

## **Endogenous Market Structure: Over-the-Counter versus Exchange Trading**

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**SUMMARY:** This paper investigates the factors influencing traders' preferences for over-the-counter (OTC) or centralized markets, focusing on how trader heterogeneity and information shape market and counterparty choices. The study finds that OTC trading is more attractive when idiosyncratic asset values and private information dominate, as it allows for better learning and liquidity compared to centralized markets. OTC trading can remain advantageous even in competitive markets with lower price volatility. The paper also explores the feedback effects between market size and composition, leading to either single-venue or coexisting OTC and centralized markets.

**METHOD:** The research employs a theoretical framework to model trader behavior and market dynamics. The sample includes various asset classes, and the data are derived from theoretical models and empirical evidence.

### **KEY FINDINGS:**

- OTC trading is favored when idiosyncratic components dominate asset values and private information is imprecise.
- OTC trading can remain attractive in competitive markets with lower price volatility.
- Trader heterogeneity significantly influences market structure and counterparty choices.
- Feedback effects between market size and composition can lead to single-venue or coexisting OTC and centralized markets.

**IMPLICATION FOR TRADING:** Understanding the conditions under which OTC trading is more beneficial can help traders and market designers optimize their strategies. For practitioners, this research suggests that the choice between OTC and centralized markets depends on the nature of the asset and the information environment. Market regulators may also use these insights to design more effective trading venues and policies that balance the benefits of both OTC and centralized markets.

# Endogenous Market Structure: Over-the-Counter versus Exchange Trading\*

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## Abstract

For many assets, traders favor either over-the-counter (OTC) or centralized markets. This paper examines how trader heterogeneity and information shape endogenous market and counterparty choices. OTC trading is more attractive when idiosyncratic components dominate common components in asset values and private information is imprecise, because targeted counterparties can then improve both learning and liquidity relative to the centralized market. OTC trading can remain attractive even when the centralized market is competitive or exhibits lower price volatility. Traders' endogenous entry decisions create a size-composition feedback and generate rich market structures. When the market-size feedback dominates, equilibrium concentrates in a single venue, either OTC or centralized; when the composition effect is strong and the market is imperfectly competitive, OTC and centralized markets can coexist. The same trade-off and entry feedback determine whether restricting OTC transactions raises or lowers welfare.

**KEYWORDS:** Imperfect competition, Over-the-counter markets, Exchanges, Price impact, Learning from prices, Endogenous market structure

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# 1 Introduction

Over-the-counter (OTC) markets are an essential alternative to centralized trading in financial markets. In OTC markets, traders interact bilaterally and transact on pair-specific terms, whereas centralized markets bring many traders together and clear their demands and supplies at a common market price. Certain assets, such as corporate bonds, credit derivatives, and interest-rate derivatives, are traded predominantly over the counter, while many equities are traded largely in centralized markets. Other asset classes, including foreign exchange, are actively traded in both types of venues. These differences persist even in markets with substantial trading volume, suggesting that the level of trading activity alone does not determine where trading takes place. The design of trading venues has also become an important regulatory issue. For example, the Dodd-Frank Act and MiFID II/MiFIR require certain standardized derivatives to be traded on regulated exchanges or other organized trading venues. These reforms seek to move some transactions from over the counter toward more centralized trading, raising the question of when such centralization improves market outcomes.<sup>1</sup> This paper therefore asks what attracts traders to OTC trading when centralized trading is available, what equilibrium market structure emerges when traders choose their venue, and what the resulting welfare implications are.

Existing studies have identified several motives for OTC trading, including the opportunity to search for favorable prices, the value of opaque trading for informed traders, and the customizability of securities and trading terms. Complementing these insights, this paper focuses on two other dimensions central to market quality: liquidity and learning from prices. Traders care not only about the prices at which they transact, but also about how much they can trade without moving prices substantially and what information market prices convey about asset values. A common intuition is that centralized markets should perform better on both dimensions: a larger pool of participants provides greater liquidity and aggregates information from more traders. We challenge this conventional intuition and show when OTC trading improves both liquidity and learning relative to centralized trading.

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<sup>1</sup>Empirical evidence is mixed. Organized electronic trading can improve liquidity and execution quality (Loon and Zhong, 2016; Benos, Payne, and Vasios, 2020). At the same time, decentralized and selective trading remains prevalent: voice, intermediated, and RFQ trading coexist in corporate bonds (Hendershott and Madhavan, 2015; Biais and Green, 2019; O'Hara and Zhou, 2021). Holden et al. (2021) show that introducing a parallel OTC market attracted most trading away from a centralized limit-order book and reduced transaction costs for medium and large trades.

The key force in this paper is trader heterogeneity. When traders differ in their asset values or information, whom a trader interacts with matters for trading outcomes. OTC trading can exploit this heterogeneity by allowing traders to target a preferred counterparty, potentially providing both greater liquidity and better learning than trading against an aggregate market that includes traders with not only offsetting but also competing trading needs. By contrast, when traders are homogeneous, there is no benefit from targeting whom to trade with, and the broader participation in the centralized market becomes more attractive. Heterogeneity therefore plays a central role in traders’ market choices and in equilibrium market structure that emerges from those choices. These mechanisms are broadly consistent with empirical evidence linking OTC trading to liquidity, information frictions, and heterogeneous trading and hedging needs for some asset classes.<sup>2</sup>

To formalize these ideas, we study a two-period economy in which traders endogenously choose a trading venue. Traders are uncertain about the value of a risky asset but receive a private signal. Traders’ asset values contain a common component (e.g., fundamental asset value, common trading pressure) and an idiosyncratic component (e.g., exposures to outside portfolios, customer order flow), with the latter allowed to be heterogeneously correlated across traders; information precision may also differ across traders. At  $t = 0$ , each trader chooses between a centralized market and an OTC market and, if entering the OTC market, also chooses a counterparty. Traders know other traders’ types—that is, how their asset values are correlated and how precise their signals are—but not their signal realizations when making these choices.<sup>3</sup> OTC counterparties are determined through pairwise-stable matching, and each matched pair trades bilaterally at a pair-specific price at  $t = 1$ .<sup>4</sup> The resulting

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<sup>2</sup>Empirical evidence across OTC markets points to the importance of those components. In the interbank market, banks with stronger hedging motives form over-the-counter relationships with counterparties with offsetting liquidity needs (Afonso, Kovner, and Schoar, 2013); larger trades migrated disproportionately from a centralized limit-order book to an OTC market in foreign exchange (Holden et al., 2021); and large and information-sensitive corporate-bond trades remain disproportionately in voice trading (O’Hara and Zhou, 2021). See also Section 4.2.

<sup>3</sup>In practice, counterparty selection based on such information can be implemented through brokers, dealers, and other intermediaries that match or internalize customer order flow (Goldstein and Hotchkiss, 2020; Jurkatis et al., 2023; Moore, Schrimpf, and Sushko, 2016). RFQ protocols provide another analogue by allowing customers to selectively contact a subset of dealers based on their characteristics (Hendershott and Madhavan, 2015; Riggs et al., 2020). See also Section 4.1.

<sup>4</sup>Our model abstracts from frictions *before trading*, such as search costs and bid-ask spreads, and focuses on potential inefficiencies upon meeting a counterparty arising from price impact and learning from prices. Such pre-trade frictions may change traders’ incentives to choose either venue,

equilibrium market structure must be immune to individual or pairwise deviations in market and counterparty choices.

In the centralized market, at  $t = 1$ , all participating traders submit demand schedules and trade at a single market-clearing price, following the canonical uniform price auction framework (Klemperer and Meyer, 1989; Kyle, 1989; Vives, 2011; Rostek and Weretka, 2012). We model each OTC bilateral trade using the same uniform price auction. Holding the trading mechanism fixed across venues isolates the difference between broad participation in the centralized market and selective bilateral interaction in the OTC market, while using endogenous price impact and learning from prices as key statistics consistently across these venues.

Given this framework, we provide three sets of results on individual market choice, equilibrium market structure, and welfare implications.

First, what determines a trader’s choice between OTC and centralized trading? The key trade-off is between market size and whom the trader trades against. Holding market composition fixed, a centralized market aggregates signals from more traders and absorbs a marginal demand increase with a smaller price change, thereby improving learning and liquidity. A suitable OTC counterparty, however, can provide more information about the trader’s idiosyncratic value—which tends to be aggregated out in the centralized-market price—and more price-elastic counterparty demand—which lowers price impact and improves liquidity. Hence, the benefit of targeting a preferred counterparty need not come at a cost of either dimension. OTC trading becomes attractive when the idiosyncratic value component is sufficiently large and private information is sufficiently imprecise (Theorem 1). A larger idiosyncratic component strengthens the gain from trading with a counterparty whose trading needs are more opposed to the trader’s own, while lower information precision makes learning from prices more valuable. These forces can support OTC trading even when the centralized market is competitive or when OTC trading offers neither a more favorable expected price nor lower price volatility (Corollaries 1 and 2).

Heterogeneous information precision also shapes whom traders prefer as counterparties and which traders have stronger incentives to enter each venue. A more informed counterparty improves learning but lowers liquidity. This learning-liquidity trade-off is absent when market size or asset-value composition alone changes (Proposition 1). Facing this trade-off, a relatively uninformed trader places greater weight

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but the effects studied in this paper would remain present. See Section 6.1.

on learning and therefore prefers an informed counterparty, whereas a sufficiently informed trader places greater weight on liquidity and prefers an uninformed counterparty. Therefore, conditional on entering the OTC market, sufficiently heterogeneous precision can generate cross-type matching between informed and uninformed traders (Proposition 4). Once market choice is endogenous, however, informed traders with strong liquidity concerns choose the centralized market rather than remain in cross-type OTC pairs, so such matching does not arise in equilibrium (Theorem 4). Endogenous market choice therefore generates informational segmentation and limits information transmission from informed to uninformed traders.

Second, what market structure emerges when all traders choose their trading venues? Endogenous entry creates a feedback between individual market choice and market characteristics (Proposition 3). Entering the centralized market increases its size, but it also changes trader composition and hence the average asset-value correlation. These two impacts work in opposite directions. When the market-size impact dominates, each entry strengthens subsequent traders' incentives to enter the same venue, pushing participation toward a corner market structure. For intermediate information precision, this feedback can support multiple equilibria: an OTC-only market and a centralized-only market. When the composition impact is strong, the feedback reverses: entry into the centralized market raises its average correlation, makes aggregate trading needs more similar to those of subsequent traders, and thereby increases the price impact they face, making the market less attractive. This opposing feedback can support coexistence of the OTC and centralized markets. Thus, even with symmetric traders, endogenous market choice can generate various market structures (Theorem 2). Pairwise heterogeneity in asset-value correlations suffices; traders themselves can remain symmetric.

We emphasize the distinct roles of the size-composition trade-off in individual market choice and equilibrium market structure. Individual market choice compares the *levels* of market size and average correlation across the centralized and OTC markets, taking these market characteristics as given. Endogenous market structure instead depends on the *impacts* of market entry on these characteristics: each trader's entry changes market size and composition, and hence the relative strength of the two effects for subsequent traders. The distinction is especially clear in the competitive-market limit. OTC trading can arise regardless of whether the market is imperfectly or perfectly competitive (Corollary 1). By contrast, coexistence of the two venues is

an imperfect-competition phenomenon: in the competitive-market limit, the opposing composition impact vanishes and hence cannot sustain both venues ([Corollary 3](#)).

Lastly, what are the welfare implications of OTC market availability? Closing the OTC market shifts traders into the centralized market, creating size and composition externalities on traders already there. The welfare effect need not coincide with traders’ individual market-choice incentives. When multiple corner equilibria exist, an OTC-only equilibrium can arise even though a coordinated or policy-induced transition to the centralized market makes all traders better off. Conversely, when OTC trading by all traders who have a choice is the unique equilibrium, closing the OTC market makes those traders worse off, while its effect on traders captive in the centralized market depends on how information precision shapes the relative strength of the size-composition externalities. At sufficiently low precision, the adverse composition effect outweighs the gain from greater market size and makes even these traders worse off, generating a Pareto loss. Thus, there is no unambiguous welfare ranking of OTC and centralized markets ([Theorem 3](#)). Policy interventions that discourage or promote OTC trading must account for both coordination in market participation and the externalities that induced entry imposes on other traders.

**Related Literature.** A growing theoretical literature examines the liquidity and informational properties of OTC trading. Existing theories show that OTC liquidity conditions can generate endogenous price dispersion and liquidity premia and can affect efficiency in either direction across different settings ([Duffie, Garleanu, and Pedersen, 2005](#); [Weill, 2008](#); [Malamud and Rostek, 2017](#); [Colliard, Foucault, and Hoffmann, 2021](#); [Hugonnier, Lester, and Weill, 2022](#); [Maurin, 2022](#), among others). A related strand studies how private information is aggregated or transmitted in OTC markets and how these informational channels shape bilateral trading terms and market outcomes ([Duffie, Malamud, and Manso, 2014](#); [Golosov, Lorenzoni, and Tsyvinski, 2014](#); [Cujean and Praz, 2015](#); [Babus and Kondor, 2018](#); [Li and Song, 2021](#); [Pinter, Wang, and Zou, 2022](#)). Together, these literatures establish liquidity and information as important determinants of decentralized market outcomes. This raises a broader question of how these forces affect the attractiveness of OTC trading relative to more centralized alternatives—the central question of this paper.

Several papers explain the persistence of OTC trading through mechanisms different from ours. Opacity or limited counterparty access can preserve informational rents or strengthen incentives for information acquisition ([Madhavan, 1995](#); [Glode and](#)

Opp, 2020; Baldauf, Mollner, and Williams, 2026),<sup>5</sup> while decentralization can affect gains from trade through equilibrium demand elasticities or counterparty composition (Viswanathan and Wang, 2002; Malamud and Rostek, 2017; Peivandi and Vohra, 2021) and alter *ex ante* portfolio composition (Geromichalos and Herrenbrueck, 2016). Closest to us, Liu, Vogel, and Zhang (2018) and Baldauf and Mollner (2024) study liquidity and information jointly in dealer markets, showing how broader dealer contact trades off stronger competition against greater information leakage. In contrast, without an exogenous dealer-client distinction, pairwise interdependence in our model allows targeting a particular OTC counterparty to improve both learning and liquidity relative to centralized markets, rather than generating a trade-off between them.

Related work, including some discussed above, endogenizes traders’ venue choices and studies how these choices shape equilibrium market structure through different externality mechanisms. Bolton, Santos, and Scheinkman (2016) and Lee and Wang (2025) show how informed and uninformed traders sort across OTC and centralized markets, generating informational composition externalities. Others isolate thickness or counterparty-pool effects: Vayanos and Wang (2007) generate a positive market-size externality through search liquidity, while Dugast, Uslu, and Weill (2022) show how heterogeneous trading capacity of OTC participants creates gains from intermediation. Closest to us, Boyarchenko, Lucca, and Veldkamp (2021) and Babus and Parlatore (2022) study how clients’ choices of dealers shape each dealer’s local market through size, information, and price impact. Our paper instead endogenizes *all* traders’ market choices without captive dealers and adds a bilateral counterparty-selection margin. Heterogeneity in pairwise asset-value correlations creates a novel cross-venue composition linkage: a trader’s venue choice changes not only the centralized market characteristics but also the set of bilateral matches available in OTC trading. This linkage generates differences in liquidity and learning across coexisting venues and enriches the size-composition impact governing market structure.

A complementary literature studies the endogenous formation of OTC markets themselves. OTC matching or network connectivity can endogenously arise from information acquisition, intermediation incentives, or counterparty risk (Duffie, Malamud, and Manso, 2014; Babus and Hu, 2017; Farboodi, 2023). A related strand studies directed counterparty choice in dealer markets, where one side selects counter-

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<sup>5</sup>By contrast, Acharya and Bisin (2014) show that opacity about aggregate positions in OTC markets can generate counterparty-risk externalities and reduce the attractiveness of OTC trading.

parties based on trading terms, execution quality, information, or other counterparty characteristics (Zhu, 2012; Lester, Rocheteau, and Weill, 2015; Li and Song, 2021). Others emphasize two-sided matching based on complementary trading needs (Wang, 2022; Chang and Zhang, 2021). Our OTC matching mechanism belongs to the last class. We contribute to this literature by embedding such counterparty matching in a setting with a centralized-market outside option. As a result, the OTC pool is itself endogenous to venue choice. Matches that arise conditional on OTC participation therefore need not survive once centralized trading is available, changing OTC matching outcomes and the associated liquidity, learning, and efficiency.<sup>6</sup>

## 2 Model

We consider a static economy in which two trading venues open simultaneously for the same assets: a centralized market where participating traders' demands clear together and an over-the-counter (OTC) market where matched pairs trade bilaterally. At  $t = 0$  (the entry period), traders choose a market and, if entering the OTC market, a counterparty. At  $t = 1$  (the trading period), they submit demands and trade in their chosen market. Figure 1 summarizes the timing; details follow.

**Strategic Traders.** There are  $I < \infty$  large strategic traders (such as hedge funds, insurance companies, or institutional dealers). The finite number of traders suggests that the markets in this economy are imperfectly competitive: the traders' behavior can influence market outcomes, and they internalize such impacts when optimizing their market entry and trading strategies.

Each trader  $i$  trades a risky asset against a riskfree asset (numeraire) and has quadratic utility over the quantity  $q^i$  of the risky asset traded, net of payment  $pq^i$ , where  $p$  is the price in the market where he participates.

$$u^i(q^i, p; \tilde{\theta}^i) = \tilde{\theta}^i q^i - \frac{\mu}{2}(q^i)^2 - pq^i. \quad (1)$$

Here,  $\mu > 0$  is the common risk-aversion coefficient, and  $\tilde{\theta}^i$  is trader  $i$ 's individual valuation of the risky asset. The value of the numeraire is normalized to one. The individual asset value  $\tilde{\theta}^i$  is uncertain and follows a Gaussian distribution  $\tilde{\theta}^i \sim \mathcal{N}(0, \sigma_\theta^2)$ , where the common prior mean  $E[\tilde{\theta}]$  is normalized to zero. We assume that the prior

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<sup>6</sup>While Atkeson, Eisfeldt, and Weill (2015) do not model an endogenous centralized-market outside option, they show how entry and exit from an OTC market change the availability and composition of counterparties and hence search liquidity.

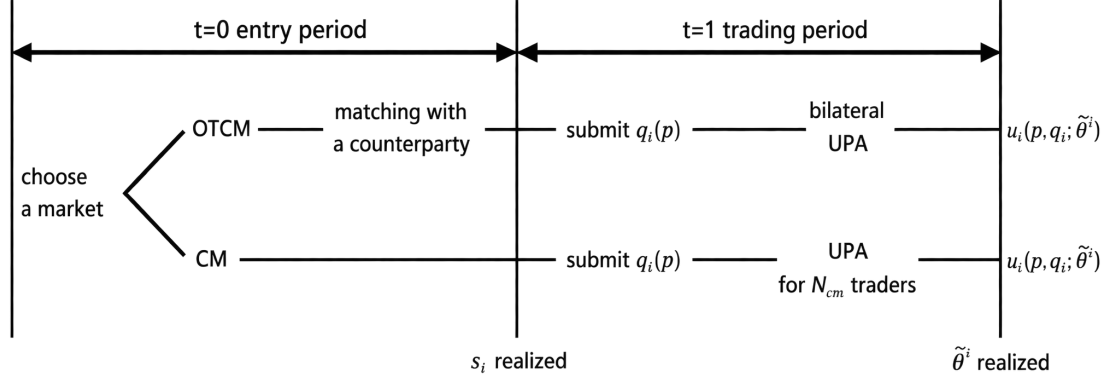


Figure 1: Timing of the economy: entry at  $t = 0$  and trading at  $t = 1$ . OTCM denotes the over-the-counter market, CM the centralized market, and UPA the uniform price auction. After choosing a market and, if applicable, a counterparty, trader  $i$  receives a private signal  $s^i$  before trading. Here,  $q^i(p)$  is trader  $i$ 's demand at price  $p$ , and  $N_{cm}$  is centralized-market size. At the end of the economy,  $\tilde{\theta}^i$  is realized and utility  $u^i$  is evaluated at  $\tilde{\theta}^i$ ,  $q^{i,*} = q^i(p^*)$ , and  $p^*$ .

(marginal) distribution of  $\tilde{\theta}^i$  is the same for all traders  $i$ .<sup>7,8</sup>

**Interdependent Asset Values.** One of the key assumptions of our framework is that traders' individual asset values  $(\tilde{\theta}^i)_{i \in I}$  are *interdependent*. In particular, as Section 4 will show, various market structures arise in equilibrium because of the rich interdependence in asset values allowed by this model.

We adopt a general Gaussian structure for  $(\tilde{\theta}^i)_i$ , with a correlation matrix denoted by  $\Sigma_\theta = (\tilde{\rho}_{ij})_{i,j \in I}$  where  $\tilde{\rho}_{ij} = \text{Corr}[\tilde{\theta}^i, \tilde{\theta}^j]$ . The interdependence of  $(\tilde{\theta}^i)_i$  is captured by two components:

$$\tilde{\theta}^i = \omega_{cv}^i \theta + \omega_{iv}^i \delta^i \quad \forall i \in I, \quad (2)$$

where  $\theta$  is a *common value* component in all traders' asset values,  $\delta^i$  is an *idiosyncratic value* component for each trader  $i$ , and  $(\omega_{cv}^i, \omega_{iv}^i) \in [0, 1]^2$  are weights such

<sup>7</sup>The quadratic (mean-variance) utility and Gaussian distribution are standard assumptions in imperfectly competitive market models: see Kyle (1989), Vives (2011), and Rostek and Weretka (2012). Equilibrium characterization outside the quadratic-Gaussian setting is a well-known challenge in the literature. Glebkin, Malamud, and Teguia (2023) and Du and Zhu (2017) study nonlinear-demand equilibria, respectively, with no private information and in bilateral trading. Nonlinear equilibria also arise with bounded action spaces due to budget, short-selling, or collateral constraints. We focus on a setting with linear equilibrium to accommodate arbitrary market size and private information.

<sup>8</sup>The homogeneous mean and variance of asset values capture identical *ex ante* trading needs across all traders. One can allow traders to have *ex ante* heterogeneous trading needs by introducing trader-specific means  $E[\tilde{\theta}^i]$  and variances  $\text{Var}[\tilde{\theta}^i]$ . Such heterogeneity and the resulting *ex ante* surplus  $E[\tilde{\theta}^i] - E[\tilde{\theta}^j]$  can strengthen the benefits of OTC trading, when traders already prefer the OTC market to the centralized market in the absence of such *ex ante* surplus (Theorem 1).

that  $(\omega_{cv}^i)^2 + (\omega_{iv}^i)^2 = 1$ . The common component may reflect fundamental asset values, whereas the idiosyncratic component may capture traders' individual liquidation needs, outside-portfolio exposures, or, for dealers, customer order flows.

The common and idiosyncratic components follow a joint normal distribution with  $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$  and  $(\delta^i)_i \sim \mathcal{N}(0, \sigma_\theta^2 \Sigma_\delta)$ . We assume that the idiosyncratic components are independent of the common component, but they are *correlated* with one another. In particular, we denote the correlation matrix of  $(\delta^i)_i \in \mathbb{R}^I$  by

$$\Sigma_\delta = (\text{Corr}[\delta^i, \delta^j])_{i,j} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1I} \\ \rho_{12} & 1 & \cdots & \rho_{2I} \\ \vdots & & \ddots & \vdots \\ \rho_{1I} & \rho_{2I} & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{I \times I}. \quad (3)$$

The correlations  $\rho_{ij}$  can be *heterogeneous across trader pairs*  $(i, j)$ , which is the key heterogeneity in this paper. The sum of off-diagonal correlations in each row is assumed to be zero: i.e.,  $\sum_{j \neq i} \rho_{ij} = 0$  for all  $i$ .<sup>9</sup> This correlation structure encompasses a wide range of settings in the literature: a common value  $\tilde{\theta}^i = \theta$  for all  $i$  (Kyle, 1989); independent private values  $\tilde{\theta}^i = \delta^i$  with  $\rho_{ij} = 0$  for all pairs  $(i, j)$ ; and imperfectly correlated values with homogeneous correlations  $\tilde{\rho}_{ij} = \tilde{\rho}$  for all pairs  $(i, j)$  (Vives, 2011) or with heterogeneous correlations  $\{\tilde{\rho}_{ij}\}_{i,j}$  across pairs (Rostek and Weretka, 2012), while maintaining homogeneous weights  $(\omega_{cv}^i, \omega_{iv}^i)$  for all traders  $i$ .<sup>10</sup>

Example 1 illustrates a simple and intuitive setting that we often focus on throughout this paper, although more general settings are also explored.

**Example 1 (Two Trader Groups).** We consider two groups of traders, called *buyers and sellers*, each of size  $I/2$ .<sup>11</sup> Suppose all traders adopt the same weights  $(\omega_{cv}, \omega_{iv})$  in their asset values (2) and their idiosyncratic components are correlated:  $\text{Corr}[\delta^i, \delta^j] = \rho < 0$  for any buyer-seller pair and  $\text{Corr}[\delta^i, \delta^j] = \rho_o = -\frac{I}{I-2}\rho > 0$  for

<sup>9</sup>This assumption is imposed to prevent the heterogeneity in idiosyncratic-component correlations from being an additional source of “common value” and to clearly separate two components in Eq. (2). Consequently, we can also interpret the common component  $\theta$  as an aggregate uncertainty in traders' asset values, more general than the idea that it equals the asset's fundamental value.

<sup>10</sup>Much of the literature on uniform price auctions, including these papers, focuses on symmetric traders: see Definition 2. Rostek and Yoon (2018), Manzano and Vives (2021), and Lou and Rahi (2023) allow for asymmetric market environments, but they focus on a single centralized exchange without considering traders' endogenous market entry into alternative trading venues.

<sup>11</sup>Our model does not explicitly designate buyers or sellers. The names are introduced for convenience: the two groups have negatively correlated idiosyncratic components, and thus, without loss of generality, we call one group buyers and the other sellers.

pairs in the same group. The traders' *total asset values*  $(\tilde{\theta}^i)_i$  are correlated as:

$$\Sigma_\theta = \frac{1}{\omega_{cv}^2 + \omega_{iv}^2} \begin{bmatrix} \omega_{cv}^2 + \omega_{iv}^2 \rho_o & \omega_{cv}^2 + \omega_{iv}^2 \rho \\ \omega_{cv}^2 + \omega_{iv}^2 \rho & \omega_{cv}^2 + \omega_{iv}^2 \rho_o \end{bmatrix} = \begin{bmatrix} \tilde{\rho}_+ & \tilde{\rho}_- \\ \tilde{\rho}_- & \tilde{\rho}_+ \end{bmatrix}, \quad (4)$$

where each block submatrix has dimension  $(I/2) \times (I/2)$ . The within-group correlation is larger than the cross-group correlation ( $\tilde{\rho}_+ > \tilde{\rho}_-$ ), and when  $\omega_{iv}$  is sufficiently larger than  $\omega_{cv}$ , traders in different groups have opposite trading incentives for the risky asset ( $\tilde{\rho}_- < 0$ ): hence the labels buyers and sellers.

The interdependence of traders' asset values in Eq. (4) is *symmetric*, in the sense that the set of correlations in each row is the same,  $\{\tilde{\rho}_{ik}\}_{k \neq i} = \{\tilde{\rho}_{jk}\}_{k \neq j}$  for any  $i \neq j$ : essentially the rows of  $\Sigma_\theta$  are permutations of each other. However, the correlations  $\tilde{\rho}_{ij}$  are still heterogeneous across pairs of traders  $(i, j)$ . Sections 4 and 5 will revisit this example to study the implications of heterogeneous correlations across pairs, separately from other sources of asymmetry across traders.  $\square$

**Information.** Trader  $i$  is uncertain about his own asset value  $\tilde{\theta}^i$  and instead receives a private signal  $s^i = \tilde{\theta}^i + \varepsilon^i$ , where  $\varepsilon^i \sim \mathcal{N}(0, (\sigma^i)^2 \sigma_\theta^2)$  is independent noise. The information precision  $\phi^i = (\sigma^i)^{-2}$  can differ across traders. The signal  $s^i$  is realized and privately observed by trader  $i$  at the beginning of the trading period (Figure 1). Other traders' signals  $(s^j)_{j \neq i}$  are not observed by trader  $i$  until the economy ends.

Heterogeneity across traders, which is crucial for the paper's results, can arise in both the correlation structure  $\Sigma_\theta = \text{Corr}[(\tilde{\theta}^i)_i] \in \mathbb{R}^{I \times I}$  and the information precisions  $(\phi^i)_i \in \mathbb{R}^I$ . We refer to trader  $i$ 's correlation profile and information precision  $\phi^i$  jointly as his *type*; for example, buyers and sellers in Example 1 have different types.<sup>12</sup> Notably, a trader's type does not include the realization of the private signal  $s^i$ , unlike the conventional notion of type in incomplete-information games. Traders' types and the prior distributions of asset values and signals are common knowledge.<sup>13</sup>

<sup>12</sup>Our assumption that traders know others' types is weaker than assuming that they know others' identities. Counterparty choice depends only on type—the relevant correlations in  $\Sigma_\theta$  and information precision  $\phi^j$ —so traders are indifferent between any two counterparties of the same type, regardless of identity. Accordingly, our framework does not study “relationship trading” based on counterparty identity, even though such relationships are important in some markets.

<sup>13</sup>In illiquid markets, trades are often dominated by large institutions that have better knowledge of the markets and other participants. The number of relevant agents can indeed be small, as documented in Avellaneda and Cont (2010), Oehmke and Zawadowski (2017), and Hendershott et al. (2020). Nevertheless, complete knowledge of other traders' types is imposed for simplicity; the economic mechanisms remain present when traders instead have partial but sufficiently precise information about potential counterparties' types. See also Section 4.1.

