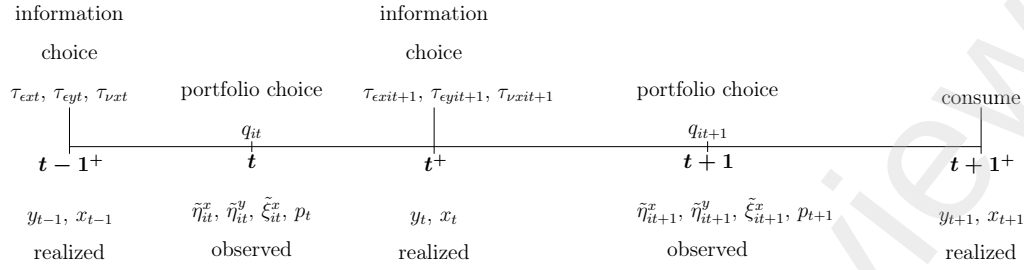


Fig. 1. Life cycle of investors born at the start of period t . Generation- t investors are born at the beginning of period t (marked as $t - 1^+$) and live for two periods: period t and period $t + 1$. In period t , investors first observe realized past dividend innovations y_{t-1} and demand shocks x_{t-1} , then make information choices $\tau_{ext}, \tau_{eyt}, \tau_{vxt}$. After receiving private signals $\tilde{\eta}_{it}^x, \tilde{\eta}_{it}^y$ and observing the market-clearing price p_t , investors choose the optimal portfolio holding q_{it} . At the end of period t (denoted t^+), the period- t dividend innovation y_t and demand shock x_t are realized. Investors repeat information acquisition and portfolio decisions in period $t + 1$, and consume all terminal wealth at $t + 1^+$.

$\hat{\mathcal{I}}_{it}^+]$ subject to the information capacity constraint

$$\gamma_{ey} \hat{\tau}_{eyit+1} + \gamma_{ex} \hat{\tau}_{exit+1} + \gamma_{vx} \hat{\tau}_{vxit+1} \leq K. \quad (2)$$

The information constraint is assumed to be linear in signal precision, that is the marginal cost of an extra unit of signal precision is the same at all signal precision levels.⁷ This assumption helps to isolate the benefits of learning about future demand shocks, i.e. the future uncertainty channel, from a cost reduction channel. Another popular choice of information constraint in the literature is a convex one, so that the marginal cost of learning within one

⁷The linear information capacity constraint is common in the literature; see Kacperczyk et al. (2016) among others.

signal is increasing as investors allocate more precision to that signal. In that case, the possibility to learn about future demand shocks today and tomorrow makes learning about demand shocks effectively cheaper, because investors can smooth learning costs. This cost reduction channel works in the same direction of the benefit increase through the future uncertainty channel, and will make the results even stronger. For the purpose of demonstrating the future uncertainty channel clearly, here I use a linear constraint.

When making portfolio choices in period t , $\mathcal{G}_t$ -investor i also has to take into consideration future choices in period $t + 1$ and thus chooses q_{it} and q_{it+1} ⁸ to maximize the expected utility $\mathbb{E}[-\exp(-\rho\tilde{W}_{it+1+}) \mid \mathcal{I}_{it}]$, subject to the budget constraint

$$\tilde{W}_{it+1+} = r^2 W_0 + r q_{it}(\tilde{d}_t + \alpha p_{t+1} - p_t r) + \hat{q}_{it+1}(\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r). \quad (3)$$

Notice the difference between $\hat{q}_{it+1}$ and q_{it+1} . While $\hat{q}_{it+1}$ is the actual investment that $\mathcal{G}_t$ -investor i makes at the portfolio choice stage of period $t + 1$, based on all previous dividends and signal realizations, $\hat{q}_{it+1}$ is what $\mathcal{G}_t$ -investor i predict herself to hold in period t based on only the information that is available to her at the portfolio choice stage in period t .

⁸Technically, $\mathcal{G}_t$ -investor i also needs to include three other choice variables, namely τ_{exit+1} , τ_{eyit+1} and $\tau_{\nu xit+1}$. They can be omitted, however, because of two reasons. Firstly, we only solve portfolio choices in a functional form of conditional expectations and variances of future payoffs. Although those moments depend on the optimal information choices in period $t + 1$, that functional form does not. Secondly, solving for the optimal information choices in period $t + 1$ is also useless since the investor will make better information choices after d_t is realized at the beginning of period $t + 1$. The same reasoning goes for the information choice problem at the beginning of period t .

Finally, in the beginning of period t , $\mathcal{G}_t$ -investor i makes period- t information choices τ_{eyt} , τ_{ext} , and $\tau_{\nu xt}$ to maximize her ex ante utility $\mathbb{E}[-\exp(-\rho\tilde{W}_{it+1+}) \mid \mathcal{I}_{t-1}^+]$ subject to the information capacity constraint

$$\gamma_{ey}\tau_{eyt} + \gamma_{ex}\tau_{ext} + \gamma_{\nu x}\tau_{\nu xt} \leq K. \quad (4)$$

2.1.4. Information structure

The equilibrium price is a linear function of the signals. Since signals are unbiased, the price can be expressed as a linear function of shocks:

$$p_t = A_t + B_t d_{t-1} + C_t \tilde{y}_t + D_t \tilde{x}_t + E_t \tilde{x}_{t+1}. \quad (5)$$

This is different from existing literature in that there is an extra term of the future demand shock, $\tilde{x}_{t+1}$, in the equilibrium price. For future demand shocks to enter the equilibrium price relation, we need two conditions. Firstly, signals about future demand shocks must exist and can be acquired. Otherwise, as in Farboodi and Veldkamp (2020) which modeled a long-lived asset and signals about current dividend and demand shocks, the future demand shock $\tilde{x}_{t+1}$ does not enter the equilibrium price relation. Secondly, the asset has to be long-lived ($\alpha = 1$). Otherwise when $\alpha = 0$, $\tilde{d}_t$ and $\tilde{d}_{t+1}$ are different assets, and the demand shocks about $\tilde{d}_{t+1}$ does not affect the equilibrium price of $\tilde{d}_t$ directly, i.e. $\tilde{x}_{t+1}$ does not appear in the equilibrium price, yet it could indirectly make a difference through price coefficients due to information acquisition.

Define the price-to-current-noise ratio to be $\zeta_{1t} \equiv C_t/D_t$, and the price-to-future noise ratio to be $\zeta_{2t} \equiv C_t/E_t$, where C_t is the price sensitivity to dividend innovation, and D_t and E_t are the price sensitivity to current and future demand shocks, respectively, in the equilibrium

price (5). A transformation of the equilibrium price p_t can generate an unbiased signal about current dividend shock. The two generations observe the same price but extract out of it different information about $\tilde{y}_t$. In period t , $\mathcal{G}_t$ -investor i extracts a signal $\tilde{\eta}_{it}^p = \tilde{y}_t + \tilde{\nu}_{it}^p$, where $\tilde{\nu}_{it}^p = (\tilde{x}_t - \mathbb{E}[\tilde{x}_t \mid \tilde{\eta}_{it}^x])/\zeta_{1t} + (\tilde{x}_{t+1} - \mathbb{E}[\tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x])/\zeta_{2t} \sim N(0, \tau_{pt}^{-1})$, with the variance of the price signal being $\tau_{pt}^{-1} = \zeta_{1t}^{-2}V_{xt} + \zeta_{2t}^{-2}V_{xt}^\xi$.⁹ $\mathcal{G}_{t-1}$ -investor i extracts a signal $\tilde{\eta}_{it}^p = \tilde{y}_t + \tilde{\nu}_{it}^p$, where $\tilde{\nu}_{it}^p \equiv (\tilde{x}_t - \mathbb{E}[\tilde{x}_t \mid \tilde{\eta}_{it}^x, \tilde{\xi}_{it-1}^x])/\zeta_{1t} + (\tilde{x}_{t+1} - \mathbb{E}[\tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x])/\zeta_{2t} \sim N(0, \hat{\tau}_{pit}^{-1})$, with the variance of the price signal being $\hat{\tau}_{pit}^{-1} = \zeta_{1t}^{-2}\hat{V}_{xit} + \zeta_{2t}^{-2}\hat{V}_{xit}^\xi$. To proceed, this article defines an important concept of *price informativeness*.

DEFINITION 1. *Price informativeness for investor i is defined by precision of the price signal given the dividend realization and the investor's information about demand shocks. Mathematically, in period t , the price informativeness is $\text{Var}^{-1}(\tilde{\nu}_{it}^p \mid \tilde{\eta}_{it}^x, \tilde{\xi}_{it}^x)$ for $\mathcal{G}_t$ and $\text{Var}^{-1}(\tilde{\nu}_{it}^p \mid \tilde{\eta}_{it}^x, \tilde{\xi}_{it-1}^x, \tilde{\xi}_{it}^x)$ for $\mathcal{G}_{t-1}$ investors.*

To summarize, each investor processes all available information and obtains two signals – the dividend signal and the price signal – concerning the dividend innovation. Private knowledge about $\tilde{x}_t$ affects their ability to extract information about the dividend innovation from the equilibrium price. In other words, it affects price informativeness. Investor can learn more from the price either when they learn more about $\tilde{x}_t$ so that they can better extract information from the price, or other investors have learned more about $\tilde{x}_t$ so that there is not much noise contained in the price. The acquired signal $\tilde{\xi}_{it}^x$

⁹ V_{xt} , V_{xt}^ξ , $\hat{V}_{xit}$ and $\hat{V}_{xit}^\xi$ are defined in the appendix. The subscript i is dropped from them and τ_{pt} because in period t , $\mathcal{G}_t$ -investors are ex ante identical.

about future demand shocks will affect next period's learning abilities and the marginal value of learning.

Let $\mathcal{F}_{t-1}^+$ be the information set that contains all past realizations of dividend innovations, demand shocks and equilibrium prices. The two generations have different information sets at different choice stages, which can be summarized by Table 1.

Table 1. Information sets of the two generations in Period t

Generation	Information Choice Stage	Portfolio Choice Stage
$\mathcal{G}_t$	$\mathcal{I}_{t-1}^+ = \mathcal{F}_{t-1}^+$	$\mathcal{I}_{it} = \{\mathcal{F}_{t-1}^+, \eta_{it}^y, \eta_{it}^x, \xi_{it}^x, p_t\}$
$\mathcal{G}_{t-1}$	$\hat{\mathcal{I}}_{it-1}^+ = \{\mathcal{F}_{t-1}^+, \xi_{it-1}^x\}$	$\hat{\mathcal{I}}_{it} = \{\mathcal{F}_{t-1}^+, \xi_{it-1}^x, \hat{\eta}_{it}^y, \hat{\eta}_{it}^x, \hat{\xi}_{it}^x, p_t\}$

$\mathcal{G}_t$ -investors and $\mathcal{G}_{t-1}$ -investors are endowed with heterogeneous information sets. Specifically, the knowledge regarding $\tilde{x}_t$ acquired by $\mathcal{G}_{t-1}$ -investors in period $t-1$ persists and exerts influence on their optimal decisions in the subsequent period t .

$\mathcal{F}_{t-1}^+$ can be interpreted as what an outsider knows about the market at the beginning of period t . Ex ante, $\mathcal{G}_t$ -investors know as much as an outsider. At the portfolio choice stage, however, they have heterogeneous information due to received private signals. After the current dividend innovation is realized, signals about the current dividend innovation and demand shock no longer assist in future decision making. Only the signal realization of next period's demand shock carries on and affects future choices.

2.1.5. Benchmark model

The benchmark, corresponding to the standard specification in the existing literature, shuts down information acquisition on future demand shocks. It

is denoted by the superscript B in this article. If the precision of the future demand signal $\tau_{\nu x}$ is zero, or the cost of learning about future demand shocks $\gamma_{\nu x}$ is infinitely large, the main model collapses to the benchmark model.

2.2. Market Clearing and Equilibrium

This article introduces information acquisition on future demand shocks to the noisy rational expectations equilibrium. See Appendix A for a formal definition. The risky asset price clears the market when the aggregate demand equals the aggregate supply:

$$\int_{i \in \mathcal{G}_t} q_{it} di + \int_{i \in \mathcal{G}_{t-1}} \hat{q}_{it} - q_{it-1} di = \bar{x}_t + \tilde{x}_t + \int_{i \in \mathcal{G}_{t-2}} \hat{q}_{it-1} di. \quad (6)$$

2.3. Model solutions

2.3.1. Portfolio choices

For notational simplicity, denote the expected period- t payoff and the expected period- $(t+1)$ payoff conditional on $\mathcal{G}_t$ -investor i 's interim information set in period t as $\mathcal{E}_{it} \equiv \mathbb{E}[\tilde{d}_t + \alpha p_{t+1} \mid \mathcal{I}_{it}] - p_t r$ and $\mathcal{E}_{2i} \equiv \mathbb{E}[\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r \mid \mathcal{I}_{it}]$, respectively, and their variance as $\mathcal{V}_t \equiv \mathbb{V}ar(\tilde{d}_t + \alpha p_{t+1} \mid \mathcal{I}_{it})$ and $\mathcal{V}_2 \equiv \mathbb{V}ar(\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r \mid \mathcal{I}_{it})$, respectively. The covariance of the period- t payoff and the period- $(t+1)$ payoff conditional on $\mathcal{I}_{it}$ is denoted as $\mathcal{C} \equiv \mathbb{C}ov(\tilde{d}_t + \alpha p_{t+1}, \tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r \mid \mathcal{I}_{it})$. These conditional moments are derived in Appendix B.1.1 and Appendix B.1.2. The portfolio choice of $\mathcal{G}_t$ -investor i can then be written as

$$q_{it} = \frac{1}{\rho r} \left(\frac{\mathcal{V}_2}{\mathcal{V}_t \mathcal{V}_2 - \mathcal{C}^2} \mathcal{E}_{it} - \frac{\mathcal{C}}{\mathcal{V}_t \mathcal{V}_2 - \mathcal{C}^2} \mathcal{E}_{2i} \right). \quad (7)$$

If the conditional expectation about future excess payoffs ($\mathcal{E}_{2i}$) changes, the individual demand curve undergoes a parallel shift. But when the conditional

variance of future excess payoff becomes smaller or the conditional covariance of current and future excess payoffs becomes larger, the slope of the demand curve would become less steep.

$\mathcal{G}_{t-1}$ -investor i faces a simple one-shot portfolio choice problem, which has an optimal solution

$$\hat{q}_{it} = \frac{\hat{\mathcal{E}}_{it}}{\rho \hat{\mathcal{V}}_{it}}, \quad (8)$$

where $\hat{\mathcal{E}}_{it} \equiv \mathbb{E}[\tilde{d}_t + \alpha p_{t+1} \mid \hat{\mathcal{I}}_{it}] - p_t r$ and $\hat{\mathcal{V}}_{it} \equiv \text{Var}(\tilde{d}_t + \alpha p_{t+1} \mid \hat{\mathcal{I}}_{it})$.

Static case. In the static case where $\alpha = 0$, no portfolio rebalance is involved. There is no generation difference and each period t is an identical snapshot from the timeline. It is worth noticing is that the covariance term $\mathcal{C}_t = 0$. The reasoning is as follows. Conditional on $\mathcal{I}_{it}$, the random components in $\tilde{d}_t - p_t$ is $\tilde{y}_t \mid \tilde{\eta}_{it}^y, \tilde{\eta}_{it}^p$ and in $\tilde{d}_{t+1} - p_{t+1}$ are $\tilde{y}_{t+1}, \tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x$ and $\tilde{x}_{t+2}$, respectively. They are orthogonal (see the proof of Lemma 4 in the appendix). This effectively separates the two portfolio choices of investors. Therefore, the optimal portfolio choices for $\mathcal{G}_t$ - and $\mathcal{G}_{t-1}$ -investors are

$$q_{it} = \frac{\mathbb{E}[\tilde{d}_t \mid \mathcal{I}_{it}] - p_t r}{\rho r \text{Var}(\tilde{d}_t \mid \mathcal{I}_{it})} \quad \text{and} \quad \hat{q}_{it} = \frac{\mathbb{E}[\tilde{d}_t \mid \hat{\mathcal{I}}_{it}] - p_t r}{\rho \text{Var}(\tilde{d}_t \mid \hat{\mathcal{I}}_{it})},$$

respectively.

2.3.2. Equilibrium price relation

For simplicity, define $\bar{\chi}_{yt} \equiv \Omega_t V_{yt}/r + \int_{i \in \mathcal{G}_{t-1}} \hat{V}_{yit}/\hat{\mathcal{V}}_{it} \, di$ and $\bar{\chi}_{xt} \equiv \Omega_t V_{xt}/r + \int_{i \in \mathcal{G}_{t-1}} \hat{V}_{xit}/\hat{\mathcal{V}}_{it} \, di$ (refer to Appendix B.2 for details). In equilibrium, the

period- t price coefficients are recursively characterized by

$$A_t = \left(1 - \frac{\bar{\Sigma}_t}{2r} \frac{\mathcal{C}}{\mathcal{V}_t \mathcal{V}_2 - \mathcal{C}^2}\right) \frac{\mu}{r - \alpha g} - \frac{\bar{\Sigma}_t \rho}{r} \bar{x}_t + \frac{1}{r} \left(\alpha + \frac{\bar{\Sigma}_t}{2r} \frac{\mathcal{C}}{\mathcal{V}_t \mathcal{V}_2 - \mathcal{C}^2}\right) A_{t+1} - \frac{\bar{\Sigma}_t}{2r^2} \frac{\mathcal{C}}{\mathcal{V}_t \mathcal{V}_2 - \mathcal{C}^2} \alpha A_{t+2}, \quad (9)$$

$$B_t = \frac{g}{r - \alpha g}, \quad (10)$$

$$C_t = \frac{1}{r - \alpha g} (1 - \bar{\Sigma}_t \bar{\chi}_{yt} \tau_y), \quad (11)$$

$$D_t = -\frac{\rho \bar{\Sigma}_t}{r} + \frac{1}{r - \alpha g} \bar{\Sigma}_t \bar{\zeta}_{1t}^\dagger \tau_x,$$

$$E_t = \frac{1}{r} \left[\frac{\bar{\Sigma}_t}{2r} \frac{\mathcal{C}}{\mathcal{V}_t \mathcal{V}_2 - \mathcal{C}^2} V_{xt}^\xi \tau_{\nu xt} + \alpha (1 - \bar{\Sigma}_t \bar{\chi}_{xt} \tau_x) \right] D_{t+1} + \frac{1}{r - \alpha g} \bar{\Sigma}_t \bar{\zeta}_{2t}^\dagger \tau_x. \quad (12)$$

B_t is the constant dividend growth rate, and governs the level of the equilibrium price together with A_t . C_t is the price sensitivity to dividend innovation. D_t and E_t regulate the amount of current and future demand shocks contained in the price, respectively. $\bar{\zeta}_{1t}^\dagger$ and $\bar{\zeta}_{2t}^\dagger$ are the market-wide average future-uncertainty-adjusted dividend-to-total-uncertainty-weighted signal-to-current-noise and signal-to-future-noise ratios, defined by (B.43) and (B.44), respectively.