## 2.1 Centralized Market (CM) Mechanism

We model the centralized market as a canonical uniform price auction with divisible assets (Klemperer and Meyer, 1989; Kyle, 1989). A trader  $i$  who enters the centralized market submits a (net) demand schedule  $q^i(p) : \mathbb{R} \rightarrow \mathbb{R}$  (i.e., a set of limit orders) that maximizes his expected utility. As is standard, optimization over demand schedules  $q^i(\cdot)$  is equivalent to pointwise optimization over demand quantities  $q^i \in \mathbb{R}$  for each possible realization of price  $p \in \mathbb{R}$ :

$$\max_{q^i(\cdot) : \mathbb{R} \rightarrow \mathbb{R}} E[u^i(q^i, p) | s^i] = \max_{\{q^i(p) \in \mathbb{R}; \forall p \in \mathbb{R}\}} \{E[\tilde{\theta}^i | s^i, p] q^i - p q^i - \frac{\mu}{2} (q^i)^2\}. \quad (5)$$

The centralized market clears all participating traders' demands at a price at which the total demand equals zero:  $\sum_i q^i(p) = 0$ .<sup>14</sup> The equilibrium allocation is determined by the demand schedules already submitted,  $q^i = q^i(p)$  at the market-clearing price  $p$ . If  $q^i > 0$ , the trader is a buyer; if  $q^i < 0$ , the trader is a seller.

## 2.2 Over-the-Counter (OTC) Market Mechanism

As illustrated in Figure 1, upon entering the OTC market, traders form a *pairwise-stable* matching such that no two traders can both strictly gain by leaving their current matches and forming a new pair. More precisely, each trader ranks other traders based on their types—information precision and correlations—and these rankings determine a pairwise-stable matching by Irving's (1985) algorithm. If a trader who entered the OTC market fails to find a counterparty (e.g., when an odd number of traders enters), he leaves the economy with no additional trading opportunity in either market.

After the OTC matching is determined, traders receive their private signals about asset values.<sup>15</sup> In the trading period  $t = 1$ , each bilateral trade is conducted as a uniform price auction, as in the centralized market. Two traders simultaneously submit their demand schedules  $q^i(p) : \mathbb{R} \rightarrow \mathbb{R}$  as functions of to-be-realized price  $p \in \mathbb{R}$  and solve the optimization problem (5). The pair-specific market-clearing price is determined by  $q^i(p) + q^j(p) = 0$ ; see ft. 14.

We adopt the same trading mechanism—uniform price auctions—in both the cen-

<sup>14</sup>If no such price exists or multiple such prices exist, trading in the centralized market fails and all participating traders leave the economy with zero quantities  $q^i = 0$ .

<sup>15</sup>The particular timing of private signal realization ensures that equilibrium consists of linear demands in the trading period. If traders' signals are realized before their market or counterparty choices, this can lead to truncated Gaussian distributions for asset values and signals within each market and OTC pair that may fail to support linear demands in equilibrium. See also ft. 7.

tralized market and OTC bilateral trades to isolate the implications of all-to-all trading in the centralized market versus bilateral trading in the OTC market from those of different trading mechanisms. See [Section 6.1](#) for discussion.

## 2.3 Market Choice and Trading

**Strategies.** At  $t = 0$ , each trader  $i$  chooses which market to enter,  $m^i \in \{OTC, CM\}$ , and a counterparty type  $\tau^i$  if he enters the OTC market. When a trader chooses  $m^i = CM$ , his counterparty choice is denoted by  $\tau^i = \emptyset$  for convenience. At  $t = 1$ , the trader chooses his demand schedule  $q^i(\cdot; m^i, \tau^i)$  in the market or pair  $(m^i, \tau^i)$  in which he participates. Therefore, trader  $i$ 's strategy is  $\{(m^i, \tau^i), q^i(\cdot; m^i, \tau^i)\}$ .

**Equilibrium.** [Definition 1](#) provides three equilibrium conditions: (i) a Bayesian Nash equilibrium in each uniform price auction in the trading period, (ii) no incentive for any trader to deviate individually from his market choice, and (iii) no profitable pairwise deviation from the centralized market to the OTC market or from one OTC pair to another. For each trader  $i$ ,  $E[u^i(m^i, \tau^i)]$  denotes the *ex ante* utility from choosing  $(m^i, \tau^i)$ , given the distribution of other traders across markets in the equilibrium market structure.

**Definition 1 (Equilibrium).** *A profile of all traders' strategies  $\{(m^i, \tau^i), q^i(\cdot; m^i, \tau^i)\}_i$  is an equilibrium if and only if*

(i) *the profile of demand schedules  $\{q^i(\cdot; m^i, \tau^i)\}_i$  (Eq. (5)) constitutes a Bayesian Nash equilibrium in the centralized market and each OTC pair;*

(ii) *no trader in the OTC market strictly gains from deviating to the centralized market: i.e., for any  $i$  such that  $m^i = OTC$ ,*

$$E[u^i(m^i, \tau^i)] \geq E[u^i(CM, \emptyset)];$$

(iii) *no pair of traders can both strictly gain by jointly deviating from their current markets or pairs and forming a new OTC pair: i.e., for any  $i$  and  $j \neq i$ ,*

$$E[u^i(m^i, \tau^i)] \geq E[u^i(OTC, j)] \quad \text{or} \quad E[u^j(m^j, \tau^j)] \geq E[u^j(OTC, i)].$$

We make three remarks on [Definition 1](#). First, condition (iii) applies even when one or both traders are initially in the centralized market. This ensures that OTC matches are immune both to the entry of one or more traders from the centralized market and to changes in counterparty choices within the OTC market. Second, a market

structure where all traders choose the centralized market,  $m^i = CM$  for all  $i$ , is not a *trivial* equilibrium arising simply from the absence of other traders in the OTC market (condition (iii)). A pair of traders can jointly deviate from the centralized market to the OTC market whenever doing so is profitable for both.<sup>16</sup>

Third, the market structure where only the OTC market opens,  $m^i = OTC$  for all  $i$ , is a trivial equilibrium. To focus on nontrivial equilibria, we note that this market structure yields the same equilibrium outcomes as any alternative structure in which one matched pair  $(i, j)$  is in the centralized market instead, while all other pairs remain in the OTC market. This equivalence arises because the centralized market and OTC pairs employ the same trading mechanism in our framework. Accordingly, the OTC-only market structure is regarded as an equilibrium only if every equivalent market structure—where one matched pair is treated as the centralized market—satisfies conditions (ii) and (iii).<sup>17</sup>

**Roadmap and Special Cases.** [Definition 1](#) separates equilibrium into a trading-period problem and a market-structure problem. [Section 3](#) solves for the Bayesian Nash equilibrium in the trading period, taking market structure  $\{(m^i, \tau^i)\}_i$  as given (part (i)). The solution determines traders' *ex ante* utilities as functions of market structure and highlights the key effects—*liquidity* and *learning*—that govern their market and counterparty choices. [Sections 4](#) and [5](#) then use these utilities to endogenize equilibrium market structure (parts (ii) and (iii)) and study how it responds to asset and trader characteristics. To highlight the key mechanisms, [Section 4](#) first focuses on a simpler setting with symmetric traders.

**Definition 2 (Symmetric Traders).** *Traders are symmetric if they*

- (i) *are equally informed:*  $\phi^i = \phi$  is the same for all  $i$ ;
- (ii) *assign the same asset-value weights:*  $(\omega_{cv}^i, \omega_{iv}^i) = (\omega_{cv}, \omega_{iv})$  for all  $i$ .

By [Definition 2](#), symmetric traders still exhibit heterogeneity across pairwise correlations (i.e.,  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{ij}$  can differ across pairs  $(i, j)$ ) and individual correlation profiles (i.e.,  $\{\rho_{ij}\}_{j \neq i}$  can differ across traders  $i$ ), while part (ii) ensures that the av-

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<sup>16</sup>There is no incentive to deviate unilaterally from the centralized market to the OTC market because, holding other traders' strategies fixed, such a deviation leaves the trader unmatched.

<sup>17</sup>An alternative is to introduce  $L \geq 2$  captive traders in the centralized market, which prevents the OTC-only structure from being a trivial equilibrium. We do not adopt this approach because assumptions about the captive traders—their asset-value correlations with non-captive traders and the precision of their private signals—affect equilibrium outcomes and welfare quantitatively, although our qualitative results extend to settings with captive traders in either or both markets.

average correlation  $\bar{\rho} = \frac{1}{I-1} \sum_{j \neq i} \text{Corr}[\tilde{\theta}^i, \tilde{\theta}^j] = \omega_{cv}^2$  is common to all traders. [Section 4.1](#) contrasts the implications for OTC matching of these forms of correlation heterogeneity among symmetric traders with those of heterogeneity in average correlations. [Section 5](#) considers further implications of trader asymmetry.

### 3 Equilibrium in Uniform Price Auctions

This section determines traders' optimal trading strategies and a Bayesian Nash equilibrium (hereafter, equilibrium) in the trading period, taking as given a market structure  $\{(m^i, \tau^i)\}_i$ . For convenience, we call each bilateral pair and the centralized market “a market.” Suppose there are  $N$  traders in a market:  $N = 2$  in an OTC pair and  $N \in [3, I]$  in the centralized market.

Each trader  $i$  in the market faces the following pointwise first-order condition:

$$E[\tilde{\theta}^i | s^i, p] - \mu q^i = p + \lambda^i q^i \quad \forall p \in \mathbb{R}, \quad (6)$$

where  $\lambda^i = \frac{dp}{dq^i}$  is the trader's *price impact*, which measures the price change following a marginal increase in the trader's demand, and the *expected asset value*  $E[\tilde{\theta}^i | s^i, p] = c_\theta^i E[\tilde{\theta}] + c_s^i s^i + c_p^i p$  is determined by the joint equilibrium distribution of  $(\tilde{\theta}^i, s^i, p)$ .

We make two observations from the first-order condition (6). First, a larger price impact  $\lambda^i$  means that a marginal increase in demand raises the price more, increasing the payment on all units purchased. Price impact therefore acts as a marginal transaction cost,  $\lambda^i q^i$  in Eq. (6), and reduces optimal demand relative to the competitive-market demand.<sup>18</sup> This demand reduction captures endogenous *market illiquidity*. Trader  $i$ 's price impact  $\lambda^i$  equals the slope of the inverse residual supply, i.e.,

$$\lambda^i = -\left(\sum_{j \neq i} \frac{\partial q^j(\cdot)}{\partial p}\right)^{-1} = \left(\sum_{j \neq i} \frac{1 - c_p^j}{\mu + \lambda^j}\right)^{-1} \geq 0, \quad (7)$$

Second, trader  $i$ 's expected asset value is conditional on both his private signal  $s^i$  and *to-be-realized* price  $p$ . His demand schedule  $q^i(\cdot)$  therefore incorporates the information in each potential price realization, as if he observed the price before submitting his demand. Thus, even in this static model, traders *learn asset values from prices*, and price informativeness affects *ex ante* utilities, trading strategies, and

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<sup>18</sup>Following the convention, we focus on equilibrium with positive price impacts for all traders, which guarantees their second-order conditions. In the competitive-market limit as  $N \rightarrow \infty$ , all traders' price impacts converge to zero. Thus, perfect and imperfect competition can be studied through an equilibrium comparison rather than by imposing price taking. See [Corollaries 1 and 3](#).

market and counterparty choices.<sup>19</sup> The expected asset value  $E[\tilde{\theta}^i | s^i, p]$  is obtained by applying the Projection Theorem to the equilibrium price distribution,

$$p = \left( \sum_i \frac{1 - c_p^i}{\mu + \lambda^i} \right)^{-1} \sum_i \frac{c_\theta^i E[\tilde{\theta}] + c_s^i s^i}{\mu + \lambda^i}. \quad (8)$$

Because the remaining equilibrium characterization is standard and largely technical, we relegate it to the Appendix. [Proposition 6](#) in [Appendix A.2](#) shows that price impact (liquidity) and expected asset value (learning from price) are sufficient statistics for trading-period equilibrium outcomes and endogenizes both statistics.<sup>20</sup> Trader  $i$ 's *ex ante* equilibrium utility is written as

$$E[u^i] = \frac{1}{2\mu} \underbrace{\left( 1 - \left( \frac{\lambda^i}{\mu + \lambda^i} \right)^2 \right)}_{\text{liquidity effect}} \underbrace{\text{Var}[E[\tilde{\theta}^i | s^i, p] - p]}_{\text{learning effect}}. \quad (9)$$

Eq. (9) maps the two sufficient statistics of trading-period equilibrium—price impact and expected asset value—into the trader's *ex ante* utility and decomposes it into the liquidity and learning effects. First, the *liquidity effect* captures the benefit of lower price impact. A larger price impact  $\lambda^i$  lowers the liquidity effect, and thus, lowers the equilibrium utility (9), holding the learning effect fixed. Second, the *learning effect* is captured by the variance of the marginal trading surplus,  $\text{Var}[E[\tilde{\theta}^i | s^i, p] - p]$ . Learning from price distorts the primitive trading surplus,  $\text{Var}[s^i - \bar{s}]$ , where  $\bar{s} = \frac{1}{N} \sum_{j \in N} s^j$ , because the equilibrium price aggregates market participants' signals (and reflects  $\bar{s}$  in symmetric markets). When price inference shifts  $E[\tilde{\theta}^i | s^i, p]$  toward  $p$  and away from  $s^i$ , it compresses  $E[\tilde{\theta}^i | s^i, p] - p$  relative to  $s^i - \bar{s}$  and lowers utility, holding the liquidity effect fixed. Conversely, learning can increase utility by moving the expected asset value away from the price and inducing more aggressive trading on  $s^i$ . The key determinant is the conditional price informativeness  $\text{Corr}[\tilde{\theta}^i, p | s^i]$ , the additional information in price beyond the trader's private signal.

Given the centralized- and OTC-market characteristics, each trader's liquidity and

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<sup>19</sup>This contingent learning makes transparency rules irrelevant in our framework. The centralized market and OTC pairs clear independently, and observing realized prices after trades settle does not affect equilibrium. See [Section 6.2](#) for discussion of dynamic extensions.

<sup>20</sup>While general equilibrium existence and uniqueness are not established, the literature proves them in special cases: [Rostek and Weretka \(2012\)](#) for symmetric traders ([Definition 2](#)) and [Manzano and Vives \(2021\)](#) for two asymmetric traders ( $N = 2$ ). [Corollaries 4 and 5](#) in [Appendix A.1](#) restate these results without proof. [Proposition 7](#) in [Appendix A.2](#) establishes existence and local uniqueness in the two-group environment with bounded heterogeneity in precision and asset-value weights, extending the literature to asymmetric settings in which prices need not fully reveal private signals.

learning effects are interdependent and jointly govern his market and counterparty choices. These choices in turn influence market size and trader composition, creating externalities on other traders' liquidity and learning effects. Before closing this equilibrium loop in [Section 4](#), we first present a simple example illustrating how the centralized and OTC markets generate different liquidity and learning effects and the resulting benefits of OTC trading ([Section 3.1](#)), and then conduct comparative statics on the key market characteristics governing these effects ([Section 3.2](#)).

### 3.1 Motivating Example

**Example 1-Cont'd (Learning and Liquidity Effects in Market Choices).** In the two-group environment of [Example 1](#), suppose for simplicity that a sufficiently large number  $N$  of traders, with equal numbers of buyers and sellers, are in the market and let

$$\tilde{\theta}^i = \begin{cases} \theta + \delta & \text{if } i \text{ is a buyer,} \\ \theta - \delta & \text{if } i \text{ is a seller,} \end{cases} \quad s^i = \tilde{\theta}^i + \varepsilon^i.$$

The centralized-market price aggregates all traders' signals and approximates the common component  $\theta$ , while averaging out the idiosyncratic component  $\delta$ :

$$p_{cm} \stackrel{\text{info.}}{\approx} \frac{1}{N} \sum_{j \neq i} (\tilde{\theta}_j + \varepsilon^j) \approx \theta.$$

Here,  $\stackrel{\text{info.}}{\approx}$  indicates informational equivalence. If a buyer trades over the counter with a seller, the bilateral price is informationally equivalent to the seller's signal:

$$p_{otc} \stackrel{\text{info.}}{\approx} s^j = (\theta - \delta) + \varepsilon^j.$$

(i) *Learning effect.* The centralized-market price is informative mainly about  $\theta$ , whereas the OTC price is informative about  $\theta - \delta$ . When the idiosyncratic component dominates,  $\text{Var}[\delta] > \text{Var}[\theta]$ , the latter provides better information about the buyer's dominant asset-value component  $\delta$ . Formally,  $\text{Corr}[\tilde{\theta}^i, p_{otc} | s^i] = \text{Corr}[\theta + \delta, \frac{\theta - \delta}{2} | s^i] \propto (\text{Var}[\theta] - \text{Var}[\delta]) < 0$  and  $\text{Corr}[\tilde{\theta}^i, p_{cm} | s^i] = \text{Corr}[\theta + \delta, \theta | s^i] \propto \text{Var}[\theta] > 0$ . The negative conditional correlation in the OTC pair moves the buyer's expected asset value  $E[\tilde{\theta}^i | s^i, p_{otc}]$  away from the price and induces more aggressive trading on  $s^i$ . Thus, when the idiosyncratic component dominates, the targeted OTC pair provides a larger learning effect than the centralized market.

(ii) *Liquidity effect.* Although the centralized market has more participants, Eq. (7) shows that price impact also depends on other traders' demand elasticities. In the buyer-seller OTC pair, a high price signals a high  $\theta - \delta$  and, when  $\text{Var}[\delta] > \text{Var}[\theta]$ , a low  $\delta$ . The buyer therefore reduces demand both because the price is high and because his expected asset value  $E[\theta + \delta | s^i, p_{otc}]$  is low. By contrast, a high centralized-market price signals a high  $\theta$ , so the increase in  $E[\theta + \delta | s^i, p_{cm}]$  partly offsets the direct price effect on demand. Targeted OTC trading can therefore generate more price-elastic demand and a higher liquidity effect despite its smaller market size.  $\square$

The example highlights three central insights underlying our results. First, the key benefit of OTC trading arises from targeting a preferred counterparty whose trading needs generate better learning, greater trading surplus, more elastic demand, or lower price impact—or all of them—than the aggregate centralized market. Counterparty rankings therefore matter not only for OTC matching outcomes but also for comparisons of liquidity and learning across venues (Propositions 2-4 and Theorem 4). Second, these benefits require heterogeneous interdependence across traders: if potential OTC counterparties resemble the aggregate centralized market, targeting provides no advantage.<sup>21</sup> Theorems 1 and 2 formalize this role of heterogeneous interdependence. Lastly, OTC trading can improve learning and liquidity simultaneously, so the two effects need not generate a trade-off. Proposition 1 next examines when these effects move in the same or opposite directions in general settings.

### 3.2 Endogenous Liquidity and Learning Effects

This section formalizes how the liquidity and learning effects vary with three market characteristics: market size  $N$ , average asset-value correlation  $\bar{\rho}_N$ , and private-information precision  $\phi$ .

**Proposition 1 (Liquidity and Learning Effects: Symmetric Market).** Consider a market with  $N \leq I$  symmetric traders whose average correlations  $\bar{\rho}_N = \frac{1}{N-1} \sum_{j \neq i, j \in N} \tilde{\rho}_{ij}$  are the same for all  $i \in N$ .<sup>22</sup> A trader's *ex ante* utility  $E[u^i]$  in-

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<sup>21</sup>For instance, with independent private values ( $\rho_{ij} = 0$  for all  $(i, j)$ ), neither venue generates learning from price, and hence, there is no amplification or offsetting of demand elasticities through learning. With a pure common value ( $\text{Var}[\delta] = 0$ ; Kyle, 1989), the centralized market aggregates information about the same common component  $\theta$  from more traders, and its larger number of participants also provides a liquidity advantage. The dominance of centralized-market trading extends naturally to settings with homogeneous correlations (Vives, 2011). See Corollary 3.

<sup>22</sup>Proposition 1 adopts a stronger symmetry assumption than Definition 2, when  $2 < N < I$ : all

creases as

- (i) the number of traders in the market  $N$  increases;
- (ii) the average correlation  $\bar{\rho}_N$  decreases; or
- (iii) the precision  $\phi$  increases if and only if  $\phi < \phi^*(N, \bar{\rho}_N)$  for  $\phi^*(N, \bar{\rho}_N) \in [0, \infty]$ .

Parts (i) and (ii) formalize the two forces illustrated in the motivating example in [Section 3.1](#). A larger market improves both learning and liquidity by aggregating more signals and lowering price impact, while a lower average correlation increases trading surplus and makes other traders' demands more price-elastic. Thus, market size and average correlation move the liquidity and learning effects in the same direction.

Information precision can create a learning-liquidity trade-off (part (iii)). When  $\bar{\rho}_N > 0$ , both effects increase with precision; when  $\bar{\rho}_N < 0$ , higher precision improves learning but worsens liquidity. Greater precision reduces traders' reliance on price, weakening the effect of price inference on demand elasticities: this improves liquidity when price inference dampens demand elasticities ( $\bar{\rho}_N > 0$ ), but worsens it when price inference amplifies them ( $\bar{\rho}_N < 0$ ). [Figure 2](#) illustrates the latter case and the resulting liquidity-learning trade-off. The marginal learning benefit dominates the liquidity cost if and only if  $\phi < \phi^*(N, \bar{\rho}_N)$ . [Proposition 5](#) in [Appendix A.1](#) extends this trade-off to bilateral trading between traders with heterogeneous precisions.

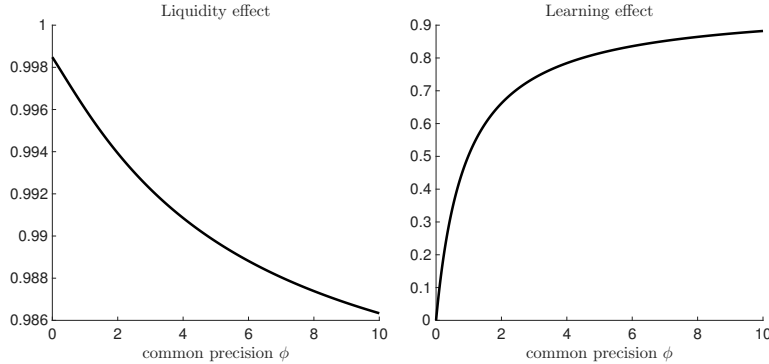


Figure 2: The liquidity and learning effects with respect to information precision  $\phi$ . The two effects create a trade-off in the *ex ante* utility when  $\bar{\rho}_N < 0$  ([Proposition 1](#) (iii)).  $N = 8$  and  $\bar{\rho}_N = -0.1$ .

Importantly, [Proposition 1](#) treats market size and average correlation as given. With endogenous entry, market and counterparty choices jointly determine those

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rows of the corresponding  $(N \times N)$  submatrix of  $\Sigma_\delta$  have the same sum of elements. The common sum need not be zero unless  $N = I$ . For instance, [Example 1](#) satisfies this stronger symmetry, although the assumption applies more generally.

characteristics in the centralized market  $(N_{cm}, \bar{\rho}_{cm})$  and OTC pairwise correlations  $\tilde{\rho}_{ij}$ , creating an endogenous trade-off between market size and correlation. [Section 4](#) studies this interaction and shows how the resulting trade-off depends on precision  $\phi$ .

## 4 Endogenous Market Structure

This section endogenizes market structure, focusing on symmetric traders. With common information precision, the key market characteristics identified in [Proposition 1](#) are the centralized-market size and average correlation and the pool of pairwise correlations available in the OTC market. We now endogenize these characteristics.

### 4.1 Pairwise-Stable OTC Counterparty Matching

As a first step toward characterizing the entry-period equilibrium ([Definition 1](#) (ii) and (iii)), we characterize traders' counterparty rankings and pairwise-stable matching within the OTC market. [Proposition 2](#) shows how pairwise asset-value correlations determine stable matching. The result requires only common information precision, not the common asset-value weights imposed by [Definition 2](#).

**Proposition 2 (Pairwise-Stable OTC Matching).** Suppose traders in the OTC market have common information precision ( $\phi^i = \phi$  for all  $i$ ). An OTC matching is pairwise stable if and only if it can be generated by the following sequential procedure: match a pair with the lowest negative correlation  $\tilde{\rho}_{ij} = \min_{(k,\ell)} \tilde{\rho}_{k\ell}$ , remove the matched pair, and repeat. A pairwise-stable OTC matching exists for any  $\Sigma_\theta$  and for any submatrix corresponding to OTC participation, and if  $\tilde{\rho}_{ij} \neq \tilde{\rho}_{ik}$  for every triple of distinct traders  $i, j, k$ , it is unique.

Three observations follow. First, when traders' types differ only in correlations, [Proposition 1](#) (ii) implies a common criterion for counterparty rankings: every trader prefers a lower pairwise asset-value correlation. A more negatively correlated counterparty has trading needs that are more likely to offset the trader's own and improves both learning and liquidity.

Second, although [Irving's \(1985\)](#) roommate-matching algorithm does not generally guarantee the existence of a pairwise-stable matching, [Proposition 2](#) establishes that a stable OTC matching exists under arbitrary interdependence in traders' asset values. This robust existence result follows from the common ranking criterion. Because every trader prefers a lower pairwise correlation and the pairwise index is symmetric ( $\tilde{\rho}_{ij} =$

$\tilde{\rho}_{ji}$ ), a pair with the lowest correlation among any set of remaining traders consists of mutual most-preferred counterparties. This property rules out cycles in first-choice rankings and allows the stable matching to be constructed sequentially.<sup>23,24</sup>

Endogenous market choice further disciplines equilibrium by preventing anticipated matching or trading failures from persisting. Conditional on entering the OTC market, failures can arise for two reasons. First, if an odd number of traders enter, at least one remains unmatched. Second, even when two traders are paired, trade does not occur if their asset values are positively correlated: their optimal demands become inelastic and no market-clearing price exists (Corollary 5 in Appendix A.1). A trader who anticipates either failure instead enters the centralized market.

Third, the result highlights the role of our assumption that traders know potential counterparties' types. A trader's preferred counterparty need not choose him in return. Under the sequential direct-search interpretation of Proposition 2, he then searches for another counterparty until successfully matched or no potential counterparty remains. Complete information about counterparty types allows us to represent this sequential search process—which may capture voice trading with zero or low search costs in practice—as a static matching problem with *ex ante* counterparty and market choices. Traders anticipate potential matching failures and therefore may be matched in equilibrium with a less-preferred counterparty.<sup>25</sup>

Examples 1 and 2 illustrate how mutual counterparty choice and anticipated matching failures interact with two forms of correlation heterogeneity: cross-pair heterogeneity under cross-trader symmetry and cross-trader heterogeneity.

**Example 1-Cont'd (Cross-Pair Heterogeneity).** Recall that Example 1 has common cross-group and within-group correlations  $\tilde{\rho}_-$  and  $\tilde{\rho}_+$ . Thus, the correlation structure (4) exhibits cross-pair heterogeneity  $\tilde{\rho}_- \neq \tilde{\rho}_+$  while maintaining cross-trader symmetry across the rows of  $\Sigma_\theta$ . By Proposition 2, the qualitatively unique stable OTC matching exists and pairs every buyer with a seller. These natural counterparties

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<sup>23</sup>This argument need not hold under heterogeneous information precisions, because the learning-liquidity trade-off arises uniquely in its comparative static (Proposition 1 (iii)). A trader's own precision determines which effect dominates and can therefore generate asymmetric criteria for ranking counterparties of different precision types. See Section 5.

<sup>24</sup>The stable matching may not be unique if some traders are indifferent between two or more counterparties, but it does not affect the qualitative results on the endogenous market structure.

<sup>25</sup>Sequential counterparty search can be relatively rapid in some OTC markets and may therefore involve low search costs. For example, intermediation chains are short in U.S. municipal bonds (Li and Schurhoff, 2019), while estimated median search delays are below five minutes in UK government bonds (Pinter, Uslu, and Wijnandts, 2019).

have offsetting exposures and trading needs, allowing lower liquidity costs and better learning about uncertain exposures.

In any pairwise-stable matching, every OTC pair  $(i, j)$  has the same correlation  $\tilde{\rho}_{ij} = \tilde{\rho}_-$ . Consequently, all traders obtain the same *ex ante* utility conditional on entering the OTC market and face the same incentive problem in choosing the OTC and centralized markets. More generally, the proof of [Proposition 2](#) shows that under cross-trader symmetry—when all rows of  $\Sigma_\theta$  are permutations of one another—all matched OTC pairs have the same minimum correlation.  $\square$

**Example 2 (Cross-Trader Heterogeneity).** Suppose  $I = 6$  traders enter the OTC market and their asset values are correlated as follows:

$$\Sigma_\theta = \left[ \begin{array}{ccc|ccc} 1 & 0.5 & 0.7 & -0.5 & -0.3 & -0.4 \\ 0.5 & 1 & 0 & -0.1 & -0.3 & -0.1 \\ 0.7 & 0 & 1 & -0.4 & -0.2 & -0.1 \\ \hline -0.5 & -0.1 & -0.4 & 1 & 0.6 & 0.4 \\ -0.3 & -0.3 & -0.2 & 0.6 & 1 & 0.2 \\ -0.4 & -0.1 & -0.1 & 0.4 & 0.2 & 1 \end{array} \right]. \quad (10)$$

As in [Example 1](#), the traders can be partitioned into buyers and sellers with opposite trading directions, but the strength of these exposures now varies across traders and violates [Definition 2](#) (ii). Buyer 1 has the strongest average exposures: his average correlations with the other buyers and sellers are 0.6 and  $-0.4$ , respectively. The corresponding averages are 0.25 and  $-0.5/3$  for buyer 2 and 0.35 and  $-0.7/3$  for buyer 3. Thus, buyer 2 has the weakest average exposures.

[Proposition 2](#) sequentially constructs the three buyer-seller pairs highlighted in red in [Eq. \(10\)](#), whose pairwise correlations are  $-0.5$ ,  $-0.3$ , and  $-0.1$ . Although buyer 3 has stronger average correlation than buyer 2, he is matched with seller 6 at the least negative correlation  $-0.1$ , and therefore obtains the lowest *ex ante* utility from OTC trading among the matched traders.

Buyer 3 would prefer seller 4, but seller 4 does not prefer him in return. Mutual counterparty choice therefore leaves buyer 3 with a less-preferred match, which may induce him to enter the centralized market once market choice is endogenous. More generally, under cross-trader heterogeneity, a trader’s “weakness” as a potential counterparty depends on the entire pattern of bilateral correlations and counterparty rankings, not on his own average correlation or even his own correlation profile alone.

Identifying weak counterparties therefore requires the full correlation matrix.  $\square$

The static pairwise-stable matching mechanism also has a direct interpretation in actual OTC markets: it can represent counterparty matching through brokers, dealers, and platforms, while abstracting from explicit client-dealer roles. Dealers often locate offsetting trades rather than hold positions to limit inventory exposure; corporate-bond evidence shows active search for offsetting trades and relationships with clients who repeatedly absorb dealer inventory, while FX dealers internalize staggered offsetting client flows on their own books (Moore, Schrimpf, and Sushko, 2016; Goldstein and Hotchkiss, 2020; Jurkatis et al., 2023). Selective counterparty matching is particularly explicit in upstairs and crossing markets, such as the Paris Bourse and Liquidnet, where brokers use knowledge of clients’ latent trading interests to locate counterparties and trading platforms identify institutions with opposite-side orders (Bessembinder and Venkataraman, 2004; Ready, 2014). Repeated bilateral relationships also shape subsequent counterparty choice (Jurkatis et al., 2023), supporting our assumption that traders can target counterparties according to their types.

## 4.2 Market Choice: When the OTC Market Opens?

When would traders prefer to trade over the counter rather than in the centralized market? When would the OTC market open in equilibrium? To answer these questions, we begin with the market choices of the last pair of traders, assuming all others have entered the centralized market. This isolates the individual venue-choice problem and provides a necessary condition for any equilibrium in which the OTC market opens. Propositions 1 and 2 identify the relevant trade-off: the centralized market offers greater market size, while targeted OTC trading offers a lower pairwise correlation. Table 1 summarizes these two market characteristics.

|                     | OTCM                                                                           |   | CM                                                                                  |
|---------------------|--------------------------------------------------------------------------------|---|-------------------------------------------------------------------------------------|
| number of traders   | 2                                                                              | < | $N_{cm} = I$                                                                        |
| average correlation | $\min_{j \neq i} \tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}$ | < | $\bar{\rho}_{cm} = \frac{1}{I-1} \sum_{j \neq i} \tilde{\rho}_{ij} = \omega_{cv}^2$ |

Table 1: Trade-off between market size and correlations in the market choices of the last pair of traders. If the two traders choose the centralized market, all  $I$  traders participate in the centralized market and the OTC market does not open. Alternatively, they jointly enter the OTC market and form a bilateral trading pair. Here,  $\rho_{min} = \min_{j \neq i} \rho_{ij}$  denotes the minimum idiosyncratic-component correlation between trader  $i$  and any other trader.

[Theorem 1](#) characterizes a necessary and sufficient condition under which the correlation effect dominates the size effect and hence the last pair of traders prefer OTC trading to centralized-market trading.<sup>26</sup>

**Theorem 1 (When the OTC Market Opens?).** Suppose that traders are symmetric and  $\rho_{\min} = \min_{(i,j)} \rho_{ij} < 0$ . Subject to the existence of trading-period equilibrium ([Corollaries 4](#) and [5](#)), no entry-period equilibrium consists solely of the centralized market if and only if

$$\omega_{iv}^2 > \hat{\kappa}(I, \rho_{\min}) \quad \text{and} \quad \phi < \hat{\phi}(I, \omega_{iv}^2, \rho_{\min}). \quad (11)$$

The set of  $(\omega_{iv}^2, \phi)$  satisfying condition (11) is nonempty if and only if  $\rho_{\min} < -\frac{I-2}{2(I-1)}$ .

The first condition on  $\omega_{iv}^2$  in [Theorem 1](#) implies that the OTC market opens in equilibrium when traders' asset values are sufficiently heterogeneous, with a large idiosyncratic-value component. In this case, the average correlations differ substantially between the OTC pair and the centralized market, so matching with a preferred counterparty with offsetting trading needs creates a larger surplus. The correlation effect is further strengthened by low precision, as shown by the second condition in [Theorem 1](#). With less precise private information, traders rely more heavily on prices for inference, magnifying the role of correlation differences in the trade-off with market size. Thus, low precision, *a characteristic of traders*, and heterogeneous asset values, *a characteristic of assets*, jointly determine when the OTC market opens through traders' endogenous participation.<sup>27</sup>

The predictions in [Theorem 1](#) are qualitatively aligned with several empirical patterns in financial markets. In the U.S. overnight interbank market, [Afonso, Kovner, and Schoar \(2013\)](#) find that banks with stronger hedging motives rely more on relationships with counterparties whose liquidity needs are negatively correlated with

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<sup>26</sup>The size-correlation mechanism in [Theorem 1](#) is related to [Rostek and Weretka \(2015\)](#), who show that greater participation need not improve welfare when market size and average correlation vary together. They study a given trading market, whereas we apply this mechanism across OTC and centralized markets, incorporating the role of signal precision, endogenous market structure, and welfare implications ([Theorems 1-4](#)).

<sup>27</sup>[Proposition 8](#) in [Appendix A.2](#) separately compares the learning and liquidity effects across markets. For sufficiently low  $\phi$  and  $\rho_{\min}$ , there exist cutoffs  $\hat{\kappa}_V \leq \hat{\kappa} \leq \hat{\kappa}_\lambda$  such that the OTC market provides better learning, higher *ex ante* utility, and greater liquidity when  $\omega_{iv}^2$  exceeds  $\hat{\kappa}_V$ ,  $\hat{\kappa}$ , and  $\hat{\kappa}_\lambda$ , respectively. This implies that learning is the first force favoring OTC trading: the OTC market can dominate only in learning or in both learning and liquidity, but not in liquidity alone, when traders are symmetric. [Example 4](#) in [Appendix A.2](#) shows that heterogeneity in  $(\omega_{cv}^i, \omega_{iv}^i)$  can reverse this implication.

their own. Corporate-bond dealers match customers with offsetting trades (Choi, Huh, and Shin, 2024), while upstairs brokers locate unexpressed liquidity for large block trades (Bessembinder and Venkataraman, 2004). Consistent with such targeting being particularly valuable for large trades, Holden et al. (2021) find that larger trades migrated disproportionately from a centralized limit-order book to a newly introduced parallel OTC market in the Chinese interbank FX market.

The information channel in Theorem 1 is also aligned with empirical patterns. In U.S. corporate bonds, O’Hara and Zhou (2021) find that large and information-sensitive trades remain disproportionately in voice trading and that electronic RFQ trading declines around rating downgrades. More broadly, dealer relationships and connectivity become more valuable during stressed or information-intensive periods (Di Maggio, Kermani, and Song, 2017; Czech and Pinter, 2026). These patterns do not directly identify private-signal precision, but they illustrate that the value of OTC trading with selective counterparties depends on the information environment.

Our binary distinction between OTC and centralized trading is a tractable representation of a finer spectrum in financial markets, ranging from selective counterparty contact to broad multilateral participation. Empirically, electronic RFQ trading lies between these endpoints: it increases competition relative to bilateral voice trading while preserving selective dealer contact and bilateral execution (Hendershott and Madhavan, 2015; O’Hara and Zhou, 2021). Within RFQ trading, customers typically contact only a few dealers, with larger orders sent to fewer dealers (Riggs et al., 2020), while organized swap venues offer both multi-dealer RFQs and central limit-order books (Benos, Payne, and Vasios, 2020).<sup>28</sup> Thus, the forces in Theorem 1 can be interpreted along this spectrum: broader participation increases the market-size advantage, while narrower access captures greater benefits from targeting particular counterparties.

Corollaries 1 and 2 provide additional implications that may be useful in understanding financial markets through the lens of our theoretical framework.

**Corollary 1 (OTC Trading with a Competitive Centralized Market).** Suppose  $I \rightarrow \infty$  symmetric traders and  $\rho_{\min} = \min_{(i,j)} \rho_{ij} < 0$ . Subject to the existence of

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<sup>28</sup>Related theory endogenizes the breadth of dealer contact in RFQ and related search protocols. Baldauf and Mollner (2024) show that broader dealer contact trades off stronger competition against greater information leakage, while Yueshen and Zou (2026) show that contacting fewer dealers can strengthen dealers’ incentives to respond. Our mechanism provides a complementary channel in a stylized RFQ interpretation.

trading-period equilibrium, no equilibrium structure consists solely of the centralized market if and only if

$$\omega_{iv}^2 > \widehat{\kappa}_\infty = 1 - \rho_{\min} - \sqrt{(1 - \rho_{\min})^2 - 2} \quad \text{and} \quad \phi < \widehat{\phi}(\omega_{iv}^2, \rho_{\min}).$$

The set of  $(\omega_{iv}^2, \phi)$  satisfying these conditions is nonempty if and only if  $\rho_{\min} < -1/2$ .

**Corollary 2 (Price Difference across OTC and Centralized Markets).** In the setting of [Theorem 1](#), the last pair of traders  $(i, j)$  prefer the OTC market and, simultaneously, the equilibrium prices satisfy  $E[p_{otc}] = E[p_{cm}]$  and  $Var[p_{otc}] > Var[p_{cm}]$  if  $\rho_{\min} \in (-1 + (\frac{2}{I})^{1/2}, -\frac{I-2}{2(I-1)})$ ,

$$\omega_{iv}^2 > \widehat{\kappa}_V(I, \rho_{\min}) \geq \widehat{\kappa}(I, \rho_{\min}), \quad \text{and} \quad \phi < \widehat{\phi}(I, \omega_{iv}^2, \rho_{\min}).$$

[Corollary 1](#) shows that traders with sufficiently strong incentives to learn about heterogeneous asset values and uncertain exposures may prefer the OTC market even when the centralized market is perfectly liquid and thus dominates in terms of liquidity. [Corollary 2](#) further shows that learning and liquidity provide motives for OTC trading beyond the common intuition that traders search for better prices in the OTC market. Even when the OTC price distribution is less favorable, these forces can increase trading quantities and surplus for any price realization. Thus, price-based measures of liquidity or information discovery may not fully capture traders' incentives to trade in each market.

We conclude this section by noting that the proof of [Theorem 1](#) extends the analysis from the last pair of traders to any marginal trader, allowing for an arbitrary centralized-market size  $N_{cm}$ , average correlation  $\bar{\rho}_{cm} = \frac{1}{N_{cm}-1} \sum_{j \neq i; m_j = CM} \tilde{\rho}_{ij}$ , and OTC pairwise correlation  $\tilde{\rho}_{ij}$ , which need not equal  $I$ ,  $\omega_{cv}^2$ , and  $\omega_{cv}^2 + \omega_{iv}^2 \rho_{\min}$ , respectively. The necessary and sufficient condition for a marginal trader to prefer the OTC market to the centralized market generalizes condition (11) to

$$\bar{\rho}_{cm} > \frac{N_{cm} - 2}{2(N_{cm} - 1)} + \tilde{\rho}_{ij} \quad \text{and} \quad \phi < \widehat{\phi}(N_{cm}, \bar{\rho}_{cm}, \tilde{\rho}_{ij}), \quad (12)$$

under a stronger symmetry assumption: the  $N_{cm} \leq I$  traders in the centralized market have a symmetric correlation structure, as represented by equal row sums in the corresponding  $(N_{cm} \times N_{cm})$  submatrix of  $\Sigma_\theta$ ; see also [ft. 22](#). The proofs of [Corollaries 1](#) and [2](#) use condition (12) for an arbitrary marginal trader, rather than condition (11) for the last pair of traders.

### 4.3 Equilibrium Market Structure and Coexisting Venues

This section closes the equilibrium loop by endogenizing the centralized-market size  $N_{cm}$  and average correlation  $\bar{\rho}_{cm}$ . [Section 4.2](#) characterized a marginal trader’s market choice taking these characteristics as given. Once entry is endogenous, however, a trader who enters the centralized market changes both its size and composition, and hence the trade-off faced by subsequent traders. This entry externality determines whether market choices reinforce or weaken one another.

[Corollary 3](#) first identifies special cases in which this feedback disappears or becomes one-sided, thereby ruling out coexistence of the two markets.

**Corollary 3 (Necessary Conditions for Coexistence of Two Markets).** Equilibrium consists of only one market, either the OTC market ( $N_{cm} = 2$ )<sup>29</sup> or the centralized market ( $N_{cm} = I$ ), if any of the following conditions holds:

- (i) traders’ asset values are homogeneously correlated:  $\tilde{\rho}_{ij} = \tilde{\rho}$  for all  $(i, j)$ .
- (ii) traders are perfectly informed or perfectly uninformed:  $\phi \rightarrow \infty$  or  $\phi \rightarrow 0$ .
- (iii) the centralized market is competitive:  $I \rightarrow \infty$ .

The first two cases of [Corollary 3](#) eliminate the feedback from previous entry decisions to subsequent traders’ market choices. Under homogeneous correlations in part (i), the correlation advantage of targeted OTC trading disappears,  $\bar{\rho}_{cm} = \bar{\rho}_{otc} = \tilde{\rho}$ , leaving the larger centralized market to dominate. In part (ii), although the size-correlation trade-off remains, extreme information precision makes one side of the trade-off always dominate regardless of previous entry decisions. By contrast, in the competitive-market limit of part (iii), the feedback remains and becomes reinforcing: entry changes the relevant centralized-market participation margin  $N_{cm}/I$  without generating an offsetting change in average correlation  $\bar{\rho}_{cm}$ , so a profitable deviation from one market to the other weakly strengthens the same market choice for subsequent traders.<sup>30</sup> In each case, market-choice incentives become one-sided along the entry sequence, yielding a “bang-bang” solution in which only one market opens. The proof of [Corollary 3](#) formally characterizes the necessary and sufficient condition for

<sup>29</sup>Recall that we use a centralized market of size  $N_{cm} = 2$  and an OTC pair interchangeably. See the discussion following [Definition 1](#).

<sup>30</sup>In our framework, the strength of this reinforcement vanishes in the limit of  $I \rightarrow \infty$  because private-signal noises  $\{\varepsilon^i\}_i$  are independent and identically distributed for all  $i$ . If the model were extended to allow correlated noises arising from shared information sources, the reinforcement mechanism remains strict in the competitive-market limit while changes in  $\bar{\rho}_{cm}$  remain absent.

each corner solution in the competitive-market limit. Thus, coexistence of the two venues is an imperfect-competition phenomenon ([Theorem 2](#) below).

Outside these cases, the key question is whether imperfect competition allows the induced change in  $\bar{\rho}_{cm}$  to reverse the reinforcing market-size feedback. If so, coexistence can arise; otherwise, the corner structure persists. [Proposition 3](#) provides sufficient conditions for each case, treating  $N_{cm}$  as continuous.

**Proposition 3 (Entry Cutoff Feedback).** Consider the setting of [Example 1](#) with buyer-seller correlation  $\rho$ . The idiosyncratic-value weight satisfies  $\omega_{iv}^2 > \underline{\omega}_{iv}^2(I, \rho)$ , so that the entry cutoff  $\hat{\phi}(N_{cm}, \rho, \omega_{iv}^2)$  from condition (12) exists for all  $N_{cm} \leq I$ . For sufficiently low correlation  $\rho < \rho_L(I) < 0$ , there exist cutoffs  $\underline{\omega}_{iv}^2(I, \rho) < \omega_{iv,L}^2(I, \rho) < \omega_{iv,H}^2(I, \rho) \leq 1$  and  $I_H < \infty$  such that

- (i) if  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ , then  $\frac{d\hat{\phi}}{dN_{cm}} < 0$  for all  $N_{cm}$ .
- (ii) if  $\omega_{iv}^2 > \omega_{iv,H}^2(I, \rho)$  and if  $I \leq I_H$ , then  $\frac{d\hat{\phi}}{dN_{cm}} > 0$  for all  $N_{cm}$ .

[Proposition 3](#) focuses on the setting of [Example 1](#), not because this setting is qualitatively essential, but because it allows  $\bar{\rho}_{cm}$  to be expressed explicitly as a function of  $N_{cm}$ , making the joint feedback quantitatively tractable. We first elaborate on this mechanism quantitatively in the example below and then explain why it extends qualitatively to more general settings.

**Example 1-Cont'd (Two Trader Groups: Entry Feedback).** Suppose  $I < \infty$  is even. Recall that within-group correlation satisfies  $\rho_o = -\frac{I}{I-2}\rho$  so that  $\sum_{j \neq i} \rho_{ij} = 0$  holds for all  $i$  as assumed. By [Proposition 2](#), marginal traders who enter the OTC market form only buyer-seller pairs. When  $\frac{I-N_{cm}}{2}$  buyer-seller pairs enter the OTC market and  $N_{cm}$  traders enter the centralized market, the average correlation in the centralized market can be expressed as a function of  $N_{cm}$ :

$$\bar{\rho}_{cm}(N_{cm}) = \frac{1}{N_{cm} - 1} \left[ \left( \frac{N_{cm}}{2} - 1 \right) \tilde{\rho}_+ + \frac{N_{cm}}{2} \tilde{\rho}_- \right] = \omega_{cv}^2 + \frac{I - N_{cm}}{(N_{cm} - 1)(I - 2)} \omega_{iv}^2 \rho. \quad (13)$$

The average correlation  $\bar{\rho}_{cm}(N_{cm})$  increases with its size  $N_{cm}$ . Eq. (13) therefore shows how market entry jointly endogenizes  $N_{cm}$  and  $\bar{\rho}_{cm}$ , generating two opposing effects on the entry cutoff: when  $\hat{\sigma}^2 = 1/\hat{\phi}$ ,

$$\frac{d\hat{\sigma}^2}{dN_{cm}} = \underbrace{\frac{\partial \hat{\sigma}^2}{\partial N_{cm}}}_{(+)} + \underbrace{\frac{\partial \hat{\sigma}^2}{\partial \bar{\rho}_{cm}}}_{(-)} \underbrace{\frac{(-\omega_{iv}^2 \rho)(I - 1)}{(I - 2)(N_{cm} - 1)^2}}_{(+)}. \quad (14)$$

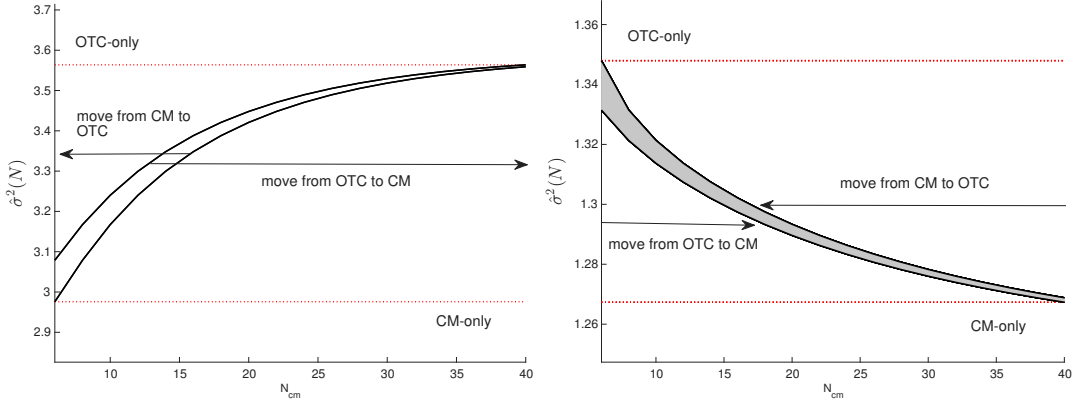


Figure 3: Endogenous market structure.  $\omega_{iv}^2 = 0.65$  (left),  $\omega_{iv}^2 = 0.75$  (right),  $\rho = -0.95, I = 40$ . The two panels illustrate the two cases of Proposition 3 using the noise-variance cutoff  $\hat{\sigma}^2 = 1/\hat{\phi}$ . In the left panel (Panel A),  $\hat{\sigma}^2(N_{cm})$  increases and  $\hat{\phi}(N_{cm})$  decreases with  $N_{cm}$ , so market entry is reinforcing and only corner market structures arise. In the right panel (Panel B),  $\hat{\sigma}^2(N_{cm})$  decreases with  $N_{cm}$ , which sustains coexistence of the two markets: in particular, the centralized-market size  $N_{cm}$  in equilibrium lies in the shaded region.

The first term  $\frac{\partial \hat{\sigma}^2}{\partial N_{cm}}$  generates reinforcing entry, whereas the second term  $\frac{\partial \hat{\sigma}^2}{\partial \bar{\rho}_{cm}}$  can reverse it. The latter becomes stronger as  $(-\omega_{iv}^2 \rho)$  increases, yielding the sufficient conditions in Proposition 3 (ii); by contrast, for sufficiently small  $\omega_{iv}^2$ , the market-size effect dominates as in part (i). Figure 3 illustrates the two cases.  $\square$

The mechanism in Eq. (14) is not specific to the two-group structure. More negatively correlated pairs sort into the OTC market (Proposition 2, condition (12)), leaving the centralized market tilted toward more positively correlated traders. This composition effect is stronger when correlations are more heterogeneous, either because  $\omega_{iv}^2$  is larger or because  $\Sigma_\delta$  is more dispersed. Hence, analogous feedback can arise under more general correlation structures, including asymmetric ones, as long as traders have common information precision.

Theorem 2 translates these two cutoff patterns into sufficient conditions for three types of equilibrium candidates: only the OTC market opens ( $N_{cm}^* = 2$ ), only the centralized market opens ( $N_{cm}^* = I$ ), and the OTC and centralized markets coexist, allowing an interior equilibrium size  $N_{cm}^* \in (2, I)$  satisfying  $\hat{\sigma}^2(N_{cm}^* + 1) < \sigma^2 < \hat{\sigma}^2(N_{cm}^*)$ . See the gray region in Figure 3.

**Theorem 2 (Endogenous Market Structure).** Consider the setting of Example 1 where  $I \leq I_H$  is even,  $\rho < \rho_L(I)$ , and  $\omega_{iv}^2 > \underline{\omega}_{iv}^2(I, \rho)$ . Conditional on the existence

of a pure-strategy equilibrium, the following properties hold:<sup>31</sup>

- (i) (*Single-venue equilibrium*) If  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ , every equilibrium has a single venue. In particular, only the OTC market opens if  $\phi < \phi_{min}$ , only the centralized market opens if  $\phi > \phi_{max}$ , and multiple single-venue equilibria arise if  $\phi_{min} < \phi < \phi_{max}$ .
- (ii) (*Coexisting-venue equilibrium*) If  $\omega_{iv}^2 > \omega_{iv,H}^2(I, \rho)$  and  $\phi_{min} < \phi < \phi_{max}$ , every equilibrium has both the centralized and OTC markets open. Only the OTC market opens if  $\phi < \phi_{min}$ , while only the centralized market opens if  $\phi > \phi_{max}$ .

[Theorem 2](#) relies on the global monotonicity of  $\hat{\phi}(N)$ , established in [Proposition 3](#), and therefore provides relatively strong sufficient conditions for single-venue equilibria, coexistence, and multiplicity. [Figure 4](#) illustrates that the cutoff can instead be nonmonotonic when  $\omega_{iv,L}^2(I, \rho) < \omega_{iv}^2 < \omega_{iv,H}^2(I, \rho)$ . Such nonmonotonicity can arise naturally when cross-trader correlations are more dispersed, as can occur under asymmetric asset-value weights. More negatively correlated OTC pairs remain in the OTC market at smaller  $N_{cm}$ , but as  $N_{cm}$  increases further, their reallocation to the centralized market can make the composition effect more likely to dominate.

[Figure 4](#) provides a further implication of the size-composition feedback. In particular, mixed equilibria near the peak of the cutoff  $\hat{\sigma}^2(N_{cm})$  are *fragile*. While they sustain substantial centralized-market participation (over 20 in the figure), a small increase in noise variance can eliminate these equilibria and leave only the OTC market, producing a large discontinuous collapse in centralized-market participation. This pattern is reminiscent of episodes such as the October 2014 U.S. Treasury flash rally and the March 2020 Treasury-market disruption, which occurred during information-intensive or highly uncertain periods. Liquidity on centralized electronic venues deteriorated abruptly in both episodes; during the March 2020 disruption, bilateral trading activity also increased sharply ([Fleming et al., 2022](#)). Existing explanations emphasize changes in market making and electronic liquidity

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<sup>31</sup>Discreteness in  $N_{cm}$  can generate cycles of profitable deviations and prevent a pure-strategy equilibrium from existing: even when  $\hat{\sigma}^2(N_{cm} + 2) < \sigma^2 < \hat{\sigma}^2(N_{cm})$  rules out pairwise deviations at  $N_{cm}$ , a profitable unilateral OTC-to-centralized-market deviation at  $N_{cm}$ —so the centralized-market size changes from  $N_{cm}$  to  $N_{cm} + 1$ —induces the counterparty to also follow, leading to an unstable market size of  $N_{cm} + 2$ . Numerical simulations nevertheless support existence under the theorem’s conditions: in part (i),  $E[u_{cm}^i(3)] < E[u_{cm}^i(4)]$ ; in part (ii),  $E[u_{cm}^i(N_{cm} + 1)] < E[u_{cm}^i(N_{cm} + 2)]$  whenever  $\hat{\sigma}^2(N_{cm} + 2) < \sigma^2 < \hat{\sigma}^2(N_{cm})$ . We have not been able to prove these additional inequalities, and the theorem remains conditional on equilibrium existence.

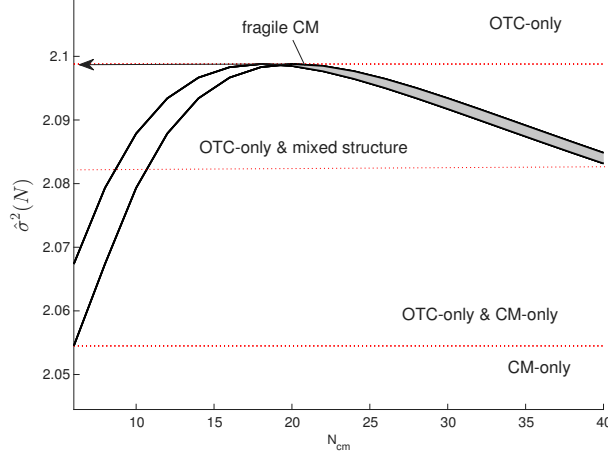


Figure 4: Endogenous market structure.  $\omega_{iv}^2 = 0.69, \rho = -0.95, I = 40$ . When the noise-variance cutoff  $\hat{\sigma}^2 = 1/\hat{\phi}$  is nonmonotone, the set of equilibrium market structures expands. When precision is low (high  $\sigma^2$ ), only the OTC market opens; as precision increases, multiplicity arises, supporting either one single-venue and one mixed-venue equilibrium or both single-venue equilibria. When precision is high (low  $\sigma^2$ ), only the centralized market opens.

provision (Bouveret et al., 2015); liquidity-driven asset sales and dealer balance-sheet constraints (Vissing-Jorgensen, 2021; He, Nagel, and Song, 2022). The mechanism here suggests a complementary, empirically testable channel in which relatively small changes in the information environment are amplified by endogenous venue choice through the size-composition feedback. We do not pursue these implications further, but the mechanism may provide a useful framework for studying liquidity fragility, abrupt changes in market structure, and related policy or market-design questions.

#### 4.4 Welfare Implications of OTC Market Availability

The endogenous market structures in the previous section raise a natural question: would restricting OTC trading be socially beneficial by concentrating traders in the centralized market, or can it destroy gains from targeted counterparty choice and lower welfare? This question is relevant to market design that has moved in both directions. Post-crisis reforms shifted standardized OTC derivatives toward organized trading and central clearing, while newer platforms such as MarketAxess, Moscow Exchange, and Tradeweb preserve OTC features such as counterparty search, selective dealer contact, and bilateral negotiation (see also ft. 1). Because traders' venue choices change market size and composition for other traders, and hence, liquidity and learning they face, the equilibrium market structure need not maximize welfare.

**Theorem 3** studies this question in an extension of [Example 1](#). Among  $I < \infty$  symmetric traders,  $L > 2$  traders are captive in the centralized market, with equal numbers of captive buyers and sellers, while the remaining  $I - L$  traders endogenously choose between the centralized and OTC markets. As in [Theorem 2](#), equilibrium market structure endogenously determines the centralized-market size  $N_{cm}^* \in [L, I]$ . For simplicity, an “OTC-only” equilibrium refers to the structure where all endogenous traders enter the OTC market, even though captive traders remain in the centralized market. We define total welfare as the sum of all traders’ *ex ante* utilities:

$$W \equiv \sum_{\{i:m^i=OTC\}} E[u_{otc}^i] + \sum_{\{i:m^i=CM\}} E[u_{cm}^i].$$

The captive traders allow us to distinguish the effect of OTC closure on traders whose venue changes under this intervention from the externality imposed on traders already in the centralized market. We therefore ask whether OTC closure produces a Pareto improvement, a Pareto loss, or a trade-off between these two groups.

**Theorem 3 (Welfare Effects of OTC Market Closure).** Consider the setting of [Example 1](#) with  $L > 2$  captive traders in the centralized market. Suppose  $\rho < \rho_L(I)$  and  $\omega_{iv}^2 > \underline{\omega}_{iv}^2(I, \rho)$ , and let  $\phi_{min} = \min_{N \in [L, I]} \hat{\phi}(N)$  and  $\phi_{max} = \max_{N \in [L, I]} \hat{\phi}(N)$ .

- (i) (*Unique OTC-only equilibrium*) Suppose  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ ,  $\phi < \phi_{min}$ , and the corner equilibrium with  $N_{cm}^* = L$  is the unique equilibrium. There exist cutoffs  $0 < \phi_C < \phi_W < \phi_{min}$  such that closing the OTC market lowers the utility of captive traders if and only if  $\phi < \phi_C$  and lowers total welfare if and only if  $\phi < \phi_W$ , whereas it lowers the utility of endogenous traders for all  $\phi < \phi_{min}$ .
- (ii) (*Multiple corner equilibria*) Suppose  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ ,  $\phi_{min} < \phi < \phi_{max}$ , and both corner equilibria exist. Closing the OTC market, in the equilibrium where it opens, strictly increases all traders’ utilities and hence total welfare.
- (iii) (*Coexisting-venue equilibrium*) Suppose  $\omega_{iv}^2 > \omega_{iv,H}^2(I, \rho)$ ,  $I < I_H$ ,  $\phi_{min} < \phi < \phi_{max}$ , and the coexisting equilibrium with  $N_{cm}^* \in (L, I)$  is the unique equilibrium. Closing the OTC market strictly lowers all traders’ utilities and hence total welfare.