Static case. When $\alpha = 0$, the equilibrium price relation greatly simplifies. Firstly, $E_t = 0$. This means that here $\tilde{x}_{t+1}$ does not directly enter the equilibrium price at t . This is because $\tilde{d}_t$ and $\tilde{d}_{t+1}$ are different assets, and the demand shock to $\tilde{d}_{t+1}$ does not affect the equilibrium price of $\tilde{d}_t$ directly, but only indirectly through price coefficients due to information acquisition. Secondly, $\mathcal{C}_t = 0$, that is the current and future excess payoffs are conditionally

independent. Thirdly, $\mathcal{G}_{t-1}$ -investors make homogeneous period- t information choices. The price coefficients then become

$$\begin{aligned} A_t &= \frac{1}{r} (\mu - \bar{\Sigma}_t \rho \bar{x}_t), \quad B_t = \frac{g}{r}, \quad C_t = \frac{1}{r} \left[1 - \frac{r+1}{2r} \bar{\Sigma}_t \tau_y \right], \\ D_t &= \frac{1}{r} \left[-\rho \bar{\Sigma}_t + \frac{r+1}{2r} \bar{\Sigma}_t \zeta_t \tau_x \right]. \end{aligned} \quad (13)$$

2.3.3. Inforamtion choices

Ex ante expected utilities. In period t , $\mathcal{G}_{t-1}$ -investors are in the second period of their life. the objective function is $\mathbb{E}[-\exp(-\rho \tilde{W}_{it+}) \mid \hat{\mathcal{I}}_{it-1}^+]$ and the constraint of the information choice problem is very similar to (2), with only a change of subscript t . This yields an objective for investor $i \in \{\mathcal{G}_{t-1}\}$ that is increasing and concave in precision of the fundamental signal and the current demand signal: where $\mu_{\hat{\mathcal{E}}_t}$ and $\hat{\Sigma}_{t-1+}$ are defined by (B.18) and (B.17), respectively. Notice that in the static model, investors optimally choose not to learn about future demand shocks. Thus $\hat{\tau}_{\nu xit+1}$ does not affect the ex ante utility.

$$\begin{aligned} \mathbb{E}[\exp(-\rho r W_{it+}) \mid \hat{\mathcal{I}}_{it-1}^+] &= -|\hat{\Sigma}_{t-1+}/\hat{\mathcal{V}}_{it}|^{-1/2} \times \\ &\exp(-\rho r W_{it-1+}) \exp\left(-\frac{\mu_{\hat{\mathcal{E}}_t}^2}{2\hat{\Sigma}_{t-1+}}\right). \end{aligned} \quad (14)$$

For $\mathcal{G}_t$ -investors, the information choice problem is to maximize $\mathbb{E}[-\exp(-\rho \tilde{W}_{it+1+}) \mid \mathcal{I}_{t-1}^+]$ subject to (4). This yields an objective for $i \in \{\mathcal{G}_t\}$ that is increasing and concave in precisions of the signals of the current dividend innovation, the current demand shock and the future

demand shock:

$$\begin{aligned} \mathbb{E} \left[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+ \right] &= -\exp(-\rho r^2 W_0) \times \\ &\quad |I - 2\Sigma F|^{-\frac{1}{2}} \times \sqrt{\hat{\mathcal{V}}_{it+1} / (\hat{\mathcal{V}}_{it+1} + \Sigma_{\hat{Z}_{t+1}})} \times \\ &\quad \exp\left(\frac{1}{2} G' (I - 2\Sigma F)^{-1} \Sigma G + J\right), \end{aligned} \quad (15)$$

where $\hat{\mathcal{V}}_{it+1}$, $\Sigma_{\hat{Z}_{t+1}}$, Σ , F , G and J are defined in the appendix.

Static case. Information choices in the dynamic model are very complex. To develop intuitions, here I derive the analytical solution of information choices in the static model. To understand how investors would allocate precision to signals about the current dividend innovation and the current demand shock, it is convenient to first define the concept of learning index.

DEFINITION 2. *Learning index for a signal is defined as the increase in ex ante marginal utility associated with an extra unit of precision assigned to this signal.*

For $\mathcal{G}_{t-1}$ -investors, the learning index for the signals about the current dividend innovation and the current demand shock are $\partial \widehat{EU} / \partial \hat{\tau}_{eyit}$ and $\partial \widehat{EU} / \partial \hat{\tau}_{exit}$, respectively. The learning indices are functions of current precision choices $\hat{\tau}_{eyit}$ and $\hat{\tau}_{exit}$, the signal precision $\tau_{\nu xt-1}$, and the realization of the signal about current demand shocks received in the previous period, ξ_{it-1}^x . Since all investors are ex ante identical, the signal precision $\tau_{\nu xt-1}$ is the same across $\mathcal{G}_{t-1}$ -investors. However, signal realizations ξ_{it-1}^x are private information to investors, introducing heterogeneity in second-period learning indices. The effects of ξ_{it-1}^x on learning indices for signals about

the current dividend innovation and the current demand shock are in such a proportional way that the ratio of them does not depend on learning indices. This observation can be summarized by the following lemma.

LEMMA 1 (Proportionality of second-period learning indices). *The ratio of second-period learning indices for the current dividend innovation and the current demand shock is $\hat{L}_{\epsilon xit}/\hat{L}_{\epsilon yit} = \zeta_t^2$.*

To better characterize the benefit of learning about future demand shocks, it is useful to study it when learning about different shocks is equally costly. The equal-cost assumption is stated below for later reference.

ASSUMPTION 1. *Acquiring signals about current dividend innovation, current and future demand shocks is equally costly, that is $\gamma_{\epsilon y} = \gamma_{\epsilon x} = \gamma_{\nu x}$.*

The information choices of $\mathcal{G}_{t-1}$ -investors can then be summarized by the following proposition.

PROPOSITION 1 (Second-period information choice). *In period t , $\mathcal{G}_{t-1}$ -investors*

- (a) *optimally choose not to learn about future demand shocks, that is $\hat{\tau}_{\nu xit} = 0$; and*
- (b) *[under Assumption 1] only learn about current dividend innovation regardless of the realization of signal about $\tilde{x}_t$ they received in the previous period, that is $\hat{\tau}_{\epsilon yit} = K/\gamma_{\epsilon y}$ and $\hat{\tau}_{\epsilon xit} = 0$.*

What $\mathcal{G}_{t-1}$ -investors have learned in the past about today does not affect their information choice for today. It should be noted, however, the portfolio

choice and payoffs are affected. This arises because they do have an additional signal about today's asset. Their conditional mean and variance about today's asset are different than those of $\mathcal{G}_t$ -investors.

In the first period, the proportionality of learning indices for signals about the current dividend innovation and the current shock preserves, as is summarized in Lemma 2.

LEMMA 2 (Proportionality of first-period learning indices). *The ratio of first-period learning indices for the current dividend innovation and the current demand shock only depends on the signal-to-noise ratio, that is $L_{\epsilon_{xit}}/L_{\epsilon_{yit}} = \zeta_t^2$.*

The first-period information choices can be summarized by Proposition 2.

PROPOSITION 2 (First-period information choice). *Under Assumption 1, $\mathcal{G}_t$ -investors optimally choose to only learn about future demand shocks, that is $\tau_{\epsilon_{yt}} = \tau_{\epsilon_{xt}} = 0$ and $\tau_{\nu_{xt}} = K/\gamma_{\nu x}$.*

Because $\mathcal{G}_t$ -investors are identical at the information choice stage of period t , they will make the same choice of precision, and thus will have the same learning indices. Compared with the benchmark with no future demand shocks, the learning indices have changed because the option to learn about future demand shocks affects $\bar{\Sigma}_t$, the across-generation average of posterior variance of $\tilde{y}_t$, in two ways. Firstly, $\mathcal{G}_t$ -investors optimally choose to only acquire signals about $\tilde{x}_{t+1}$ and not learn about any current shocks. Secondly, the learning indices of $\mathcal{G}_{t-1}$ -investors are affected by their observed private signal $\tilde{\xi}_{it}^x$. Their portfolio choices change accordingly. Changes in learning indices of the old generation spill over to the young generation through $\bar{\Sigma}_t$.

Compared with learning about $\tilde{x}_{t+1}$ in period $t + 1$, learning about it in period t has an extra bite: it decreases the variance of future excess payoffs conditional on today's information.

3. Results

With a long-lived risky asset, learning about future demand shocks affects the market via two mechanisms: the front-running channel and the future uncertainty channel. To see how the novel future uncertainty channel works more clearly, this section studies the static model and set $\alpha = 0$ to shut down the front-running channel. In the dynamic model where $\alpha = 1$, the front-running channel works in the same direction as the future uncertainty channel and will strengthen the results in the static model.

3.1. Benchmark model

In the benchmark model, investors do not have access to signals about future demand shocks. The results are shown in Appendix C. When $\gamma_{\nu x} \rightarrow \infty$ in the main model, information capacity constraints make learning future demand shocks prohibitively costly, so investors optimally abstain from acquiring such information. Accordingly, the main model collapses into the benchmark model. When learning future demand shocks is sufficiently costly, the main model converges to the benchmark model.

PROPOSITION 3. *There exists a threshold $\bar{\gamma}_{\nu x}$ such that for all $\gamma_{\nu x} > \bar{\gamma}_{\nu x}$, the results in Proposition 11 hold for the full main model.*

3.2. Price reveals less fundamental information

When we grant investors another source for non-fundamental information, the informational content of price will be changed accordingly. We now examine this issue by looking at the signal-to-noise ratio, the average posterior precision about dividend innovation and price informativeness.

Understanding how the signal-to-noise ratio $|\zeta_t|$ changes with model parameters helps us figure out the information quality in the market. In the benchmark model and the second-period information choice of the main model, $|\zeta_t|$ summarizes all relevant informational contents in the market; together with parameters associated with learning costs (γ_{ey} , γ_{ex} , γ_{vx} and K), the signal-to-noise ratio $|\zeta_t|$ determines how investors allocate signal precision in Benchmark I and in the second period of the main model.

Investors have a prior belief about the dividend innovations. The average posterior precision about the dividend innovation $\bar{\Sigma}_t^{-1}$ is the sum of prior precision of the dividend innovations, precision of dividend signal and precision of the price signal. It is a function of the risk-free rate, the risk-aversion coefficient, prior precision of dividend innovation and demand shock, and the signal-to-noise ratio $|\zeta_t|$. $\bar{\Sigma}_t^{-1}$ and $|\zeta_t|$ often comove with changes in parameters, as is shown in Figure 2.

LEMMA 3. *The market-wide average posterior precision for the dividend innovation $\bar{\Sigma}_t^{-1}$ comoves positively with the signal-to-noise ratio $|\zeta_t|$ whenever variations in $|\zeta_t|$ stem from shifts in the parameters K , γ_{ex} , γ_{ey} , γ_{vx} , or τ_y .*

PROPOSITION 4. *With access to information about future demand shock,*

(a) *the signal-to-noise ratio $|\zeta_t|$ and the market-wide average posterior*

precision about the dividend innovation $\bar{\Sigma}_t^{-1}$ increase with information processing capacity K and prior precision of demand shocks τ_x ;

(b) Assumes that the learning cost associated with demand shocks are the same today and tomorrow, i.e. $\gamma_{\epsilon x} = \gamma_{\nu x} = \gamma_x$. Then there exists $\bar{K}$, $\underline{\gamma}_x$ and γ_x^* such that when $K > \bar{K}$ and $\gamma_x < \underline{\gamma}_x$, $|\zeta_t|$ and $\bar{\Sigma}_t^{-1}$ decrease with γ_x if $\gamma_x < \gamma_x^*$ and increase with γ_x if $\gamma_x > \gamma_x^*$. When $K \leq \bar{K}$ or $\gamma_x \geq \underline{\gamma}_x$, $|\zeta_t|$ and $\bar{\Sigma}_t^{-1}$ increase with γ_x . Mathematically,

$$\text{sign} \frac{\partial |\zeta_t|}{\partial \gamma_x} = \text{sign} \frac{\partial \bar{\Sigma}_t^{-1}}{\partial \gamma_x} \begin{cases} -, & \text{if } K > \bar{K}, \gamma_x < \underline{\gamma}_x, \gamma_x < \gamma_x^*, \\ +, & \text{otherwise.} \end{cases}$$

PROPOSITION 5. Compared with Benchmark I, the signal-to-noise ratio $|\zeta_t|$ decreases when investors have an extra source of information on future demand shocks. The same results apply to average posterior precision $\bar{\Sigma}_t^{-1}$.

THEOREM 1 (Price informativeness). With access to information about future demand shocks, price informativeness decreases.

Figure 3a plots price informativeness against the information processing capacity. For the two periods in the life of investors, the equilibrium price is not equally informative. In the second period, price is more informative because she has required a signal about second-period demand shocks in her first period. Although the price contains less dividend innovations, her ability to extract other investors' private information out of the equilibrium price is higher, and the latter effect dominates. However, in the first period, the ability to extract information from price is not changed. Since price contains less fundamental information, it is less informative for her.

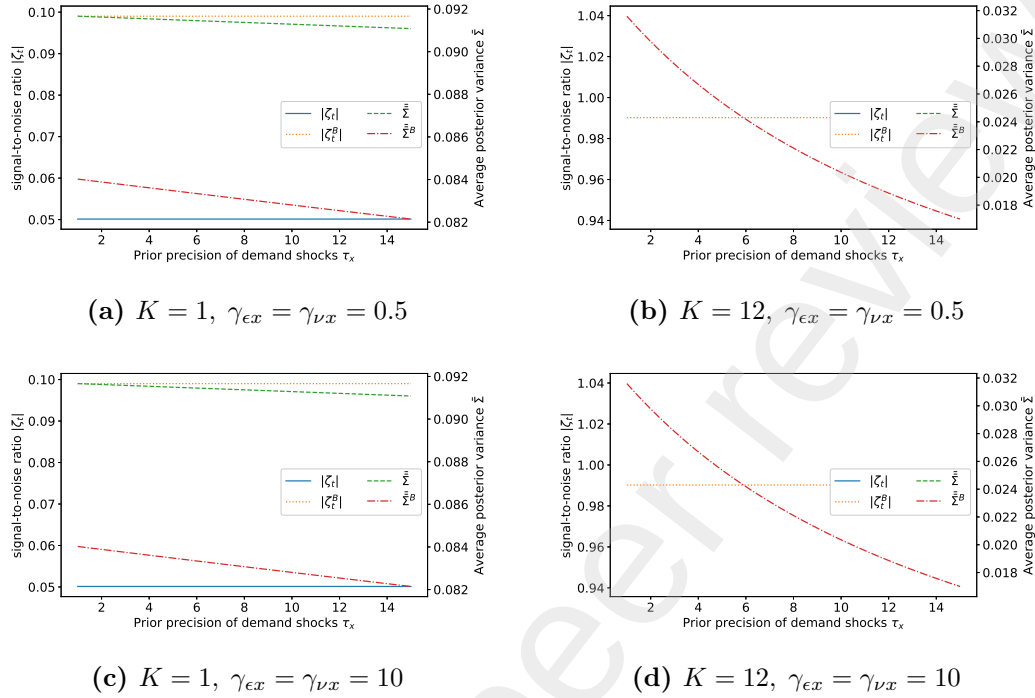
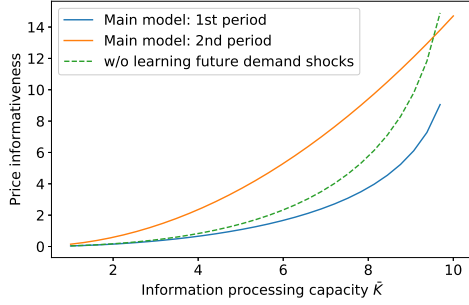


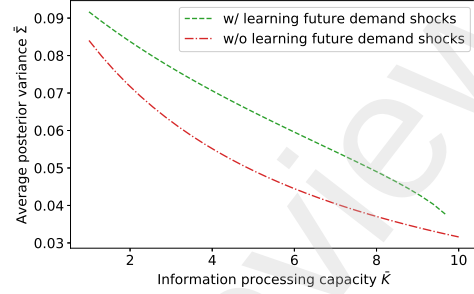
Fig. 2. The signal-to-noise ratio and the average posterior precision.

These figures plot how the signal-to-noise ratio $|\zeta_t|$ and the average posterior precision $\bar{\Sigma}_t$ change with K and learning costs for demand shocks. Relative to the benchmark model with no information acquisition on future demand shocks, the equilibrium signal-to-noise ratio declines while the average posterior variance rises. In both models, the signal-to-noise ratio rises and average posterior variance falls as information processing capacity K increases.

Figure 3b plots the average posterior variance of the dividend innovation against the information processing capacity. On the aggregate level, less fundamental information is acquired in the market. As we can see here, the average posterior variance of dividend innovations is larger when investors can learn about future demand shocks. As a result, risk premium is higher.



(a) $K = 1, \gamma_{\epsilon x} = \gamma_{\nu x} = 10$



(b) $K = 12, \gamma_{\epsilon x} = \gamma_{\nu x} = 10$

Fig. 3. Impact of learning about future demand shocks on price informativeness and risk premium. Panel (a) plots price informativeness against the information processing capacity. In general, price informativeness rises with information processing capacity. Investors across heterogeneous life stages hold distinct information sets, which generates dispersion in their capacity to recover dividend signals from equilibrium prices. Panel (b) depicts the average posterior variance of dividend innovations as a function of information processing capacity. Because the average posterior variance of dividend innovations is positively associated with the risk premium, the figure illustrates that permitting investors to learn about future demand shocks elevates equilibrium risk premiums.

3.3. Information choices

THEOREM 2. *When it is possible to learn from future demand shocks, investor i 's information choices change in both periods:*

- (a) [**fundamental vs. non-fundamental**] Agent i learns less about the dividend innovations today compared with the benchmark;
- (b) [**current vs. future**] Agent i learns more about the future demand shock than the current demand shock.

Since the marginal cost of allocating precision to one single signal is increasing, granting access to another signal about demand shocks to investors essentially makes demand shocks cheaper to learn. As a result, investor i will learn less about the dividend innovations today. Precision of the signal of current demand shock and that of future demand shock contribute the conditional mean and variance of $\tilde{x}_t$ in a symmetric way. Starting from the benchmark equilibrium, every unit of τ_{ext} investor i moves away, she can allocate more than one unit to τ_{vxt} . Since investor i live for two periods, allocating precision to τ_{vxt} has an extra benefit of lowering the conditional variance of excess payoff from the perspective of today, which in turn affects tomorrow's learning index.

3.4. *Ex ante utility*

In practice, high frequency traders are learning about future demand shocks. We tend to think of this as wasteful activity. But here I show that learning about future demand shocks indeed increase their ex ante utility.

PROPOSITION 6. *With access to information about future demand shocks, the ex ante utility of the young generation is improved, while that of the old generation is improved when K , τ_x and $\gamma_{ex} = \gamma_{vx}$ are small enough.*

Learning about the future demand shocks resolves the fallacy of composition. As showed in Figure 4, the ex ante utility improves through the future uncertainty channel. On the aggregate level, less fundamental information acquisition leads to higher risk premium, while price is more informative and thus investors take less risk in the second period.

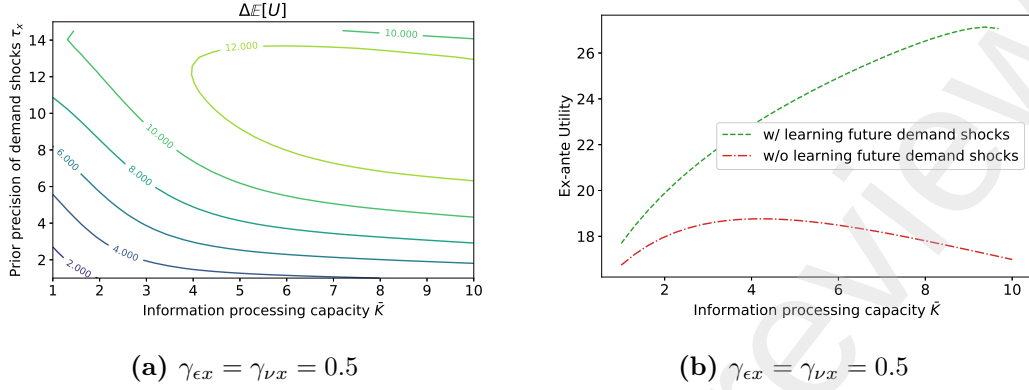


Fig. 4. Information acquisition on future demand shocks improves ex ante expected utility of investors. Panel (a) displays the contour plot of $\Delta\mathbb{E}[U] = \mathbb{E}[U] - \mathbb{E}[U^B]$ subject to the linear information constraint. Raising information processing capacity or the prior precision of demand shocks enlarges the welfare gain from learning future demand shocks. Panel (b) demonstrates that investor expected utility exhibits weaker concavity with respect to information processing capacity relative to the benchmark model.

3.5. Price volatility

From the perspective of $\mathcal{G}_{t-1}$ -investor i , the price volatility at the beginning of period t is $\mathbb{V}ar(p_t | \hat{\mathcal{I}}_{it-1}^+) = C_t^2/\tau_y + D_t^2/(\tau_x + \tau_{\nu xt-1})$. For an individual asset j , its conditional variance is $\mathbb{V}ar(p_t | \hat{\mathcal{I}}_{it-1}^+) = C_t^2/\tau_y + D_t^2/(\tau_x + \tau_{\nu xt-1})$.

Notice that each generation t' of investors only choose to learn about future demand shocks in period t' , which affects market clearing in period $t' + 1$. Recall that $\bar{\Sigma}_t^{-1} = [\tau_y + \tau_{\epsilon yt} + \zeta_t^2(\tau_x + \tau_{\epsilon xt})]/r + \tau_y + \hat{\tau}_{\epsilon yt} + \zeta_t^2(\tau_x + \hat{\tau}_{\epsilon xt} + \hat{\tau}_{\nu xt-1})$.

Comparing with the benchmark, due to acquisition of future demand signals at $t - 1^+$ by generation $\mathcal{G}_t$, $\tau_{\epsilon xt}$ or $\tau_{\epsilon yt}$ is decreased. Due to acquisition of future demand signals by generation $\mathcal{G}_{t-1}$ at $t - 1^+$, $\hat{\tau}_{\epsilon xt}$ or $\hat{\tau}_{\epsilon yt}$ is decreased,

but their information choice at $t - 2^+$ increases $\hat{\tau}_{\nu x t-1}$.

THEOREM 3 (Price volatility). *The price volatility $\text{Var}(p_t)$ changes with information processing capacity K . Given that $\partial|\zeta_t|/\partial K > 0$,*

(a) *if $\tau_x \geq \bar{\Omega}_x$, then $\partial\text{Var}(p_t)/\partial K \geq 0$;*

(b) *if $\underline{\Omega}_x < \tau_x < \bar{\Omega}_x$, then*

$$\frac{\partial\text{Var}(p_t)}{\partial K} = \begin{cases} \geq 0, & \text{if } \zeta_t \leq \underline{\zeta}_t \text{ or } \bar{\zeta}_t \leq \zeta_t < 0, \\ < 0, & \text{if } \underline{\zeta}_t < \zeta_t < \bar{\zeta}_t; \end{cases}$$

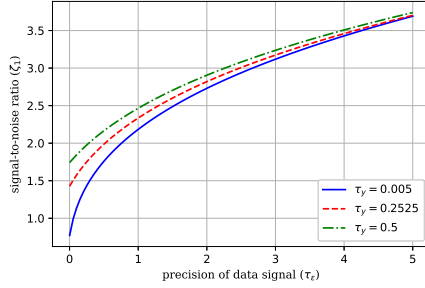
(c) *if $0 < \tau_x \leq \underline{\Omega}_x$, then*

$$\frac{\partial\text{Var}(p_t)}{\partial K} = \begin{cases} \geq 0, & \text{if } \zeta_t \leq \underline{\zeta}_t, \\ < 0, & \text{if } \underline{\zeta}_t < \zeta_t < 0, \end{cases}$$

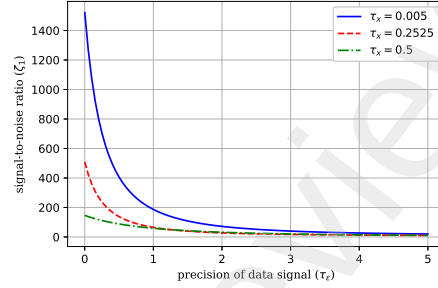
where $\bar{\Omega}_x$, $\underline{\Omega}_x$, $\bar{\zeta}_t$, and $\underline{\zeta}_t$ are defined in Appendix C.

From the perspective of the young generation $\mathcal{G}_t$, price volatility at the beginning of period t is not affected by whether learning about future demand shocks is possible.

Figure 6a shows that for given learning costs $\gamma_{\epsilon x} = \gamma_{\nu x} = 0.5$, compared with Benchmark I where learning about future demand shocks is impossible, price volatility increases when the price precision of demand shocks τ_x is low and/or the information processing capacity is high. Figure 6b plots price volatility against the information processing capacity. In each period t , there are two generations of investors. Without learning about future demand shocks, the two generations are identical. Learning about future demand

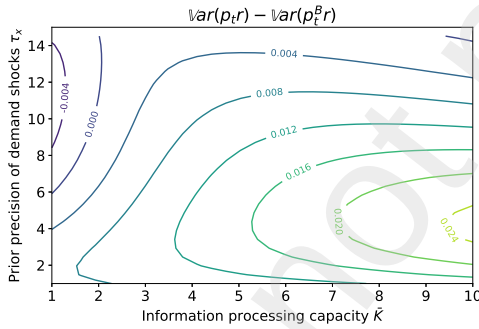


(a) $\gamma_{\epsilon x} = \gamma_{\nu x} = 0.5$

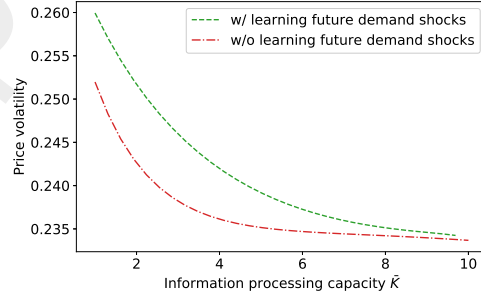


(b) $\gamma_{\epsilon x} = \gamma_{\nu x} = 0.5$

Fig. 5. Access to information about future demand shocks affects price volatility. Panel (a) reports the contour surface for the volatility wedge $\text{Var}(p_t r) - \text{Var}(p_t^B r)$. Panel (b) illustrates how equilibrium price volatility varies with investors' information processing capacity.



(a) $\gamma_{\epsilon x} = \gamma_{\nu x} = 0.5$



(b) $\gamma_{\epsilon x} = \gamma_{\nu x} = 0.5$

Fig. 6. Access to information about future demand shocks affects price volatility. Panel (a) shows the contour plot of the volatility differential $\text{Var}(p_t r) - \text{Var}(p_t^B r)$. Panel (b) traces how equilibrium price volatility varies with information processing capacity.

shocks introduces more heterogeneity on private information on two margins. On the extensive margin, the older generation will not learn about future demand shocks because they do not have a future to come, while the younger generation do. The two generations make different information choices and receive different private information. On the intensive margin, the older generation have one more signal, thus their information set is more diverse. Heterogenous private information increases price volatility $\text{Var}(p_{tr})$.

4. Applicable Scenarios

This model can be applied to a broad range of real-world scenarios where investors are able to learn about future demand shocks. Some examples include portfolio adjustment by index funds upon index recomposition and the BlackRock-BGI acquisition.¹⁰

Index recomposition. As Han et al. (2016) points out, institutional traders, e.g. index funds or ETFs, need to rebalance portfolios for non-informational reasons around index recomposition. The existence of index funds and the requirement that they replicate indices creates demand shocks that are non-fundamental to the underlying securities. On the announcement date of index recomposition, investors receive a signal of effective-date demand shocks. Sophisticated investors can even front run the market before the index recomposition announcement by making informed predictions about how index funds will have to rebalance.

¹⁰Zou (2019) used the BlackRock case to provide identification to the documented positive correlation between ETF flows and the price-to-fundamentals relation of underlying stocks.

The BGI transaction. BlackRock, Inc. announced on June 11, 2009 that it agreed to pay \$13.5 billion to acquire Barclays Global Investors (BGI), including iShares, from Barclays Bank PLC.¹¹ When the deal closed on December 1, 2009, BlackRock paid approximately \$6.65 billion in cash and issued capital stock valued at \$8.53 billion to Barclays.¹²

While Barclays sold BGI to raise funds to strengthen its balance sheet, BlackRock struck the deal with Barclays, according to its Chairman and CEO Laurence Fink, to “broaden the firm’s investment capability with passive and quantitative investment strategies, particularly exchange-traded funds, and to address client demand for outcome-oriented portfolio strategies and offer institutional and other investors a full spectrum of investment strategies, including passive and quantitative approaches”. BGI’s products complement BlackRock’s fundamental active strategies that employ a bottom-up security selection process led by portfolio managers who use quantitative and fundamental analysis and their market experience to inform their judgment. BlackRock aims at industry leadership in ETF globally. The BGI transaction will increase the scale benefits and introduce synergies between alpha and beta investments. Observing the news of the BGI transaction on June 11, sophisticated investors expect the iShares AUM to expand in the following years. For the component stocks of iShares ETFs, these are future demand

¹¹Barclays PLC Annual Report (2009), <https://home.barclays/content/dam/home-barclays/documents/investor-relations/annualreports/ar2009/2009-form-20-F.pdf> (accessed December 23, 2019).

¹²BlackRock, Inc. Annual Report (2009), <https://ir.blackrock.com/Cache/1001211956.PDF> (accessed December 23, 2019).

shocks that are not related to their firm fundamentals.

5. Conclusion

In the NREE literature, research has been done to understand how information acquisition on current demand shocks, current dividend innovations and future dividend innovations affects the market. This article tries to put together the missing piece and studies how learning about future demand shocks would affect investors' information and investment choices and their ex ante utility, as well as price informativeness and volatility.

This article shows that with access to signals about future demand shocks, investors choose to learn less about current dividend innovations and demand shocks and allocate precision to learn about future demand shocks. Learning about future demand shocks affects the financial market through the front-running channel and the future uncertainty channel. To isolate the future uncertainty channel, a set of results are derived for the static model where the asset is a series of dividend strips. If future demand shocks are independent from the demand shocks today, then future demand shocks per se do not affect today's asset price. It is *information acquisition* on future demand shocks that makes it relevant to current price through the *future uncertainty channel*. While the current equilibrium price still does not aggregate signals about future demand shocks, information acquisition on future demand shocks affects the risk premium and how the current equilibrium price aggregates signals about current shocks. The future uncertainty channel works through two forces: on the one hand, the aggregate level of dividend information acquisition decreases, which makes dividends more risky and leads to higher

required returns; on the other hand, an atomic investor is more certain about the asset payoff in the second period due to the signal about the second-period demand shocks she acquires in the first period. As a result, investors' ex ante expected utility improves. Price volatility increases due to more heterogeneous private information, and learning about future demand shocks makes price more informative in the future but less informative now.

Learning about future demand shocks is important since it breaks the fallacy of composition and improves the investors' ex ante expected utility. It extends the concept of news shock and have implications for business cycle analysis. It also breaks the conventional fallacy of composition in information economies: wider investor access to private forward information does not necessarily deteriorate aggregate market outcomes. This suggests that predictable demand-side policy signals and information disclosure on future demand shocks can improve welfare.

Appendix A. Equilibrium Definition

DEFINITION 3. *A rational expectations equilibrium in this economy consists of a sequence of information choices $\{\tau_{eyt}, \tau_{ext}, \tau_{vxt}, \hat{\tau}_{eyit+1}, \hat{\tau}_{exit+1}, \hat{\tau}_{vxit+1}\}$, risky asset prices $\{p_t\}$, and portfolio holdings $\{q_{it}, \hat{q}_{it+1}\}$ defined for all $t \in [0, \infty)$, satisfying the following conditions:*

- (a) *At the start of period t , each $\mathcal{G}_t$ -investor i selects signal precisions τ_{eyt} , τ_{ext} and τ_{vxt} to maximize ex ante utility $\mathbb{E}[-\exp(-\rho\tilde{W}_{it+1}) \mid \mathcal{I}_{t-1}^+]$ subject to the information capacity constraint (4);*
- (b) *In the portfolio choice stage of period t , each $\mathcal{G}_t$ -investor i allocates q_{it}*

to the risky asset to maximize $\mathbb{E}[-\exp(-\rho\tilde{W}_{it+1+}) \mid \mathcal{I}_{it}]$, taking asset prices and realized signals as given, subject to the budget constraint (3);

(c) At the beginning of period $t + 1$, each $\mathcal{G}_t$ -investor i chooses signal precisions $\hat{\tau}_{\epsilon yit+1}$, $\hat{\tau}_{\epsilon xit+1}$ and $\hat{\tau}_{\nu xit+1}$ to maximize $\mathbb{E}[-\exp(-\rho\tilde{W}_{it+1+}) \mid \hat{\mathcal{I}}_{it}^+]$, conditioning on private signals acquired in period t , subject to the information capacity constraint (2);

(d) In the portfolio choice stage of period $t + 1$, each $\mathcal{G}_t$ -investor i holds $\hat{q}_{it+1}$ in the risky asset to maximize $\mathbb{E}[-\exp(-\rho\tilde{W}_{it+1+}) \mid \hat{\mathcal{I}}_{it+1}]$, treating current and lagged signal realizations together with asset prices as exogenous, subject to the budget constraint (1);

(e) The risky asset price clears the market via the equality of aggregate demand and aggregate supply:

$$\int_{i \in \mathcal{G}_t} q_{it} \, di + \int_{i \in \mathcal{G}_{t-1}} \hat{q}_{it} - q_{it-1} \, di = \bar{x}_t + \tilde{x}_t + \int_{i \in \mathcal{G}_{t-2}} \hat{q}_{it-1} \, di. \quad (\text{A.1})$$

Appendix B. Model Solutions

Appendix B.1. Investors

This section works out investors' optimal information and portfolio choices, taking price coefficients and market aggregates as given. Without loss of generality, the model is solved in period t . Note that although A_{t+1} depends on $\tilde{x}_t$ (B.46), which is not $\mathcal{I}_{t-1}^+$ - or $\mathcal{I}_{it}$ -measurable, the solution to the optimization problems faced by investors are not affected.

Appendix B.1.1. Bayesian updating

At the beginning of period t , all $\mathcal{G}_t$ -investors are identical and thus hold the same belief about period- t payoffs.

Their information choices, $\tau_{\epsilon xt}$, $\tau_{\epsilon yt}$ and $\tau_{\nu xt}$, are also identical (we drop the i subscript for simplicity). Since the signal realizations $\mathcal{G}_t$ -investors receive in period t are different, at the t -period portfolio choice stage, they hold heterogeneous beliefs about t -period payoffs. Some useful conditional variances that we can use to simplify notation is the conditional variance of dividend, current and future demand shocks in current period.

$$V_{yt} \equiv \mathcal{V}(\tilde{d}_t \mid \mathcal{I}_{it}) = 1/(\tau_y + \tau_{\epsilon yt} + \tau_{pt}), \quad (\text{B.1})$$

$$V_{xt} \equiv \mathcal{V}(\tilde{x}_t \mid \mathcal{I}_{it}) = 1/(\tau_x + \tau_{\epsilon xt}), \quad (\text{B.2})$$

$$V_{xt}^\xi \equiv \mathcal{V}(\tilde{x}_{t+1} \mid \mathcal{I}_{it}) = 1/(\tau_x + \tau_{\nu xt}). \quad (\text{B.3})$$

On the aggregate level, the period- $(t + 1)$ price coefficients are $\mathcal{I}_{it}$ -measurable since there is a continuum of investors and thus the price coefficients depend on the distributions of shocks, not the realizations. Therefore, for $\mathcal{G}_t$ -investor i , at period- t portfolio choice stage, the uncertainty in the period- t excess payoff $\tilde{d}_t + \alpha p_{t+1} - p_t r$ comes from $\tilde{y}_t$, $\tilde{y}_{t+1}$, $\tilde{x}_t$, $\tilde{x}_{t+1}$ and $\tilde{x}_{t+2}$, and the uncertainty in the period- $(t + 1)$ excess payoff $\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r$ comes from $\tilde{y}_t$, $\tilde{y}_{t+1}$, $\tilde{y}_{t+2}$, $\tilde{x}_t$, $\tilde{x}_{t+1}$, $\tilde{x}_{t+2}$ and $\tilde{x}_{t+3}$. Thus the corresponding conditional expectation and variance of the period- t excess

payoff are

$$\begin{aligned}\mathcal{V}_t &\equiv \mathbb{V}ar(\tilde{d}_t + \alpha p_{t+1} - rp_t \mid \mathcal{I}_{it}) \\ &= (1 + \alpha B_{t+1})^2 V_{yt} + \alpha C_{t+1}^2 \tau_y^{-1} + \alpha D_{t+1}^2 V_{xt}^\xi + \alpha E_{t+1}^2 \tau_x^{-1},\end{aligned}\quad (\text{B.4})$$

$$\begin{aligned}\mathcal{E}_{it} &\equiv \mathbb{E}[\tilde{d}_t + \alpha p_{t+1} - rp_t \mid \mathcal{I}_{it}] \\ &= (1 + \alpha B_{t+1}) \left[\mu + gd_{t-1} + V_{yt} (\tau_{eyt} \tilde{\eta}_{it}^y + \tau_{pit} \tilde{\eta}_{it}^p) \right] + \alpha A_{t+1} \\ &\quad + \alpha D_{t+1} V_{xt}^\xi \tau_{\nu xt} \tilde{\xi}_{it}^x - rp_t.\end{aligned}\quad (\text{B.5})$$

For $\mathcal{G}_{t-1}$ -investors, however, the signal realizations they receive in period $t-1$ are different, which adds another layer of heterogeneity to their information set. For $\mathcal{G}_{t-1}$ -investor i at the period- t portfolio choice stage, conditional variances of the current dividend innovation, the current demand and the future demand shock are

$$\hat{V}_{yit} \equiv \mathbb{V}ar(\tilde{d}_t \mid \hat{\mathcal{I}}_{it}) = 1/(\tau_y + \hat{\tau}_{eyit} + \hat{\tau}_{pit}), \quad (\text{B.6})$$

$$\hat{V}_{xit} \equiv \mathbb{V}ar(\tilde{x}_t \mid \hat{\mathcal{I}}_{it}) = 1/(\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1}), \quad (\text{B.7})$$

$$\hat{V}_{xit}^\xi \equiv \mathbb{V}ar(\tilde{x}_{t+1} \mid \hat{\mathcal{I}}_{it}) = 1/(\tau_x + \hat{\tau}_{\nu xit}), \quad (\text{B.8})$$

For $\mathcal{G}_{t-1}$ -investor i at period- t portfolio choice stage, the uncertainty in the period- t excess payoff $\tilde{d}_t + \alpha p_{t+1} - p_t r$ comes from $\tilde{y}_t$, $\tilde{y}_{t+1}$, $\tilde{x}_t$, $\tilde{x}_{t+1}$ and $\tilde{x}_{t+2}$, and the corresponding conditional expectation and variance of are

$$\begin{aligned}\hat{\mathcal{V}}_{it} &\equiv \mathbb{V}ar(\tilde{d}_t + \alpha p_{t+1} - rp_t \mid \hat{\mathcal{I}}_{it}) \\ &= (1 + \alpha B_{t+1})^2 \hat{V}_{yit} + \alpha C_{t+1}^2 \tau_y^{-1} + \alpha D_{t+1}^2 \hat{V}_{xit}^\xi + \alpha E_{t+1}^2 \tau_x^{-1},\end{aligned}\quad (\text{B.9})$$

$$\begin{aligned}\hat{\mathcal{E}}_{it} &\equiv \mathbb{E}[\tilde{d}_t + \alpha p_{t+1} - rp_t \mid \hat{\mathcal{I}}_{it}] \\ &= (1 + \alpha B_{t+1}) \left[\mu + gd_{t-1} + \hat{V}_{yit} (\hat{\tau}_{eyit} \tilde{\eta}_{it}^y + \hat{\tau}_{pit} \tilde{\eta}_{it}^p) \right] + \alpha A_{t+1} \\ &\quad + \alpha D_{t+1} \hat{V}_{xit}^\xi \hat{\tau}_{\nu xit} \tilde{\xi}_{it}^x - rp_t.\end{aligned}\quad (\text{B.10})$$

To obtain the expectation and variance of period- $(t + 1)$ excess payoffs conditional on $\hat{\mathcal{I}}_{it+1}$, $\hat{V}_{it+1}$ and $\hat{E}_{it+1}$, we only need to change the time subscript t in (B.9) and (B.10) to $t + 1$. Notice that $\hat{V}_i$ is $\mathcal{I}_{t-1}^+$ -measurable, but $\hat{V}_{it+1}$ is only $\mathcal{I}_{it}$ -measurable, not $\mathcal{I}_{t-1}^+$ -measurable. This is left to verify after we have derived the ex-ante expected utility.

Appendix B.1.2. Optimal portfolio choice

In this section, I will solve for the optimal portfolio choice of $\mathcal{G}_t$ -investor i in both period t and $t + 1$. The solution will apply to any period t .

In period $t + 1$, $\mathcal{G}_t$ -investor i faces the standard one-shot portfolio choice problem, and the solution is

$$\hat{q}_{it+1} = \hat{\mathcal{E}}_{it+1} / (\rho \hat{\mathcal{V}}_{it+1}). \quad (\text{B.11})$$

This solution is taken as given in period t . But note that because the uncertainty in $\hat{\mathcal{E}}_{it+1}$ is not fully resolved at the portfolio choice stage of period t , the realization of $\hat{q}_{it+1}$ is not known, while its distribution is.

To solve the portfolio choice problem for $\mathcal{G}_t$ -investor i in period t , plug in (3) to the expected utility $\mathbb{E}[-\exp(-\rho \tilde{W}_{it+1}) \mid \mathcal{I}_{it}]$, which is maximized by choosing q_{it} . The expected utility can be simplified to

$$\begin{aligned} \mathbb{E}[-\exp(-\rho \tilde{W}_{it+1}) \mid \mathcal{I}_{it}] &= -\exp(-\rho r^2 W_0) \mathbb{E} \left[\exp \left(-\rho r q_{it} (\tilde{d}_t + \alpha p_{t+1} - r p_t) - \right. \right. \\ &\quad \left. \left. \rho \hat{q}_{it+1} (\tilde{d}_{t+1} + \alpha p_{t+2} - r p_{t+1}) \right) \mid \mathcal{I}_{it} \right]. \end{aligned}$$

To proceed, we first summarize the properties of the covariance term in the following lemma.