For traders who endogenously choose their venue, the welfare effects in parts (i) and (iii) are aligned with their equilibrium market-choice incentives. In part (i), the unique OTC corner equilibrium, where the OTC market yields a higher utility than

the centralized market with any size (Figure 3A), implies that forcing endogenous traders into the centralized market makes them worse off. In part (iii), the unique coexisting-venue equilibrium likewise implies that closing the OTC market lowers the utility of both OTC traders and traders already in the centralized market (Figure 3B). Part (ii) differs because of a coordination problem. Although the OTC corner can be sustained as an equilibrium, a coordinated or policy-induced transition to the centralized-market corner makes every trader better off—even those who chose the OTC market. Marginal market-choice incentives can therefore sustain an equilibrium that is Pareto dominated by another equilibrium allocation.<sup>32</sup>

The captive traders introduce a second distinction between the equilibrium and welfare analyses. Theorem 2 compares the size and composition impacts of centralized-market entry at the marginal trader’s indifference cutoff  $\hat{\phi}$ , where the sign of the entry-cutoff feedback coincides locally with the sign of the utility change. Theorem 3 instead evaluates these same impacts at the primitive precision  $\phi$ , which generally differs from  $\hat{\phi}$ . The relative strength of the two impacts can therefore change with precision. Part (i) illustrates this mechanism. At the indifference margin  $\hat{\phi}$ , the favorable market-size impact dominates, so captive traders benefit when OTC closure enlarges the centralized market. As precision falls, however, Proposition 1 (iii) implies that the relative importance of liquidity and learning changes, and so does the strength of the composition impact. If the liquidity effect is more important for the response to precision ( $\phi > \phi^*$ , where  $\phi^*$  is the liquidity-learning cutoff from Proposition 1 (iii)), the adverse composition impact weakens as  $\phi$  moves below  $\hat{\phi}$ . At sufficiently low precision, learning instead becomes relatively more important ( $\phi < \phi^*$ ), and the adverse composition impact strengthens. The proof of part (i) shows that this strengthened composition effect can eventually outweigh the market-size gain as  $\phi$  becomes sufficiently close to zero, making captive traders worse off after OTC market closure, thereby producing a Pareto loss. For precision close to the indifference cutoff, OTC closure creates a trade-off between captive and OTC traders.

There is therefore no unambiguous answer as to which trading mechanism policy-

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<sup>32</sup>The utility comparisons underlying these three cases are, respectively,  $E[u_{otc}^i] > E[u_{cm}^i(I)]$  in part (i),  $E[u_{cm}^i(I)] > E[u_{otc}^i] > E[u_{cm}^i(L)]$  in part (ii), and  $E[u_{cm}^i(N_{cm}^*)] > E[u_{otc}^i] > E[u_{cm}^i(I)]$  in part (iii). These comparisons also show that the proof applies more generally than the stated conditions of the theorem. For instance, in part (i),  $\rho < \rho_L(I)$  and  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$  are needed for uniqueness of  $\phi_C$  and  $\phi_W$ ; existence of these cutoffs requires only that  $\phi_{\min}$  and  $\phi_{\max}$  are attained at the two corner structures.

makers should promote. Appropriate regulation and market design must account for traders’ heterogeneity, the available public and private information, and the endogenous composition of trader types across venues.<sup>33</sup>

To conclude Section 4, Table 2 places the welfare results (Panel B; Theorem 3) alongside the market-choice results developed earlier (Panel A; Theorem 1).

| Panel A. Individual Market Choice |            |             | Panel B. Welfare-Improving Intervention |                 |               |
|-----------------------------------|------------|-------------|-----------------------------------------|-----------------|---------------|
|                                   | Low $\phi$ | High $\phi$ |                                         | Very low $\phi$ | Low $\phi$    |
| Low $\omega_{iv}^2$               | CM         | CM          | Very low $\omega_{iv}^2$                | Keep OTC        | Shut down OTC |
| High $\omega_{iv}^2$              | OTC        | CM          | Low $\omega_{iv}^2$                     | Keep OTC        | Keep OTC      |

Table 2: Individual market choice versus welfare effects of closing the OTC market. Panel A presents the joint conditions on  $\omega_{iv}^2$  and  $\phi$  under which an individual trader chooses the OTC market (“OTC”) or the centralized market (“CM”). Conditional on some traders choosing the OTC market, Panel B presents the joint conditions under which closing the OTC market raises total welfare (“Shut down OTC”) or lowers total welfare (“Keep OTC”) relative to the equilibrium market structure.

The two panels reflect related but distinct size-composition trade-offs. Theorem 1 is a *level comparison*: a marginal trader compares the larger size of the centralized market with the more favorable pairwise correlation available through targeted OTC trading, taking both market characteristics as given. Theorem 2 then endogenizes these characteristics and the resulting equilibrium market structure, comparing how entry *impacts* its size and composition for subsequent traders at the indifference margin. Theorem 3 examines the same size-composition impacts of trader reallocation at the primitive precision.

The distinction between levels and impacts is particularly important for correlation. Individual venue choice compares the full correlation difference between a preferred OTC counterparty and the centralized-market average, whereas market entry impacts the centralized market’s average correlation only incrementally while simultaneously increasing its size: in Eq. (14),  $\frac{\partial \bar{p}_{cm}}{\partial N_{cm}} < \frac{\partial N_{cm}}{\partial N_{cm}} = 1$ . The correlation advantage that induces a trader to choose the OTC market can therefore be substantially larger than the correlation impact generated by reallocating that trader, as reflected in the

<sup>33</sup>These results complement existing theories through a different mechanism. Glode and Opp (2020) and Geromichalos and Herrenbrueck (2016) show that eliminating decentralized trading can lower welfare by weakening incentives for information acquisition or *ex ante* liquidity provision, while Acharya and Bisin (2014) and Lee and Wang (2025) show that greater centralization can improve welfare by mitigating counterparty-risk or adverse-selection externalities. Kirilenko (2000) further shows that an interior market structure can be optimal and need not coincide with the equilibrium structure.

different parameter regions for  $\omega_{iv}^2$  and  $\phi$  (“Low” and “Very low”) in Table 2. Consequently, the same market-size and correlation forces can yield different implications for individual venue choice, endogenous market structure, and the welfare effects of market intervention.

## 5 Market Structure with Asymmetric Precision

This section relaxes the trader symmetry assumed in Section 4. Rather than providing a complete characterization of equilibrium market structure, we focus on how asymmetry changes traders’ market- and counterparty-choice incentives, with particular attention to heterogeneous precision.<sup>34</sup> In particular, when traders are differently informed, which types are more attracted to each market?

Proposition 1 highlights a distinctive feature of information precision. An increase in precision  $\phi$  can move the liquidity and learning effects in opposite directions, whereas changes in the other market characteristics,  $N$  and  $\bar{\rho}_N$ , move them in the same direction. This learning-liquidity trade-off has an important implication for market and counterparty choice. Even holding fixed the composition of other traders across the two markets, a marginal trader’s own information precision can affect both his ranking of counterparty types and his ranking of the two trading venues. Example 3 first illustrates this implication for counterparty rankings.

**Example 3 (Two Trader Groups with Heterogeneous Precisions).** Suppose there are two information-precision types: informed traders with  $\phi_I = 1/\sigma_I^2$  and uninformed traders with  $\phi_U = 1/\sigma_U^2 < \phi_I$ . Combined with the buyer-seller groups in Example 1, this yields four trader types: informed buyers, informed sellers, uninformed buyers, and uninformed sellers. The number of traders need not be the same across these four types. How do these traders rank potential counterparties?

Proposition 5 in Appendix A.1 characterizes the underlying mechanism. Fixing trader  $i$ ’s own precision  $\phi^i$ , an increase in the counterparty’s precision  $\phi^j$  improves trader  $i$ ’s learning but worsens his liquidity. Which effect dominates depends on trader  $i$ ’s own precision. A sufficiently uninformed trader values the learning benefit

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<sup>34</sup>Alternatively, two examples explore the implications of heterogeneous asset-value weights. Example 2 in Section 4.1 considers OTC matching under cross-trader heterogeneity induced by heterogeneous weights and shows that traders with sufficiently large and opposing exposures on average can be less attractive as counterparties, depending on the pattern of the entire correlation matrix. Example 4 in Appendix A.2 considers a marginal trader’s market choice and shows that an additional liquidity-learning trade-off can arise with this form of heterogeneity.

more and therefore prefers an informed counterparty, whereas a sufficiently informed trader values the liquidity benefit more and therefore prefers an uninformed counterparty. Hence, when precision heterogeneity is sufficiently large, counterparty rankings become asymmetric: informed traders prefer uninformed counterparties, while uninformed traders prefer informed counterparties. When precision heterogeneity is small, by contrast, the two types rank counterparty precision in the same direction.  $\square$

Heterogeneous precision therefore introduces a learning-liquidity trade-off through variation in the trader's own precision  $\phi^i$ . How does this potentially asymmetric ranking affect pairwise-stable OTC matching and endogenous market participation? [Proposition 4](#) and [Theorem 4](#) answer this question in the setting of [Example 3](#).

**Proposition 4 (OTC Matching with Heterogeneous Precisions).** Suppose that all traders enter the OTC market in the setting of [Example 3](#). The pairwise-stable matching in the OTC market induces cross-precision pairs if and only if  $\phi_I$  is sufficiently high and sufficiently larger than  $\phi_U$ .

The restriction to two precision types guarantees the existence of pairwise-stable OTC matching.<sup>35</sup> OTC matching is either positively assortative (*same-precision* matching) or negatively assortative (*cross-precision* matching). [Figure 5A](#) presents the regions supporting each matching outcome, conditional on all traders entering the OTC market. When informed traders have sufficiently high precision (small  $\sigma_I$ ) and uninformed traders have sufficiently low precision (large  $\sigma_U$ ), cross-precision matching is stable. Outside this region, same-precision matching arises. When both types have high precision, liquidity considerations dominate and both types prefer uninformed counterparties; informed traders, anticipating that they are less preferred, are therefore matched with other informed traders. Similarly, when both types have low precision, learning considerations dominate and both prefer informed counterparties, again yielding same-precision matching.

[Theorem 4](#), however, shows that cross-precision OTC matching is not sustained once market choice is endogenous.

**Theorem 4 (Liquidity Provision toward Informed Traders).** Suppose the set-

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<sup>35</sup>With a richer set of information precisions, nonmonotonicity of *ex ante* utility in the counterparty's precision can generate circular preferences. With only two precision types, however, each trader chooses only between an informed and an uninformed counterparty, so circular preferences cannot arise. Combined with the buyer-seller structure, this guarantees the existence of a pairwise-stable OTC matching.

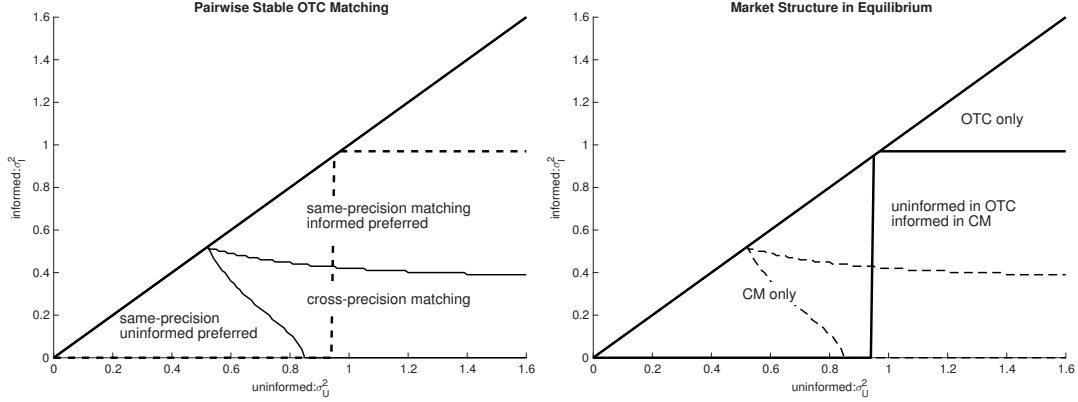


Figure 5: Pairwise-stable OTC matching and equilibrium market structure. The  $x$ -axis is the inverse of uninformed traders' precision ( $\sigma_U^2 = 1/\phi_U$ ), while the  $y$ -axis is the inverse of informed traders' precision ( $\sigma_I^2 = 1/\phi_I$ ). Panel A (solid lines) shows the regions supporting same-precision and cross-precision matching, conditional on all traders entering the OTC market. Panel B endogenizes traders' market entry, and the solid lines delineate the regions in which OTC-only, CM-only, or coexisting-venue equilibria arise.

ting of [Example 3](#) with two precision types  $\phi_I > \phi_U$  and  $L = I - 4 \geq 2$  captive traders in the centralized market, consisting of an equal number of buyers and sellers, all with high precision  $\phi_I$ . The remaining four traders consist of one buyer and one seller of each precision type. Whenever  $(\phi_I, \phi_U)$  supports cross-precision pairing among the four traders conditional on their entering the OTC market, the high-precision buyer and seller have a strict incentive to jointly move to the centralized market.

Conditional on entering the OTC market, a sufficiently informed trader prefers an uninformed counterparty who acts as a liquidity provider. [Theorem 4](#) shows, however, that once market choice is endogenous, the centralized market provides more attractive liquidity than an uninformed OTC counterparty, so the informed trader prefers entering the centralized market to remaining in a cross-precision OTC pair. Cross-precision OTC matching therefore disappears in equilibrium. Uninformed traders who remain in the OTC market match with other uninformed traders. The OTC market consequently attracts either only uninformed traders or both precision types, with same-precision matching in the latter case ([Figure 5B](#)). Endogenous market choice therefore changes not only who enters each venue but also which counterparty relationships can be sustained within the OTC market.

This endogenous sorting has an informational implication. Cross-precision OTC matching could facilitate information transmission between informed and uninformed traders; its disappearance therefore generates informational segmentation across pre-

cision types. In this sense, endogenous market entry can aggravate information asymmetry across traders.<sup>36</sup> More broadly, the result suggests that liquidity and information aggregation in OTC markets should not be studied in isolation from endogenous market choices.

## 6 Conclusion and Discussion

This paper contributes to the literature on OTC and centralized trading in three ways. First, we identify trader heterogeneity as a key force shaping venue choice: bilateral OTC trading can improve both liquidity and learning by allowing traders to target counterparties whose trading needs and information are particularly valuable, even when a centralized market offers broader participation. Second, we characterize the endogenous market structure that arises when traders choose their trading venues and, in OTC markets, their counterparties. The interaction between market size and trader composition can generate OTC-only, centralized-only, or coexisting market structures and can also make equilibrium market structure fragile or informationally segmented. Finally, we show that the welfare effects of OTC market availability depend on the externalities created by endogenous entry. Centralizing trade can improve welfare when the gains from greater market size outweigh adverse composition effects and traders' liquidity considerations outweigh their learning considerations, but can reduce welfare when changes in composition sufficiently worsen liquidity and learning. The relative merits of OTC and centralized trading therefore cannot be assessed independently of trader heterogeneity and endogenous participation.

### 6.1 Alternative OTC Market Frameworks

The OTC market framework we adopt is not essential to the broader role of counterparty heterogeneity or its potential to improve learning and liquidity. Alternative theoretical frameworks model several aspects of OTC trading differently. One promi-

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<sup>36</sup>Under alternative OTC mechanisms, cross-type information transmission need not remain segmented: random matching permits cross-type interactions, while dealers may seek informed order flow to use acquired information in subsequent trades (Pinter, Wang, and Zou, 2022; Li and Song, 2021). The latter mechanism also relies on information about traders' types, as in our framework: uninformed traders who know who is better informed may provide liquidity to attract them. The key difference is that these studies emphasize dynamic information use, which is absent from our static model, whereas our mechanism emphasizes endogenous venue choice, which allows informed traders to leave such cross-precision OTC relationships for the centralized market.

nent approach in the literature emphasizes random counterparty search and matching frictions, with trading delays or failures constituting a form of OTC illiquidity (Duffie, Garleanu, and Pedersen, 2005; Vayanos and Weill, 2008; Cuijean and Praz, 2015; Atkeson, Eisfeldt, and Weill, 2015; Hugonnier, Lester, and Weill, 2022). If the stable matching in our model were replaced by random matching, the benefit of targeted counterparty selection would be weakened, but heterogeneity would still affect expected OTC trading opportunities and hence venue choice. Another modeling choice concerns the pricing mechanism. Dealer or market-maker pricing can generate adverse-selection and discriminatory-pricing rents (Lee and Wang, 2025); these forces would change how surplus and trading costs are reflected in transaction terms without eliminating our mechanism, as heterogeneity across counterparties or dealers could still strengthen the benefits of selective OTC trading. A third extension is to replace pairwise interaction with a trading network, in which liquidity also depends on network position and connectivity (Malamud and Rostek, 2017; Babus and Kondor, 2018; Colliard, Foucault, and Hoffmann, 2021). These additional features would add further margins through which market structure matters without overturning the role of counterparty heterogeneity.

## 6.2 Extension to Dynamic Markets

Given the dynamic nature of financial markets, a natural question is how the static mechanism in this paper extends over time. A dynamic extension would introduce two additional forces. First, the asset-value components,  $\theta$  and  $\delta^i$ , which are exogenous primitives in our static framework, would become endogenous objects generated by the dynamics of common asset returns and trader-specific liquidation needs. Second, current trades change future positions and trading needs, making liquidity and learning state-dependent and feeding back into current trading and venue-choice incentives. Existing work develops complementary parts of this problem. Rostek, Wu, and Yoon (2026) study a nonrecursive centralized-market equilibrium in which intertemporal liquidity and dynamic learning are jointly determined. Maurin (2022) studies feedback between current and future liquidity in OTC markets, while Duffie, Malamud, and Manso (2014), Li and Song (2021), and Pinter, Wang, and Zou (2022) show how information acquired in one interaction affects subsequent trading. Building directly on the static venue-choice framework developed in this paper, Wu (2026) extends it dynamically by endogenizing asset-value and belief dynamics and allowing

venue choices common to all traders to vary across rounds, while abstracting from intertemporal liquidity feedback to isolate the within-round liquidity and learning forces characterized here. A natural next step is to combine this dynamic venue-choice margin with intertemporal feedback in liquidity, learning, and market composition.

The same intertemporal linkages also make transparency a dynamic market-design variable. Disclosure in either venue changes the information available in later rounds and can therefore affect future liquidity and venue choice. Existing models largely study this channel within a given venue: [Back, Liu, and Teguia \(2020\)](#) study disclosure of earlier OTC transactions, [Cespa and Foucault \(2014\)](#) study selective access to past prices sold by a centralized exchange, and [Ordin and Sverchkov \(2026\)](#) study voluntary disclosure in RFQ trading. A dynamic version of our framework could instead study transparency jointly with endogenous venue choice. Such an extension would introduce additional informational incentives absent from the static model, including a dynamic motive for informed traders to preserve private information. This force may interact with the liquidity and learning incentives characterized here and alter how traders sort across venues over time.

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# A Appendix

## A.1 Proofs

**Corollary 4 (Equilibrium Existence and Uniqueness: Symmetric Traders).** Suppose traders are symmetric (Definition 2). We define  $\bar{\rho} = \frac{1}{I-1} \sum_{j \neq i} \tilde{\rho}_{ij}$  that is common for all traders. A symmetric equilibrium exists and is unique if and only if

$$-\frac{1}{I-1} < \bar{\rho} < \frac{(I-2)^2 - 2(I-1)\sigma^2 + \sqrt{I^2(I-2)^2 + 4(I-1)^2\sigma^4}}{2(I-1)(I-2)}.$$

The *ex ante* equilibrium utility of trader  $i$  is  $E[u^i] = \frac{\mu+2\lambda}{2(\mu+\lambda)^2} c_s^2 \text{Var}[s^i - \bar{s}]$ , when

$$\begin{aligned} c_s &= \frac{1 - \bar{\rho}}{1 + \sigma^2 - \bar{\rho}}, & c_p &= \frac{I\sigma^2\bar{\rho}}{(1 + \sigma^2 - \bar{\rho})(1 + (I-1)\bar{\rho})}, & \text{and} \\ \lambda &= \frac{\mu(1 + \sigma^2 - \bar{\rho})(1 + (I-1)\bar{\rho})}{(I-1)(1 - \bar{\rho})(1 + \sigma^2 + (I-1)\bar{\rho}) - (1 + \sigma^2 - \bar{\rho})(1 + (I-1)\bar{\rho})}. \end{aligned}$$

**Corollary 5 (Equilibrium Existence and Uniqueness: Bilateral Trades).** Suppose two asymmetric traders. An equilibrium exists and is unique if and only if

$$\tilde{\rho}_{ij} = \text{Corr}[\tilde{\theta}^i, \tilde{\theta}^j] < 0.$$

The *ex ante* equilibrium utility of trader  $i$  is  $E[u^i] = \frac{\mu+2\lambda^i}{2(\mu+\lambda^i)^2} (\frac{1-c_p^i}{\mu+\lambda^i} + \frac{1-c_p^j}{\mu+\lambda^j})^{-2} \text{Var}[\frac{(1-c_p^j)c_s^i}{\mu+\lambda^j} s^i - \frac{(1-c_p^i)c_s^j}{\mu+\lambda^j} s^j]$ , when for each  $i = 1, 2$  and  $j \neq i$ ,

$$\begin{aligned} c_s^i &= \frac{(1 - \tilde{\rho}_{ij}) - \frac{\tilde{\rho}_{ij}(1+\tilde{\rho}_{ij})(\sigma_i^2 - \sigma_j^2)}{2(1+\sigma_i^2+\tilde{\rho}_{ij})(1+\sigma_j^2+\tilde{\rho}_{ij})}}{(1 + \sigma_i^2 - \tilde{\rho}_{ij}) - \frac{\tilde{\rho}_{ij}(\sigma_i^2 - \sigma_j^2)}{2(1+\sigma_j^2+\tilde{\rho}_{ij})}}, & c_p^i &= \frac{2\tilde{\rho}_{ij}\sigma_i^2}{(1 + \tilde{\rho}_{ij})((1 + \sigma_i^2 - \tilde{\rho}_{ij}) - \frac{\tilde{\rho}_{ij}(\sigma_i^2 - \sigma_j^2)}{2(1+\sigma_j^2+\tilde{\rho}_{ij})})}, & \text{and} \\ \lambda^i &= \mu \frac{(1 + \tilde{\rho}_{ij})(2(1 + \sigma_i^2 + \tilde{\rho}_{ij})(1 + \sigma_j^2 - \tilde{\rho}_{ij}) + \tilde{\rho}_{ij}(\sigma_i^2 - \sigma_j^2))}{-2\tilde{\rho}_{ij}[2\sigma_1^2\sigma_2^2 + (1 + \tilde{\rho}_{ij})(\sigma_1^2 + \sigma_2^2)]}. \end{aligned}$$

**Proof of Proposition 1 (Liquidity and Learning Effects: Symmetric Market).**

This proof, and all proofs in Appendix, assume  $\mu = 1$  and  $\sigma_\theta^2 = 2$  without loss of generality.

Corollary 4 simplifies the two components in the *ex ante* utility (9) in a closed form:

$$\text{liquidity effect} = 1 - \left(\frac{\lambda}{1 + \lambda}\right)^2 = 1 - \left(\frac{(1 + \sigma^2 - \bar{\rho})(1 + (N-1)\bar{\rho})}{(N-1)(1 - \bar{\rho})(1 + \sigma^2 + (N-1)\bar{\rho})}\right)^2, \quad (15)$$

$$\text{learning effect} = \text{Var}[E[\tilde{\theta}^i | s^i, p] - p] = \frac{N-1}{N} (1 + \sigma^2 - \bar{\rho}) c_s^2 = \frac{N-1}{N} \frac{(1 - \bar{\rho})^2}{1 + \sigma^2 - \bar{\rho}}. \quad (16)$$

Eqs. (15)-(16) allow us to conduct explicit comparative statics of two effects with respect to market size  $N$ , average correlation  $\bar{\rho}$ , and information precision  $\phi = 1/\sigma^2$ .

Differentiating the liquidity effect (15) with respect to  $N$ ,  $\bar{\rho}$ , and  $\sigma^2$  gives

$$\begin{aligned} \text{w.r.t. } N : & 2 \left( \frac{(1 + \sigma^2 - \bar{\rho})^2 (1 + (N - 1)\bar{\rho})}{(N - 1)(1 - \bar{\rho})^2 (1 + \sigma^2 + (N - 1)\bar{\rho})} \right) \frac{\sigma^2 + (1 + (N - 1)\bar{\rho})^2}{(N - 1)^2 (1 + \sigma^2 + (N - 1)\bar{\rho})^2} > 0, \\ \text{w.r.t. } \bar{\rho} : & -2 \left( \frac{(1 + \sigma^2 - \bar{\rho})(1 + (N - 1)\bar{\rho})}{(N - 1)^2 (1 - \bar{\rho})(1 + \sigma^2 + (N - 1)\bar{\rho})} \right) \frac{N\sigma^2 (1 + \sigma^2 + (N - 1)\bar{\rho}^2)}{(1 - \bar{\rho})^2 (1 + \sigma^2 + (N - 1)\bar{\rho})^2} < 0, \\ \text{w.r.t. } \sigma^2 : & -2 \left( \frac{(1 + \sigma^2 - \bar{\rho})(1 + (N - 1)\bar{\rho})^2}{(N - 1)^2 (1 - \bar{\rho})^2 (1 + \sigma^2 + (N - 1)\bar{\rho})} \right) \frac{N\bar{\rho}}{(1 + \sigma^2 + (N - 1)\bar{\rho})^2} > 0 \text{ iff } \bar{\rho} < 0. \end{aligned}$$

Similarly, differentiating the learning effect (16) with respect to  $N$ ,  $\bar{\rho}$ , and  $\sigma^2$  gives

$$\begin{aligned} \text{w.r.t. } N : & \frac{1}{N^2} \frac{(1 - \bar{\rho})^2}{1 + \sigma^2 - \bar{\rho}} > 0, \\ \text{w.r.t. } \bar{\rho} : & -\frac{N - 1}{N} \frac{(1 - \bar{\rho})(1 + 2\sigma^2 - \bar{\rho})}{(1 + \sigma^2 - \bar{\rho})^2} < 0, \\ \text{w.r.t. } \sigma^2 : & -\frac{N - 1}{N} \frac{(1 - \bar{\rho})^2}{(1 + \sigma^2 - \bar{\rho})^2} < 0. \end{aligned}$$

Therefore, the *ex ante* utility (9) increases, with no trade-off between liquidity and learning, as the market size increases (part (i)), as the average correlation decreases (part (ii)), or when  $\bar{\rho} > 0$ , as the precision increases (part (iii));  $\phi^* \rightarrow \infty$  if  $\bar{\rho} > 0$ ).

On the other hand, when  $\bar{\rho} < 0$ , an increase in precision lowers the liquidity effect while it increases the learning effect, and thus, it creates a trade-off. We suppose  $\bar{\rho} < 0$  and find the necessary and sufficient condition under which the trade-off gives a concave *ex ante* utility with respect to  $\phi \in [0, \infty)$ , and hence,  $\phi^* < \infty$  (part (iii)).

$$\frac{\partial E[u^i]}{\partial \sigma^2} = - \left( 1 - \frac{(1 + \sigma^2 - \bar{\rho})^2 (1 + (N - 1)\bar{\rho})^2 (1 + \sigma^2 - (N + 1)\bar{\rho})}{(N - 1)^2 (1 - \bar{\rho})^2 (1 + \sigma^2 + (N - 1)\bar{\rho})^3} \right) \frac{N - 1}{N} \frac{(1 - \bar{\rho})^2}{(1 + \sigma^2 - \bar{\rho})^2}.$$

The partial implies that, given  $(N, \bar{\rho})$ ,  $\frac{\partial E[u^i]}{\partial \sigma^2} < 0$  if and only if the inverse-precision  $\sigma^2$  satisfies the following inequality:

$$\frac{(N - 1)^2 (1 - \bar{\rho})^2}{(1 + (N - 1)\bar{\rho})^2} > \frac{(1 + \sigma^2 - \bar{\rho})^2 (1 + \sigma^2 - (N + 1)\bar{\rho})}{(1 + \sigma^2 + (N - 1)\bar{\rho})^3}. \quad (17)$$

The right hand side of Eq. (17) is decreasing as  $\sigma^2$  increases, because

$$\frac{\partial(\text{the r.h.s. of (17)})}{\partial \sigma^2} = \frac{2N(1 + \sigma^2 - \bar{\rho})\bar{\rho}(2(1 + \sigma^2) - (N + 2)\bar{\rho})}{(1 + \sigma^2 + (N - 1)\bar{\rho})^4} < 0,$$

Also, inequality (17) holds when  $\sigma^2 \rightarrow \infty$ . Therefore,  $\frac{\partial E[u^i]}{\partial \sigma^2} < 0$  for any  $\sigma^2 > \bar{\sigma}^2$  and for some  $\bar{\sigma}^2 \in [0, \infty)$ . The cutoff  $\bar{\sigma}^2$  is strictly positive if and only if Eq. (17) is violated at  $\sigma^2 = 0$ , or equivalently, if and only if other characteristics  $(N, \bar{\rho})$  satisfy

$$-\frac{1}{N - 1} < \bar{\rho} < -\frac{N - 2}{N^2 - 3N + 4} < 0. \quad (18)$$

The first inequality is from the positive definiteness of  $\text{Var}[(\tilde{\theta}^i)_i]$ . Condition (18) induces a non-empty interval for  $\bar{\rho}$  for any  $N \geq 2$ . Defining  $\phi^* = 1/\bar{\sigma}^2$ , we conclude that  $\frac{\partial E[u^i]}{\partial \phi} > 0$  for all  $\phi < \phi^*$  and  $\frac{\partial E[u^i]}{\partial \phi} < 0$  for all  $\phi > \phi^*$ , and that  $\phi^* < \infty$  if and only if  $(N, \bar{\rho})$  satisfies condition (18). The proof is complete. ■

**Proof of Proposition 2 (Pairwise-Stable OTC Matching).** By Proposition 1 (ii), the *ex ante* utility of trader  $i$  from a bilateral OTC trade is strictly decreasing in the correlation with his counterparty:  $\frac{\partial E[u^i]}{\partial \rho_{ij}} < 0$  for any  $\phi$ . Hence, for any two counterparties  $j$  and  $k$ ,  $\tilde{\rho}_{ij} < \tilde{\rho}_{ik} < 0$  if and only if  $E[u_{otc}^i(\tau^i = j)] > E[u_{otc}^i(\tau^i = k)]$ . We now show that an OTC matching is pairwise stable if and only if it can be generated by the sequential procedure proposed in the proposition. Given its equivalence with sequentially constructed matching, the uniqueness of pairwise-stable matching, when  $\tilde{\rho}_{ij} \neq \tilde{\rho}_{ik}$  for all distinct traders  $i, j, k$ , is immediate.