LEMMA 4. *At the t -period portfolio choice stage, the covariance of the t -period payoff and the $(t+1)$ -period payoff of the risky asset for $\mathcal{G}_t$ -investor i is*

$$\begin{aligned}\mathcal{C}_t &\equiv \text{Cov}\left(\hat{\mathcal{E}}_{it+1}(\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1}r), \tilde{d}_t + \alpha p_{t+1} - p_t r \mid \mathcal{I}_{it}\right) \\ &= (2\gamma_1\delta_2\gamma_2^2 + \delta_1\gamma_2\gamma_2^\dagger)V_{yt} + [\gamma_1\delta_3(\gamma_3 + \gamma_3^\dagger) + \delta_1\gamma_3\gamma_3^\dagger]\tau_y + \\ &\quad [\gamma_1\delta_4(\gamma_4 + \gamma_4^\dagger) + \delta_1\gamma_4\gamma_4^\dagger]V_{xt}^\xi + [\gamma_1\delta_5(\gamma_5 + \gamma_5^\dagger) + \delta_1\gamma_5\gamma_5^\dagger]\tau_x. \quad (\text{B.12})\end{aligned}$$

For the static model, $\mathcal{C}_t = 0$.

Proof. For $\mathcal{G}_t$ -investor i at her portfolio choice stage, the t -period payoff of the risky asset is

$$\tilde{d}_t + \alpha p_{t+1} \mid \mathcal{I}_{it} = \delta_1 + \delta_2 \tilde{y}_t \mid \tilde{\eta}_{it}^y, \tilde{\eta}_{it}^p + \delta_3 \tilde{y}_{t+1} + \delta_4 \tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x + \delta_5 \tilde{x}_{t+2},$$

where $\delta_1 = (1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1}$, $\delta_2 = 1 + \alpha B_{t+1}$, $\delta_3 = \delta_3 = \alpha C_{t+1}$, $\delta_4 = \alpha D_{t+1}$ and $\delta_5 = \alpha E_{t+1}$.

The $(t+1)$ -period excess payoff is

$$\begin{aligned}\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1}r \mid \mathcal{I}_{it} &= \gamma_1 + \gamma_2 \tilde{y}_t \mid \tilde{\eta}_{it}^y, \tilde{\eta}_{it}^p + \gamma_3^\dagger \tilde{y}_{t+1} + \gamma_4^\dagger \tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x + \gamma_5^\dagger \tilde{x}_{t+2} + \\ &\quad \alpha(C_{t+2} \tilde{y}_{t+2} + E_{t+2} \tilde{x}_{t+3}),\end{aligned}$$

where $\gamma_1 = (1 + \alpha B_{t+2})(\mu + g\mu + g^2 d_{t-1}) - r B_{t+1}(\mu + d_{t-1}) - r A_{t+1} + \alpha A_{t+2}$, $\gamma_2 = g(1 + \alpha B_{t+2}) - r B_{t+1}$, $\gamma_3^\dagger = \gamma_3^\dagger = 1 + \alpha B_{t+2} - r C_{t+1}$, $\gamma_4^\dagger = -r D_{t+1}$ and $\gamma_5^\dagger = \alpha D_{t+2} - r E_{t+1}$.

The expected period- $(t+1)$ excess payoff conditional on $\hat{\mathcal{I}}_{it+1}$ is a random variable at the portfolio choice stage of period t , denoted by $\hat{\mathcal{E}}_{it+1} \mid \mathcal{I}_{it}$. From (B.10), we have that

$$\begin{aligned}\hat{\mathcal{E}}_{it+1} &\equiv (1 + \alpha B_{t+2})\left[\mu + g \tilde{d}_t + \hat{V}_{yit+1}(\hat{\tau}_{eyit+1} \tilde{\eta}_{it+1}^y + \hat{\tau}_{pit+1} \tilde{\eta}_{it+1}^p)\right] + \\ &\quad \alpha A_{t+2} + \alpha D_{t+2} \hat{V}_{xit+1}^\xi \hat{\tau}_{\nu xit+1} \tilde{\xi}_{it+1}^x - p_{t+1}r.\end{aligned}$$

Recall that the period- $t+1$ price signal for $\mathcal{G}_t$ -investor i can be written as

$$\begin{aligned}\tilde{\eta}_{it+1}^p &= \tilde{y}_{t+1} + \frac{1}{\zeta_{1t+1}} \left(\tilde{x}_{t+1} - \mathbb{E}[\tilde{x}_{t+1} \mid \tilde{\eta}_{it+1}^x, \tilde{\xi}_{it}^x] \right) \\ &\quad + \frac{1}{\zeta_{2t+1}} \left(\tilde{x}_{t+2} - \mathbb{E}[\tilde{x}_{t+2} \mid \tilde{\xi}_{it+1}^x] \right) \\ &= \tilde{y}_{t+1} + \frac{1}{\zeta_{1t+1}} \left(\tilde{x}_{t+1} - \hat{V}_{xit+1} \left(\hat{\tau}_{\epsilon xit+1} \tilde{\eta}_{it+1}^x + \tau_{\nu xt} \tilde{\xi}_{it}^x \right) \right) \\ &\quad + \frac{1}{\zeta_{2t+1}} \left(\tilde{x}_{t+2} - \hat{V}_{xit+1}^\xi \hat{\tau}_{\nu xit+1} \tilde{\xi}_{it+1}^x \right).\end{aligned}$$

From the above two equations, we know

$$\hat{\mathcal{E}}_{it+1} \mid \mathcal{I}_{it} = \gamma_1 + \gamma_2 \tilde{y}_t \mid \tilde{\eta}_{it}^y, \tilde{\eta}_{it}^p + \gamma_3 \tilde{y}_{t+1} + \gamma_4 \tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x + \gamma_5 \tilde{x}_{t+2},$$

where

$$\begin{aligned}\gamma_3 &= (1 + \alpha B_{t+2}) \hat{V}_{yit+1} (\hat{\tau}_{\epsilon yit+1} + \hat{\tau}_{pit+1}) - r C_{t+1}, \\ \gamma_4 &= \alpha D_{t+2} \hat{V}_{xit+1}^\xi \hat{\tau}_{\nu xit+1} - r D_{t+1} + (1 + \alpha B_{t+2}) \hat{V}_{yit+1} \hat{\tau}_{pit+1} \hat{V}_{xit+1} \tau_x / \zeta_{1t+1}, \\ \gamma_5 &= (1 + \alpha B_{t+2}) \hat{V}_{yit+1} \hat{\tau}_{pit+1} \hat{V}_{xit+1}^\xi \tau_x / \zeta_{2t+1} - r E_{t+1}.\end{aligned}$$

Using the assumption that $\tilde{y}_t$, $\tilde{y}_{t+1}$, $\tilde{x}_{t+1}$ and $\tilde{x}_{t+2}$ are zero-mean and mutually independent, it follows that all third moments and cross moments are zero. Thus

$$\begin{aligned}\mathcal{C}_t &\equiv \text{Cov} \left(\hat{\mathcal{E}}_{it+1} (\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r), \tilde{d}_t + \alpha p_{t+1} - p_t r \mid \mathcal{I}_{it} \right) \\ &= (2\gamma_1 \delta_2 \gamma_2 + \delta_1 \gamma_2 \gamma_2^\dagger) V_{yt} + [\gamma_1 \delta_3 (\gamma_3 + \gamma_3^\dagger) + \delta_1 \gamma_3 \gamma_3^\dagger] \tau_y + \\ &\quad [\gamma_1 \delta_4 (\gamma_4 + \gamma_4^\dagger) + \delta_1 \gamma_4 \gamma_4^\dagger] V_{xt}^\xi + [\gamma_1 \delta_5 (\gamma_5 + \gamma_5^\dagger) + \delta_1 \gamma_5 \gamma_5^\dagger] \tau_x.\end{aligned}$$

The stationary solution of B_t gives that $\gamma_2 = (1 + \alpha B_{t+2})g - r B_{t+1} = 0$.

In the static model where $\alpha = 0$, we have $\mathcal{C}_t = 0$. $\square$

On the other hand, the *ex ante* choice of precision of signals in the next period is irrelevant absent of realizations of those signals. In other words, $\tau_{\epsilon xit+1}$, $\tau_{\epsilon yit+1}$, $\tau_{\nu xit+1}$ do not enter the objective function and thus do not affect q_{it} and q_{it+1} if investors are still at the portfolio choice stage of period t . We summarize it in the following lemma.

LEMMA 5. *At the portfolio choice stage of period t , $\mathcal{G}_t$ -investor i does not need to consider information choice of next period.*

Therefore, the optimization problem can be simplified to

$$\max_{q_{it}} r q_{it} \mathcal{E}_{it} - \frac{\rho}{2} r^2 q_{it}^2 \mathcal{V}_t - r \mathcal{C}_t \frac{1}{\hat{\mathcal{V}}_{it+1}} q_{it},$$

which implies

$$q_{it} = \frac{\mathcal{E}_{it}}{\rho r \mathcal{V}_t} - \frac{\mathcal{C}_t}{\rho r \mathcal{V}_t \hat{\mathcal{V}}_{it+1}}. \quad (\text{B.13})$$

In the static case ($\alpha = 0$), $\mathcal{C}_t = 0$, which greatly simplifies the portfolio choices to

$$q_{it} = \frac{\mathbb{E}[\tilde{d}_t | \mathcal{I}_{it}] - p_t r}{\rho r \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}, \quad \hat{q}_{it} = \frac{\mathbb{E}[\tilde{d}_t | \hat{\mathcal{I}}_{it}] - p_t r}{\rho r \text{Var}(\tilde{d}_t | \hat{\mathcal{I}}_{it})}. \quad (\text{B.14})$$

Appendix B.1.3. Optimal Information Choices

In this section, I calculate the *ex ante* expected utility of $\mathcal{G}_t$ - and $\mathcal{G}_{t-1}$ -investors at the beginning of period t when they have not made any period- t information or portfolio choice, and then solve their information choice problems, that is to maximize the *ex ante* expected utility subject to the information capacity constraint. The *ex ante* expected utility of $\mathcal{G}_{t-1}$ -investor i is summarized by the following proposition.

PROPOSITION 7. *At the beginning of period t , the ex ante utility of $\mathcal{G}_{t-1}$ is*

$$\mathbb{E}[\exp(-\rho r W_{it+}) \mid \hat{\mathcal{I}}_{it-1+}] = -\exp(-\rho r W_{it-1+}) |\hat{\Sigma}_{t-1+} / \hat{\mathcal{V}}_{it}|^{-1/2} \times \exp(-\mu_{\hat{\mathcal{E}}_t}^2 / (2\hat{\Sigma}_{t-1+})). \quad (\text{B.15})$$

where $\mu_{\hat{\mathcal{E}}_t}$ and $\hat{\Sigma}_{t-1+}$ are defined by (B.18) and (B.17), respectively.

Proof.

Step 1: Calculate $\mathbb{E}[-\exp(-\rho W_{it+}) \mid \hat{\mathcal{I}}_{it}]$

Conditional on $\hat{\mathcal{I}}_{it}$, the end-of-period- t wealth W_{it+} is log-normally distributed, and thus $\mathbb{E}[-\exp(-\rho W_{it+}) \mid \hat{\mathcal{I}}_{it}] = -\exp(-\rho \mathbb{E}[W_{it+} \mid \hat{\mathcal{I}}_{it}] + (\rho^2/2)\text{Var}(W_{it+} \mid \hat{\mathcal{I}}_{it}))$. Combining the budget constraint $W_{it+} = rW_{it-1+} + \hat{q}_{it}(\tilde{d}_t + \alpha p_{t+1} - p_t r)$ with the portfolio choice of $\mathcal{G}_{t-1}$ -investors (8), we have $\mathbb{E}[W_{it+} \mid \hat{\mathcal{I}}_{it}] = rW_{it-1+} + \hat{\mathcal{E}}_{it}^2 / (\rho \hat{\mathcal{V}}_{it})$ and $\text{Var}(W_{it+} \mid \hat{\mathcal{I}}_{it}) = \hat{\mathcal{E}}_{it}^2 / (\rho^2 \hat{\mathcal{V}}_{it})$. Therefore,

$$\mathbb{E}[-\exp(-\rho W_{it+}) \mid \hat{\mathcal{I}}_{it}] = -\exp(-\rho r W_{it-1+}) \exp\left(-\frac{\hat{\mathcal{E}}_{it}^2}{2\hat{\mathcal{V}}_{it}}\right). \quad (\text{B.16})$$

Step 2: Calculate $\mathbb{E}[-\exp(-\rho W_{it+}) \mid \hat{\mathcal{I}}_{it-1+}]$

By law of iterated expectations,

$$\begin{aligned} \mathbb{E}[-\exp(-\rho W_{it+}) \mid \hat{\mathcal{I}}_{it-1+}] &= \mathbb{E}\left[\mathbb{E}[-\exp(-\rho W_{it+}) \mid \hat{\mathcal{I}}_{it}] \mid \hat{\mathcal{I}}_{it-1+}\right] \\ &= -\exp(-\rho r W_{it-1+}) \mathbb{E}\left[\exp\left(-\frac{\hat{\mathcal{E}}_{it}^2}{2\hat{\mathcal{V}}_{it}}\right) \mid \hat{\mathcal{I}}_{it-1+}\right]. \end{aligned}$$

Let $\mu_{\hat{\mathcal{E}}_t} \equiv \mathbb{E}[\hat{\mathcal{E}}_{it} \mid \hat{\mathcal{I}}_{it-1}^+]$ and $\hat{\Sigma}_{t-1+} \equiv \text{Var}(\tilde{d}_t + \alpha p_{t+1} - p_t r \mid \hat{\mathcal{I}}_{it-1}^+)$. Conditional on $\hat{\mathcal{I}}_{it-1+}$, $\hat{\mathcal{E}}_{it}$ is still normally distributed. By law of total variance, $\hat{\Sigma}_{t-1+} = \mathbb{E}[\hat{\mathcal{V}}_{it} \mid \mathcal{I}_{it-1}^+] + \text{Var}(\hat{\mathcal{E}}_{it} \mid \mathcal{I}_{it-1}^+)$. Since $\hat{\mathcal{V}}_{it}$ is $\mathcal{I}_{it-1}^+$ -measurable, we have $\text{Var}(\hat{\mathcal{E}}_{it} \mid$

$\mathcal{I}_{it-1}^+) = \hat{\Sigma}_{t-1+} - \hat{\mathcal{V}}_{it}$. And the total variance

$$\begin{aligned}\hat{\Sigma}_{t-1+} &\equiv \mathbb{V}ar\left(\tilde{d}_t + \alpha p_{t+1} - p_t r \mid \hat{\mathcal{I}}_{it-1+}\right) \\ &= \left[(1 + \alpha B_{t+1} - r C_t)^2 + \alpha C_{t+1}^2\right] \tau_y + r^2 D_t^2 (\tau_x + \tau_{\nu x t-1})^{-1} + \\ &\quad \left[(\alpha D_{t+1} - r E_t)^2 + \alpha E_{t+1}^2\right] \tau_x.\end{aligned}\quad (\text{B.17})$$

To compute the mean of $\hat{\mathcal{E}}_{it}$, by law of iterated expectations,

$$\begin{aligned}\mu_{\hat{\mathcal{E}}_t} &\equiv \mathbb{E}[\hat{\mathcal{E}}_{it} \mid \hat{\mathcal{I}}_{it-1+}] = \mathbb{E}[\tilde{d}_t + \alpha p_{t+1} - p_t r \mid \hat{\mathcal{I}}_{it-1+}] \\ &= \frac{r}{r - \alpha g} \mu + \alpha A_{t+1} - r A_t - r D_t V_{xt-1}^{\xi} \tau_{\nu x t-1} \xi_{it-1}^x.\end{aligned}\quad (\text{B.18})$$

To work out $\mathbb{E}[\exp(-\hat{\mathcal{E}}_{it}^2/(2\hat{\mathcal{V}}_{it})) \mid \hat{\mathcal{I}}_{it-1+}]$, it is useful to write it in terms of the mean-zero variable $\hat{\mathcal{E}}_{it} - \mu_{\hat{\mathcal{E}}_t}$ ¹³:

$$\begin{aligned}&\mathbb{E}\left[\exp\left(-\frac{\hat{\mathcal{E}}_{it}^2}{2\hat{\mathcal{V}}_{it}}\right) \mid \hat{\mathcal{I}}_{it-1+}\right] \\ &= \mathbb{E}\left[\exp\left(-\frac{(\hat{\mathcal{E}}_{it} - \mu_{\hat{\mathcal{E}}_t})^2}{2\hat{\mathcal{V}}_{it}} - \frac{(\hat{\mathcal{E}}_{it} - \mu_{\hat{\mathcal{E}}_t})\mu_{\hat{\mathcal{E}}_t}}{\hat{\mathcal{V}}_{it}} - \frac{\mu_{\hat{\mathcal{E}}_t}^2}{2\hat{\mathcal{V}}_{it}}\right) \mid \hat{\mathcal{I}}_{it-1+}\right].\end{aligned}$$

Evaluating this expectation yields

$$\mathbb{E}\left[\exp\left(-\frac{\hat{\mathcal{E}}_{it}^2}{2\hat{\mathcal{V}}_{it}}\right) \mid \hat{\mathcal{I}}_{it-1+}\right] = \left|\frac{\hat{\Sigma}_{t-1+}}{\hat{\mathcal{V}}_{it}}\right|^{-1/2} \exp\left(-\frac{\mu_{\hat{\mathcal{E}}_t}^2}{2\hat{\Sigma}_{t-1+}}\right).$$

□

¹³See Veldkamp (2011). The general formula for the mean of the exponential of a multivariate quadratic form of normal variables is the following: If $z \sim N(0, \Sigma)$, F , G and H are non-random, then $\mathbb{E}[\exp(z' F z + G' z + H)] = |I - 2\Sigma F|^{-1/2} \exp\left((1/2)G'(I - 2\Sigma F)^{-1}\Sigma G + H\right)$.

The key feature of (B.15) is that period- t information choices only enter through $\hat{\mathcal{V}}_{it}$, which greatly simplifies the optimization problem, as stated in the following lemma.

LEMMA 6. *At the beginning of period t , the information choice facing $\mathcal{G}_{t-1}$ -investor i is equivalent to*

$$\begin{aligned} \max_{\{\hat{\tau}_{eyit}, \hat{\tau}_{exit}, \hat{\tau}_{vxit}\}} -\hat{\mathcal{V}}_{it}^{\dagger} \equiv & -\alpha D_{t+1}^2 (\tau_x + \hat{\tau}_{vxit})^{-1} \\ & - \frac{r^2}{(r - \alpha g)^2} \left[\tau_y + \hat{\tau}_{eyit} + \left(\frac{1}{\zeta_{1t}^2 (\tau_x + \hat{\tau}_{exit} + \tau_{vx t-1})} + \right. \right. \\ & \left. \left. \frac{1}{\zeta_{2t}^2 (\tau_x + \hat{\tau}_{vxit})} \right)^{-1} \right]^{-1}. \end{aligned} \quad (\text{B.19})$$

subject to information capacity constraint.