(If) The proof is by induction on the number of traders. If no negatively correlated pair exists, the empty matching is pairwise stable. Otherwise, let  $(i_1, j_1)$  be a pair with the lowest negative correlation:  $\tilde{\rho}_{i_1 j_1} = \min_{\{(i,j): \tilde{\rho}_{ij} < 0\}} \tilde{\rho}_{ij}$ . Let  $\mathcal{M}'$  be a matching generated by the same procedure after removing  $i_1$  and  $j_1$ . By the induction hypothesis,  $\mathcal{M}'$  is pairwise stable among the remaining traders. Consider  $\mathcal{M} = \mathcal{M}' \cup \{(i_1, j_1)\}$ . No two traders within  $\mathcal{M}'$  can both become strictly better off by forming another pair, by the pairwise stability of  $\mathcal{M}'$ . Moreover, neither  $i_1$  nor  $j_1$  strictly prefers another trader to the assigned counterparty, because  $\tilde{\rho}_{i_1 j_1} \leq \tilde{\rho}_{i_1 k}$  and  $\tilde{\rho}_{i_1 j_1} \leq \tilde{\rho}_{j_1 k}$  for all  $k \neq i_1, j_1$ . Hence,  $\mathcal{M}$  is pairwise stable.

(Only if) Let  $\mathcal{M}$  be any pairwise-stable OTC matching. If no negatively correlated pair exists, the claim is immediate. Otherwise, let  $\underline{\rho} = \min_{\{(i,j): \tilde{\rho}_{ij} < 0\}} \tilde{\rho}_{ij}$ . The matching  $\mathcal{M}$  must contain a pair with correlation  $\underline{\rho}$ . If  $\mathcal{M}$  contains no such pair, then traders  $i$  and  $j$  such that  $\tilde{\rho}_{ij} = \underline{\rho}$  are either unmatched or matched at a strictly higher correlation. They both are strictly better off by forming the pair  $(i, j)$ , contradicting pairwise stability. Remove a pair in  $\mathcal{M}$  attaining  $\underline{\rho}$ . The subset of  $\mathcal{M}$ , the remaining traders, is again pairwise stable; otherwise, a profitable deviation among the remaining traders would contradict the stability of  $\mathcal{M}$ . Repeating this argument shows that the sequential procedure generates  $\mathcal{M}$  under an appropriate tie-breaking rule.

(Circular strict preferences cannot arise) Suppose that there exist three distinct traders  $i, j$ , and  $k$  whose counterparty rankings are circular:  $i$  prefers  $j$  to  $k$ ,  $j$  prefers  $k$  to  $i$ , and  $k$  prefers  $i$  to  $j$ . By Proposition 1 (ii), this implies that  $\tilde{\rho}_{ij} \leq \tilde{\rho}_{ik} < 0$ ,  $\tilde{\rho}_{jk} \leq \tilde{\rho}_{ji} < 0$ , and  $\tilde{\rho}_{ki} \leq \tilde{\rho}_{kj} < 0$ . Hence, all correlations among these three traders must be the same:  $\tilde{\rho}_{ij} = \tilde{\rho}_{jk} = \tilde{\rho}_{ki}$ . The traders are indifferent between the two other traders in their counterparty choices.

(Stable OTC matching under cross-pair heterogeneity and cross-trader homogeneity) Sup-

pose that all rows of  $\Sigma_\theta$  are permutations of each other. We show that while this cross-trader symmetry still allows cross-pair heterogeneity, all matched OTC pairs have the common minimum correlation:  $\tilde{\rho}_{ij} = \min_{k \neq i} \tilde{\rho}_{ik} = \rho_{\min}$ . The proof is by contradiction. Suppose there is an OTC pair  $(i, j)$  such that  $\tilde{\rho}_{ij} \gtrsim \rho_{\min}$ . Since the correlation profiles are the same for all traders, there exists another trader  $k \neq i, j$  such that  $\tilde{\rho}_{ik} = \rho_{\min}$ . If trader  $k$ 's current counterparty does not have the minimum correlation, traders  $i$  and  $k$  have a strict incentive to deviate from their current pairs, so that the given OTC matching is not pairwise stable. Hence, trader  $k$ 's current pair  $(k, l)$  must satisfy  $\tilde{\rho}_{kl} = \rho_{\min}$ . Therefore, trader  $k$  has at least two other traders  $i$  and  $l$  with the minimum correlation  $\rho_{\min}$ . By the symmetry, the correlation profile of trader  $i$ ,  $\{\tilde{\rho}_{im}\}_{m \neq i}$ , contains two or more  $\rho_{\min}$ , in that there exists another trader  $m \neq i, j, k$  such that  $\tilde{\rho}_{im} = \rho_{\min}$ . Repeating the same argument, the current matched counterparty for trader  $m$  must have the minimum correlation, which implies that there are three or more  $\rho_{\min}$  in the correlation profile  $\{\tilde{\rho}_{im}\}_{m \neq i}$ . Eventually, all pairs of traders in the OTC market have the same correlation  $\tilde{\rho}_{ij} = \rho_{\min}$ . This contradicts the cross-pair heterogeneity:  $\tilde{\rho}_{ij} \gtrsim \rho_{\min}$  for some  $i$  and  $j \neq i$ . ■

**Proof of Theorem 1 (When the OTC Market Opens?).** The market structure with the centralized market alone is an equilibrium if and only if the last trader prefers the centralized market, assuming all  $I - 1$  other traders enter the centralized market. This is equivalent to

$$E[u_{cm}^i(I)] \geq \max_{j \neq i} E[u_{otc}^i(\tau^i = j)], \quad \forall i. \quad (19)$$

If condition (19) is violated for a trader  $i$ , his best counterparty  $j$  also prefers the OTC market, given their symmetry  $E[u_{cm}^j(I)] = E[u_{cm}^i(I)]$  and  $E[u_{otc}^j(\tau^j = i)] = E[u_{otc}^i(\tau^i = j)]$ , and hence, a pairwise deviation occurs. From Proposition 1 (ii),  $E[u_{otc}^i(\tau^i = j)]$  is maximized at a counterparty  $j$  whose asset value is most negatively correlated with trader  $i$ 's value,  $\tilde{\rho}_{ij} = \min_{k \neq i} \tilde{\rho}_{ik}$ . If the minimum correlation is nonnegative,  $E[u_{otc}^i(\tau^i = k)] = 0$  for all  $k \neq i$  so that Eq. (19) immediately holds.

Now, suppose  $\tilde{\rho}_{ij} = \min_{k \neq i} \tilde{\rho}_{ik} < 0$ . Under the symmetry assumption, the expected utilities are characterized in a closed form (Corollaries 4 and 5).

$$E[u_{cm}^i(I)] = \frac{(I-1)^2(1-\bar{\rho})^2(1+\sigma^2 + (I-1)\bar{\rho})^2 - (1+\sigma^2 - \bar{\rho})^2(1+(I-1)\bar{\rho})^2}{I(I-1)(1+\sigma^2 + (I-1)\bar{\rho})^2(1+\sigma^2 - \bar{\rho})},$$

$$E[u_{otc}^i(\tau^i = j)] = \frac{-2\sigma^2\tilde{\rho}_{ij}(1+\sigma^2 - \tilde{\rho}_{ij}^2)}{(1+\sigma^2 + \tilde{\rho}_{ij})^2(1+\sigma^2 - \tilde{\rho}_{ij})}.$$

Comparing the equilibrium utility in two markets and using the positive definiteness of  $\Sigma_\theta$ ,  $E[u_{cm}^i(I)] \geq E[u_{otc}^i(\tau^i = j)]$  holds if and only if the following inequality holds:

$$f(\sigma^2; I, \bar{\rho}, \tilde{\rho}_{ij}) \geq 0, \quad (20)$$

when  $f(\sigma^2)$  is a fifth-order polynomial,

$$\begin{aligned} f(\sigma^2; I, \bar{\rho}, \tilde{\rho}_{ij}) &= ((I-2)(1+\sigma^2 + (I-2)\bar{\rho} - (I-1)\bar{\rho}^2) - 2(I-1)\sigma^2\bar{\rho}) \\ &\quad \times (1+\sigma^2 + (I-2)\bar{\rho} - (I-1)\bar{\rho}^2)(1+\sigma^2 + \tilde{\rho}_{ij})^2(1+\sigma^2 - \tilde{\rho}_{ij}) \\ &\quad + (I-1)(1+\sigma^2 + (I-1)\bar{\rho})^2(1+\sigma^2 - \bar{\rho})2\sigma^2\tilde{\rho}_{ij}(1+\sigma^2 - \tilde{\rho}_{ij}^2). \end{aligned} \quad (21)$$

To solve the inequality (20), let us denote the polynomial function (21) by  $f(\sigma^2; I, \bar{\rho}, \tilde{\rho}_{ij}) = \sum_{k=0}^5 a_k(\sigma^2)^k$ . Lemma 1 shows a property of the coefficients  $(a_0, a_1, a_2, a_3, a_4, a_5)$ , that is useful for determining the number of positive solutions for  $f(\sigma^2) = 0$ .

**Lemma 1.** *Suppose  $I \geq 3$  and  $-1 < \tilde{\rho}_{ij} < 0 \leq \bar{\rho} < 1$ . If  $a_n \geq 0$ , then  $a_{n-1} > 0$ , for all  $n = 1, \dots, 5$ . Moreover,  $a_0 > 0$ .*

**Proof.** See Appendix A.3 for this technical proof. ■

Suppose  $-1 < \tilde{\rho}_{ij} < 0 \leq \bar{\rho} < 1$ . From Lemma 1, if the highest-power coefficient  $a_5 = (I-2-2(I-1)(\bar{\rho}-\tilde{\rho}_{ij}))$  is nonnegative, all other coefficients  $a_k, k = 0, \dots, 4$ , are also positive and do not change their signs. Hence, by the Descartes' Sign Rule, there is no positive solution  $\sigma^2 > 0$  for  $f(\sigma^2) = 0$ , i.e.,  $f(\sigma^2) > 0$  holds for all  $\sigma^2 > 0$ . By Eq. (20),  $E[u_{cm}^i(I)] > E[u_{otc}^i(\tau^i = j)]$  holds for any  $\phi = 1/\sigma^2 > 0$ . On the other hand, if  $a_5 = (I-2-2(I-1)(\bar{\rho}-\tilde{\rho}_{ij})) < 0$ , Lemma 1 shows that there exists only one sign change in the sequence of coefficients. By the Descartes' Sign Rule, there exists a unique positive solution  $\hat{\sigma}^2(I, \tilde{\rho}_{ij}, \bar{\rho}) > 0$  such that  $f(\hat{\sigma}^2) = 0$ , and consequently,  $E[u_{cm}^i(I)] \geq E[u_{otc}^i(\tau^i = j)]$  holds if and only if  $\sigma^2 \leq \hat{\sigma}^2(I, \tilde{\rho}_{ij}, \bar{\rho})$ . When  $\bar{\rho} = 1$  or  $\rho_{ij} = -1$ , the definition of  $\{a_k\}_k$  in the proof of Lemma 1 shows that  $a_5, a_4, a_3, a_2 < 0$  and  $a_1 = a_0 = 0$ . Then,  $f(\sigma^2) < 0$  for all  $\sigma^2 > 0$ , and thus,  $E[u_{cm}^i(I)] < E[u_{otc}^i(\tau^i = j)]$  holds for all  $\sigma^2 > 0$ .

The necessary and sufficient condition for  $E[u_{cm}^i(I)] < E[u_{otc}^i(\tau^i = j)]$  is therefore

$$\frac{I-2}{2(I-1)} + \tilde{\rho}_{ij} < \bar{\rho} \quad \text{and} \quad \sigma^2 > \hat{\sigma}^2(I, \tilde{\rho}_{ij}, \bar{\rho}), \quad (22)$$

when the cutoff  $\hat{\sigma}^2(I, \tilde{\rho}_{ij}, \bar{\rho}) \geq 0$  satisfies the equality if and only if  $\tilde{\rho}_{ij} = -1$  or  $\bar{\rho} = 1$ ; and  $\hat{\sigma}^2(I, \tilde{\rho}_{ij}, \bar{\rho}) = +\infty$  if and only if  $\frac{I-2}{2(I-1)} + \tilde{\rho}_{ij} \geq \bar{\rho}$  or  $\tilde{\rho}_{ij} \geq 0$ . We remark that the derivation of condition (22) treats  $I, \tilde{\rho}_{ij}, \bar{\rho}$  as independent variables. Consequently, the same condition (22) applies to *any marginal trader*. In the rest of the proof, we translate the condition (22) into two cases: (i) a marginal trader after  $N_{cm} - 1$  other traders have entered the centralized market and (ii) the last trader when all other traders are in the centralized market.

(Part (i): A marginal trader facing the centralized-market size  $N_{cm} \leq I$ ) When a centralized market contains  $N_{cm} - 1 < I$  other traders, we suppose that the post-entry centralized

market is symmetric and let

$$\bar{\rho}_{cm} = \frac{1}{N_{cm} - 1} \sum_{k \neq i; m^k = CM} \tilde{\rho}_{ik} = \omega_{cv}^2 + \omega_{iv}^2 \frac{1}{N_{cm} - 1} \sum_{k \neq i; m^k = CM} \rho_{ik}.$$

When  $N_{cm} = I$ , we get  $\bar{\rho}_{cm} = \omega_{cv}^2$ ; however, whenever  $N_{cm} < I$ ,  $\bar{\rho}_{cm} \neq \omega_{cv}^2$ . Provided that  $0 \leq \bar{\rho}_{cm} < 1$ , the marginal trader prefers the OTC pair over the centralized market if and only if condition (22), replacing  $I$  by  $N_{cm}$  and  $\bar{\rho}$  by  $\bar{\rho}_{cm}$ , holds:

$$\bar{\rho}_{cm} > \frac{N_{cm} - 2}{2(N_{cm} - 1)} + \tilde{\rho}_{ij} \quad \text{and} \quad \sigma^2 > \hat{\sigma}^2(N_{cm}, \tilde{\rho}_{ij}, \bar{\rho}_{cm}), \quad (23)$$

where the cutoff  $\hat{\sigma}^2(N_{cm}, \tilde{\rho}_{ij}, \bar{\rho}_{cm})$  is the unique nonnegative solution of Eq. (21) when  $(\bar{\rho}_{cm}, \tilde{\rho}_{ij})$  satisfies the former correlation condition.

(Part (ii): *The last trader's entry, when the OTC market opens*) When  $N_{cm} = I$ , and hence,  $\bar{\rho}_{cm} = \omega_{cv}^2$  and  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{ij}$ , condition (22) is equivalently rewritten as

$$\omega_{iv}^2 > -\frac{I - 2}{2(I - 1)} \frac{1}{\rho_{ij}} \quad \text{and} \quad \phi = \frac{1}{\sigma^2} < \frac{1}{\hat{\sigma}^2(I, \tilde{\rho}_{ij}, \bar{\rho})} = \hat{\phi}(\omega_{iv}^2, \Sigma).$$

The cutoff is  $\hat{\phi}(\omega_{iv}^2, \Sigma) = \infty$  if and only if  $\omega_{iv}^2 = 1$  and  $\rho_{ij} = -1$  (so that  $\tilde{\rho}_{ij} = -1$ ), whereas  $\hat{\phi}(\omega_{iv}^2, \Sigma) = 0$  if and only if  $\omega_{iv}^2 \leq -\frac{I-2}{2(I-1)} \frac{1}{\rho_{ij}}$  (so that  $\bar{\rho} \leq \frac{I-2}{2(I-1)} + \tilde{\rho}_{ij}$ ) or  $\omega_{iv}^2 \leq \frac{1}{1-\rho_{ij}}$  (so that  $\tilde{\rho}_{ij} \geq 0$ ). Otherwise,  $\hat{\phi}(\omega_{iv}^2, \Sigma) \in (0, \infty)$ . Let us define

$$\hat{\kappa}(I, \Sigma) = \max\left\{-\frac{I - 2}{2(I - 1)} \frac{1}{\rho_{ij}}, \frac{1}{1 - \rho_{ij}}\right\} > 0, \quad (24)$$

so that  $\hat{\phi}(\omega_{iv}^2, \Sigma) > 0$  whenever  $\omega_{iv}^2 > \hat{\kappa}(I, \Sigma)$ . The cutoff  $\hat{\kappa}(I, \Sigma)$  is smaller than one if and only if  $\rho_{ij} < -\frac{I-2}{2(I-1)}$ . We note that when  $N_{cm} < I$ , the cutoff  $\hat{\kappa}(N_{cm}, \Sigma)$  on  $\omega_{iv}^2$  depends on the entire profile of correlations  $\{\rho_{ik}\}_{m^k=CM}$ , not only on the centralized-market size  $N_{cm}$  and the OTC-pair correlation  $\rho_{ij}$ . The proof is complete. ■

### Proof of Corollary 1 (OTC Trading with a Competitive Centralized Market).

Suppose  $N_{cm} \rightarrow \infty$ . From Theorem 1,  $E[u_{cm}^i] \geq E[u_{otc}^i(\tau^i = j)]$  holds in the limit if and only if the competitive-market counterpart of Eq. (20) holds. Specifically, the leading-order terms in Eq. (20) are of order  $O(N_{cm}^3)$ . Dividing all coefficients by  $N_{cm}^3$  and taking  $N_{cm} \rightarrow \infty$  yields the following condition:

$$0 \leq ((1 - \bar{\rho}_{cm})^2 + 2\tilde{\rho}_{ij})\sigma^6 + ((1 - \bar{\rho}_{cm})^2(3 + \tilde{\rho}_{ij}) + (2 - \bar{\rho}_{cm} - \tilde{\rho}_{ij}^2)2\tilde{\rho}_{ij})\sigma^4 + ((1 - \bar{\rho}_{cm})^2(3 + 2\tilde{\rho}_{ij} - \tilde{\rho}_{ij}^2) + (1 - \bar{\rho}_{cm})(1 - \tilde{\rho}_{ij}^2)2\tilde{\rho}_{ij})\sigma^2 + (1 - \bar{\rho}_{cm})^2(1 - \tilde{\rho}_{ij})(1 + \tilde{\rho}_{ij})^2. \quad (25)$$

Because Lemma 1 applies to the limit as  $N_{cm} \rightarrow \infty$ , we obtain that if  $((1 - \bar{\rho}_{cm})^2 + 2\tilde{\rho}_{ij}) \geq 0$  then all other coefficients of the r.h.s. of Eq. (25), as a polynomial function of  $\sigma^2$ , are positive. Then, the inequality (25) holds for any  $\sigma^2 > 0$ . If  $((1 - \bar{\rho}_{cm})^2 + 2\tilde{\rho}_{ij}) < 0$  and

$\tilde{\rho}_{ij} = -1$ , Eq. (25) is simplified to

$$0 \leq -(2 - (1 - \bar{\rho}_{cm})^2)\sigma^6 - 2(1 - \bar{\rho}_{cm})\bar{\rho}_{cm}\sigma^4,$$

which fails to hold for all  $0 \leq \bar{\rho}_{cm} \leq 1$  and  $\sigma^2 > 0$ . If  $((1 - \bar{\rho}_{cm})^2 + 2\tilde{\rho}_{ij}) < 0$  and  $\bar{\rho}_{cm} = 1$ , Eq. (25) is simplified to

$$0 \leq 2\tilde{\rho}_{ij}\sigma^6 + (1 - \tilde{\rho}_{ij}^2)2\tilde{\rho}_{ij}\sigma^4$$

which fails to hold for all  $-1 \leq \tilde{\rho}_{ij} < 0$  and  $\sigma^2 > 0$ . If  $((1 - \bar{\rho}_{cm})^2 + 2\tilde{\rho}_{ij}) < 0$ ,  $\tilde{\rho}_{ij} > -1$ , and  $\bar{\rho}_{cm} < 1$ , by Lemma 1, there is one sign change in the r.h.s of Eq. (25) given that the constant coefficient is positive. Hence, there exists a unique positive solution  $\hat{\sigma}_\infty^2 > 0$  such that the equality of (25) holds. We therefore conclude that the marginal trader prefers the OTC market to the centralized market if and only if

$$(1 - \bar{\rho}_{cm})^2 + 2\tilde{\rho}_{ij} < 0 \quad \text{and} \quad \sigma^2 > \hat{\sigma}_\infty^2 > 0. \quad (26)$$

where  $\hat{\sigma}_\infty^2 = 0$  if and only if  $\tilde{\rho}_{ij} = -1$  or  $\bar{\rho}_{cm} = 0$ . When  $N_{cm} = I$ , and thus, when  $\bar{\rho}_{cm} = \omega_{cv}^2$ , and  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}$ , this condition simplifies as in the corollary. ■

**Proof of Corollary 2 (Price Difference across OTC and Centralized Markets).**

Let  $p_{otc}$  and  $p_{cm}$  be the equilibrium prices (8) in the OTC and centralized markets, respectively.  $E[p_{otc}] = E[p_{cm}] = E[\tilde{\theta}]$  always holds. Corollaries 4 and 5 characterize these prices (8) in closed form and derive the price variances as follows:

$$Var[p_{otc}] = \frac{(1 + \tilde{\rho}_{ij})^2}{2(1 + \sigma^2 + \tilde{\rho}_{ij})} \quad \text{and} \quad Var[p_{cm}] = \frac{(1 + (N_{cm} - 1)\bar{\rho}_{cm})^2}{N_{cm}(1 + \sigma^2 + (N_{cm} - 1)\bar{\rho}_{cm})}.$$

Differentiating  $Var[p_{cm}]$  with respect to  $\bar{\rho}_{cm}$  shows that  $Var[p_{cm}]$  is increasing in  $\bar{\rho}_{cm}$ ,

$$\frac{\partial Var[p_{cm}]}{\partial \bar{\rho}_{cm}} = \frac{(N_{cm} - 1)(1 + (N_{cm} - 1)\bar{\rho}_{cm})(\sigma^2 + (1 + \sigma^2 + (N_{cm} - 1)\bar{\rho}_{cm}))}{N_{cm}(1 + \sigma^2 + (N_{cm} - 1)\bar{\rho}_{cm})^2} > 0.$$

Given this monotonicity, we now show that  $Var[p_{otc}] > Var[p_{cm}]$  if and only if

$$\bar{\rho}_{cm} < \rho_V^{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}), \quad (27)$$

where  $\rho_V^{cm}$  is the largest solution of  $Var[p_{otc}] = Var[p_{cm}]$ , a quadratic equation of  $\bar{\rho}_{cm}$ . Because  $Var[p_{cm}] = 0$  at  $\bar{\rho}_{cm} = -\frac{1}{N_{cm}-1}$ , the lower bound induced by the positive definite  $\Sigma_\theta$ , it suffices to show that the cutoff  $\rho_V^{cm}$  is smaller than the upper bound of the equilibrium-existence interval in the centralized market (Corollary 4). Here is its proof: Because  $Var[p_{otc}]$  and  $Var[p_{cm}]$  are increasing in  $\tilde{\rho}_{ij}$  and  $\bar{\rho}_{cm}$ , respectively, the cutoff  $\rho_V^{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$  is increasing in  $\tilde{\rho}_{ij}$ . Therefore,  $\rho_V^{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}) < \rho_V^{cm}(N_{cm}, \sigma^2, 0)$  for all  $\tilde{\rho}_{ij} < 0$  satisfying the equilibrium-existence condition in the OTC market (Corollary 5). Additionally,  $\rho_V^{cm}(N_{cm}, \sigma^2, 0) < \frac{N_{cm}-2}{2(N_{cm}-1)}$  holds. This can be seen by  $Var[p_{cm}] = \frac{N_{cm}}{2(N_{cm}+2\sigma^2)}$

evaluated at  $\bar{\rho}_{cm} = \frac{N_{cm}-2}{2(N_{cm}-1)}$  being strictly larger than  $Var[p_{otc}] = \frac{1}{2(1+\sigma^2)}$  at  $\tilde{\rho}_{ij} = 0$ . From [Corollary 4](#), the upper bound of the existence condition is strictly larger than  $\frac{N_{cm}-2}{2(N_{cm}-1)}$  for any  $N_{cm} > 2$ . This shows that  $\rho_V^{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$  lies in the existence interval.

When  $N_{cm} = I$ , and thus, when  $\bar{\rho}_{cm} = \omega_{cv}^2$ , and  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}$ , condition (27) is equivalently rewritten as a condition on  $(\omega_{iv}^2, \rho_{min}, \sigma^2, I)$ :

$$\begin{aligned} & \sigma^2 [I(2 - \omega_{iv}^2(1 - \rho_{min}))^2 - 2(I - (I - 1)\omega_{iv}^2)^2] \\ & > -\omega_{iv}^2(2 - \omega_{iv}^2(1 - \rho_{min}))(I - (I - 1)\omega_{iv}^2)(I(1 + \rho_{min}) - 2). \end{aligned} \quad (28)$$

We now recall that the last trader prefers the OTC trade over the centralized market if and only if  $\omega_{iv}^2 > \hat{\kappa}(I, \Sigma)$  and  $\sigma^2 > \hat{\sigma}^2(I, \Sigma)$  hold (condition (11); [Theorem 1](#)). Therefore, equilibrium contains the OTC market even though the OTC price has a larger variance if and only if conditions (11) and (28) jointly hold.

Instead of completely solving Eq. (28), we provide its sufficient condition in primitive terms. The r.h.s. of Eq. (28) is nonpositive for all  $\omega_{iv}^2 \in [0, 1]$  if and only if  $I(1 + \rho_{min}) - 2 \geq 0$  or equivalently  $\rho_{min} \geq -1 + 2/I$ . Additionally, if the l.h.s. of Eq. (28) is positive, condition (28) does not bound  $\sigma^2 > 0$ , beyond  $\sigma^2 > \hat{\sigma}^2(I, \Sigma)$  from condition (11). Evaluating this coefficient at  $\omega_{iv}^2 = 1$  gives  $I(1 + \rho_{min})^2 - 2$ , which is positive for any  $\rho_{min} > -1 + \sqrt{2/I}$ , while evaluating at  $\omega_{iv}^2 = 0$  gives  $4I - 2I^2 < 0$  for any  $I$  and  $\rho_{min}$ . Because the coefficient is a quadratic function of  $\omega_{iv}^2$ , when  $\rho_{min} > -1 + \sqrt{2/I}$ , there exists a unique root  $\omega_{iv,V}^2(I, \rho_{min})$  in  $(0, 1)$ . Therefore, whenever  $\rho_{min} > -1 + \sqrt{2/I} > -1 + 2/I$  and  $\omega_{iv}^2 > \omega_{iv,V}^2$ , condition (28) becomes redundant. Combined with condition (11), this completes the construction of the sufficient condition in this corollary. ■

### Proof of Corollary 3 (Necessary Conditions for Coexistence of Two Markets).

The proofs of parts (i) and (ii) are immediate from [Theorem 1](#). This proof focuses on part (iii), by taking  $(N_{cm}, I) \rightarrow \infty$  at a fixed ratio  $\eta = N_{cm}/I \in [0, 1]$ .

We remark that this limit is conceptually different from the one in [Corollary 1](#), where we take  $N_{cm} \rightarrow \infty$  (and thus  $I \rightarrow \infty$  as well) holding *fixed* the relevant correlations  $\tilde{\rho}_{ij}$  and  $\bar{\rho}_{cm}$ . To characterize the equilibrium market structure in the limit, it is necessary to take  $(N_{cm}, I) \rightarrow \infty$  allowing the correlations to vary as functions of  $(N_{cm}, I)$ . Nevertheless, rather than deriving this joint limit independently, we prove this corollary by establishing the equivalence between these two limiting procedures.

Let us denote Eq. (25) by  $f_\infty(\sigma^2; \bar{\rho}_{cm}, \tilde{\rho}_{ij}) \geq 0$ . For any  $\eta \in (0, 1]$ , the finite-market condition (20) can be expressed as

$$f(\sigma^2; I, \bar{\rho}_{cm}, \tilde{\rho}_{ij}) = I^3 \eta^3 f_\infty(\sigma^2; \bar{\rho}_{cm}, \tilde{\rho}_{ij}) + O(I^2).$$

By definition,  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}$  and  $\bar{\rho}_{cm} = \omega_{cv}^2 + O(I^{-1})$ . From Eq. (25), all coefficients

of  $f_\infty(\sigma^2)$  are polynomial functions of  $(\bar{\rho}_{cm}, \tilde{\rho}_{ij})$ , and thus,

$$f_\infty(\sigma^2; \bar{\rho}_{cm}, \tilde{\rho}_{ij}) = f_\infty(\sigma^2; \omega_{cv}^2, \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}) + O(I^{-1}).$$

Consequently, the  $O(I^{-1})$ -dependence of the correlations affects only lower-order terms without altering the leading-order condition:

$$\begin{aligned} f(\sigma^2; I, \bar{\rho}_{cm}, \tilde{\rho}_{ij}) &= I^3 \eta^3 [f_\infty(\sigma^2; \omega_{cv}^2, \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}) + O(I^{-1})] + O(I^2) \\ &= I^3 \eta^3 f_\infty(\sigma^2; \omega_{cv}^2, \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}) + O(I^2). \end{aligned}$$

This shows the equivalence between the limiting procedures in [Corollaries 1](#) and [3](#).