Proof. From (B.18) and (B.17), it is clear that $\mu_{\hat{\varepsilon}_t}$ and $\hat{\Sigma}_{t-1+}$ are functions of model primitives, price coefficients, precision choice and signal realization of last period's future demand signal, but have nothing to do with this period's information choices, $\hat{\tau}_{eyit}$, $\hat{\tau}_{exit}$ and $\hat{\tau}_{vxit}$. The information choices enter the *ex ante* utility (B.15) only through $\hat{\mathcal{V}}_{it}$. Therefore, maximizing the *ex ante* utility amounts to minimizing $\hat{\mathcal{V}}_{it}$, or maximizing $-\hat{\mathcal{V}}_{it}$. Using (B.9), and disposing of terms that only contains model primitives and price coefficients, one gets the equivalent objective function $\hat{\mathcal{V}}_{it}^{\dagger}$. $\square$

PROPOSITION 8. *The optimal period- t information choice for $\mathcal{G}_{t-1}$ -investor i exists and is unique. Furthermore, it is $\mathcal{I}_{t-1}^+$ -measurable and thus homogeneous for all $\mathcal{G}_{t-1}$ -investors.*

Proof. To begin with, let me calculate the marginal utility with respect to

$\hat{\tau}_{eyit}$, $\hat{\tau}_{exit}$ and $\hat{\tau}_{\nu xit}$.

$$\begin{aligned}
\hat{L}_{eyit} &\equiv -\frac{\partial \hat{\mathcal{V}}_{it}^{\dagger}}{\partial \hat{\tau}_{eyit}} \\
&= r^2(r - \alpha g)^{-2} \times \\
&\quad \left[\tau_y + \hat{\tau}_{eyit} + \left[\frac{1}{\zeta_{1t}^2(\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1})} + \frac{1}{\zeta_{2t}^2(\tau_x + \hat{\tau}_{\nu xit})} \right]^{-1} \right]^{-2}, \\
\hat{L}_{exit} &\equiv -\frac{\partial \hat{\mathcal{V}}_{it}^{\dagger}}{\partial \hat{\tau}_{exit}} \\
&= r^2(r - \alpha g)^{-2} \zeta_{1t}^{-2} (\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1})^{-2} \times \\
&\quad \left[\left[\frac{1}{\zeta_{1t}^2(\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1})} + \frac{1}{\zeta_{2t}^2(\tau_x + \hat{\tau}_{\nu xit})} \right] \right]^{-2} \times \\
&\quad \left[\tau_y + \hat{\tau}_{eyit} + \left[\frac{1}{\zeta_{1t}^2(\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1})} + \frac{1}{\zeta_{2t}^2(\tau_x + \hat{\tau}_{\nu xit})} \right]^{-1} \right]^{-2}, \\
\hat{L}_{\nu xit} &\equiv -\frac{\partial \hat{\mathcal{V}}_{it}^{\dagger}}{\partial \hat{\tau}_{\nu xit}} \\
&= \alpha D_{t+1}^2 (\tau_x + \hat{\tau}_{\nu xit})^{-2} + r^2(r - \alpha g)^{-2} \zeta_{1t}^{-2} (\tau_x + \hat{\tau}_{\nu xit})^{-2} \times \\
&\quad \left[\left[\frac{1}{\zeta_{1t}^2(\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1})} + \frac{1}{\zeta_{2t}^2(\tau_x + \hat{\tau}_{\nu xit})} \right] \right]^{-2} \times \\
&\quad \left[\tau_y + \hat{\tau}_{eyit} + \left[\frac{1}{\zeta_{1t}^2(\tau_x + \hat{\tau}_{exit} + \tau_{\nu xt-1})} + \frac{1}{\zeta_{2t}^2(\tau_x + \hat{\tau}_{\nu xit})} \right]^{-1} \right]^{-2}.
\end{aligned}$$

It is straightforward to verify that $\partial \hat{\mathcal{V}}_{it}^{\dagger} / \partial \hat{\tau}_{eyit}$, $\partial \hat{\mathcal{V}}_{it}^{\dagger} / \partial \hat{\tau}_{exit}$, $\partial \hat{\mathcal{V}}_{it}^{\dagger} / \partial \hat{\tau}_{\nu xit} > 0$. Computing the second-order partial derivatives further yields $\partial^2 \hat{\mathcal{V}}_{it}^{\dagger} / \partial \hat{\tau}_{eyit}^2$, $\partial^2 \hat{\mathcal{V}}_{it}^{\dagger} / \partial \hat{\tau}_{exit}^2$, $\partial^2 \hat{\mathcal{V}}_{it}^{\dagger} / \partial \hat{\tau}_{\nu xit}^2 < 0$. As a result, $\hat{\mathcal{V}}_{it}^{\dagger}$ and hence the *ex ante* utility function given in Eq.B.15 are strictly increasing and concave with respect to the information choices $\hat{\tau}_{eyit}$, $\hat{\tau}_{exit}$, and $\hat{\tau}_{\nu xit}$. As the information capacity constraint is linear, the solution to the period- t

information choice problem for generation- t investors ($\mathcal{G}_{t-1}$) exists and is unique.

The F.O.C.'s for the constrained optimization problem is

$$\hat{L}_{eyit} = \lambda \gamma_{ey}, \quad (\text{B.20})$$

$$\hat{L}_{exit} = \lambda \gamma_{ex}, \quad (\text{B.21})$$

$$\hat{L}_{\nu xit} = \lambda \gamma_{\nu x}, \quad (\text{B.22})$$

$$\gamma_{ey} \hat{\tau}_{eyit} + \gamma_{ex} \hat{\tau}_{exit} + \gamma_{\nu x} \hat{\tau}_{\nu xit} = K. \quad (\text{B.23})$$

Dividing (B.20) by (B.21), one can express $\hat{\tau}_{\nu xit}$ in terms of $\hat{\tau}_{exit}$, and dividing (B.20) by (B.22), one can express $\hat{\tau}_{exit}$ in terms of $\hat{\tau}_{\nu xit}$ and $\hat{\tau}_{exit}$. Then plugging them back into (B.23) pins down $\hat{\tau}_{exit}$. Because the F.O.C.'s depend only on model primitives, price coefficients and $\tau_{\nu xt-1}$, and $\tau_{\nu xt-1}$ is by definition $\mathcal{I}_{t-1}^+$ -measurable, period- t information choices of $\mathcal{G}_{t-1}$ -investor i is also $\mathcal{I}_{t-1}^+$ -measurable. $\square$

This means that despite the fact that $\mathcal{I}_{it-1}^+$, the information set $\mathcal{G}_{t-1}$ -investors hold at the beginning of period t is heterogeneous, the information that can be used for information choice is still homogeneous. Information choices for both periods are determined in the very beginning, and investors within the same generation all make the same information choices for both periods.

LEMMA 7. *The uncertainty in the variance of excess payoff resolves earlier than the realization of corresponding shocks. More specifically,*

- (1) $\mathcal{V}_t$, $\mathcal{V}_2$ and $\mathcal{C}$ are $\mathcal{I}_{t-1}^+$ -measurable;
- (2) $\hat{\mathcal{V}}_{it+1}$ is $\mathcal{I}_{t-1}^+$ -measurable.

PROPOSITION 9. *The ex ante utility of $\mathcal{G}_t$ is*

$$\begin{aligned}
& \mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+] \\
&= - \left| \frac{\hat{\Sigma}_{t+}}{\hat{\mathcal{V}}_{it+1}} \right|^{-1/2} \left| 1 - 2(\tau_x^{-1} + \tau_{\nu xt}^{-1})F \right|^{-1/2} \times \\
& \quad \exp \left(-\rho r^2 W_{it-1+} + \frac{5\mathcal{C}_t^2}{8\mathcal{V}_t \hat{\mathcal{V}}_{it+1}^2} - \frac{\mathcal{C}_t^2}{8\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}^2} \right) \times \\
& \quad \exp \left(-\frac{1}{2} \frac{G^2}{(\tau_x^{-1} + \tau_{\nu xt}^{-1})^{-1} - 2F} + H \right), \tag{B.24}
\end{aligned}$$

where F , G , and H are defined by (B.26), (B.27), and (B.28), respectively.

Proof. Plugging in the optimal portfolio choices into the *ex ante* expected utility, we have

$$\begin{aligned}
& \mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+] \\
&= -\exp(-\rho r^2 W_0) \mathbb{E} \left[\exp \left(- \left(\frac{\mathcal{E}_{it}}{\mathcal{V}_t} - \frac{\mathcal{C}_t}{\mathcal{V}_t \hat{\mathcal{V}}_{it+1}} \right) (\tilde{d}_t + \alpha p_{t+1} - p_t r) \right. \right. \\
& \quad \left. \left. - \frac{\hat{\mathcal{E}}_{it+1}}{\hat{\mathcal{V}}_{it+1}} (\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1} r) \right) \right].
\end{aligned}$$

According to Lemma 9, $\hat{\mathcal{V}}_{it+1}$ is $\mathcal{I}_{it}$ -measurable. Thus the exponent term in the above expression is a quadratic form of normal variables under $\mathcal{I}_{it}$. Then we derive the *ex ante* expected utility in two steps.

Step 1: Calculate $\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \hat{\mathcal{I}}_{it+1}]$

Conditional on $\hat{\mathcal{I}}_{it+1}$, W_{it+1+} is a normal random variable. Thus

$$\begin{aligned}
& \mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \hat{\mathcal{I}}_{it+1}] \\
&= -\exp \left(-\rho \mathbb{E}[W_{it+1+} \mid \hat{\mathcal{I}}_{it+1}] + \frac{\rho^2}{2} \text{Var}(W_{it+1+} \mid \hat{\mathcal{I}}_{it+1}) \right). \tag{B.25}
\end{aligned}$$

Recall that $W_{it+1+} = rW_{it+} + \hat{q}_{it+1}(\tilde{d}_{t+1} + \alpha p_{t+2} - p_{t+1}r)$. Plugging the optimal portfolio, we have $\mathbb{E}[W_{it+1+} | \hat{\mathcal{I}}_{it+1}] = rW_{it+} + \hat{\mathcal{E}}_{it+1}^2/(\rho\hat{\mathcal{V}}_{it+1})$ and $\text{Var}(W_{it+1+} | \hat{\mathcal{I}}_{it+1}) = \hat{\mathcal{E}}_{it+1}^2/(\rho^2\hat{\mathcal{V}}_{it+1})$. Plugging them back into (B.25), we have

$$\mathbb{E} \left[-\exp(-\rho W_{it+1+}) | \hat{\mathcal{I}}_{it+1} \right] = -\exp(-\rho r W_{it+}) \exp \left(-\frac{\hat{\mathcal{E}}_{it+1}^2}{2\hat{\mathcal{V}}_{it+1}} \right).$$

Step 2: Calculating $\mathbb{E}[-\exp(-\rho W_{it+1+}) | \mathcal{I}_t]$.

Applying (7) to period $t+1$, one has

$$\mathbb{E} \left[-\exp(-\rho W_{it+1+}) | \mathcal{I}_t^+ \right] = -\exp(-\rho r W_{it+}) \left| \frac{\hat{\Sigma}_{t+}}{\hat{\mathcal{V}}_{it+1}} \right|^{-1/2} \exp \left(-\frac{\mu_{\hat{\mathcal{E}}_{t+1}}^2}{2\hat{\Sigma}_{t+}} \right).$$

On the one hand, note that $\hat{\Sigma}_{t+}$ is $\mathcal{I}_{t-1}^+$ -measurable, and that $\mu_{\hat{\mathcal{E}}_{t+1}}$ is $\mathcal{I}_{it}$ -measurable. On the other hand, $W_{it+} = rW_{it-1+} + q_{it}(\tilde{d}_t + \alpha p_{t+1} - p_t r)$, and $q_{it} = \mathcal{E}_{it}/(\rho r \mathcal{V}_t) - \mathcal{C}_t/(\rho r \mathcal{V}_t \hat{\mathcal{V}}_{it+1})$. Thus

$$\mathbb{E} \left[\exp(-\rho r W_{it+}) | \mathcal{I}_t \right] = \exp(-\rho r^2 W_{it-1+}) h_{it},$$

where the rightmost item h_{it} is the expectation of the exponential of a linear combination of random variables, that is $h_{it} = \exp \left(-\mathcal{E}_{it}^2/(2\mathcal{V}_t) + \mathcal{C}_t \mathcal{E}_{it}/(2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}) + \mathcal{C}_t^2/(2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}^2) \right)$. Therefore, by law of iterated expectations,

$$\begin{aligned} & \mathbb{E} \left[-\exp(-\rho W_{it+1+}) | \mathcal{I}_t \right] \\ &= \mathbb{E} \left[\mathbb{E} \left[-\exp(-\rho W_{it+1+}) | \mathcal{I}_t^+ \right] | \mathcal{I}_t \right] \\ &= - \left| \frac{\hat{\Sigma}_{t+}}{\hat{\mathcal{V}}_{it+1}} \right|^{-1/2} \exp \left(-\rho r^2 W_{it-1+} + \frac{\mathcal{C}_t^2}{2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}^2} \right) \times \\ & \quad \exp \left(-\frac{\mathcal{E}_{it}^2}{2\mathcal{V}_t} + \frac{\mathcal{C}_t \mathcal{E}_{it}}{2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}} - \frac{\mu_{\hat{\mathcal{E}}_{t+1}}^2}{2\hat{\Sigma}_{t+}} \right). \end{aligned}$$

Step 3: Calculate $\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+, \xi_{it}^x]$.

Conditional on $\mathcal{I}_{t-1}^+$ and ξ_t^x , randomness in the above equation only comes from $\mathcal{E}_{it}$. Let $\mu_{\mathcal{E}_t}^\xi \equiv \mathbb{E}[\mathcal{E}_{it} \mid \mathcal{I}_{t-1}^+, \xi_{it}^x]$ and $\Sigma_{t-1+}^\xi \equiv \text{Var}(\tilde{d}_t + \alpha p_{t+1} - p_t r \mid \mathcal{I}_{t-1}^+, \xi_{it}^x)$.

$$\begin{aligned}\mu_{\mathcal{E}_t}^\xi &\equiv \mathbb{E}[\mathcal{E}_{it} \mid \mathcal{I}_{t-1}^+, \xi_t^x] = \mathbb{E}[\tilde{d}_t + \alpha p_{t+1} - p_t r \mid \mathcal{I}_{t-1}^+, \xi_t^x] \\ &= \frac{r}{r - \alpha g} \mu + \alpha A_{t+1} - r A_t + (\alpha D_{t+1} - r E_t) V_{xt}^\xi \tau_{\nu xt} \xi_{it}^x.\end{aligned}$$

And the total variance

$$\begin{aligned}\Sigma_{t-1+}^\xi &\equiv \text{Var}(\tilde{d}_t + \alpha p_{t+1} - p_t r \mid \hat{\mathcal{I}}_{it-1+}) \\ &= (\alpha D_{t+1} - r E_t)^2 (\tau_x + \tau_{\nu xt})^{-1} + \\ &\quad [r^2 D_t^2 + \alpha E_{t+1}^2] \tau_x + [(1 + \alpha B_{t+1} - r C_t)^2 + \alpha C_{t+1}^2] \tau_y.\end{aligned}$$

It is useful to write $\mathcal{E}_{it}$ in terms of mean-zero variable $\mathcal{E}_{it} - \mu_{\mathcal{E}_t}^\xi$, that is

$$-\frac{\mathcal{E}_{it}^2}{2\mathcal{V}_t} = -\frac{(\mathcal{E}_{it} - \mu_{\mathcal{E}_t}^\xi)^2}{2\mathcal{V}_t} - \frac{(\mathcal{E}_{it} - \mu_{\mathcal{E}_t}^\xi)\mu_{\mathcal{E}_t}^\xi}{\mathcal{V}_t} - \frac{(\mu_{\mathcal{E}_t}^\xi)^2}{2\mathcal{V}_t}.$$

Therefore,

$$\begin{aligned}&\mathbb{E}\left[\exp\left(-\frac{\mathcal{E}_{it}^2}{2\mathcal{V}_t} + \frac{C_t \mathcal{E}_{it}}{2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}}\right) \mid \mathcal{I}_{t-1}^+\right] \\ &= \mathbb{E}\left[\exp\left(-\frac{(\mathcal{E}_{it} - \mu_{\mathcal{E}_t}^\xi)^2}{2\mathcal{V}_t} + \left(\frac{C_t}{2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}} - \frac{\mu_{\mathcal{E}_t}^\xi}{\mathcal{V}_t}\right)(\mathcal{E}_{it} - \mu_{\mathcal{E}_t}^\xi) - \frac{(\mu_{\mathcal{E}_t}^\xi)^2}{2\mathcal{V}_t} + \frac{C_t \mu_{\mathcal{E}_t}^\xi}{2\mathcal{V}_t \hat{\mathcal{V}}_{it+1}}\right) \mid \mathcal{I}_{t-1}^+\right] \\ &= \exp\left(\frac{C_t^2}{8\hat{\mathcal{V}}_{it+1}^2} \left(\frac{1}{\mathcal{V}_t} - \frac{1}{\Sigma_{t-1+}^\xi}\right)\right) \exp\left(-\frac{(\mu_{\mathcal{E}_t}^\xi)^2}{2\Sigma_{t-1+}^\xi} - \frac{C_t \mu_{\mathcal{E}_t}^\xi}{2\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}}\right).\end{aligned}$$

Putting them together, one has

$$\begin{aligned}&\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+, \xi_{it}^x] \\ &= \mathbb{E}\left[\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{it}] \mid \mathcal{I}_{t-1}^+, \xi_{it}^x\right] \\ &= -\left|\frac{\hat{\Sigma}_{t+}}{\hat{\mathcal{V}}_{it+1}}\right|^{-1/2} \exp\left(-\rho r^2 W_{it-1+} + \frac{5C_t^2}{8\mathcal{V}_t \hat{\mathcal{V}}_{it+1}^2} - \frac{C_t^2}{8\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}^2}\right) \exp(f),\end{aligned}$$

where

$$f = -\frac{\mu_{\hat{\mathcal{E}}_{t+1}}^2}{2\hat{\Sigma}_{t+}} - \frac{(\mu_{\mathcal{E}_t}^\xi)^2}{2\Sigma_{t-1+}^\xi} - \frac{c_t \mu_{\mathcal{E}_t}^\xi}{2\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}}.$$

Step 4: Calculate $\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+]$.

Conditional on $\mathcal{I}_{t-1}^+$, the only randomness in f comes from $\mu_{\hat{\mathcal{E}}_{t+1}}$ and $\mu_{\mathcal{E}_t}^\xi$, both of which are functions of ξ_{it}^x . Let $M = r\mu/(r - \alpha g) + \alpha A_{t+1} - rA_t$. Then

$$f = F^2 \xi_{it}^x + G \xi_{it} + H,$$

where

$$F = -\frac{1}{2} (V_{xt}^\xi \tau_{\nu xt})^2 \left[\hat{\Omega}_{t+} r^2 D_{t+1}^2 + \Omega_{t-1+}^\xi (\alpha D_{t+1} - r E_t)^2 \right], \quad (\text{B.26})$$

$$G = V_{xt}^\xi \tau_{\nu xt} \left[\hat{\Omega}_{t+} M r D_{t+1} - \Omega_{t-1+}^\xi (\alpha D_{t+1} - r E_t) \left(M - \frac{C_t}{2\hat{\mathcal{V}}_{it+1}} \right) \right], \quad (\text{B.27})$$

$$H = -\frac{M^2}{2\hat{\Sigma}_{t+}} - \frac{M^2}{2\Sigma_{t-1+}^\xi} - \frac{C_t M}{2\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}}. \quad (\text{B.28})$$

The *ex ante* utility for $\mathcal{G}_t$ investors is

$$\begin{aligned} & \mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+] \\ &= \mathbb{E} \left[\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+, \xi_{it}^x] \mid \mathcal{I}_{t-1}^+ \right] \\ &= - \left| \frac{\hat{\Sigma}_{t+}}{\hat{\mathcal{V}}_{it+1}} \right|^{-1/2} \left| 1 - 2(\tau_x^{-1} + \tau_{\nu xt}^{-1}) F \right|^{-1/2} \exp \left(-\frac{1}{2} \frac{G^2}{(\tau_x^{-1} + \tau_{\nu xt}^{-1})^{-1} - 2F} + H \right) \\ & \quad \exp \left(-\rho r^2 W_{it-1+} + \frac{5C_t^2}{8\mathcal{V}_t \hat{\mathcal{V}}_{it+1}^2} - \frac{C_t^2}{8\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}^2} \right). \end{aligned}$$

□

One can do a strictly increasing transformation of the *ex ante* utility:

$$\begin{aligned} V &\stackrel{\text{def}}{=} -2 \ln \left[\frac{-\mathbb{E}[-\exp(-\rho W_{it+1+}) \mid \mathcal{I}_{t-1}^+]}{\exp(-\rho r^2 W_0)} \right] \\ &= \ln H_1 + \ln H_2 + H_3 + H_4, \end{aligned} \quad (\text{B.29})$$

where

$$H_1 = \frac{\hat{\Sigma}_{t+}}{\hat{\mathcal{V}}_{it+1}}, \quad (\text{B.30})$$

$$H_2 = 1 + V_{xt}^\xi \frac{\tau_{\nu xt}}{\tau_x} \left[\hat{\Omega}_{t+} r^2 D_{t+1}^2 + \Omega_{t-1+}^\xi (\alpha D_{t+1} - r E_t)^2 \right], \quad (\text{B.31})$$

$$H_3 = -\frac{5\mathcal{C}_t^2}{4\mathcal{V}_t \hat{\mathcal{V}}_{it+1}^2} + \frac{\mathcal{C}_t^2}{4\Sigma_{t-1+}^\xi \hat{\mathcal{V}}_{it+1}^2}, \quad (\text{B.32})$$

$$H_4 = \frac{G^2}{(\tau_x^{-1} + \tau_{\nu xt}^{-1})^{-1} - 2F} - H. \quad (\text{B.33})$$

Appendix B.2. Market Clearing

In period t , $\mathcal{G}_t$ -investors make the same information choices, while $\mathcal{G}_{t-1}$ -investors choose precision heterogeneously, dependent on what they already learned in period $t-1$ about the period- t demand shock. This effectively makes the aggregation of signals and expected payoffs for the two generations different.