Substituting  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}$  and  $\bar{\rho}_{cm} = \omega_{cv}^2$  into Eq. (25) shows that  $\hat{\sigma}_\infty^2(\omega_{iv}^2, \rho_{min})$  is independent of the fraction  $\eta$  of traders in the centralized market. Hence, previous traders' entry does not affect a subsequent trader's market choice, so the limiting equilibrium consists of a single venue. Under the tie-breaking rule favoring the OTC market at indifference, the equilibrium is unique for all  $(\rho_{min}, \omega_{iv}^2, \sigma^2)$ . It consists only of the OTC market under the same condition as the last trader's OTC-entry condition in [Corollary 1](#). ■

**Proof of Proposition 3 (Entry Cutoff Feedback).** In the setting of [Example 1](#), where  $N_{cm}$  and  $\bar{\rho}_{cm}(N_{cm})$  vary together, we analyze their net effect on the entry cutoff  $\hat{\sigma}^2(N_{cm}) = \hat{\sigma}^2(N_{cm}, \tilde{\rho}_{ij}, \bar{\rho}_{cm}(N_{cm}))$  (Eq. (23)). We omit  $\tilde{\rho}_{ij}$  from the input variables, because it stays the same regardless of traders' market entry ([Proposition 2](#)). With the cross- and within-group idiosyncratic-value correlations  $\rho < 0$  and  $-\frac{I\rho}{I-2} > 0$ ,

$$\bar{\rho}_{cm}(N_{cm}) = 1 - \omega_{iv}^2 + \omega_{iv}^2 \rho \frac{I - N_{cm}}{(N_{cm} - 1)(I - 2)}. \quad (29)$$

(*Global existence of  $\hat{\sigma}^2(N_{cm})$* ) We first show that  $\omega_{iv}^2 > -\frac{I-2}{2(I-1)\rho}$  and  $\rho < -\frac{I-2}{2(I-1)}$  must hold for  $\hat{\sigma}^2(N_{cm})$  to exist for all  $N_{cm} \in [4, I]$ . Recall that the condition on  $\omega_{iv}^2$  in [Theorem 1](#) arises from requiring the leading coefficient  $a_5$  of Eq. (21) to be negative.<sup>37</sup> Replacing  $I$  by  $N_{cm}$  and substituting  $(\tilde{\rho}_{ij}, \bar{\rho}_{cm})$  as functions of  $(\rho, \omega_{iv}^2, I, N_{cm})$  gives us

$$a_5 = (N_{cm} - 2) - 2(N_{cm} - 1)(\bar{\rho}_{cm} - \tilde{\rho}_{ij}) = (N_{cm} - 2)(1 + \frac{2(I-1)}{I-2} \rho \omega_{iv}^2). \quad (30)$$

Therefore,  $a_5 < 0$  if and only if  $\omega_{iv}^2 > -\frac{I-2}{2(I-1)\rho}$ . Since  $\omega_{iv}^2 \leq 1$ , this also requires  $\rho < -\frac{I-2}{2(I-1)}$ . These conditions are independent of  $N_{cm}$ , and under them  $\hat{\sigma}^2(N_{cm})$  exists for all  $N_{cm}$ .

(*Equivalence between comparative statics of  $E[u_{cm}^i(N_{cm})]$  and  $\hat{\sigma}(N_{cm})$* ) When it exists, the cutoff  $\hat{\sigma}^2(N_{cm})$  is the unique positive solution of

$$E[u_{cm}^i(\hat{\sigma}^2; N_{cm})] - E[u_{otc}^i(\hat{\sigma}^2)] = 0. \quad (31)$$

<sup>37</sup>Eq. (21) is derived without requiring  $I = N_{cm}$  or  $\sum_{j \neq i} \rho_{ij} = 0$ , and thus, it can apply to any symmetric centralized market, including the setting of [Example 1](#).

Applying the Implicit Function Theorem to Eq. (31) gives

$$\left( \frac{\partial E[u_{cm}^i]}{\partial \sigma^2} - \frac{\partial E[u_{otc}^i]}{\partial \sigma^2} \right) \frac{d\hat{\sigma}^2}{dN_{cm}} = - \left( \frac{dE[u_{cm}^i]}{dN_{cm}} - \frac{dE[u_{otc}^i]}{dN_{cm}} \right) = - \frac{dE[u_{cm}^i]}{dN_{cm}}.$$

From the proof of [Theorem 1](#), the utility-difference (the l.h.s. of Eq. (31)) crosses zero from above, i.e.,  $\frac{\partial E[u_{cm}^i]}{\partial \sigma^2} - \frac{\partial E[u_{otc}^i]}{\partial \sigma^2} < 0$  at  $\sigma^2 = \hat{\sigma}^2$ . This proves that  $\frac{d\hat{\sigma}^2}{dN_{cm}}$  and  $\frac{dE[u_{cm}^i]}{dN_{cm}}$  have the same signs, and thus, these comparative statics are interchangeable.

Additionally, to decompose the effect of each variable, we apply the Implicit Function Theorem to Eq. (31) with respect to  $N_{cm}$  holding  $\bar{\rho}_{cm}$  fixed or to  $\bar{\rho}_{cm}$  holding  $N_{cm}$  fixed.

$$\begin{aligned} \text{w.r.t. } N_{cm} : \quad & \left( \frac{\partial E[u_{cm}^i]}{\partial \sigma^2} - \frac{\partial E[u_{otc}^i]}{\partial \sigma^2} \right) \frac{\partial \hat{\sigma}^2}{\partial N_{cm}} = - \frac{\partial E[u_{cm}^i]}{\partial N_{cm}}, \\ \text{w.r.t. } \bar{\rho}_{cm} : \quad & \left( \frac{\partial E[u_{cm}^i]}{\partial \sigma^2} - \frac{\partial E[u_{otc}^i]}{\partial \sigma^2} \right) \frac{\partial \hat{\sigma}^2}{\partial \bar{\rho}_{cm}} = - \frac{\partial E[u_{cm}^i]}{\partial \bar{\rho}_{cm}}. \end{aligned}$$

From [Proposition 1](#), the expected utility  $E[u_{cm}^i]$  in the centralized market is increasing in  $N_{cm}$  and decreasing in  $\bar{\rho}_{cm}$ . Consequently,  $\frac{\partial \hat{\sigma}^2}{\partial N_{cm}} > 0$  and  $\frac{\partial \hat{\sigma}^2}{\partial \bar{\rho}_{cm}} < 0$ . Combined with  $\frac{d\bar{\rho}_{cm}}{dN_{cm}}$  obtained from Eq. (29), the net effect on the entry cutoff is decomposed into

$$\frac{d\hat{\sigma}^2}{dN_{cm}} = \underbrace{\frac{\partial \hat{\sigma}^2}{\partial N_{cm}}}_{(+)} + \underbrace{\frac{\partial \hat{\sigma}^2}{\partial \bar{\rho}_{cm}}}_{(-)} \underbrace{\frac{(-\omega_{iv}^2)(I-1)}{(I-2)(N_{cm}-1)^2}}_{(+)}. \quad (32)$$

The remainder of this proof provides sufficient conditions under which either the effect of  $N_{cm}$  or that of  $\bar{\rho}_{cm}$  dominates in the trade-off in Eq. (32).

(Part (i): Sufficient condition that  $\frac{d\hat{\sigma}^2}{dN_{cm}} > 0$  for all  $N_{cm} \leq I$ ) Recall that the cutoff  $\hat{\sigma}^2$  exists only when  $\omega_{iv}^2 > \underline{\omega}_{iv}^2(I, \rho) = -\frac{I-2}{2(I-1)\rho}$  and that  $\hat{\sigma}^2 \rightarrow \infty$  as  $\omega_{iv}^2 \downarrow \underline{\omega}_{iv}^2(I, \rho)$ . We use this limit to construct a desired sufficient condition. Namely, we show that the scaled cutoff  $(\frac{\omega_{iv}^2}{\underline{\omega}_{iv}^2(I, \rho)} - 1)\hat{\sigma}^2$  is strictly increasing in  $N_{cm}$  in the limit of  $\omega_{iv}^2 \downarrow \underline{\omega}_{iv}^2(I, \rho)$  when  $\rho$  is sufficiently low. Then, by continuity, there exists  $\rho_L(I) < 0$  and  $\omega_{iv,L}^2(I, \rho) > 0$  such that if  $\rho < \rho_L(I)$  and  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ , then  $\frac{d\hat{\sigma}^2}{dN_{cm}} > 0$  for all  $N_{cm}$ .

For simplicity, we denote the buyer-seller correlation in the limit by  $\tilde{\rho}_0 = 1 - \underline{\omega}_{iv}^2(I, \rho) + \underline{\omega}_{iv}^2(I, \rho)\rho$ . We also define  $\delta = (\frac{\omega_{iv}^2}{\underline{\omega}_{iv}^2(I, \rho)} - 1) > 0$ , which converges to zero as  $\omega_{iv}^2 \downarrow \underline{\omega}_{iv}^2(I, \rho)$ . From Eq. (30), the leading coefficient  $a_5$  of  $f(\sigma^2)$  converges to zero from below as  $\delta \downarrow 0$ . Also, in that limit, the fourth-order coefficient is, near  $\delta = 0$ ,

$$a_4 = a_4^0 + O(\delta) = \frac{N_{cm} - 2}{4} [(N_{cm} - 1)(1 + 2\tilde{\rho}_0)^3 + \frac{1}{N_{cm} - 1} + 2(1 - \tilde{\rho}_0)] + O(\delta),$$

and the limit  $a_4^0$  is positive. The remaining coefficients  $a_3, \dots, a_0$  are bounded as  $\delta \downarrow 0$ . Since  $\hat{\sigma}^2 \rightarrow +\infty$ , dividing Eq. (21) by  $\hat{\sigma}^8$  gives

$$a_5 \hat{\sigma}^2 + a_4 + \frac{a_3}{\hat{\sigma}^2} + \frac{a_2}{(\hat{\sigma}^2)^2} + \frac{a_1}{(\hat{\sigma}^2)^3} + \frac{a_0}{(\hat{\sigma}^2)^4} = 0,$$

which becomes, as  $\delta \downarrow 0$ ,  $-(N_{cm} - 2)\delta\hat{\sigma}^2 + a_4^0 + O(\delta) = 0$ . Therefore, near  $\delta = 0$ ,

$$\delta\hat{\sigma}^2 = \frac{a_4^0}{(N_{cm} - 2)} + O(\delta) = \frac{1}{4}[(N_{cm} - 1)(1 + 2\tilde{\rho}_0)^3 + \frac{1}{N_{cm} - 1} + 2(1 - \tilde{\rho}_0)] + O(\delta). \quad (33)$$

Differentiating  $\delta\hat{\sigma}^2$  with respect to  $N_{cm}$  gives

$$\frac{\partial(\delta\hat{\sigma}^2)}{\partial N_{cm}} = \frac{1}{4}[(1 + 2\tilde{\rho}_0)^3 - \frac{1}{(N_{cm} - 1)^2}] + O(\delta).$$

Therefore,  $\frac{\partial(\delta\hat{\sigma}^2)}{\partial N_{cm}} > 0$  for all  $N_{cm} \geq 4$  if and only if  $1 + 2\tilde{\rho}_0 > 3^{-2/3}$ , or equivalently,

$$\rho < -\frac{I - 2}{(2I - 1) - 3^{-2/3}(I - 1)} = \rho_\infty(I).$$

The correlation cutoff  $\rho_\infty(I)$  lies in  $(-1, 0)$  for any  $I \geq 4$ . Thus, if  $\rho < \rho_\infty(I)$ , the limit of  $\frac{\partial(\delta\hat{\sigma}^2)}{\partial N_{cm}}$  is uniformly and strictly positive for all  $N_{cm} \in [4, I]$ . Because Eq. (21) is regular at  $\delta = 0$ , by the implicit-function theorem,  $\delta\hat{\sigma}^2$  and  $\partial(\delta\hat{\sigma}^2)/\partial N_{cm}$  are continuous in  $(N_{cm}, \delta)$  near  $\delta = 0$ . The compactness of  $[4, I]$  implies that when  $\delta < \bar{\delta}_L(I, \rho)$ , there exists  $\varepsilon_L(I, \rho) > 0$  such that if  $\underline{\omega}_{iv}^2(I, \rho) < \omega_{iv}^2 < \bar{\omega}_{iv}^2(I, \rho) + \varepsilon_L(I, \rho)$  then  $\frac{\partial\hat{\sigma}^2}{\partial N_{cm}} > 0$  for all  $N_{cm} \in [4, I]$ .

(Part (ii): *Sufficient condition that  $\frac{d\hat{\sigma}^2}{dN_{cm}} < 0$  for all  $N_{cm} \leq I$* ) We consider the opposite boundary  $\omega_{iv}^2 = 1$  to construct a desired sufficient condition. The continuity argument, as in part (i), extends the constructed condition at  $\omega_{iv}^2 = 1$  to a neighborhood  $\omega_{iv}^2 > \omega_{iv,H}^2(I, \rho)$  for some  $\omega_{iv,H}^2(I, \rho) < 1$ . At  $\omega_{iv}^2 = 1$ , we have  $\tilde{\rho}_{ij} = \rho$  and  $\bar{\rho}_{cm}(N_{cm}) = \rho \frac{I - N_{cm}}{(N_{cm} - 1)(I - 2)}$ . Let  $\hat{\sigma}_1^2(N_{cm}, \rho)$  denote the cutoff evaluated at  $\omega_{iv}^2 = 1$  (Eq. (21)). Given the equivalent comparative statics of  $E[u_{cm}^i]$  and  $\hat{\sigma}(N_{cm})$ , we show that for sufficiently negative  $\rho < \rho_H(I) < 0$ ,

$$\frac{dE[u_{cm}^i]}{dN_{cm}} < 0 \quad (34)$$

holds at the cutoff  $\sigma^2 = \hat{\sigma}_1^2(N_{cm}, \rho)$  for all  $N_{cm} \in [4, I]$ .

**Corollary 4** characterizes  $E[u_{cm}^i]$  in a closed form. In particular, at the lowest feasible cross-group correlation  $\rho = -\frac{I-2}{I}$ , the average correlation is  $\bar{\rho}_{cm} = -\frac{I - N_{cm}}{I(N_{cm} - 1)}$ . We evaluate  $E[u_{cm}^i]$  at this correlation, to take into account  $\bar{\rho}_{cm}$  varying with  $N_{cm}$ :

$$E[u_{cm}^i] = \frac{N_{cm} - 1}{N_{cm}} \frac{(\frac{N_{cm}(I-1)}{I(N_{cm}-1)})^2}{\sigma^2 + \frac{N_{cm}(I-1)}{I(N_{cm}-1)}} \left( 1 - \frac{(\sigma^2 + \frac{N_{cm}(I-1)}{I(N_{cm}-1)})^2}{(I-1)^2(\sigma^2 + \frac{N_{cm}}{I})^2} \right). \quad (35)$$

Differentiating Eq. (35) with respect to  $N_{cm}$  and multiplying by its positive denominator shows that  $\frac{dE[u_{cm}^i]}{dN_{cm}} < 0$  if and only if  $P(\sigma^2) < 0$ , when  $P(\sigma^2)$  is defined as

$$\begin{aligned}
P(\sigma^2) = & -I^5(I-2)(N_{cm}-1)^3(\sigma^2)^4 \\
& -I^3N_{cm}(N_{cm}-1)^2(3I^2N_{cm}-3I^2-6IN_{cm}+2I-2N_{cm}^2+6N_{cm})(\sigma^2)^3 \\
& -I^2(I-1)N_{cm}^2(N_{cm}-1)(3IN_{cm}^2-6IN_{cm}-2I-9N_{cm}^2+14N_{cm})(\sigma^2)^2 \\
& +I(I-1)^2N_{cm}^3(2I-N_{cm}^3+9N_{cm}^2-10N_{cm})\sigma^2 \\
& +2(I-1)^3N_{cm}^5.
\end{aligned} \tag{36}$$

For  $I \geq N_{cm} \geq 4$ , the cubic coefficient is strictly negative, whereas the quadratic coefficient is nonpositive, zero only if  $I = N_{cm} = 4$ . Since the coefficients of  $P(\sigma^2)$  have signs either  $(-, -, -, +, +)$  or  $(-, -, -, -, +)$ , by the Descartes' rule of signs,  $P(\sigma^2) = 0$  has a unique positive solution  $\sigma_D^2(I, N_{cm})$ . Also,  $P(\sigma^2)$  crosses zero from above, given its negative leading coefficient and positive  $P(0)$ . Therefore,  $\frac{dE[u_{cm}^i]}{dN_{cm}} < 0$  if and only if  $\sigma^2 > \sigma_D^2(I, N_{cm})$ .

To prove Eq. (34) at the minimum correlation  $\rho = -\frac{I-2}{I}$ , it suffices to show that

$$\hat{\sigma}_1^2(N_{cm}, -\frac{I-2}{I}) > \sigma_D^2(I, N_{cm}) \quad \forall 4 \leq N_{cm} \leq I. \tag{37}$$

The following lemma provides a sufficient condition for Eq. (37).

**Lemma 2.**  $\hat{\sigma}_1^2(N_{cm}, -\frac{I-2}{I}) > \sigma_D^2(I, N_{cm})$  holds for any  $I \leq \hat{I} = 24$  and  $N_{cm} \in [4, I]$ .<sup>38</sup>

**Proof.** See Appendix A.3 for this technical proof. ■

From Lemma 2,  $\frac{dE[u_{cm}^i]}{dN_{cm}} < 0$  holds at  $\sigma^2 = \hat{\sigma}_1^2(N_{cm}, \rho)$  for all  $4 \leq N_{cm} \leq I \leq 24$ , when  $\rho = -\frac{I-2}{I}$  and  $\omega_{iv}^2 = 1$ . Since the expected utility  $E[u_{cm}^i]$  and the entry cutoff  $\hat{\sigma}_1^2(N_{cm}, \rho)$  are continuous in  $\rho$ , there exists  $\rho_H(I) > -\frac{I-2}{I}$  such that  $\frac{dE[u_{cm}^i]}{dN_{cm}} < 0$  holds for all  $4 \leq N_{cm} \leq I \leq 24$ , whenever  $-\frac{I-2}{I} < \rho < \rho_H(I)$ . Furthermore, by the continuity with respect to  $\omega_{iv}^2$ ,  $\frac{dE[u_{cm}^i]}{dN_{cm}} < 0$  continues to hold in a neighborhood of  $\omega_{iv}^2 = 1$ , which is denoted by  $(\omega_{iv,H}^2(I, \rho), 1)$ . This completes the proof of part (ii).

Given  $\rho_\infty(I)$  defined in the proof of part (i) and  $\rho_H(I)$  defined in the proof of part (ii), we set  $\rho_L(I) \leq \min\{\rho_\infty(I), \rho_H(I)\}$  and complete the proof of the proposition. ■

**Proof of Theorem 2 (Endogenous Market Structure).** Equilibrium market structure, which is captured by  $N_{cm}$  in the setting of Example 1, requires three conditions (Definition 1): (1) a pairwise deviation of an OTC pair to the centralized market is not profitable, i.e.,  $E[u_{otc}^i] \geq E[u_{cm}^i(N_{cm} + 2)]$ ; (2) a pairwise deviation from the centralized market to the OTC market is not profitable, i.e.,  $E[u_{cm}^i(N_{cm})] > E[u_{otc}^i]$ ; and (3) an individual deviation from the OTC market to the centralized market is not profitable, i.e.,

<sup>38</sup>Numerical calculations suggest that the bound in the lemma is considerably conservative. We numerically find that Eq. (37) holds for all  $N_{cm} \in [4, I]$  and all  $I \leq 1205$ , while it fails for some  $N_{cm}$  when  $I > 1205$ . This lemma analytically constructs a sufficient but not necessary bound  $\hat{I} = 24$ .

$E[u_{otc}^i] \geq E[u_{cm}^i(N_{cm} + 1)]$ . We adopt the tie-breaking rule that a trader who is indifferent between the two venues chooses the OTC market. An individual deviation from the centralized market to the OTC market is never profitable because the deviating trader cannot form an OTC pair without another trader's joint deviation.

Conditional on the existence of a pure-strategy equilibrium, this theorem constructs market structures from conditions (1)-(2) for any equilibrium that exists. Condition (3) may eliminate a candidate market structure and prevent equilibrium existence, but it cannot restore a market structure ruled out by conditions (1)-(2).

**Theorem 1** and **Proposition 3** provide the individual entry cutoff  $\hat{\sigma}^2(N_{cm}, \tilde{\rho}_{ij}, \bar{\rho}_{cm})$  and its comparative statics as  $(N_{cm}, \bar{\rho}_{cm})$  jointly vary. The pairwise-stable OTC pairs have the common correlation  $\tilde{\rho}_{ij} = \omega_{cv}^2 + \omega_{iv}^2 \rho_{min}$  regardless of traders' market-entry choices (**Proposition 2**), so we can take  $\tilde{\rho}_{ij}$  fixed. Hence, we use the simplified notation  $\hat{\sigma}^2(N_{cm})$ . **Theorem 1** shows that  $E[u_{cm}^i(N_{cm})] > E[u_{otc}^i]$  if and only if  $\sigma^2 < \hat{\sigma}^2(N_{cm})$ .

(Part (i): *Single-venue equilibrium*) Suppose  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ . By **Proposition 3** (i), we obtain  $\frac{d\hat{\sigma}^2(N_{cm})}{dN_{cm}} > 0$  for all  $N_{cm}$ . It is immediate to rule out an equilibrium in which the two venues coexist. Such an equilibrium necessarily requires  $E[u_{cm}^i(N_{cm} + 2)] \leq E[u_{otc}^i] < E[u_{cm}^i(N_{cm})]$  for conditions (1) and (2), and equivalently,  $\hat{\sigma}^2(N_{cm} + 2) \leq \sigma^2 < \hat{\sigma}^2(N_{cm})$  for some  $N_{cm}$ . However, this contradicts that  $\hat{\sigma}^2(N_{cm})$  is increasing in  $N_{cm}$ . Therefore, every equilibrium that exists must have either  $N_{cm} = 2$  or  $N_{cm} = I$ .

Given that  $\hat{\sigma}^2(N_{cm})$  is increasing,  $\phi_{min} = 1/\hat{\sigma}^2(I)$  and  $\phi_{max} = 1/\hat{\sigma}^2(2)$ . If  $\sigma^2 \geq \hat{\sigma}^2(I)$ , then  $\sigma^2 \geq \hat{\sigma}^2(N_{cm})$  for all  $N_{cm}$ . Traders prefer the OTC market regardless of other  $N_{cm}$  traders' entry to the centralized market, and thus, every equilibrium consists only of the OTC market. Similarly, if  $\sigma^2 < \hat{\sigma}^2(2)$ , then  $\sigma^2 < \hat{\sigma}^2(N_{cm})$  for all  $N_{cm}$ , and thus, equilibrium consists only of the centralized market. When  $\hat{\sigma}^2(2) \leq \sigma^2 < \hat{\sigma}^2(I)$ , the pairwise-deviation conditions do not select between the two corner market structures.

(Part (ii): *Coexisting-venue equilibrium*) Suppose  $\omega_{iv}^2 > \omega_{iv,H}^2(I, \rho)$ . By **Proposition 3** (ii), we obtain  $\frac{d\hat{\sigma}^2(N_{cm})}{dN_{cm}} < 0$  for all  $N_{cm}$ . This implies that  $\phi^{min} = 1/\hat{\sigma}^2(2)$  and  $\phi^{max} = 1/\hat{\sigma}^2(I)$ . If  $\sigma^2 \geq \hat{\sigma}^2(2)$ , then  $\sigma^2 \geq \hat{\sigma}^2(N_{cm})$  for all  $N_{cm}$ . The pairwise-deviation conditions rule out both an interior equilibrium and the centralized-only market structure, and thus, when it exists, every equilibrium consists only of the OTC market. Similarly, if  $\sigma^2 < \hat{\sigma}^2(I)$ , both an interior equilibrium and the OTC-only structure are ruled out, so equilibrium consists only of the centralized market. When  $\hat{\sigma}^2(I) \leq \sigma^2 < \hat{\sigma}^2(2)$ , neither corner market structure can be an equilibrium. Consequently, if an equilibrium exists, it must have an interior centralized-market size,  $2 < N_{cm} < I$ . This completes the proof. ■

**Proof of Theorem 3 (Welfare Effects of OTC Market Closure).** We let  $W(N_{cm}) = (I - N_{cm})E[u_{otc}^i] + N_{cm}E[u_{cm}^i(N_{cm})]$  be the total welfare at a centralized-market size  $N_{cm} \in$

$[N, I]$ . The welfare after the OTC market closure is  $W^{shut} = W(I)$ . Instead of proving parts (i)–(iii) sequentially, we first address parts (ii) and (iii), as their proofs extend beyond the parameter sets assumed in the theorem.

(Part (ii): *Multiple corner equilibria*) Suppose  $\phi_{min} = \hat{\phi}(I)$  and  $\phi_{max} = \hat{\phi}(N)$  in the interval  $N_{cm} \in [N, I]$ . The additional condition assumed in part (ii),  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$ , guarantees the global monotonicity of  $\hat{\phi}(N_{cm})$  for all  $N_{cm} \in [N, I]$  (Proposition 3 (i)), which is not necessary for this proof. When  $\phi_{min} < \phi < \phi_{max}$ ,  $E[u_{cm}^i(I)] > E[u_{otc}^i] > E[u_{cm}^i(N)]$ . Hence, closing the OTC market increases the expected utilities of the  $N$  captive traders ( $E[u_{cm}^i(I)] > E[u_{cm}^i(N)]$ ) and those of the  $(I - N)$  traders who choose the OTC market ( $E[u_{cm}^i(I)] > E[u_{otc}^i]$ ). The total welfare strictly increases.

(Part (iii): *Unique coexisting-venue equilibrium*) Suppose that the unique equilibrium market structure has both markets open ( $N_{cm} \in (N, I)$ ). Pairwise stability implies  $E[u_{cm}^i(N_{cm})] > E[u_{otc}^i]$ . The equilibrium uniqueness also implies  $E[u_{otc}^i] > E[u_{cm}^i(I)]$ . This can be seen by contradiction: If  $E[u_{otc}^i] \leq E[u_{cm}^i(I)]$ , the centralized-only market structure also becomes an equilibrium, as seen in the proof of part (i). Consequently,  $E[u_{cm}^i(N_{cm})] > E[u_{otc}^i] > E[u_{cm}^i(I)]$  holds. This implies that closing the OTC market lowers the expected utility of all traders, both  $N$  captive traders ( $E[u_{cm}^i(N_{cm})] > E[u_{cm}^i(I)]$ ) and  $(I - N)$  traders who endogenously choose a market ( $E[u_{otc}^i] > E[u_{cm}^i(I)]$ ), and lowers the total welfare. We remark that this result holds whenever the coexisting-venue equilibrium exists and it is the unique equilibrium. The condition of Proposition 3 (ii) is sufficient but not necessary.

(Part (i): *Unique OTC-only equilibrium*:  $\omega_{iv}^2 < \omega_{iv,L}^2(I, \rho)$  and  $\phi < \phi_{min} = 1/\hat{\sigma}^2(I)$ ). First, closing the OTC market makes all (I-N) OTC-market traders worse off:

$$E[u_{otc}^i] > E[u_{cm}^i(I)]. \quad (38)$$

The proof is immediate: If  $E[u_{otc}^i] \leq E[u_{cm}^i(I)]$ , the centralized-only market structure becomes an equilibrium, contradicting the uniqueness of the OTC-only equilibrium.

Second, we compare the captive traders' expected utilities:  $E[u_{cm}^i(N)]$  in equilibrium and  $E[u_{cm}^i(I)]$  after closing the OTC market. We first consider two extreme values of precision  $\phi \in [0, \phi_{min}]$ . At  $\phi = \phi_{min} = \hat{\phi}(I)$ ,  $E[u_{cm}^i(I)] = E[u_{otc}^i]$  by definition, and  $E[u_{otc}^i] > E[u_{cm}^i(N)]$  because  $\phi_{min} < \hat{\phi}(N)$  (Proposition 3 (i)). This implies

$$E[u_{cm}^i(I)] > E[u_{cm}^i(N)] \quad \text{at } \phi = \phi_{min}. \quad (39)$$

At the other end point  $\phi \downarrow 0$ , equivalently when  $\sigma^2 \rightarrow \infty$ , Corollary 4 gives us

$$E[u_{cm}^i(N_{cm})] = \frac{1}{\sigma^2} (2(1 - \bar{\rho}_{cm}(N_{cm})) - \frac{N_{cm}}{N_{cm} - 1}) + O((\sigma^2)^{-2}).$$

By substituting  $\bar{\rho}_{cm}(N_{cm})$  and evaluating  $E[u_{cm}^i(N_{cm})]$  at  $N_{cm} = I$  and  $N_{cm} = N$ ,

$$E[u_{cm}^i(I)] - E[u_{cm}^i(N)] = \frac{I - N}{(N - 1)\sigma^2} \left( \frac{1}{I - 1} + \frac{2\omega_{iv}^2\rho}{I - 2} \right) + O((\sigma^2)^{-2}).$$

The global existence of cutoff  $\hat{\phi}(N_{cm})$  requires  $\omega_{iv}^2 > \underline{\omega}_{iv}^2(I, \rho) = -\frac{I-2}{2(I-1)\rho}$ . This implies that  $\frac{1}{I-1} + \frac{2\omega_{iv}^2\rho}{I-2} < 0$  and thus

$$E[u_{cm}^i(I)] < E[u_{cm}^i(N)] \quad \text{for sufficiently small } \phi. \quad (40)$$

By continuity, Eqs. (39) and (40) imply that there exists at least one  $\phi_C \in (0, \phi_{min})$  such that  $E[u_{cm}^i(I; \phi_C)] = E[u_{cm}^i(N; \phi_C)]$ .