When aggregated, the noise in signals cancels out. The price signals of $\mathcal{G}_t$ -investors aggregate to

$$\begin{aligned} \int_{i \in \mathcal{G}_t} \tilde{\eta}_{it}^p \, di &= \int_{i \in \mathcal{G}_t} \frac{1}{C_t} \left(p_t - A_t - B_t d_{t-1} - D_t \mathbb{E}[\tilde{x}_t \mid \tilde{\eta}_{it}^x] - E_t \mathbb{E}[\tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x] \right) \, di \\ &= \frac{1}{2} \left[\frac{1}{C_t} (p_t - A_t - B_t d_{t-1}) - \frac{1}{\zeta_{1t}} V_{xt} \tau_{\epsilon xt} \tilde{x}_t - \frac{1}{\zeta_{2t}} V_{xt}^\xi \tau_{\nu xt} \tilde{x}_{t+1} \right], \end{aligned} \quad (\text{B.34})$$

Then the expected period- t excess payoffs for $\mathcal{G}_t$ -investors aggregate to

$$\begin{aligned} \int_{i \in \mathcal{G}_t} \mathcal{E}_{it} \, di &= \frac{1}{2} \left\{ (1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} + (1 + \alpha B_{t+1}) V_{yt} \tau_{\epsilon yt} \tilde{y}_t + \right. \\ &\quad \alpha D_{t+1} (1 - V_{xt}^\xi \tau_x) \tilde{x}_{t+1} - p_t r + (1 + \alpha B_{t+1}) V_{yt} \tau_{pt} \times \\ &\quad \left. \left[\frac{1}{C_t} (p_t - A_t - B_t d_{t-1}) - \frac{1}{\zeta_{1t}} V_{xt} \tau_{\epsilon xt} \tilde{x}_t - \frac{1}{\zeta_{2t}} V_{xt}^\xi \tau_{\nu xt} \tilde{x}_{t+1} \right] \right\}. \end{aligned} \quad (\text{B.35})$$

For $\mathcal{G}_{t-1}$ -investors, heterogeneity in conditional variances makes things more complicated:

$$\begin{aligned}
& \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{\mathcal{V}}_{it}} \tilde{\eta}_{it}^p di \\
&= \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{\mathcal{V}}_{it}} \frac{1}{C_t} \left(p_t - A_t - B_t d_{t-1} - D_t \mathbb{E}[\tilde{x}_t \mid \tilde{\eta}_{it}^x, \tilde{\xi}_{it-1}^x] - \right. \\
&\quad \left. E_t \mathbb{E}[\tilde{x}_{t+1} \mid \tilde{\xi}_{it}^x] \right) di \\
&= \frac{1}{2} \left[\hat{\omega}_{pt} \frac{1}{C_t} (p_t - A_t - B_t d_{t-1}) - \frac{1}{\zeta_{1t}} \hat{\mathcal{Z}}_{1t} \tilde{x}_t - \frac{1}{\zeta_{2t}} \hat{\mathcal{Z}}_{2t} \tilde{x}_{t+1} \right], \tag{B.36}
\end{aligned}$$

where

$$\begin{aligned}
\hat{\mathcal{Z}}_{1t} &\equiv 2 \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{\mathcal{V}}_{it}} (1 - \hat{V}_{xit} \tau_x) di, \\
\hat{\mathcal{Z}}_{2t} &\equiv 2 \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{\mathcal{V}}_{it}} (1 - \hat{V}_{xit}^\xi \tau_x) di, \\
\hat{\omega}_{pt} &\equiv 2 \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{\mathcal{V}}_{it}} di
\end{aligned}$$

are known by the market since the distributions of $\hat{\tau}_{exit}$ and $\hat{\tau}_{\nu exit}$ are known. Define the average posterior precision of period- t excess payoff to be $\hat{\Omega}_t \equiv 2 \int_{i \in \mathcal{G}_{t-1}} \frac{1}{\hat{\mathcal{V}}_i} di$, the average ratio of the posterior variance of current demand shock to the posterior variance of period- t excess payoff to be $\hat{\chi}_{xt}^\xi \equiv 2 \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{xit}^\xi}{\hat{\mathcal{V}}_{it}} di$, and $\hat{\omega}_{yt} \equiv 2 \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{eyit}}{\hat{\mathcal{V}}_{it}} di$. Then the risk premium of $\mathcal{G}_{t-1}$ -investors aggregate to

$$\begin{aligned}
\int_{i \in \mathcal{G}_{t-1}} \frac{\hat{\mathcal{E}}_{it}}{\hat{\mathcal{V}}_{it}} di &= \frac{1}{2} \left\{ \hat{\Omega}_t [(1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} - p_t r] + \right. \\
&\quad (1 + \alpha B_{t+1}) \hat{\omega}_{yt} \tilde{y}_t + \alpha D_{t+1} (\hat{\Omega}_t - \hat{\chi}_{xt} \tau_x) \tilde{x}_{t+1} + \\
&\quad (1 + \alpha B_{t+1}) \left[\hat{\omega}_{pt} \frac{1}{C_t} [p_t - A_t - B_t d_{t-1}] - \frac{1}{\zeta_{1t}} \hat{\mathcal{Z}}_{1t} \tilde{x}_t - \right. \\
&\quad \left. \left. \frac{1}{\zeta_{2t}} \hat{\mathcal{Z}}_{2t} \tilde{x}_{t+1} \right] \right\}. \tag{B.37}
\end{aligned}$$

Recall that the market clearing condition is

$$\bar{x}_t + \tilde{x}_t + \int_{i \in \mathcal{G}_{t-2}} \hat{q}_{it-1} di = \int_{i \in \mathcal{G}_t} q_{it} di + \int_{i \in \mathcal{G}_{t-1}} (\hat{q}_{it} - q_{it-1}) di.$$

Plugging in portfolio choices, multiplying ρ on both sides and moving $\frac{\mathcal{C}_t}{r\mathcal{V}_t\hat{\mathcal{V}}_{it+1}}$ to LHS, one has

$$\begin{aligned} & \rho(\bar{x}_t + \tilde{x}_t) + \int_{i \in \mathcal{G}_t} \frac{\mathcal{C}_t}{r\mathcal{V}_t\hat{\mathcal{V}}_{it+1}} di + \int_{i \in \mathcal{G}_{t-1}} \left(\frac{\mathcal{E}_{it-1}}{r\mathcal{V}_{t-1}} - \frac{\mathcal{C}_{t-1}}{r\mathcal{V}_{t-1}\hat{\mathcal{V}}_{it}} \right) di + \\ & \int_{i \in \mathcal{G}_{t-2}} \frac{\hat{\mathcal{E}}_{it-1}}{\hat{\mathcal{V}}_{it-1}} di \\ & = \int_{i \in \mathcal{G}_t} \frac{\mathcal{E}_{it}}{r\mathcal{V}_t} di + \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{\mathcal{E}}_{it}}{\hat{\mathcal{V}}_{it}} di. \end{aligned}$$

Let's work out RHS and LHS separately. RHS contains constants, p_t , $\tilde{y}_t$, $\tilde{x}_t$ and $\tilde{x}_{t+1}$:

$$\begin{aligned} \text{RHS} = & \frac{1}{2r} \Omega_t \left\{ (1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} + (1 + \alpha B_{t+1}) V_{yt} \tau_{eyt} \tilde{y}_t + \right. \\ & \alpha D_{t+1} (1 - V_{xt}^\xi \tau_x) \tilde{x}_{t+1} - p_t r + (1 + \alpha B_{t+1}) V_{yt} \tau_{pt} \times \\ & \left[\frac{1}{C_t} (p_t - A_t - B_t d_{t-1}) - \frac{1}{\zeta_{1t}} V_{xt} \tau_{ext} \tilde{x}_t - \frac{1}{\zeta_{2t}} (1 - V_{xt}^\xi \tau_x) \tilde{x}_{t+1} \right] \Big\} + \\ & \frac{1}{2} \left\{ \hat{\Omega}_t [(1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} - p_t r] + \right. \\ & (1 + \alpha B_{t+1}) \hat{\omega}_{yt} \tilde{y}_t + \alpha D_{t+1} (\hat{\Omega}_t - \hat{\chi}_{xt} \tau_x) \tilde{x}_{t+1} + \\ & \left. (1 + \alpha B_{t+1}) \left[\hat{\omega}_{pt} \frac{1}{C_t} [p_t - A_t - B_t d_{t-1}] - \frac{1}{\zeta_{1t}} \hat{\mathcal{Z}}_{1t} \tilde{x}_t - \frac{1}{\zeta_{2t}} \hat{\mathcal{Z}}_{2t} \tilde{x}_{t+1} \right] \right\}. \end{aligned}$$

Collecting terms for $\tilde{y}_t$, $\tilde{x}_t$ and $\tilde{x}_{t+1}$, we have

$$\begin{aligned}
\text{RHS} = & \frac{1}{2r}\Omega_t \left\{ (1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} - p_t r + \right. \\
& \left. (1 + \alpha B_{t+1})V_{yt}\tau_{pt} \left[\frac{1}{C_t}(p_t - A_t - B_t d_{t-1}) \right] \right\} + \\
& \frac{1}{2} \left\{ \hat{\Omega}_t [(1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} - p_t r] + \right. \\
& \left. (1 + \alpha B_{t+1}) \left[\hat{\omega}_{pt} \frac{1}{C_t} [p_t - A_t - B_t d_{t-1}] \right] \right\} + \\
& (1 + \alpha B_{t+1})\bar{\omega}_{yt}\tilde{y}_t - (1 + \alpha B_{t+1})\frac{1}{\zeta_{1t}}\bar{Z}_{1t}\tilde{x}_t + \\
& \left[\alpha D_{t+1}(\bar{\Sigma}_t^{-1} - \bar{\chi}_{xt}\tau_x) - (1 + \alpha B_{t+1})\frac{1}{\zeta_{2t}}\bar{Z}_{2t} \right] \tilde{x}_{t+1}, \tag{B.38}
\end{aligned}$$

where

$$\begin{aligned}
\bar{\omega}_{yt} &= \frac{1}{2r}\Omega_t V_{yt}\tau_{eyt} + \frac{1}{2}\hat{\omega}_{yt}, & \bar{\chi}_{xt} &= \frac{1}{2r}\Omega_t V_{xt}^\xi + \frac{1}{2}\hat{\chi}_{xt}, \\
\bar{Z}_{2t} &= \frac{1}{2r}\Omega_t V_{yt}\tau_{pt}(1 - V_{xt}^\xi \tau_x) + \frac{1}{2}\hat{Z}_{2t}, & \bar{Z}_{1t} &= \frac{1}{2r}\Omega_t V_{yt}\tau_{pt}V_{xt}\tau_{ext} + \frac{1}{2}\hat{Z}_{1t}.
\end{aligned}$$

LHS only contains constants and $\tilde{x}_t$:

$$\begin{aligned}
\text{LHS} = & \rho(\bar{x}_t + \tilde{x}_t) + \frac{1}{2}\hat{\Omega}_{t+1}\frac{\mathcal{C}_t}{r\mathcal{V}_t} - \frac{1}{2}\hat{\Omega}_t\frac{\mathcal{C}_{t-1}}{r\mathcal{V}_{t-1}} + (1 + \alpha B_t)\bar{\omega}_{yt-1}\tilde{y}_{t-1} + \\
& \frac{1}{2r}\Omega_{t-1} \left\{ (1 + \alpha B_t)(\mu + g d_{t-2}) + \alpha A_t - p_{t-1}r + \right. \\
& \left. (1 + \alpha B_t)V_{yt-1}\tau_{pt-1} \left[\frac{1}{C_{t-1}}(p_{t-1} - A_{t-1} - B_{t-1}d_{t-2}) \right] \right\} + \\
& \frac{1}{2} \left\{ \hat{\Omega}_{t-1} [(1 + \alpha B_t)(\mu + g d_{t-2}) + \alpha A_t - p_{t-1}r] + \right. \\
& \left. (1 + \alpha B_t) \left[\hat{\omega}_{pt-1} \frac{1}{C_{t-1}} [p_{t-1} - A_{t-1} - B_{t-1}d_{t-2}] \right] \right\} - \\
& (1 + \alpha B_t)\frac{1}{\zeta_{1t-1}}\bar{Z}_{1t-1}\tilde{x}_{t-1} + \\
& \left[\alpha D_t(\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}\tau_x) - (1 + \alpha B_t)\frac{1}{\zeta_{2t-1}}\bar{Z}_{2t-1} \right] \tilde{x}_t.
\end{aligned}$$

Notice that at the portfolio choice stage of period t , $\tilde{y}_{t-1}$ and $\tilde{x}_{t-1}$ are constants.

For notational simplicity, let

$$\begin{aligned}\bar{A}_t = & \frac{1}{2}\hat{\Omega}_{t+1}\frac{\mathcal{C}_t}{r\mathcal{V}_t} - \frac{1}{2}\hat{\Omega}_t\frac{\mathcal{C}_{t-1}}{r\mathcal{V}_{t-1}} + \frac{1}{2r}\Omega_{t-1}\left\{(1+\alpha B_t)(\mu + gd_{t-2}) + \alpha A_t - p_{t-1}r + \right. \\ & \left. (1+\alpha B_t)V_{yt-1}\tau_{pt-1}\left[\frac{1}{C_{t-1}}(p_{t-1} - A_{t-1} - B_{t-1}d_{t-2})\right]\right\} + \\ & \frac{1}{2}\left\{\hat{\Omega}_{t-1}[(1+\alpha B_t)(\mu + gd_{t-2}) + \alpha A_t - p_{t-1}r] + \right. \\ & \left. (1+\alpha B_t)\left[\hat{\omega}_{pt-1}\frac{1}{C_{t-1}}[p_{t-1} - A_{t-1} - B_{t-1}d_{t-2}]\right]\right\} + \\ & (1+\alpha B_t)\bar{\omega}_{yt-1}\tilde{y}_{t-1} - (1+\alpha B_t)\frac{1}{\zeta_{1t-1}}\bar{Z}_{1t-1}\tilde{x}_{t-1}.\end{aligned}$$

Then

$$\text{LHS} = \bar{A}_t + \rho\bar{x}_t + \rho\tilde{x}_t + \alpha D_t[\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}\tau_x]\tilde{x}_t - (1+\alpha B_t)\frac{1}{\zeta_{2t-1}}\bar{Z}_{2t-1}\tilde{x}_t, \quad (\text{B.39})$$

Then we try to match coefficients of (B.38) and (B.39) with the following linear equilibrium price relation:

$$p_t = A_t + B_t d_{t-1} + C_t \tilde{y}_t + D_t \tilde{x}_t + E_t \tilde{x}_{t+1}.$$

Define the average market-wide posterior precision as $\bar{\Sigma}_t^{-1} \equiv \frac{1}{2}(\Omega_t/r + \hat{\Omega}_t)$.

Move (B.39) to RHS and $\bar{\Sigma}_t^{-1}p_t r$ to LHS, and then multiply by $\bar{\Sigma}_t/r$ on both

sides. This yields

$$\begin{aligned}
p_t r = & \frac{1}{r} \bar{\Sigma}_t \left\{ \frac{1}{2r} \Omega_t \left[(1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} + \right. \right. \\
& (1 + \alpha B_{t+1}) V_{yt} \tau_{pt} \left[\frac{1}{C_t} (p_t - A_t - B_t d_{t-1}) \right] \left. \right] + \\
& \frac{1}{2} \left[\hat{\Omega}_t \left[(1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} \right] + \right. \\
& (1 + \alpha B_{t+1}) \left[\hat{\omega}_{pt} \frac{1}{C_t} [p_t - A_t - B_t d_{t-1}] \right] \left. \right] + \\
& (1 + \alpha B_{t+1}) \bar{\omega}_{yt} \tilde{y}_t - (1 + \alpha B_{t+1}) \frac{1}{\zeta_{1t}} \bar{Z}_{1t} \tilde{x}_t + \\
& \left[\alpha D_{t+1} (\bar{\Sigma}_t^{-1} - \bar{\chi}_{xt} \tau_x) - (1 + \alpha B_{t+1}) \frac{1}{\zeta_{2t}} \bar{Z}_{2t} \right] \tilde{x}_{t+1} - \\
& \bar{A} - \rho \tilde{x}_t - \alpha D_t [\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1} \tau_x] \tilde{x}_t + (1 + \alpha B_t) \frac{1}{\zeta_{2t-1}} \bar{Z}_{2t-1} \tilde{x}_t \left. \right\}.
\end{aligned}$$

It is easy to see that at the coefficients of p_t and $B_t d_{t-1}$ on RHS only differ in sign. Define $\bar{\omega}_{pt} = \frac{1}{2r} \Omega_t V_{yt} \tau_{pt} + \frac{1}{2} \hat{\omega}_{pt}$. Again, move the p_t term to LHS and scale down so that $p_t r$ on LHS has coefficient 1:

$$\begin{aligned}
p_t r = & \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \times \frac{1}{r} \bar{\Sigma}_t \left\{ \right. \\
& \bar{\Sigma}_t^{-1} \left[(1 + \alpha B_{t+1})(\mu + g d_{t-1}) + \alpha A_{t+1} \right] + \\
& (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} (-A_t - B_t d_{t-1}) + \\
& (1 + \alpha B_{t+1}) \bar{\omega}_{yt} \tilde{y}_t - (1 + \alpha B_{t+1}) \frac{1}{\zeta_{1t}} \bar{Z}_{1t} \tilde{x}_t + \\
& \left[\alpha D_{t+1} (\bar{\Sigma}_t^{-1} - \bar{\chi}_{xt}^\xi \tau_x) - (1 + \alpha B_{t+1}) \frac{1}{\zeta_{2t}} \bar{Z}_{2t} \right] \tilde{x}_{t+1} - \\
& \bar{A} - \rho \tilde{x}_t - \alpha D_t [\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}^\xi \tau_x] \tilde{x}_t + (1 + \alpha B_t) \frac{1}{\zeta_{2t-1}} \bar{Z}_{2t-1} \tilde{x}_t \left. \right\}.
\end{aligned} \tag{B.40}$$

B_t :

From (B.40), it is easy to identify B_t , the coefficient before d_{t-1} :

$$B_t = \frac{1}{r} \bar{\Sigma}_t \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \times \left[\bar{\Sigma}_t^{-1} (1 + \alpha B_{t+1}) g - (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} B_t \right].$$

Moving all B_t terms to the left hand side, we have

$$B_t = \left[1 + \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \frac{1}{r} \bar{\Sigma}_t (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right]^{-1} \times \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \frac{1}{r} g (1 + \alpha B_{t+1}).$$

Since

$$\left[1 + \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \frac{1}{r} \bar{\Sigma}_t (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right]^{-1} \times \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} = 1,$$

it simplifies to

$$B_t = \frac{1}{r} g (1 + \alpha B_{t+1}). \quad (\text{B.41})$$

This is a linear difference equation and has a stationary solution¹⁴

$$B = \frac{g}{r - \alpha g}.$$

A_t :

From (B.40), the constant term in the equilibrium price relation is:

$$A_t = \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \times \frac{1}{r} \bar{\Sigma}_t \left[\bar{\Sigma}_t^{-1} (1 + \alpha B_{t+1}) \mu + \alpha A_{t+1} - (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} A_t - \bar{A}_t \right].$$

¹⁴The stationary solution is obtained by setting $B_t = B$, $\forall t$ in (B.41) and solving for B .