We now show the uniqueness of  $\phi_C \in (0, \phi_{min})$ . Then, we get  $E[u_{cm}^i(I)] < E[u_{cm}^i(N)]$  for all  $0 < \phi < \phi_C$  and  $E[u_{cm}^i(I)] > E[u_{cm}^i(N)]$  for all  $\phi_C < \phi < \phi_{min}$ . The condition of [Proposition 3](#) (i) is used for this step, while it is not necessary for the existence of  $\phi_C$ . We adopt the same limit argument as in the proof of [Proposition 3](#) (i). Recall  $\delta = \frac{\omega_{iv}^2}{\omega_{iv}^2(I, \rho)} - 1 > 0$  and  $\tilde{\rho}_0 = 1 - \underline{\omega}_{iv}^2(I, \rho) + \underline{\omega}_{iv}^2(I, \rho)\rho$ . As  $\delta \downarrow 0$ , we have  $\omega_{iv}^2 \downarrow \underline{\omega}_{iv}^2(I, \rho)$ . The centralized-market expected utility  $E[u_{cm}^i(N_{cm})]$  expands to

$$E[u_{cm}^i(N_{cm})] = \frac{1}{\sigma^2} \left( 2(1 - \bar{\rho}_{cm}(N_{cm})) - \frac{N_{cm}}{N_{cm} - 1} \right) + \frac{1}{(\sigma^2)^2} A(N_{cm}) + O((\sigma^2)^{-3}),$$

where

$$A(N_{cm}) = -\frac{N_{cm} - 1}{N_{cm}} (1 - \bar{\rho}_{cm})^3 + \frac{(1 + (N_{cm} - 1)\bar{\rho}_{cm})^2 (1 + (2N_{cm} - 1)\bar{\rho}_{cm})}{N_{cm}(N_{cm} - 1)}.$$

By the definition of  $\delta$ ,

$$E[u_{cm}^i(I)] - E[u_{cm}^i(N)] = -\frac{(I - N)\delta}{(N - 1)(I - 1)\sigma^2} + \frac{A(I) - A(N)}{(\sigma^2)^2} + O((\sigma^2)^{-3}). \quad (41)$$

Fixing  $\delta\sigma^2$  and taking  $\delta \downarrow 0$  gives the limit of Eq. (41) as

$$\frac{I - N}{(I - 1)(N - 1)} \frac{1}{(\delta\sigma^2)^2} \left\{ \frac{1}{4} ((I - 1)(N - 1)(1 + 2\tilde{\rho}_0)^3 + 1 + \frac{1}{I - 1} + \frac{1}{N - 1} - 2\tilde{\rho}_0) - \delta\sigma^2 \right\}. \quad (42)$$

The convergence is uniform for any compact subset of  $\delta\sigma^2 \in (0, \infty)$ . From Eq. (42), we have a unique solution of  $E[u_{cm}^i(I)] - E[u_{cm}^i(N)] = 0$  in this limit:

$$\delta\sigma_C^2 = \frac{1}{4} ((I - 1)(N - 1)(1 + 2\tilde{\rho}_0)^3 + 1 + \frac{1}{I - 1} + \frac{1}{N - 1} - 2\tilde{\rho}_0) > 0. \quad (43)$$

We now verify that  $\phi_C = 1/\sigma_C^2$  lies in  $(0, \phi_{min})$ . Substituting  $N_{cm} = I$  into Eq. (33) in the proof of [Proposition 3](#) (i) gives us, for sufficiently small  $\delta \downarrow 0$ ,

$$\delta\sigma_C^2 - \delta\hat{\sigma}^2(I) \rightarrow \frac{N - 2}{4} ((I - 1)(1 + 2\tilde{\rho}_0)^3 - \frac{1}{N - 1}).$$

Since  $(1 + 2\tilde{\rho}_0)^3 > \frac{1}{(N - 1)^2}$ , we get  $\delta\sigma_C^2 > \delta\hat{\sigma}^2(I)$  and thus  $\phi_C < \phi_{min}$ .

Because Eq. (42) has a nonzero derivative with respect to  $\delta\sigma^2$  at its root, the Implicit Function Theorem guarantees that, for a sufficiently small  $\delta > 0$ , there exists a locally unique root  $\delta\sigma_C^2(\delta)$ . We now verify the global uniqueness of  $\delta\sigma_C^2(\delta)$  in  $(\delta\hat{\sigma}^2(I), \infty)$ . Let us pick a sufficiently large  $Y \in (\delta\sigma_C^2, \infty)$  such that  $-\frac{I-N}{(N-1)(I-1)\delta\sigma^2}$  dominates the remaining higher-order terms in Eq. (41) for any  $\delta\sigma^2 > Y$ . Then, we get  $E[u_{cm}^i(I)] - E[u_{cm}^i(N)] < 0$  for all  $\delta\sigma^2 \geq Y$ . In addition, the uniform convergence in Eq. (42) implies that, for sufficiently small  $\delta$ , it has no zero in the compact set  $[\delta\hat{\sigma}^2(I), Y]$  outside a neighborhood of  $\delta\sigma_C^2$ , and has exactly one zero inside that neighborhood. Therefore, for all sufficiently small  $\delta > 0$ ,  $\phi_C \in (0, \phi_{min})$  is the globally unique solution of  $E[u_{cm}^i(I)] = E[u_{cm}^i(N)]$ .

Lastly, we consider how closing the OTC market changes the total welfare:  $W(I) - W(N)$ . Eqs. (38) and (40) give a potential trade-off between  $(I - N)$  traders who endogenously choose the OTC market and  $N$  captive traders in the centralized market. Moreover, for any  $\phi \leq \phi_C$ , we have  $W(I) - W(N) < 0$  because all traders' expected utilities, including those of captive traders, decrease by closing the OTC market. At  $\phi = \phi_{min}$ , instead,  $E[u_{otc}^i] = E[u_{cm}^i(I)]$  by the definition of  $\phi_{min}$ , and thus,  $W(I) - W(N) > 0$  by Eq. (39). Consequently, there exists at least one  $\phi_W \in (\phi_C, \phi_{min})$  such that  $W(I) = W(N)$ . In addition, from Corollary 4,

$$W(I) - W(N) = \frac{1}{\sigma^2} \frac{(I - N)(IN - I - N + 2)}{(I - 2)(I - 1)(N - 1)} ((I - 2) + 2\omega_{iv}^2 \rho(I - 1)) + O((\sigma^2)^{-2}),$$

which has a negative first-order term for all  $\omega_{iv}^2 > \underline{\omega}_{iv}^2(I, \rho) = -\frac{I-2}{2(I-1)\rho}$ , and thus,  $W(I) - W(N) < 0$  for sufficiently small  $\phi = (\sigma^2)^{-1}$ . This conclusion holds more generally than the condition of Proposition 3 (i), while the proof below for the uniqueness of  $\phi_W$  invokes that stronger condition.

To prove the uniqueness of  $\phi_W \in (\phi_C, \phi_{min})$ , we use the same limiting argument as in the proof of Proposition 3 (i). As  $\delta \downarrow 0$ , equivalently as  $\omega_{iv}^2 \downarrow \underline{\omega}_{iv}^2(I, \rho)$ ,

$$\frac{W(I) - W(N)}{\delta^2} \rightarrow \frac{I - N}{I - 1} \frac{1}{(\delta\sigma^2)^2} \left( \frac{N}{N - 1} (\delta\sigma_C^2 - \delta\sigma^2) - (I - 2)(\delta\sigma^2 - \delta\sigma_I^2) \right), \quad (44)$$

where  $\delta\sigma_I^2 = \lim_{\delta \downarrow 0} \delta\hat{\sigma}^2(I)$  with a slight abuse of notation. Eq. (44) has only one root:

$$\delta\sigma_W^2 = \frac{\frac{N}{N-1}\delta\sigma_C^2 + (I-2)\delta\sigma_I^2}{\frac{N}{N-1} + I - 2}, \quad (45)$$

which is a convex combination of  $\delta\hat{\sigma}^2(I)$  and  $\delta\sigma_C^2$  and thus lies between them. Because Eq. (44) has a nonzero derivative with respect to  $\delta\sigma^2$  at its root  $\delta\sigma_W^2$ , the Implicit Function Theorem extends the limiting result and the local uniqueness of  $\sigma_W^2(\delta)$  to sufficiently small  $\delta > 0$ . The global uniqueness of  $\delta\sigma_W^2$  is proved similarly as before. We pick a sufficiently large  $Y \in (\delta\sigma_W^2, \infty)$  so that the first order in Eq. (44) dominates the remaining term

$O((\delta\sigma^2)^{-2})$ . Because the first-order term is negative at  $\underline{\omega}_{iv}^2(I, \rho)$ , we get  $W(I) - W(N) < 0$  for all  $\delta\sigma^2 \geq Y$ , uniformly for all sufficiently small  $\delta > 0$ . In addition, the uniform convergence in (44) guarantees that for sufficiently small  $\delta$ , there is exactly one root in the compact set  $[\delta\sigma^2(I), Y]$  and  $W(I) - W(N)$  has the same sign as its limit as  $\delta \downarrow 0$ .

We remark that the limiting argument requires  $\omega_{iv,L}^2(I, \rho)$  to be chosen sufficiently close to  $\underline{\omega}_{iv}^2(I, \rho)$ , which may tighten relative to the level required in Proposition 3 (i). Taking the minimum values of  $\omega_{iv,L}^2(I, \rho)$ , both Proposition 3 (i) and Theorem 3 (i) continue to hold. ■

**Proposition 5 (Liquidity and Learning Effects: Bilateral Asymmetric Market).**

Consider a bilateral OTC pair  $(i, j)$  where equilibrium exists (Corollary 5). A trader's *ex ante* utility  $E[u^i]$  satisfies the following properties: For any  $\tilde{\rho}_{ij} \in (-1, 0)$ ,

- (i)  $\frac{\partial E[u^i]}{\partial \phi^j} > 0$  if and only if  $\phi^j < \tilde{\phi}(\phi^i, \tilde{\rho}_{ij})$  for a cutoff  $\tilde{\phi}(\phi^i, \tilde{\rho}_{ij}) > 0$ .
- (ii)  $\frac{\partial \tilde{\phi}(\phi^i, \tilde{\rho}_{ij})}{\partial \phi^i} < 0$  for all  $\phi^i > 0$ .

As  $\tilde{\rho}_{ij} \downarrow -1$ , the limit of the cutoff is  $\tilde{\phi}(\phi^i, \tilde{\rho}_{ij}) \rightarrow \infty$ .

**Proof of Proposition 5 (Liquidity and Learning Effects: Bilateral Asymmetric Market).** For convenience, this proof uses simpler notation  $\rho = \tilde{\rho}_{ij}$ ,  $\sigma^i = (\sigma^i)^2 = 1/\phi^i$ , and  $\sigma^j = (\sigma^j)^2 = 1/\phi^j$ , where, with a slight abuse of notation, we drop the power 2 from  $(\sigma^i)^2$  and  $(\sigma^j)^2$ . Using the explicit equilibrium characterization in Corollary 5, we get

$$E[u_{otc}^i] = (1 - (\frac{\lambda^i}{\mu + \lambda^i})^2) \text{Var}[E[\tilde{\theta}^i | s^i, p] - p],$$

where the liquidity and learning effects are characterized by

$$\begin{aligned} \frac{\lambda^i}{\mu + \lambda^i} &= \frac{(1 + \rho)(2((1 + \sigma^i)(1 + \sigma^j) - \rho^2) + \rho(\sigma^j - \sigma^i))}{2(1 - \rho)((1 + \sigma^i)(1 + \sigma^j) - \rho^2) + 4\rho(1 - \rho)(1 + \sigma^i + \rho) + \rho(3 - \rho)(\sigma^j - \sigma^i)}, \\ \text{Var}[E[\theta_i | s^i, p] - p] &= \frac{((1 + \sigma^i + \rho) + (1 + \sigma^j + \rho))}{16((1 + \sigma^i)(1 + \sigma^j) - \rho^2)(1 + \sigma^i + \rho)^2(1 + \sigma^j + \rho)^2} \\ &\quad \times (2(1 - \rho)((1 + \sigma^i)(1 + \sigma^j) - \rho^2) + 4\rho(1 - \rho)(1 + \sigma^i + \rho) + \rho(3 - \rho)(\sigma^j - \sigma^i))^2. \end{aligned}$$

(Part (i)) We differentiate the two effects with respect to the counterparty's noise variance  $\sigma^j$ . The liquidity effect  $\frac{1}{\mu}(1 - (\frac{\lambda^i}{\mu + \lambda^i})^2)$  increases as  $\frac{\lambda^i}{\mu + \lambda^i}$  decreases, and

$$\frac{d}{d\sigma^j} \frac{\lambda^i}{\mu + \lambda^i} = \frac{4\rho(1 + \rho)(1 + \sigma^i + \rho)((1 - \rho)(1 + \sigma^i + \rho) - \rho\sigma^i)}{(2(1 - \rho)((1 + \sigma^i)(1 + \sigma^j) - \rho^2) + 4\rho(1 - \rho)(1 + \sigma^i + \rho) + \rho(3 - \rho)(\sigma^j - \sigma^i))^2},$$

is negative for all  $\rho < 0$ . Therefore, the liquidity effect increases as  $\sigma^j$  increases. On the

other hand, the learning effect decreases as  $\sigma^j$  increases:

$$\begin{aligned} \frac{(dVar[E[\tilde{\theta}^i|s^i, p] - p]/d\sigma^j)}{Var[E[\tilde{\theta}^i|s^i, p] - p]} &= -\frac{(1 + \sigma^i + \rho)^2}{((1 + \sigma^i + \rho) + (1 + \sigma^j + \rho))((1 + \sigma^i)(1 + \sigma^j) - \rho^2)} \\ &+ \frac{2\rho(1 + \rho)(1 + \sigma^i + \rho)}{(1 + \sigma^j + \rho)(2(1 - \rho)\sigma^i\sigma^j + (1 + \rho)(2 - 3\rho)\sigma^i + (1 + \rho)(2 - \rho)\sigma^j + 2(1 + \rho)^2(1 - \rho))} \end{aligned}$$

is negative. Two partials show that there is a trade-off between trader  $i$ 's liquidity and learning effects as counterparty  $j$ 's noise variance  $\sigma^j = 1/\phi^j$  changes.

We now show how the trade-off shapes the net comparative static. By multiplying the two components, we have a closed form formula of the *ex ante* payoff:

$$E[u_{otc}^i] = \frac{-\rho(2 + \sigma^i + \sigma^j + 2\rho)(1 + \sigma^j - \rho^2)((1 + \sigma^j + \rho)\sigma^i + (1 + \sigma^i + \rho)\sigma^j)}{2(1 + \sigma^i + \rho)((1 + \sigma^i)(1 + \sigma^j) - \rho^2)(1 + \sigma^j + \rho)^2}. \quad (46)$$

When  $\rho < 0$ ,  $E[u_{otc}^i] > 0$ . Differentiating  $E[u_{otc}^i]$  with respect to  $\sigma^j$  gives us

$$\begin{aligned} \frac{\partial E[u_{otc}^i]}{\partial \sigma^j} \frac{1}{E[u_{otc}^i]} &= \frac{-2(\sigma^i\sigma^j - (1 + \rho)^2)(1 + \sigma^i + \rho)}{((1 + \sigma^i + \rho)\sigma^j + (1 + \sigma^j + \rho)\sigma^i)(\sigma^j + \sigma^i + 2 + 2\rho)(1 + \sigma^j + \rho)} \\ &+ \frac{\sigma^i\rho^2}{((1 + \sigma^i)(1 + \sigma^j) - \rho^2)(1 + \sigma^j - \rho^2)}. \end{aligned} \quad (47)$$

Fixing  $\sigma^i$  and  $\rho$ ,  $\frac{\partial E[u_{otc}^i]}{\partial \sigma^j} < 0$  if and only if  $g(\sigma^j) < 0$  when

$$\begin{aligned} g(\sigma^j) &= -2(\sigma^i\sigma^j - (1 + \rho)^2)(1 + \sigma^i + \rho)((1 + \sigma^i)(1 + \sigma^j) - \rho^2)(1 + \sigma^j - \rho^2) \\ &+ \sigma^i\rho^2((1 + \sigma^i + \rho)\sigma^j + (1 + \sigma^j + \rho)\sigma^i)(\sigma^j + \sigma^i + 2 + 2\rho)(1 + \sigma^j + \rho). \end{aligned} \quad (48)$$

Here, we denote this cubic function of  $\sigma^j > 0$  by  $g(x) = b_3x^3 + b_2x^2 + b_1x + b_0$ .

$$\begin{aligned} b_3 &= -[2(1 + \sigma^i + \rho)(1 + \sigma^i - \rho^2) + \rho^2(1 + \rho)]\sigma^i, \\ b_2 &= -(1 + \rho)[4(1 - \rho)(\sigma^i)^3 + 2(3 - 3\rho - 5\rho^2)(\sigma^i)^2 - 3\rho(1 + \rho)(2 + \rho)\sigma^i - 2(1 + \rho)^2], \\ b_1 &= (1 + \rho)[(-2 + 2\rho + 3\rho^2)(\sigma^i)^3 + 2(1 + \rho)\rho(3 + 2\rho)(\sigma^i)^2 + 2(1 + \rho)^2(3 - \rho^2)\sigma^i \\ &\quad + 4(1 + \rho)^3(1 - \rho)], \\ b_0 &= (1 + \rho)^2[2(1 + \rho)(1 - \rho)(1 + \sigma^i + \rho)(1 + \sigma^i - \rho^2) + \rho^2(\sigma^i + 2 + 2\rho)(\sigma^i)^2]. \end{aligned}$$

It is immediate that  $b_3 < 0$  and  $b_0 \geq 0$ , where the equality  $b_0 = 0$  holds if and only if  $\rho = -1$ . In addition,  $b_2 > 0$  implies  $b_1 > 0$ . To prove this, we assume  $\rho \neq -1$  and let  $z = \frac{\sigma^i}{1 + \rho} \geq 0$ . We find a linear combination of  $b_1$  and  $b_2$  that is positive for all  $z \geq 0$ :

$$\begin{aligned} \frac{b_1}{(1 + \rho)^4} - (1 - \rho - \frac{\rho}{1 - \rho}z) \frac{b_2}{(1 + \rho)^3} &= -4\rho(1 + \rho)z^4 + \frac{(1 + \rho)(-4\rho^3 + 19\rho^2 - 12\rho + 2)}{1 - \rho}z^3 \\ &+ \frac{-10\rho^4 + 13\rho^3 + 12\rho^2 - 12\rho + 6}{1 - \rho}z^2 + \frac{-3\rho^4 + 2\rho^3 + 7\rho^2 - 10\rho + 6}{1 - \rho}z + 2(1 - \rho). \end{aligned}$$

Here,  $-10\rho^4 + 13\rho^3 + 12\rho^2 \geq -11\rho^2$  and  $-3\rho^4 + 2\rho^3 + 7\rho^2 \geq 2\rho^2$ . All coefficients on the right-hand side are positive for  $-1 < \rho < 0$ , and thus, the above linear combination of  $b_1$  and  $b_2$  is positive. Combined with  $(1 - \rho - \frac{\rho}{1-\rho}z) > 0$ , this shows that  $b_2 > 0$  implies  $b_1 > 0$  when  $\rho \neq -1$ . Given this sign property and the Descartes rules,  $g(\sigma^j)$  has a unique positive root  $\tilde{\sigma}^j(\sigma^i, \rho) > 0$  for any  $\sigma^i > 0$  and  $-1 < \rho < 0$ . Because  $b_0 > 0$  and  $b_3 < 0$ ,  $g(\sigma^j)$  crosses zero from above. Consequently,  $g(\sigma^j) > 0$  and  $\frac{\partial E[u_{otc}^i]}{\partial \sigma^j} > 0$  if  $\sigma^j < \tilde{\sigma}^j(\sigma^i, \rho)$ , whereas  $g(\sigma^j) < 0$  and  $\frac{\partial E[u_{otc}^i]}{\partial \sigma^j} < 0$  if  $\sigma^j > \tilde{\sigma}^j(\sigma^i, \rho)$ . If  $\rho = -1$ , we have  $b_2 = b_1 = 0$ , which implies that  $\tilde{\sigma}^j(\sigma^i, \rho) = 0$  and  $\frac{\partial E[u_{otc}^i]}{\partial \sigma^j} < 0$  for all  $\sigma^j > 0$ .

(Part (ii)) We use the Implicit Function Theorem to examine the sign of  $\frac{d\tilde{\sigma}^j(\sigma^i, \rho)}{d\sigma^i}$ . Suppose  $\rho \neq -1$ . From the proof of part (i),  $g(\sigma^j)$  has a unique positive root and crosses zero from above: i.e.,  $\frac{\partial g}{\partial \sigma^j} < 0$  at  $\sigma^j = \tilde{\sigma}^j(\sigma^i, \rho)$ . Because  $g(\sigma^j)$  and  $\frac{\partial E[u_{otc}^i]}{\partial \sigma^j} \frac{1}{E[u_{otc}^i]}$  have the same sign for all  $\sigma^j$ , this also implies  $\frac{\partial}{\partial \sigma^j} \left( \frac{\partial E[u_{otc}^i]}{\partial \sigma^j} \frac{1}{E[u_{otc}^i]} \right) < 0$  at  $\sigma^j = \tilde{\sigma}^j(\sigma^i, \rho)$ . Differentiating Eq. (47) with respect to  $\sigma^i$  gives  $\frac{\partial}{\partial \sigma^i} \left( \frac{\partial E[u_{otc}^i]}{\partial \sigma^j} \frac{1}{E[u_{otc}^i]} \right)$  as

$$\frac{\rho^2}{((1 + \sigma^i)(1 + \sigma^j) - \rho^2)^2} - \frac{1}{(\sigma^i + \sigma^j + 2 + 2\rho)^2} - \frac{(1 + \rho)^2}{((1 + \sigma^i + \rho)\sigma^j + (1 + \sigma^j + \rho)\sigma^i)^2}.$$

Since  $((1 + \sigma^i)(1 + \sigma^j) - \rho^2) + \rho(\sigma^i + \sigma^j + 2 + 2\rho) = (1 + \sigma^i + \rho)(1 + \sigma^j + \rho) > 0$ , the first two terms are jointly negative. Therefore,  $\frac{\partial}{\partial \sigma^i} \left( \frac{\partial E[u_{otc}^i]}{\partial \sigma^j} \frac{1}{E[u_{otc}^i]} \right) < 0$  for any  $\sigma^j > 0$ , including  $\sigma^j = \tilde{\sigma}^j(\sigma^i, \rho)$ . By the Implicit Function Theorem,  $\frac{d\tilde{\sigma}^j(\sigma^i, \rho)}{d\sigma^i} < 0$  holds. ■

**Proof of Proposition 4 (OTC Matching with Heterogeneous Precisions).** Four types of traders exist in the setting of Example 3: informed buyers, informed sellers, uninformed buyers, and uninformed sellers. Since  $\tilde{\rho}_+ = \omega_{cv}^2 - \omega_{iv}^2 \frac{I}{I-2}\rho > 0$ , no trading-period equilibrium exists in a bilateral trade between two buyers or two sellers, regardless of their precision types (Corollary 5). Consequently, OTC matching consists only of buyer-seller pairs, when  $\tilde{\rho}_- < 0$ , in the correlation-type dimension. Since there are only two types in the precision-type dimension, all feasible rankings of counterparties' precision are linear, and consequently, a pairwise-stable matching exists (Irving, 1985).

By Proposition 5,  $E[u_{otc}^i]$ , as a function of the counterparty's precision  $\phi^j$ , is single-peaked, and the optimal counterparty precision  $\tilde{\phi}(\phi^i, \tilde{\rho}_-)$  is decreasing in  $\phi^i$ . Hence, when  $\phi_I$  is sufficiently high and sufficiently larger than  $\phi_U$ , informed traders prefer an uninformed counterparty, whereas uninformed traders prefer an informed counterparty. The two types mutually prefer cross-precision matching, so pairwise stability implies cross-precision pairs. Conversely, outside this region, the two types do not rank counterparty precision in opposite directions, and same-precision matching is pairwise stable. ■

**Proof of Theorem 4 (Liquidity Provision toward Informed Traders).** Recall that

$\tilde{\rho}_- = 1 - \omega_{iv}^2(1 - \rho) < 0$  and  $\tilde{\rho}_+ = 1 - \omega_{iv}^2 - \omega_{iv}^2 \frac{I}{I-2} \rho > 0$ . Continuing from the proof of [Proposition 5](#), we use simpler notation  $\sigma^i = (\sigma^i)^2$ , and  $\sigma^j = (\sigma^j)^2$ , where, with a slight abuse of notation, we drop the power 2 from  $(\sigma^i)^2$  and  $(\sigma^j)^2$ . We fix an informed buyer  $i$  throughout the proof, and thus, his own noise variance is  $\sigma^i$ . Throughout the proof, we suppose the informed trader's entry cutoff  $\hat{\sigma}(I - 2, \tilde{\rho}_-, \bar{\rho}_{cm})$  exists ([Theorem 1](#)). If it does not, informed traders strictly prefer the centralized market for every precision, and the result is immediate. Since the argument below uses only the existence itself, not the conditions ensuring it, it applies to the entire parameter region in which the cutoff exists.

From [Proposition 5](#) (i), the trader's best counterparty has noise variance  $\tilde{\sigma}^j(\sigma_I, \tilde{\rho}_-)$ , which is the unique positive solution of  $g(\sigma^j; \sigma_I, \tilde{\rho}_-) = 0$  (Eq. (48)). Given  $\sigma_U > \sigma_I$ , cross-precision matching requires  $E[u_{otc}^i(\sigma_I, \sigma_U, \tilde{\rho}_-)] \geq E[u_{otc}^i(\sigma_I, \sigma_I, \tilde{\rho}_-)]$ , and hence,  $\tilde{\sigma}^j(\sigma_I, \tilde{\rho}_-) > \sigma_I$ . Combined with [Proposition 5](#) (ii), this implies that  $\sigma_I$  must satisfy

$$\sigma_I < \check{\sigma}(\tilde{\rho}_-), \quad (49)$$

when  $\check{\sigma}(\tilde{\rho}_-)$  is the unique positive solution of  $\sigma = \tilde{\sigma}^j(\sigma, \tilde{\rho}_-)$ . To see the existence and uniqueness of  $\check{\sigma}(\tilde{\rho}_-)$ , we substitute  $\sigma^i = \sigma^j$  into the definition (48) of  $g(\cdot)$ :

$$g(\sigma; \sigma, \tilde{\rho}_-) = -2(1 + \tilde{\rho}_- + \sigma)^3[\sigma^3 + (1 - 3\tilde{\rho}_-)(1 + \tilde{\rho}_-)\sigma^2 - (1 - \tilde{\rho}_-^2)(1 + 2\tilde{\rho}_-)\sigma - (1 - \tilde{\rho}_-^2)^2].$$

The polynomial inside the brackets has positive leading and quadratic coefficients and a negative constant term for any  $\tilde{\rho}_- \in (-1, 0)$ . By Descartes' rule, it has a unique positive root. When  $\tilde{\rho}_- \downarrow -1$ , the positive root  $\check{\sigma}(\tilde{\rho}_-)$  converges to zero. Since

$$\begin{aligned} & \sigma^3 + (1 - 3\tilde{\rho}_-)(1 + \tilde{\rho}_-)\sigma^2 - (1 - \tilde{\rho}_-^2)(1 + 2\tilde{\rho}_-)\sigma - (1 - \tilde{\rho}_-^2)^2 \\ &= (\sigma - 1 - \tilde{\rho}_-)(1 + \sigma - \tilde{\rho}_-)(1 + \sigma - \tilde{\rho}_-^2) - 2\sigma^2\tilde{\rho}_-^2, \end{aligned} \quad (50)$$

the first term must be positive at its positive root:  $\check{\sigma}(\tilde{\rho}_-) > 1 + \tilde{\rho}_-$  for all  $\tilde{\rho}_- \in (-1, 0)$ . In addition, evaluating Eq. (50) at  $\sigma = 1 - \tilde{\rho}_-^2$  gives  $-4\tilde{\rho}_-(1 - \tilde{\rho}_-)^2(1 + \tilde{\rho}_-)^3 > 0$ . Because the constant term of Eq. (50) is negative, this implies that  $\check{\sigma}(\tilde{\rho}_-) < 1 - \tilde{\rho}_-^2$ .

We first show that the centralized market with  $N = I - 2$  traders yields a higher expected utility than the same-precision OTC pair whenever the common noise variance satisfies  $\sigma \leq \check{\sigma}(\tilde{\rho}_-)$ . We then extend this result to the utility comparison between the centralized market and the cross-precision OTC pair using Eq. (49).

In the centralized market with  $N = I - 2$  traders, the average correlation is  $\bar{\rho} = 1 - \omega_{iv}^2 + \omega_{iv}^2 \frac{2\rho}{N(N-1)}$ . Fixing  $\tilde{\rho}_-$ , we determine the range of  $\bar{\rho}$ . Recall that  $\rho \geq -\frac{I-2}{I} = -\frac{N}{N+2}$  must hold. Substituting  $\omega_{iv}^2 = \frac{1-\tilde{\rho}_-}{1-\rho}$  into  $\bar{\rho}$  and differentiating it with respect to  $\rho$  gives

$$\left. \frac{\partial \bar{\rho}}{\partial \rho} \right|_{\tilde{\rho}_- \text{ fixed}} = \frac{(N-2)(N+1)(\tilde{\rho}_- - 1)}{N(N-1)(1-\rho)^2} < 0.$$

Therefore, the largest possible  $\bar{\rho}$  is attained at  $\rho = -\frac{N}{N+2}$  and equals

$$\bar{\rho}^*(\tilde{\rho}_-) = \frac{N(1 + \tilde{\rho}_-) - 2}{2(N-1)}. \quad (51)$$

Because  $\rho \leq \tilde{\rho}_-$ , the smallest possible  $\bar{\rho}$ , fixing  $\tilde{\rho}_-$ , is obtained at  $\rho = \tilde{\rho}_-$  and equals  $\frac{2\tilde{\rho}_-}{N(N-1)}$ . Therefore, the range of  $\bar{\rho}$  is  $[\frac{2\tilde{\rho}_-}{N(N-1)}, \bar{\rho}^*(\tilde{\rho}_-)]$ . In this interval, the centralized-market equilibrium exists: for all  $N > 2$  and  $\tilde{\rho}_- < 0$ ,  $\frac{2\tilde{\rho}_-}{N(N-1)} > -\frac{1}{N-1}$  and  $\bar{\rho}^* < \frac{N-2}{2(N-1)}$  hold; moreover,  $\frac{N-2}{2(N-1)}$  is strictly smaller than the upper bound in [Corollary 4](#).