Moving all A_t terms to the left hand side, we have

$$A_t = \frac{1}{r} [(1 + \alpha B_{t+1})\mu - \rho \bar{x}_t - \bar{A}_t(A_{t+1}, A_{t+2}) + \alpha A_{t+1} \bar{\Sigma}_t]. \quad (\text{B.42})$$

To emphasize the dependency of $\bar{A}_t$ on A_{t+1} and A_{t+2} , I write $\bar{A}_t(A_{t+1}, A_{t+2})$ instead of $\bar{A}_t$. Plugging in the stationary solution of B_t , we have

$$A_t = \frac{\mu}{r - \alpha g} - \frac{\rho \bar{x}_t}{r} - \frac{1}{r} \bar{A}_t(A_{t+1}, A_{t+2}) + \frac{\bar{\Sigma}_t}{r} \alpha A_{t+1} \bar{\Sigma}_t.$$

C_t :

The coefficient of $\tilde{y}_t$ in (B.40) is:

$$C_t = \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \frac{1}{r} \bar{\Sigma}_t (1 + \alpha B_{t+1}) \bar{\omega}_{yt}.$$

Multiplying $\left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)$ on both sides and rearranging, we have

$$C_t = \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) (\bar{\omega}_{yt} + \bar{\omega}_{pt}).$$

Since

$$V_{yt}(\tau_{eyt} + \tau_{pt}) = 1 - V_{yt}\tau_y,$$

$$\hat{\omega}_{yt} + \hat{\omega}_{pt} = 2 \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit}(\hat{\tau}_{eyit} + \hat{\tau}_{pit})}{\hat{V}_{it}} di = \hat{\Omega}_t - 2\tau_y \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit}}{\hat{V}_{it}} di,$$

one has

$$C_t = \frac{1}{r - \alpha g} (1 - \bar{\Sigma}_t \bar{\chi}_{yt} \tau_y),$$

where

$$\bar{\chi}_{yt} \equiv \frac{1}{r} \Omega_t \frac{V_{yt}}{2} + \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit}}{\hat{V}_{it}} di.$$

D_t :

The coefficient of the $\tilde{x}_t$ term in the price relation is:

$$D_t = \frac{1}{r} \bar{\Sigma}_t \left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \times \\ \left[-\rho - \alpha D_t (\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}^\xi \tau_x) - (1 + \alpha B_{t+1}) \frac{1}{\zeta_{1t}} \bar{Z}_{1t} + (1 + \alpha B_t) \frac{1}{\zeta_{2t-1}} \bar{Z}_{2t-1} \right].$$

Multiplying $\left(1 - \frac{\bar{\Sigma}_t}{r} (1 + \alpha B_{t+1}) \bar{\omega}_{pt} \frac{1}{C_t} \right)$ on both sides, we have

$$D_t = \frac{\bar{\Sigma}_t}{r - \alpha g \zeta_{1t}} (\bar{\omega}_{pt} - \bar{Z}_{1t}) - \frac{\rho \bar{\Sigma}_t}{r} + \frac{\bar{\Sigma}_t}{r - \alpha g \zeta_{2t-1}} \bar{Z}_{2t-1} + \\ \alpha D_t \frac{\bar{\Sigma}_t}{r} \left(\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}^\xi \tau_x \right).$$

We can then solve for D_t straightforwardly:

$$D_t = \left[1 - \alpha \frac{\bar{\Sigma}_t}{r} \left(\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}^\xi \tau_x \right) \right]^{-1} \times \\ \left[\frac{\bar{\Sigma}_t}{r - \alpha g \zeta_{1t}} (\bar{\omega}_{pt} - \bar{Z}_{1t}) - \frac{\rho \bar{\Sigma}_t}{r} + \frac{\bar{\Sigma}_t}{r - \alpha g \zeta_{2t-1}} \bar{Z}_{2t-1} \right].$$

For a better understanding of $\bar{\omega}_{pt} - \bar{Z}_{1t}$, let $\chi_{yt} \equiv \Omega_t V_{yt}$, which characterizes the proportion of excess payoff uncertainty that can be explained by the uncertainty that comes from dividend innovation. When $\alpha = 0$, the risky asset is short-lived and all the uncertainty comes from dividend innovation.

Thus in this case $\chi_{yt} = 1$. Notice that $\tau_{pt} = \frac{1}{\zeta_{1t}^{-2} V_{xt} + \zeta_{2t}^{-2} V_{xt}^\xi} = \frac{\zeta_{1t}^2}{V_{xt}} - \frac{\zeta_{1t}^4 / V_{xt}^2}{\zeta_{1t}^2 / V_{xt} + \zeta_{2t}^2 / V_{xt}^\xi}$.

Thus $\zeta_{1t}^\dagger \equiv \frac{\tau_{pt} V_{xt}}{\zeta_{1t}} = \zeta_{1t} - \frac{\zeta_{1t}^3 / V_{xt}}{\zeta_{1t}^2 / V_{xt} + \zeta_{2t}^2 / V_{xt}^\xi}$ is the future-uncertainty-adjusted signal-to-current-noise ratio.

Then define the market-wide average future-uncertainty-adjusted dividend-to-total-uncertainty-weighted signal-to-current-noise ratio to be

$$\bar{\zeta}_{1t}^\dagger \equiv \frac{1}{\zeta_{1t}} \left[\frac{1}{2r} \Omega_t V_{yt} \tau_{pt} V_{xt} + \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{V}_{it}} \hat{V}_{xit} di \right]. \quad (\text{B.43})$$

When $\alpha = 0$, $\bar{\zeta}_{1t}^\dagger = \frac{r+1}{2r}\zeta_t$. Then $(\bar{\omega}_{pt} - \bar{Z}_{1t})/\zeta_{1t} = \bar{\zeta}_{1t}^\dagger \tau_x$.

E_t :

The coefficient of the term $\tilde{x}_{t+1}$ in the price relation is:

$$E_t = \frac{1}{r}\bar{\Sigma}_t \left(1 - \frac{\bar{\Sigma}_t}{r}(1 + \alpha B_{t+1})\bar{\omega}_{pt} \frac{1}{C_t} \right)^{-1} \times \\ \left[\alpha D_{t+1} \left(\bar{\Sigma}_t^{-1} - \bar{\chi}_{xt}^\xi \tau_x \right) - (1 + \alpha B_{t+1}) \frac{1}{\zeta_{2t}} \bar{Z}_{2t} \right].$$

Multiplying both sides by $\left(1 - \frac{\bar{\Sigma}_t}{r}(1 + \alpha B_{t+1})\bar{\omega}_{pt} \frac{1}{C_t} \right)$ and rearranging, we have

$$E_t = \frac{\bar{\Sigma}_t}{r - \alpha g} \frac{1}{\zeta_{2t}} (\bar{\omega}_{pt} - \bar{Z}_{2t}) + \frac{1}{r} \alpha D_{t+1} \left(1 - \bar{\Sigma}_t \bar{\chi}_{xt}^\xi \tau_x \right).$$

Notice that $\tau_{pt} = \frac{1}{\zeta_{1t}^{-2} V_{xt} + \zeta_{2t}^{-2} V_{xt}^\xi} = \frac{\zeta_{2t}^2}{V_{xt}^\xi} - \frac{\zeta_{2t}^4 / (V_{xt}^\xi)^2}{\zeta_{1t}^2 / V_{xt} + \zeta_{2t}^2 / V_{xt}^\xi}$. Thus $\zeta_{2t}^\dagger \equiv \frac{\tau_{pt} V_{xt}^\xi}{\zeta_{2t}} = \zeta_{2t} - \frac{\zeta_{2t}^3 / V_{xt}^\xi}{\zeta_{1t}^2 / V_{xt} + \zeta_{2t}^2 / V_{xt}^\xi}$ is the future-uncertainty-adjusted signal-to-current-noise ratio. Then define the market-wide average future-uncertainty-adjusted dividend-to-total-uncertainty-weighted signal-to-current-noise ratio as

$$\bar{\zeta}_{2t}^\dagger \equiv \frac{1}{\zeta_{2t}} \left[\frac{1}{2r} \Omega_t V_{yt} \tau_{pt} V_{xt}^\xi + \int_{i \in \mathcal{G}_{t-1}} \frac{\hat{V}_{yit} \hat{\tau}_{pit}}{\hat{V}_{it}} \hat{V}_{xit}^\xi di \right]. \quad (\text{B.44})$$

We can then write E_t as

$$E_t = \frac{1}{r} \alpha D_{t+1} \left(1 - \bar{\Sigma}_t \bar{\chi}_{xt}^\xi \tau_x \right) + \frac{\bar{\Sigma}_t}{r - \alpha g} \bar{\zeta}_{2t}^\dagger \tau_x. \quad (\text{B.45})$$

In the static case, we have $E_t = \bar{\Sigma}_t \frac{E_t}{C_t} \tau_x$, which implies that $E_t = 0$.

Appendix B.3. Existence of Linear Equilibrium

The price coefficients are

$$A_t = \frac{\mu}{r - \alpha g} - \frac{\rho \bar{x}_t}{r} - \frac{1}{r} \bar{A}_t(A_{t+1}, A_{t+2}) + \frac{\bar{\Sigma}_t}{r} \alpha A_{t+1} \bar{\Sigma}_t, \quad (\text{B.46})$$

$$B = \frac{g}{r - \alpha g}, \quad (\text{B.47})$$

$$C_t = \frac{1}{r - \alpha g} (1 - \bar{\Sigma}_t \bar{\chi}_{yt} \tau_y), \quad (\text{B.48})$$

$$D_t = \left[1 - \alpha \frac{\bar{\Sigma}_t}{r} \left(\bar{\Sigma}_{t-1}^{-1} - \bar{\chi}_{xt-1}^\xi \tau_x \right) \right]^{-1} \left[\frac{\bar{\Sigma}_t}{r - \alpha g} \bar{\zeta}_{1t}^\dagger \tau_x - \frac{\rho \bar{\Sigma}_t}{r} + \frac{\bar{\Sigma}_t}{r - \alpha g} \frac{1}{\zeta_{2t-1}} \bar{z}_{2t-1} \right], \quad (\text{B.49})$$

$$E_t = \frac{1}{r} \alpha D_{t+1} \left(1 - \bar{\Sigma}_t \bar{\chi}_{xt}^\xi \tau_x \right) + \frac{\bar{\Sigma}_t}{r - \alpha g} \bar{\zeta}_{2t}^\dagger \tau_x. \quad (\text{B.50})$$

Appendix C. Proof of Results

PROPOSITION 10. *In the Benchmark model, under Assumption 1:*

- (a) *An equilibrium exists if $K \leq \frac{\rho r}{r+1} \sqrt{\gamma_{ey} \gamma_{ex}}$.*
- (b) *In both periods, investors make identical information choices $\tau_{eyt}^B = \tau_{ey,it+1}^B = K/\gamma_{ey}$ and $\tau_{ext}^B = \tau_{ex,it+1}^B = 0$ if $K < \rho r \sqrt{\gamma_{ey} \gamma_{ex}}/(r+1)$, and arbitrarily allocate precision to τ_{eyt}^B and τ_{ext}^B if $K = \rho r \sqrt{\gamma_{ey} \gamma_{ex}}/(r+1)$.*
- (c) *The signal-to-noise ratio $\zeta_t^B = -K(r+1)/(\rho r \gamma_{ey})$.*
- (d) *The posterior average variance satisfy $(\bar{\Sigma}_t^B)^{-1} = (r+1)[\tau_y + (\zeta_t^B)^2 \tau_x]/r - \rho \zeta_t^B$.*

Proof of Proposition 10. In the benchmark case,

$$\frac{r+1}{r} \tau_{ext} \zeta_t^2 + \rho \zeta_t + \frac{r+1}{r} \tau_{eyt} = 0.$$

Plugging in the solution for $\tau_{\epsilon xt}^B$ and $\tau_{\epsilon yt}^B$, we have

$$\rho \zeta_t = -\frac{r+1}{r} \sqrt{\frac{K}{\gamma_{\epsilon y}} \left[1 + \frac{\gamma_{\epsilon y}}{\gamma_{\epsilon x}} \zeta_t^4 \right]} < 0, \quad (\text{C.1})$$

or

$$\left(\frac{r+1}{r} \right)^2 \frac{K}{\gamma_{\epsilon x}} \zeta_t^4 - \rho^2 \zeta_t^2 + \left(\frac{r+1}{r} \right)^2 \frac{K}{\gamma_{\epsilon y}} = 0.$$

This is a quadratic equation of ζ_t^2 . We pick the solution which is consistent with the limit. Thus

$$\zeta_t^B = -\sqrt{\frac{\rho^2 - \sqrt{\rho^4 - 4 \frac{\gamma_{\epsilon y}}{\gamma_{\epsilon x}} \left(\frac{K}{\gamma_{\epsilon y}} \right)^2 \left(\frac{r+1}{r} \right)^4}}{2 \frac{\gamma_{\epsilon y}}{\gamma_{\epsilon x}} \frac{K}{\gamma_{\epsilon y}} \left(\frac{r+1}{r} \right)^2}}.$$

□

PROPOSITION 11. *In the Benchmark model, under Assumption 1:*

- (a) *Price coefficients $C_t^B > 0$, $D_t^B < 0$.*
- (b) *The equilibrium signal-to-noise ratio $|\zeta_t|$ rises with information capacity K , falls with demand-shock learning cost $\gamma_{\epsilon x}$, and is invariant to prior precisions τ_y and τ_x .*
- (c) *Posterior average precision $(\bar{\Sigma}_t^B)^{-1}$ increases in K , the prior precision of the dividend innovation τ_y , the prior precision of the demand shock τ_x , and decreases in demand learning cost $\gamma_{\epsilon x}$.*
- (d) *A larger K raises precision assigned to both signals, $\tau_{\epsilon xt}^B$ and $\tau_{\epsilon yt}^B$. A higher $\gamma_{\epsilon x}$ increases the precision of the current dividend innovation $\tau_{\epsilon yt}^B$ and reduces the precision of the current demand shock $\tau_{\epsilon xt}^B$. Information allocations do not respond to prior uncertainty levels.*

Proof of Proposition 11. Recall that $(\bar{\Sigma}_t^B)^{-1} = \frac{r+1}{r} [\tau_y + (\zeta_t^B)^2 \tau_x] - \rho \zeta_t^B$.

Taking the partial derivative with respect to $|\zeta_t^B|$, we have

$$\frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial |\zeta_t^B|} = 2 \frac{r+1}{r} |\zeta_t^B| \tau_x + \rho > 0.$$

By chain rule, we have

$$\begin{aligned} \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial K} &= \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial |\zeta_t|} \frac{\partial |\zeta_t^B|}{\partial K} > 0, \\ \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial \gamma_{\epsilon x}} &= \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial |\zeta_t|} \frac{\partial |\zeta_t^B|}{\partial \gamma_{\epsilon x}} < 0, \\ \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial \tau_x} &= \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial |\zeta_t|} \frac{\partial |\zeta_t^B|}{\partial \tau_x} + \frac{r+1}{r} (\zeta_t^B)^2 > 0. \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{V}(p_t)}{\partial K} &= 2 \left[\frac{C_t^B}{\tau_y} \frac{\partial C_t^B}{\partial \bar{\Sigma}_t^B} \frac{\partial \bar{\Sigma}_t^B}{\partial (\bar{\Sigma}_t^B)^{-1}} \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial |\zeta_t|} \right. \\ &\quad \left. + \frac{D_t^B}{\tau_x} \left(\frac{\partial D_t^B}{\partial \bar{\Sigma}_t^B} \frac{\partial \bar{\Sigma}_t^B}{\partial (\bar{\Sigma}_t^B)^{-1}} \frac{\partial (\bar{\Sigma}_t^B)^{-1}}{\partial |\zeta_t^B|} + \frac{\partial D_t^B}{\partial |\zeta_t^B|} \right) \right] \frac{\partial |\zeta_t^B|}{\partial K}. \end{aligned}$$

Same as in the main model.

$$\frac{\partial \tau_{\epsilon yt}^B}{\partial K} = \frac{\partial \tau_{\epsilon yt}^B}{\partial K} + \frac{\partial \tau_{\epsilon yt}^B}{\partial \zeta_t^B} \frac{\partial \zeta_t^B}{\partial K}.$$

The first term

$$\frac{\partial \tau_{\epsilon yt}^B}{\partial K} = \frac{1}{2\gamma_{\epsilon y} \left(\frac{\gamma_{\epsilon y} \zeta_t^4}{\gamma_{\epsilon x}} + 1 \right) \sqrt{\frac{K}{\gamma_{\epsilon y} \left(\frac{\gamma_{\epsilon y} \zeta_t^4}{\gamma_{\epsilon x}} + 1 \right)}} > 0.$$

$$\begin{aligned}
\frac{d\tau_{\epsilon xt}^B}{dK} &= \underbrace{\frac{\partial \tau_{\epsilon xt}^B}{\partial K}}_{>0} + \underbrace{\frac{\partial \tau_{\epsilon xt}^B}{\partial \zeta_t^B}}_{<0} \underbrace{\frac{\partial \zeta_t^B}{\partial K}}_{<0} > 0, \\
\frac{d\tau_{\epsilon yt}^B}{dK} &= \underbrace{\frac{\partial \tau_{\epsilon yt}^B}{\partial K}}_{>0} + \underbrace{\frac{\partial \tau_{\epsilon yt}^B}{\partial \zeta_t^B}}_{>0} \underbrace{\frac{\partial \zeta_t^B}{\partial K}}_{<0}, \\
\frac{d\tau_{\epsilon xt}^B}{d\gamma_{\epsilon x}} &= \underbrace{\frac{\partial \tau_{\epsilon xt}^B}{\partial \gamma_{\epsilon x}}}_{>0} + \underbrace{\frac{\partial \tau_{\epsilon xt}^B}{\partial \zeta_t^B}}_{<0} \underbrace{\frac{\partial \zeta_t^B}{\partial \gamma_{\epsilon x}}}_{>0}, \\
\frac{d\tau_{\epsilon yt}^B}{d\gamma_{\epsilon x}} &= \underbrace{\frac{\partial \tau_{\epsilon yt}^B}{\partial \gamma_{\epsilon x}}}_{<0} + \underbrace{\frac{\partial \tau_{\epsilon yt}^B}{\partial \zeta_t^B}}_{>0} \underbrace{\frac{\partial \zeta_t^B}{\partial \gamma_{\epsilon x}}}_{>0};
\end{aligned}$$

$$\frac{\partial \zeta_t^B}{\partial \gamma_{\epsilon x}} = \frac{A - B}{2\sqrt{2}C},$$

where

$$\begin{aligned}
A &= \frac{\rho^2 - \sqrt{\rho^4 - \frac{4\left(\frac{r+1}{r}\right)^4 K^2}}}{\left(\frac{r+1}{r}\right)^2 K \gamma_{\epsilon x} \gamma_{\epsilon y}}, \\
B &= \frac{2\left(\frac{r+1}{r}\right)^2 K}{\gamma_{\epsilon x} \gamma_{\epsilon y} \sqrt{\rho^4 - \frac{4\left(\frac{r+1}{r}\right)^4 K^2}}}, \\
C &= \sqrt{\frac{\gamma_{\epsilon x} \left(\rho^2 - \sqrt{\rho^4 - \frac{4\left(\frac{r+1}{r}\right)^4 K^2}} \right)}{\left(\frac{r+1}{r}\right)^2 K}}.
\end{aligned}$$

$\frac{\partial \zeta_t^B}{\partial \gamma_{\epsilon x}} < 0$ always holds when $r, K, \rho, \gamma_{\epsilon x}, \gamma_{\epsilon y}$ and the equilibrium existence condition holds. $\square$

Proof of Lemma 3 (Comovement of Σ_t^{-1} and $|\zeta_t|$). The signal-to-noise ratio is $\zeta_t = [\bar{\Sigma}_t^{-1} - (r+1)\tau_y/r] / [(r+1)\zeta_t\tau_x/r - \rho]$, which leads to

$$\bar{\Sigma}_t^{-1} = \frac{r+1}{r}\tau_x\zeta_t^2 - \rho\zeta_t + \frac{r+1}{r}\tau_y. \quad (C.2)$$

Taking partial derivative with respect to ζ_t , one has

$$\frac{\partial \bar{\Sigma}_t^{-1}}{\partial |\zeta_t|} = 2\frac{r+1}{r}|\zeta_t|\tau_x + \rho > 0.$$

The information processing capacity K , learning costs $\gamma_{\epsilon x}$, $\gamma_{\epsilon y}$ and $\gamma_{\nu x}$, and the prior precision of dividend innovation τ_y affect $\bar{\Sigma}_t^{-1}$ solely through $|\zeta_t|$. Thus any change in $|\zeta_t|$ caused by K , $\gamma_{\epsilon x}$, $\gamma_{\epsilon y}$, $\gamma_{\nu x}$ or τ_y will be accompanied by a change in $\bar{\Sigma}_t^{-1}$ in the same direction. $\square$

Proof of Proposition 4. Recall that $\bar{\Sigma}_t^{-1} = \frac{1}{r}\Sigma_t + \hat{\Sigma}_t$, which leads to $\bar{\Sigma}_t^{-1} = \frac{r+1}{r}\tau_y + \frac{1}{r}\tau_{\epsilon y t} + \hat{\tau}_{\epsilon y t} + \zeta_t^2 \left(\frac{r+1}{r}\tau_x + \frac{1}{r}\tau_{\epsilon x t} + \hat{\tau}_{\epsilon x t} + \hat{\tau}_{\nu x t-1} \right)$. Plug it into (C.2), we have

$$\left(\frac{1}{r}\tau_{\epsilon x t} + \hat{\tau}_{\epsilon x t} + \hat{\tau}_{\nu x t-1} \right) \zeta_t^2 + \rho\zeta_t + \frac{1}{r}\tau_{\epsilon y t} + \hat{\tau}_{\epsilon y t} = 0. \quad (C.3)$$

This is a quadratic equation of ζ_t , which has two solutions. We pick the larger solution, since when there is no demand information, the solution converges to the unique solution in the models where there is only dividend innovation acquisition.

$$\zeta_t = \frac{-\rho + \sqrt{\rho^2 - 4 \left(\frac{1}{r}\tau_{\epsilon x t} + \hat{\tau}_{\epsilon x t} + \hat{\tau}_{\nu x t-1} \right) \left(\frac{1}{r}\tau_{\epsilon y t} + \hat{\tau}_{\epsilon y t} \right)}}{2 \left(\frac{1}{r}\tau_{\epsilon x t} + \hat{\tau}_{\epsilon x t} + \hat{\tau}_{\nu x t-1} \right)} < 0. \quad (C.4)$$

$\square$

Appendix C.1. Information Choices

Proof of Proposition 1. (a) is obvious to see since $\hat{\tau}_{\nu xit}$ does not enter the *ex ante* utility.

(b) The Lagrangian is

$$\mathcal{L} = \hat{L}_{ey}\hat{\tau}_{eyit} + \hat{L}_{ey}\zeta_t^2\hat{\tau}_{exit} - \lambda \left([\gamma_{ex}\hat{\tau}_{exit}^2 + \gamma_{ey}\hat{\tau}_{eyit}^2] - K \right).$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \hat{\tau}_{eyit}} &= \hat{L}_{ey} - 2\lambda\gamma_{ey}\hat{\tau}_{eyit} = 0, \\ \frac{\partial \mathcal{L}}{\partial \hat{\tau}_{exit}} &= \hat{L}_{ey}\zeta_t^2 - 2\lambda\gamma_{ex}\hat{\tau}_{exit} = 0, \end{aligned}$$

which yields

$$\hat{\tau}_{exit} = \frac{\gamma_{ey}}{\gamma_{ex}}\zeta_t^2\hat{\tau}_{eyit}. \quad (C.5)$$

Plugging (C.5) into the budget constraint yields the optimal information choices. $\square$

Proof of Lemma 2. To simplify calculations, one can do a strictly increasing transformation of the *ex ante* utility

$$V \stackrel{\text{def}}{=} -2 \ln \left[\frac{-\mathbb{E} [-\exp(-\rho W_{it+1+} \mid \mathcal{I}_{t-1}^+)]}{\exp(-\rho r^2 W_0)} \right] = \ln H_1 + \ln H_2 + H_3 + H_4, \quad (C.6)$$

where

$$H_1 = \frac{\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1}) + \Sigma_Z + \Sigma_a}{\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1})}, \quad (\text{C.7})$$

$$H_2 = \frac{\text{Var}(\tilde{d}_t | \mathcal{I}_{it}) + \Sigma_S}{\text{Var}(\tilde{d}_t | \mathcal{I}_{it})}, \quad (\text{C.8})$$

$$H_3 = \frac{\bar{\Sigma}_{t+1}^2 \rho^2 \bar{x}^2}{\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1}) + \Sigma_Z + \Sigma_a}, \quad (\text{C.9})$$

$$H_4 = \frac{\bar{\Sigma}_t^2 \rho^2 \bar{x}^2}{\text{Var}(\tilde{d}_t | \mathcal{I}_{it}) + \Sigma_S}. \quad (\text{C.10})$$

Notice that τ_{eyt} and τ_{ext} enter only through $\text{Var}(\tilde{d}_t | \mathcal{I}_{it})$ and Σ_S via H_2 and H_4 . Thus we only need to check whether $\frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{eyt}} = \frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{ext}} \zeta_t^2$ and $\frac{\partial \Sigma_S}{\partial \tau_{eyt}} = \frac{\partial \Sigma_S}{\partial \tau_{ext}} \zeta_t^2$ hold. Recall that

$$\text{Var}(\tilde{d}_t | \mathcal{I}_{it}) = \frac{1}{\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})}.$$

Then

$$\frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{eyt}} = -\frac{1}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2}, \quad (\text{C.11})$$

$$\frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{ext}} = -\frac{\zeta_t^2}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2}. \quad (\text{C.12})$$

Thus $\frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{eyt}} = \frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{ext}} \zeta_t^2$ holds. One has

$$\Sigma_S = \frac{b_2^2}{\tau_y} + \frac{b_3^2}{\tau_x} + \frac{\tau_{eyt} + \zeta_t^2 \tau_{ext}}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} \quad (\text{C.13})$$

Take partial derivative with respect to τ_{eyt} on both sides, one has

$$\begin{aligned} \frac{\partial \Sigma_S}{\partial \tau_{eyt}} &= \frac{2b_2}{\tau_y} \frac{\tau_y}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} - \frac{2b_3}{\tau_x} \frac{\zeta_t \tau_x}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} \\ &\quad + \frac{1}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} - 2 \frac{\tau_{eyt} + \zeta_t^2 \tau_{ext}}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^3}, \quad (C.14) \end{aligned}$$

$$\begin{aligned} \frac{\partial \Sigma_S}{\partial \tau_{ext}} &= \frac{2b_2}{\tau_y} \frac{\zeta_t^2 \tau_y}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} - \frac{2b_3}{\tau_x} \frac{\zeta_t^3 \tau_x}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} \\ &\quad + \frac{\zeta_t^2}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^2} - 2 \zeta_t^2 \frac{\tau_{eyt} + \zeta_t^2 \tau_{ext}}{[\tau_y + \tau_{eyt} + \zeta_t^2 (\tau_x + \tau_{ext})]^3}. \quad (C.15) \end{aligned}$$

Therefore $\frac{\partial \Sigma_S}{\partial \tau_{eyt}} = \frac{\partial \Sigma_S}{\partial \tau_{ext}} \zeta_t^2$ holds. $\square$

Proof of Proposition 2.

Step 1: Calculate learning indices

On the one hand, from the proof of Lemma 2, we know that the learning index associated with the signal for the current fundamental shock is

$$L_{eyt} = \partial \ln H_2 / \partial \tau_{eyt} + \partial H_4 / \partial \tau_{eyt},$$

with

$$\begin{aligned} \frac{\partial \ln H_2}{\partial \tau_{eyt}} &= \frac{1}{[\text{Var}(\tilde{d}_t | \mathcal{I}_{it}) + \Sigma_S] \text{Var}(\tilde{d}_t | \mathcal{I}_{it})} \times \\ &\quad \left[\frac{\partial \Sigma_S}{\partial \tau_{eyt}} \text{Var}(\tilde{d}_t | \mathcal{I}_{it}) - \frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{eyt}} \Sigma_S \right], \\ \frac{\partial H_4}{\partial \tau_{eyt}} &= - \frac{\bar{\Sigma}_t^2 \rho^2 \bar{x}^2}{[\text{Var}(\tilde{d}_t | \mathcal{I}_{it}) + \Sigma_S]^2} \left[\frac{\partial \Sigma_S}{\partial \tau_{eyt}} + \frac{\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it})}{\partial \tau_{eyt}} \right]. \end{aligned}$$

where $\partial \Sigma_S / \partial \tau_{eyt}$ and $\partial \text{Var}(\tilde{d}_t | \mathcal{I}_{it}) / \partial \tau_{eyt}$ are calculated by (C.14) and (C.11).

On the other hand, notice that $\tau_{\nu xt}$ enter the ex-ante utility function only through $\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1})$, Σ_Z and Σ_a via H_1 and H_3 . Thus

$$L_{\nu xit} = \partial \ln H_1 / \partial \tau_{\nu xt} + \partial H_3 / \partial \tau_{\nu xt},$$

with

$$\begin{aligned} \frac{\partial \ln H_1}{\partial \tau_{\nu xt}} &= \frac{\frac{\partial(\Sigma_Z + \Sigma_a)}{\partial \tau_{\nu xt}} \text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1}) - \frac{\partial \text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1})}{\partial \tau_{\nu xt}} (\Sigma_Z + \Sigma_a)}{[\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1}) + \Sigma_Z + \Sigma_a] \text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1})}, \\ \frac{\partial H_3}{\partial \tau_{\nu xt}} &= -\frac{\bar{\Sigma}_{t+1}^2 \rho^2 \bar{x}^2}{[\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1}) + \Sigma_Z + \Sigma_a]^2} \times \\ &\quad \left[\frac{\partial(\Sigma_Z + \Sigma_a)}{\partial \tau_{\nu xt}} + \frac{\partial \text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1})}{\partial \tau_{\nu xt}} \right]. \end{aligned}$$

Recall that

$$\text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1}) = 1 / (\tau_y + \hat{\tau}_{eyit+1} + \zeta_{t+1}^2 (\tau_x + \hat{\tau}_{exit+1} + \tau_{\nu xt})).$$

Thus

$$\frac{\partial \text{Var}(\tilde{d}_{t+1} | \hat{\mathcal{I}}_{it+1})}{\partial \tau_{\nu xt}} = -\zeta_{t+1}^2 / [\tau_y + \hat{\tau}_{eyit+1} + \zeta_{t+1}^2 (\tau_x + \hat{\tau}_{exit+1} + \tau_{\nu xt})]^2.$$

One has

$$\Sigma_Z + \Sigma_a = \frac{a_2^2}{\tau_y} + \frac{a_3^2 + c^2}{\tau_x + \tau_{\nu xt}} + \frac{\hat{\tau}_{eyit+1} + \zeta_{t+1}^2 \hat{\tau}_{exit+1}}{[\tau_y + \hat{\tau}_{eyit+1} + \zeta_{t+1}^2 (\tau_x + \hat{\tau}_{exit+1} + \tau_{\nu xt})]^2}.$$

Taking partial derivative with respect to $\tau_{\nu xt}$ on both sides, we have

$$\begin{aligned} \frac{\partial(\Sigma_Z + \Sigma_a)}{\partial \tau_{\nu xt}} &= \frac{2a_2}{\tau_y} \frac{\zeta_{t+1}^2 \tau_y}{[\tau_y + \hat{\tau}_{eyit+1} + \zeta_{t+1}^2 (\tau_x + \hat{\tau}_{exit+1} + \tau_{\nu xt})]^2} - \frac{a_3^2 + c^2}{(\tau_x + \tau_{\nu xt})^2} \\ &\quad + \frac{2a_3}{\tau_x + \tau_{\nu xt}} \left(\frac{\zeta_{t+1}}{D} - \frac{\zeta_{t+1}^3 (\tau_x + \tau_{\nu xt})}{D^2} \right) \\ &\quad + \frac{2c}{\tau_x + \tau_{\nu xt}} \left(\frac{\zeta_{t+1}}{D} - \frac{\zeta_{t+1}^3 \tau_x}{D^2} \right) - 2\zeta_{t+1}^2 \frac{\hat{\tau}_{eyit+1} + \zeta_{t+1}^2 \hat{\tau}_{exit+1}}{D^3}, \end{aligned}$$

where $D \triangleq \tau_y + \hat{\tau}_{eyit+1} + \zeta_{t+1}^2(\tau_x + \hat{\tau}_{exit+1} + \tau_{vxt})$ for notation brevity.

Step 2: Characterizing information choices with F.O.C.'s and information capacity constraint

The first-order conditions to the information choice problem are

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_{eyt}} &= L_{eyit} - 2\lambda\gamma_{ey}\tau_{eyt} = 0, \\ \frac{\partial \mathcal{L}}{\partial \tau_{ext}} &= L_{eyit}\zeta_t^2 - 2\lambda\gamma_{ex}\tau_{ext} = 0, \\ \frac{\partial \mathcal{L}}{\partial \tau_{vxt}} &= L_{vxit} - 2\lambda\gamma_{vx}\tau_{vxt} = 0.\end{aligned}$$

Dividing them against each other, one has

$$\begin{aligned}\tau_{eyt} &= \frac{\gamma_{vx}}{\gamma_{ey}} \frac{L_{eyit}}{L_{vxit}} \tau_{vxt}, \\ \tau_{ext} &= \frac{\gamma_{vx}}{\gamma_{ex}} \frac{L_{eyit}}{L_{vxit}} \zeta_t^2 \tau_{vxt}.\end{aligned}$$

Plug them back into the budget constraint (4), one has

$$\frac{\tau_{vxt}^2}{L_{vxit}^2} \left(\frac{\gamma_{vx}}{\gamma_{ey}} L_{eyit}^2 + \frac{\gamma_{vx}}{\gamma_{ex}} L_{eyit}^2 \zeta_t^4 + L_{vxit}^2 \right) = \frac{K}{\gamma_{vx}}. \quad (\text{C.16})$$

Define

$$f_{it} \equiv \frac{L_{eyit}}{L_{vxit}} = \frac{\partial \ln H_2 / \partial \tau_{eyt} + \partial H_4 / \partial \tau_{eyt}}{\partial \ln H_1 / \partial \tau_{vxt} + \partial H_3 / \partial \tau_{vxt}}. \quad (\text{C.17})$$

Then Eq.(C.16) simplifies to

$$\tau_{vxt}^2 \left(\frac{\gamma_{vx}}{\gamma_{ey}} f_{it}^2 + \frac{\gamma_{vx}}{\gamma_{ex}} \zeta_t^4 f_{it}^2 + 1 \right) = \frac{K}{\gamma_{vx}}.$$

□

Proof of Theorem 3 (Price Volatility). The unconditional price volatility is $\text{Var}(p_t) = C_t^2 \tau_y^{-1} + D_t^2 \tau_x^{-1}$. Take partial derivative with respect to K , we

have

$$\frac{\partial \text{Var}(p_t)}{\partial K} = 2 \left[\frac{C_t}{\tau_y} \frac{\partial C_t}{\partial \bar{\Sigma}_t} \frac{\partial \bar{\Sigma}_t}{\partial \zeta_t} + \frac{D_t}{\tau_x} \left(\frac{\partial D_t}{\partial \bar{\Sigma}_t} \frac{\partial \bar{\Sigma}_t}{\partial \zeta_t} + \frac{\partial D_t}{\partial \zeta_t} \right) \right] \frac{\partial \zeta_t}{\partial K}. \quad (\text{C.18})$$

We then need to determine the sign of the term inside the square brackets. To do that, we first derive the partial derivatives of C_t and D_t with respect to $\bar{\Sigma}_t$ and ζ_t . We have $\frac{\partial C_t}{\partial \bar{\Sigma}_t} = -\frac{r+1}{r}\tau_y$, $\frac{\partial D_t}{\partial \bar{\Sigma}_t} = \frac{r+1}{r}\zeta_t\tau_x - \rho$, $\frac{\partial D_t}{\partial \zeta_t} = \frac{r+1}{r}\bar{\Sigma}_t\tau_x$ and $\frac{\partial \bar{\Sigma}_t}{\partial \zeta_t} = \bar{\Sigma}_t^2 \left(\rho - 2\frac{r+1}{r}\zeta_t\tau_x \right)$. Thus the term inside square brackets becomes

$$\begin{aligned} & \frac{C_t}{\tau_y} \frac{\partial C_t}{\partial \bar{\Sigma}_t} \frac{\partial \bar{\Sigma}_t}{\partial \zeta_t} + \frac{D_t}{\tau_x} \left(\frac{\partial D_t}{\partial \bar{\Sigma}_t} \frac{\partial \bar{\Sigma}_t}{\partial \zeta_t} + \frac{\partial D_t}{\partial \zeta_t} \right) \\ &= \frac{1 - \frac{r+1}{r}\bar{\Sigma}_t\tau_y}{\tau_y} \left(-\frac{r+1}{r}\tau_y \right) \bar{\Sigma}_t^2 \left(\rho - 2\frac{r+1}{r}\zeta_t\tau_x \right) \\ & \quad + \frac{\frac{r+1}{r}\bar{\Sigma}_t\zeta_t\tau_x - \rho}{\tau_x} \left[\left(\frac{r+1}{r}\zeta_t\tau_x - \rho \right) \bar{\Sigma}_t^2 \left(\rho - 2\frac{r+1}{r}\zeta_t\tau_x \right) + \frac{r+1}{r}\bar{\Sigma}_t\tau_x \right] \\ &= \bar{\Sigma}_t^2 \left\{ \frac{r+1}{r} \left(1 - \frac{r+1}{r}\bar{\Sigma}_t\tau_y \right) \left(2\frac{r+1}{r}\zeta_t\tau_x - \rho \right) \right. \\ & \quad \left. + \left(\frac{r+1}{r}\zeta_t\tau_x - \rho \right) \left[\left(\frac{r+1}{r}\zeta_t\tau_x - \rho \right) \frac{\bar{\Sigma}_t}{\tau_x} \left(\rho - 2\frac{r+1}{r}\zeta_t\tau_x \right) + \frac{r+1}{r} \right] \right\}. \end{aligned}$$

Let

$$\begin{aligned} \mathcal{A} \equiv & \left[\frac{r+1}{r} \right] \left(1 - \left[\frac{r+1}{r} \right] \bar{\Sigma}_t\tau_y \right) \left(2 \left[\frac{r+1}{r} \right] \zeta_t\tau_x - \rho \right) \\ & + \left(\left[\frac{r+1}{r} \right] \zeta_t\tau_x - \rho \right) \times \\ & \left[\left(\left[\frac{r+1}{r} \right] \zeta_t\tau_x - \rho \right) \left(\bar{\Sigma}_t/\tau_x \right) \left(\rho - 2 \left[\frac{r+1}{r} \right] \zeta_t\tau_x \right) + \frac{r+1}{r} \right]. \end{aligned}$$

For a given parameter region, we determine the sign of $\mathcal{A}$.

For $\mathcal{A} \leq 0$, $\zeta < 0$, $\rho, \tau_x, \tau_y > 0$ and $\bar{\Sigma}_t^{-1} = \frac{r+1}{r}\tau_y + \frac{r+1}{r}\tau_x\zeta_t^2 - \rho\zeta_t > 0$ to

hold simultaneously, we must have

$$0 < \bar{\Sigma}_t \leq \frac{-2 \frac{r+1}{r} \tau_x \rho + 3 \left(\frac{r+1}{r} \right)^2 \tau_x^2 \zeta_t}{-\left(\frac{r+1}{r} \right)^2 \tau_x \tau_y \rho - \rho^3 + 2 \left(\frac{r+1}{r} \right)^3 \tau_x^2 \tau_y \zeta_t + 4 \left(\frac{r+1}{r} \right) \tau_x \rho^2 \zeta_t - 5 \left(\frac{r+1}{r} \right)^2 \tau_x^2 \rho \zeta_t^2 + 2 \left(\frac{r+1}{r} \right)^3 \tau_x^3 \zeta_t^3}.$$

Substituting the expression of $\bar{\Sigma}_t$ and solving for ζ_t , we obtain the feasible sets:

- $\{\zeta_t \mid \zeta_t < 0\}$, if $\tau_x \geq \bar{\Omega}_x$;
- $\{\zeta_t \mid \zeta_t \leq \underline{\zeta}_t \text{ or } \bar{\zeta}_t \leq \zeta_t < 0\}$, if $\underline{\Omega}_x < \tau_x < \bar{\Omega}_x$;
- $\{\zeta_t \mid \zeta_t \leq \underline{\zeta}_t\}$, if $0 < \tau_x \leq \underline{\Omega}_x$.

Since $\partial \zeta_t / \partial K < 0$, the sign of $\partial \mathbb{V}ar(p_t) / \partial K$ is:

(a) if $\tau_x \geq \bar{\Omega}_x$, then $\partial \mathbb{V}ar(p_t) / \partial K \geq 0$;

(b) if $\underline{\Omega}_x < \tau_x < \bar{\Omega}_x$, then

$$\frac{\partial \mathbb{V}ar(p_t)}{\partial K} = \begin{cases} \geq 0, & \text{if } \zeta_t \leq \underline{\zeta}_t \text{ or } \bar{\zeta}_t \leq \zeta_t < 0, \\ < 0, & \text{if } \underline{\zeta}_t < \zeta_t < \bar{\zeta}_t; \end{cases}$$

(c) If $0 < \tau_x \leq \underline{\Omega}_x$, then

$$\frac{\partial \mathbb{V}ar(p_t)}{\partial K} = \begin{cases} \geq 0, & \text{if } \zeta_t \leq \underline{\zeta}_t, \\ < 0, & \text{if } \underline{\zeta}_t < \zeta_t < 0; \end{cases}$$

where

$$\begin{aligned}\bar{\Omega}_x &\equiv 5\rho^2 / (4 \left(\frac{r+1}{r} \right)^2 \tau_y), \\ \underline{\Omega}_x &\equiv \rho^2 / (\left(\frac{r+1}{r} \right)^2 \tau_y), \\ \bar{\zeta}_t &\equiv \frac{r}{2(r+1)\tau_x} \left[-\rho + \sqrt{-4 \left(\frac{r+1}{r} \right)^2 \tau_x \tau_y + 5\rho^2} \right], \\ \underline{\zeta}_t &\equiv \frac{r}{2(r+1)\tau_x} \left[-\rho - \sqrt{-4 \left(\frac{r+1}{r} \right)^2 \tau_x \tau_y + 5\rho^2} \right].\end{aligned}$$

□

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