[Proposition 1](#) (ii) implies that the centralized-market expected utility is minimized at  $\bar{\rho} = \bar{\rho}^*(\tilde{\rho}_-)$  within its feasible range. We now show that  $1 - \tilde{\rho}_-^2 < \hat{\sigma}(N, \bar{\rho}^*, \tilde{\rho}_-)$ . By its definition (21), it is equivalent to show the following inequality at  $\sigma = 1 - \tilde{\rho}_-^2$ ,

$$f(1 - \tilde{\rho}_-^2; N, \bar{\rho}^*, \tilde{\rho}_-) > 0. \quad (52)$$

For simplicity, let us denote  $t = -\tilde{\rho}_- \in (0, 1)$ . Substituting  $\sigma = 1 - t^2$  and  $\bar{\rho}^* = \frac{N(1-t)-2}{2(N-1)}$  into Eq. (21) shows that

$$\begin{aligned} f(1 - t^2) \propto & -4(N-1)(N^2 - 10N + 8)t^4 + (-N^4 + 52N^3 - 68N^2 - 16N + 32)t^3 \\ & + 2(7N^4 - 20N^3 - 12N^2 + 40N - 16)t^2 - 4(5N^4 + 8N^3 - 22N^2 + 8)t \\ & + 8N^2(N^2 + 4N - 4), \end{aligned}$$

with the positive proportionality  $\frac{(N-2)(1-t)^4(1+t)^3}{16(N-1)^2} > 0$ . Applying the Bernstein polynomial of degree 5 rewrites this expression into a form of  $\sum_{k=0}^5 \beta_k \binom{5}{k} t^k (1-t)^{5-k}$ , where

$$\begin{aligned} \beta_0 &= 8N^2(N^2 + 4N - 4), \quad \beta_1 = \frac{4}{5}(5N^4 + 32N^3 - 18N^2 - 8), \\ \beta_2 &= \frac{1}{5}(7N^4 + 76N^3 + 4N^2 + 40N - 80), \quad \beta_3 = \frac{1}{10}(N^4 + 60N^3 + 68N^2 + 224N - 256), \\ \beta_4 &= \frac{4}{5}(3N^3 + 7N^2 + 34N - 32), \quad \beta_5 = N(N+2)(N^2 + 6N - 4). \end{aligned}$$

All these coefficients  $\{\beta_k\}_k$  are strictly positive when  $N \geq 4$ , which proves Eq. (52). Combined with  $\check{\sigma}(\tilde{\rho}_-) < 1 - \tilde{\rho}_-^2$ , we obtain  $\check{\sigma}(\tilde{\rho}_-) < \hat{\sigma}(N, \bar{\rho}^*, \tilde{\rho}_-)$ , and hence,

$$E[u_{otc}^i(\check{\sigma}, \check{\sigma}; \tilde{\rho}_-)] < E[u_{cm}^i(N; \check{\sigma}, \bar{\rho}^*)]. \quad (53)$$

To extend the result (53) to all  $\sigma_I < \check{\sigma}(\tilde{\rho}_-)$  and  $\sigma_U > \sigma_I$ , we establish two monotonicity properties (54) and (55). First,

$$\frac{\partial}{\partial \sigma^i} E[u_{cm}^i(N; \sigma^i, \bar{\rho}^*)] < 0 \quad \text{for all } \sigma^i > 0. \quad (54)$$

By [Proposition 1](#) (iii), the expected utility is decreasing in  $\sigma^i$  outside of condition (18). In fact,  $\bar{\rho}^*$  violates this condition: Since  $\tilde{\rho}_- \geq -\frac{N}{N+2}$ , by definition (51),  $\bar{\rho}^* \geq -\frac{2}{(N-1)(N+2)} > -\frac{N-2}{N^2-3N+4}$  where the second inequality holds for all  $N \geq 3$ .

Second, we show that, for any  $0 < \sigma^i \leq \check{\sigma}(\tilde{\rho}_-)$ , the trader's OTC utility with the most preferred counterparty, i.e.,  $\tilde{\sigma}^j(\sigma^i, \tilde{\rho}_-)$ , satisfies

$$\frac{d}{d\sigma^i} E[u_{otc}^i(\sigma^i, \tilde{\sigma}^j(\sigma^i, \tilde{\rho}_-); \tilde{\rho}_-)] > 0. \quad (55)$$

We again use  $t = -\tilde{\rho}_- \in (0, 1)$  and define  $A = 1 - t + \sigma^i$  and  $B = 1 - t + \sigma^j$ . Taking a derivative of  $E[u_{otc}^i(\sigma^i, \sigma^j; -t)]$  in Eq. (46) with respect to  $\sigma^i$ , holding  $\sigma^j$  fixed, gives

$$\frac{\partial E[u_{otc}^i]}{\partial \sigma^i} \frac{1}{E[u_{otc}^i]} = \frac{B[t(1-t)(A+B)^2 - 2AB(AB - (1-t)(A+B))]}{A(A+B)(AB + t(A+B))(2AB - (1-t)(A+B))}. \quad (56)$$

When this partial (56) is zero, the corresponding derivative with respect to  $\sigma^j$  (Eq. (47)) cannot be zero: Replacing  $B^2$  by a linear function of  $B$ , obtained from  $[t(1-t)(A+B)^2 - 2AB(AB - (1-t)(A+B))] = 0$ , Eq. (47) becomes

$$-\frac{2A^4 t(1-t)(1+t)[2A^3 B + t(1-t)(2A^2 + 3AB - (1-t)(A+B))]}{(2A\sigma^i - t(1-t))^2 B(A+B)(B + t(1-t))(AB + t(A+B))(2AB - (1-t)(A+B))},$$

which is negative. This implies that the partial (56) evaluated at  $\tilde{\sigma}^j(\sigma^i, \tilde{\rho}_-)$  can never be zero. Moreover, the partial (56) evaluated at  $\sigma^i = \check{\sigma}(\tilde{\rho}_-)$  is positive, because  $\check{\sigma}(\tilde{\rho}_-) = \tilde{\sigma}^j(\check{\sigma}(\tilde{\rho}_-), \tilde{\rho}_-)$  simplifies the partial into the following, and by  $\check{\sigma}(\tilde{\rho}_-) < 1 - t^2$ , it is positive:

$$\frac{\partial E[u_{otc}^i]}{\partial \sigma^i} \frac{1}{E[u_{otc}^i]} = \frac{1 - t^2 - (\sigma^i)^2}{2\sigma^i(1 + \sigma^i + t)(1 + \sigma^i - t)} > 0.$$

Therefore, for all  $0 < \sigma^i \leq \check{\sigma}(\tilde{\rho}_-)$ , Eq. (55) holds by the envelope theorem.

We now complete the comparison. By Eqs. (49), (53), and (54), and by the fact that  $\bar{\rho}^*$  is the largest possible correlation (Eq. (51)),

$$E[u_{cm}^i(N; \sigma_I, \bar{\rho})] \geq E[u_{cm}^i(N; \sigma_I, \bar{\rho}^*)] > E[u_{cm}^i(N; \check{\sigma}, \bar{\rho}^*)] > E[u_{otc}^i(\check{\sigma}, \check{\sigma}; \tilde{\rho}_-)].$$

For any  $\sigma_U > 0$ , by the definitions of  $\tilde{\sigma}^j(\cdot)$  and  $\check{\sigma}(\cdot)$ , and by Eq. (55),

$$E[u_{otc}^i(\sigma_I, \sigma_U; \tilde{\rho}_-)] \leq E[u_{otc}^i(\sigma_I, \tilde{\sigma}^j(\sigma_I, \tilde{\rho}_-); \tilde{\rho}_-)] < E[u_{otc}^i(\check{\sigma}, \check{\sigma}; \tilde{\rho}_-)].$$

Combining these inequalities yields  $E[u_{otc}^i(\sigma_I, \sigma_U; \tilde{\rho}_-)] < E[u_{cm}^i(N; \sigma_I, \bar{\rho})]$ . By symmetry, the same comparison holds for the informed seller. The proof is complete. ■

## A.2 Other Results

**Proposition 6 (Trading-Period Equilibrium).** A profile of demand schedules  $\{q^i(\cdot)\}_i$  is a linear equilibrium if and only if, for each  $i$ ,

- (i) a demand schedule  $q^i(\cdot : \lambda^i)$  maximizing trader  $i$ 's utility is

$$q^i = \frac{E[\tilde{\theta}^i | s^i, p] - p}{\mu + \lambda^i} = \frac{c_\theta^i E[\tilde{\theta}] + c_s^i s^i - (1 - c_p^i)p}{\mu + \lambda^i}, \quad (57)$$

(ii) trader  $i$ 's price impact  $\lambda^i$  equals the slope of the inverse residual supply, i.e.,

$$\lambda^i = -\left(\sum_{j \neq i} \frac{\partial q^j(\cdot)}{\partial p}\right)^{-1} = \left(\sum_{j \neq i} \frac{1 - c_p^j}{\mu + \lambda^j}\right)^{-1} \geq 0, \quad (58)$$

(iii) trader  $i$ 's expected asset value  $E[\tilde{\theta}^i | s^i, p] = c_\theta^i E[\tilde{\theta}] + c_s^i s^i + c_p^i p$  is determined by applying the Projection Theorem to equilibrium price distribution

$$p = \left(\sum_i \frac{1 - c_p^i}{\mu + \lambda^i}\right)^{-1} \sum_i \frac{c_\theta^i E[\tilde{\theta}] + c_s^i s^i}{\mu + \lambda^i}. \quad (59)$$

**Proof of Proposition 6 (Trading-Period Equilibrium).** In a centralized market with  $I \geq 2$  traders, equilibrium is characterized by two steps: each trader optimizes his demand (1) given price impact  $\lambda^i$  and posterior belief of asset value  $\tilde{\theta}^i$  (part (i)), which are correct in equilibrium, being consistent with other traders' demands (parts (ii) and (iii)).

(Step 1: Individual Optimization) For a given price impact  $\lambda^i > 0$  and inference  $E[\tilde{\theta}^i | s^i, p] = c_\theta^i E[\tilde{\theta}] + c_s^i s^i + c_p^i p$ , trader  $i$ 's first order condition (6) determines his best response (57). The second order condition,  $-\mu - 2\lambda^i < 0$ , holds, when price impact is positive.

(Step 2: Correct Price Impact and Posterior) The market clearing condition  $\sum_i q^i(\cdot) = 0$  determines equilibrium price (59) from all traders' demands in the market. The price is a linear function of traders' private signals  $(s^i)_i \in \mathbb{R}^I$ , which follow a jointly normal distribution, and thus, price also follows a normal distribution. The unconditional expectation of price equals the mean of traders' asset values:  $E[p] = E[\theta]$ . This implies that  $c_\theta^i + c_s^i + c_p^i = 1$  for each  $i$ . The inference coefficients  $\{c_s^i, c_p^i\}$  are determined by applying the Projection Theorem to the joint distribution of  $(\tilde{\theta}^i, s^i, p)$ :

$$\begin{bmatrix} c_s^i \\ c_p^i \end{bmatrix} = \begin{bmatrix} \text{Var}[s^i] & \text{Cov}[s^i, p] \\ \text{Cov}[p, s^i] & \text{Var}[p] \end{bmatrix}^{-1} \begin{bmatrix} \text{Cov}[\tilde{\theta}^i, s^i] \\ \text{Cov}[\tilde{\theta}^i, p] \end{bmatrix}. \quad (60)$$

Eq. (59) determines the price variance and its covariance with private signal, and substituting those second moments into Eq. (60) provides a fixed point for traders' inference coefficients  $\{c_s^i, c_p^i\}_i$ : For each  $i$ ,

$$c_s^i = \frac{\sum_{j,k} \frac{c_s^j}{\mu + \lambda^j} \frac{c_s^k}{\mu + \lambda^k} [(\tilde{\rho}_{jk} + (\sigma^j)^2 \mathbf{1}_{j=k}) - \tilde{\rho}_{ij}(\tilde{\rho}_{ki} + (\sigma^i)^2 \mathbf{1}_{i=k})]}{\sum_{j,k} \frac{c_s^j}{\mu + \lambda^j} \frac{c_s^k}{\mu + \lambda^k} [(1 + (\sigma^i)^2)(\tilde{\rho}_{jk} + (\sigma^j)^2 \mathbf{1}_{j=k}) - (\tilde{\rho}_{ij} + (\sigma^i)^2 \mathbf{1}_{i=j})(\tilde{\rho}_{ik} + (\sigma^i)^2 \mathbf{1}_{i=k})]}, \quad (61)$$

$$c_p^i = \frac{\sum_j \frac{1 - c_p^j}{\mu + \lambda^j} \sum_j \frac{c_s^j}{\mu + \lambda^j} [(1 + (\sigma^i)^2) \tilde{\rho}_{ij} - (\tilde{\rho}_{ij} + (\sigma^i)^2 \mathbf{1}_{i=j})]}{\sum_{j,k} \frac{c_s^j}{\mu + \lambda^j} \frac{c_s^k}{\mu + \lambda^k} [(1 + (\sigma^i)^2)(\tilde{\rho}_{jk} + (\sigma^j)^2 \mathbf{1}_{j=k}) - (\tilde{\rho}_{ij} + (\sigma^i)^2 \mathbf{1}_{i=j})(\tilde{\rho}_{ik} + (\sigma^i)^2 \mathbf{1}_{i=k})]}. \quad (62)$$

Lastly, equilibrium price impact is characterized by implicitly differentiating the market clearing condition with respect to  $q^i$ : i.e.,  $1 + \sum_{j \neq i} \frac{\partial q^j(\cdot)}{\partial p} \lambda^i = 0$  for each  $i$  or equivalently,

Eq. (7). The system of equations (7) and (61)-(62) determines inference coefficients  $\{c_s^i, c_p^i\}_i$  and price impact  $\{\lambda^i\}_i$  in equilibrium. This completes the trading-period equilibrium.

(*Ex Ante Utility from Trading-Period Equilibrium*) From his best response (57), each trader's *ex ante* expected utility from the trading period is determined as a function of price impact and inference coefficients:

$$E[u^i | s^i, p] = -pq^i + E[\tilde{\theta}^i | s^i, p]q^i - \frac{\mu}{2}(q^i)^2 = \frac{\mu + 2\lambda^i}{2(\mu + \lambda^i)^2}(E[\tilde{\theta}^i | s^i, p] - p)^2,$$

and hence, the trader's *ex ante* expected utility is

$$E[u^i] = \frac{\mu + 2\lambda^i}{2(\mu + \lambda^i)^2}E[(E[\tilde{\theta}^i | s^i, p] - p)^2] = \frac{\mu + 2\lambda^i}{2(\mu + \lambda^i)^2}\text{Var}[E[\tilde{\theta}^i | s^i, p] - p].$$

The second equality holds because  $E[E[\tilde{\theta}^i | s^i, p] - p] = 0$ , given  $E[\tilde{\theta}^i] = E[\tilde{\theta}]$  for all  $i$ . ■

**Proposition 7 (Equilibrium Existence/Local Uniqueness: Two Groups).** Consider a centralized market with two groups of equal size  $N > 1$ . Traders are symmetric within each group,  $N_1$  and  $N_2$ , and can be asymmetric across groups through asset correlations ( $\text{Corr}[\tilde{\theta}^i, \tilde{\theta}^j] = \rho_{o,1} = \rho - \delta$  for  $i, j \in N_1$  and  $\rho_{o,2} = \rho + \delta$  for  $i, j \in N_2$ ) or signal precision ( $\text{Var}[\sigma_\varepsilon^i] = \sigma_1^2 = \sigma^2 - \pi$  for  $i \in N_1$  and  $\text{Var}[\sigma_\varepsilon^i] = \sigma_2^2 = \sigma^2 + \pi$  for  $i \in N_2$ ). When cross-group correlation is  $\text{Corr}[\tilde{\theta}^i, \tilde{\theta}^j] = \rho_f < 0$  for  $i \in N_1, j \in N_2$ , there exists  $\epsilon^*(N, \rho, \rho_f, \sigma^2) > 0$  such that for any  $\|(\delta, \pi)\|_2 \leq \epsilon^*(N, \rho, \rho_f, \sigma^2)$ , an equilibrium exists, it is locally unique, and its price impact profile  $(\lambda^1(\delta, \pi), \lambda^2(\delta, \pi))$  is positive and continuously differentiable.

**Proof of Proposition 7 (Equilibrium Existence/Local Uniqueness: Two Groups).** Superscripts 1 and 2 indicate groups  $N_1$  and  $N_2$ . We first note that the system of equilibrium fixed point equation (7) and (61)-(62) can be simplified into a single-dimensional fixed point problem for the ratio of  $\frac{c_s^1}{\mu + \lambda^1}$  and  $\frac{c_s^2}{\mu + \lambda^2}$ , the weights that the price assigns to the aggregate private signals of group 1 and group 2, respectively (Eq. (59)). Let us define

$$r = \frac{c_s^1}{\mu + \lambda^1} \left( \frac{c_s^2}{\mu + \lambda^2} \right)^{-1}.$$

We assume  $\frac{c_s^2}{\mu + \lambda^2} \neq 0$ , i.e., equilibrium price is not independent of private signals of group 2. This assumption will be verified in Step 1 below. The equilibrium fixed point problem for  $\{c_{s,k}, c_{p,k}, \lambda_k\}_{k=1,2}$  is reduced to a single scalar fixed point equation for the ratio  $r \in \mathbb{R}_+$ :

$$\frac{rc_s^2}{c_s^1} \left[ 1 - \frac{1 + \frac{r\kappa^1}{c_s^1} + \frac{\kappa^2}{c_s^2}}{(N-1) - \frac{r\kappa^1}{c_s^1} + \frac{rc_s^2}{c_s^1}(N + \frac{\kappa^2}{c_s^2})} \right] - \left[ 1 - \frac{1 + r\frac{\kappa^1}{c_s^1} + \frac{\kappa^2}{c_s^2}}{(N-1) - \frac{\kappa^2}{c_s^2} + \frac{c_s^1}{rc_s^2}(N + \frac{r\kappa^1}{c_s^1})} \right] = 0, \quad (63)$$

when  $\{c_s^k(r), \kappa^k(r), c_p^k(r), \lambda^k(r)\}_{k=1,2}$  are closed form functions of  $r \in \mathbb{R}_+$ :

$$c_s^1 = \frac{\begin{pmatrix} r^2 N(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N(1 + \sigma_2^2 + (N-1)\rho_{o,2}) + 2rN^2\rho_f \\ -(r(1 + (N-1)\rho_{o,1}) + N\rho_f)(r(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N\rho_f) \end{pmatrix}}{\begin{pmatrix} (1 + \sigma_1^2)(r^2 N(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N(1 + \sigma_2^2 + (N-1)\rho_{o,2}) + 2rN^2\rho_f) \\ -(r(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N\rho_f)^2 \end{pmatrix}}, \quad (64)$$

$$c_s^2 = \frac{\begin{pmatrix} r^2 N(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N(1 + \sigma_2^2 + (N-1)\rho_{o,2}) + 2rN^2\rho_f \\ -(rN\rho_f + (1 + (N-1)\rho_{o,2}))(rN\rho_f + (1 + \sigma_2^2 + (N-1)\rho_{o,2})) \end{pmatrix}}{\begin{pmatrix} (1 + \sigma_2^2)(r^2 N(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N(1 + \sigma_2^2 + (N-1)\rho_{o,2}) + 2rN^2\rho_f) \\ -(rN\rho_f + (1 + \sigma_2^2 + (N-1)\rho_{o,2}))^2 \end{pmatrix}}, \quad (65)$$

$$\kappa^1 = \frac{N(r(N-1)\rho_{o,1} + N\rho_f)\sigma_1^2}{\begin{pmatrix} (1 + \sigma_1^2)(r^2 N(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N(1 + \sigma_2^2 + (N-1)\rho_{o,2}) + 2rN^2\rho_f) \\ -(r(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N\rho_f)^2 \end{pmatrix}}, \quad (66)$$

$$\kappa^2 = \frac{N(Nr\rho_f + (N-1)\rho_{o,2})\sigma_2^2}{\begin{pmatrix} (1 + \sigma_2^2)(r^2 N(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + N(1 + \sigma_2^2 + (N-1)\rho_{o,2}) + 2rN^2\rho_f) \\ -(rN\rho_f + (1 + \sigma_2^2 + (N-1)\rho_{o,2}))^2 \end{pmatrix}}, \quad (67)$$

$$c_p^i = \frac{\kappa^i(\frac{r}{c_s^1} + \frac{1}{c_s^2})}{1 + r\frac{\kappa^1}{c_s^1} + \frac{\kappa^2}{c_s^2}}, \quad i = 1, 2, \quad (68)$$

$$\lambda^i = \frac{\mu(1 + r\frac{\kappa^1}{c_s^1} + \frac{\kappa^2}{c_s^2})}{N(1 + (\frac{c_s^1}{rc_s^2})^{3-2i}) - (2 - c_{p,i})(1 + r\frac{\kappa^1}{c_s^1} + \frac{\kappa^2}{c_s^2})}, \quad i = 1, 2. \quad (69)$$

We define the l.h.s. of Eq. (63) by  $LHS(r; \delta, \pi)$ . The equilibrium must satisfy  $\lambda^i(r^*) > 0$  for both  $i = 1, 2$  when  $r^* > 0$  is a solution of Eq. (63):  $LHS(r^*; \delta, \pi) = 0$ .

(Step 1: Price impact  $\lambda^i(r; \delta, \pi)$  is continuous and positive) Let us introduce auxiliary parameters  $\eta \in (0, 1)$  and  $\zeta \in (0, 1)$ , guaranteeing that the set constructed below is compact. They can be arbitrarily close to one. We define an exogenous parameter,

$$\epsilon_{pd} = \eta \min \left\{ \sigma^2, 1 - \rho, \rho + \frac{1}{N-1}, \frac{\sqrt{(1 + (N-1)\rho)^2 - N^2\rho_f^2}}{N-1} \right\} > 0,$$

so that any  $(\delta, \pi)$  in a compact set  $\{(\delta, \pi) : \delta^2 + \pi^2 \leq \epsilon_{pd}^2\}$  yields a positive definite matrix  $\Sigma_\theta$  and positive noise variances  $\sigma_i^2 > 0$ . We also define a compact interval  $r \in [1-\zeta, 1+\zeta] \subset \mathbb{R}_+$ .

The denominators of  $c_s^i(r; \delta, \pi)$  and  $\kappa^i(r; \delta, \pi)$  (Eqs. (64)-(67)) are strictly positive on this compact set  $\mathcal{D}_1 = \{(r, \delta, \pi) : |r-1| \leq \zeta, \delta^2 + \pi^2 \leq \epsilon_{pd}^2\}$ , where  $\Sigma_\theta > 0$  and  $\sigma_i^2 > 0$  hold, and when  $\rho_f < 0$ . The rational functions  $c_s^i$  and  $\kappa^i$  are hence (at least twice) continuously

differentiable on  $\mathcal{D}_1$ . This can be seen by rewriting the numerator of  $c_s^1(r; \delta, \pi)$  as

$$\begin{aligned} & [(1 + (N-1)\rho_{o,1})(1 + (N-1)\rho_{o,2}) - N^2\rho_f^2] + N\sigma_2^2 \\ & + (N-1)(1 - \rho_{o,1})[(1 + (N-1)\rho_{o,2}) + r^2(1 + \sigma_1^2 + (N-1)\rho_{o,1}) + 2rN\rho_f] - rN\rho_f\sigma_1^2 \\ & \geq [(1 + (N-1)\rho_{o,1})(1 + (N-1)\rho_{o,2}) - N^2\rho_f^2] + N\sigma_2^2 \\ & + (N-1)(1 - \rho_{o,1})[2r(1 + (N-1)\rho_{o,2})^{1/2}(1 + \sigma_1^2 + (N-1)\rho_{o,1})^{1/2} + 2rN\rho_f] - rN\rho_f\sigma_1^2, \end{aligned}$$

which is positive. The inequality is from the arithmetic and geometric means, and the positivity is from  $\Sigma_\theta > 0$  and  $\rho_f < 0$ . The same argument shows  $c_s^2(r; \delta, \pi) > 0$  on  $\mathcal{D}_1$ .

Now, we define the common numerator of  $\lambda^1(r)$  and  $\lambda^2(r)$  as  $S(r; \delta, \pi)$  and each denominator as  $E_i(r; \delta, \pi)$ , in the expression of Eq. (69). Since  $c_s^i(r; \delta, \pi) > 0$ ,  $r > 0$ , and  $c_s^i(r; \delta, \pi)$  and  $\kappa^i(r; \delta, \pi)$  are  $C^2$  functions on  $\mathcal{D}_1$ ,  $S(r; \delta, \pi)$ ,  $E_1(r; \delta, \pi)$ ,  $E_2(r; \delta, \pi)$  are also  $C^2$  functions with respect to  $(r, \delta, \pi)$  on  $\mathcal{D}_1$ . However,  $E_i(r; \delta, \pi)$  may be zero (so that  $\lambda^i(r; \delta, \pi)$  is not continuous) or  $\lambda^i(r; \delta, \pi) = \mu \frac{S(r; \delta, \pi)}{E_i(r; \delta, \pi)}$  may not be positive within  $\mathcal{D}_1$ . Therefore, we find a compact subset  $\mathcal{D}_2 \subseteq \mathcal{D}_1$  where  $\lambda^i(r; \delta, \pi)$  is continuous and positive. To find such a subset  $\mathcal{D}_2$ , we first consider the symmetric benchmark. When  $(\delta, \pi) = (0, 0)$ ,

$$\begin{aligned} S(1; 0, 0) &= \frac{(1 + (N-1)\rho + N\rho_f)((N-1)(1 + \sigma^2 - \rho) + N(1 + \sigma^2 - \rho_f))}{(1 + \sigma^2 + (N-1)\rho + N\rho_f)((N-1)(1 - \rho) + N(1 - \rho_f))}, \\ E_1(1; 0, 0) &= E_2(1; 0, 0) = \frac{2 \left( \frac{(N-1)(1 + (N-1)\rho + N\rho_f)((N-1)(1 - \rho) + N(1 - \rho_f))}{(1 + \sigma^2 + (N-1)\rho + N\rho_f)((N-1)(1 - \rho) + N(1 - \rho_f))} \right)}{(1 + \sigma^2 + (N-1)\rho + N\rho_f)((N-1)(1 - \rho) + N(1 - \rho_f))}, \end{aligned}$$

which are positive given  $\rho_f < 0$  and  $\sigma^2 > 0$ . We define two nonnegative variables,

$$\begin{aligned} m_0 &= \min\{S(1; 0, 0), E_1(1; 0, 0), E_2(1; 0, 0)\} > 0, \\ M_0 &= \max_{\mathcal{D}_1} \left\{ \|\nabla_{(r, \delta, \pi)} S(r; \delta, \pi)\|_2, \|\nabla_{(r, \delta, \pi)} E_1(r; \delta, \pi)\|_2, \|\nabla_{(r, \delta, \pi)} E_2(r; \delta, \pi)\|_2 \right\} \geq 0. \end{aligned}$$

Here,  $M_0$  is well-defined because  $S(r; \delta, \pi)$  and  $E_i(r; \delta, \pi)$  are  $C^2$  functions on the compact set  $\mathcal{D}_1$ . With the new variables  $m_0$  and  $M_0$ , we define

$$\mathcal{D}_2 = \{(r, \delta, \pi) : \underline{r} = \frac{1}{1 + \alpha} \leq r \leq \bar{r} = 1 + \alpha, \delta^2 + \pi^2 \leq \alpha^2\} \subseteq \mathcal{D}_1,$$

where  $\alpha = \min\{\zeta, \epsilon_{pd}, \eta \frac{m_0}{\sqrt{2M_0}}\}$  and  $\eta \frac{m_0}{\sqrt{2M_0}} = +\infty$  if  $M_0 = 0$ . On the compact set  $\mathcal{D}_2$ ,  $\|(r-1, \delta, \pi)\|_2 \leq \sqrt{2}\alpha$ , and thus, the mean-value theorem shows that

$$S(r; \delta, \pi) \geq S(1; 0, 0) - M_0\sqrt{2}\alpha \geq (1 - \eta)m_0 > 0 \quad \text{and} \quad E_i(r; \delta, \pi) \geq (1 - \eta)m_0 > 0.$$

Therefore,  $\lambda^i(r; \delta, \pi) = \mu \frac{S(r; \delta, \pi)}{E_i(r; \delta, \pi)}$  is continuous and strictly positive on  $\mathcal{D}_2$ .

(Step 2: Sufficient condition for equilibrium existence and local uniqueness) Equilibrium

fixed point problem (63) can be rewritten as

$$LHS(r; \delta, \pi) = \frac{rc_s^2(r)}{c_s^1(r)} \left[ 1 - \frac{S(r; \delta, \pi)}{S(r; \delta, \pi) + E_2(r; \delta, \pi)} \right] - \left[ 1 - \frac{S(r; \delta, \pi)}{S(r; \delta, \pi) + E_1(r; \delta, \pi)} \right] = 0,$$

Because  $S(r; \delta, \pi)$  and  $E_i(r; \delta, \pi)$  are positive  $C^2$  functions on  $\mathcal{D}_2$ , the rational function  $LHS(r; \delta, \pi)$  is twice continuously differentiable on the same compact set. We now show that  $\frac{\partial LHS}{\partial r}(r; 0, 0)$  is non-zero at the symmetric benchmark  $r = 1$ . For simplicity, we write  $y_1 = (N - 1)(1 - \rho) + N(1 - \rho_f) > 0$  and  $y_2 = 1 + (N - 1)\rho + N\rho_f = 2N - y_1 > 0$ . Differentiating  $LHS(r; 0, 0)$  with respect to  $r$  and evaluating at  $r = 1$  gives

$$\frac{\partial LHS}{\partial r}(1; 0, 0) = \frac{2(z_0 + z_1\sigma^2 + z_2\sigma^4 + z_3\sigma^6)}{y_1^2(y_1 + (2N - 1)\sigma^2)(y_2 + \sigma^2)^2(2N - 1)^2},$$

where the denominator is strictly positive and

$$\begin{aligned} z_0 &= y_1^3 y_2^2 (2N^2 - 2N + 1) > 0, \\ z_1 &= \frac{(N - 1)(1 - \rho)}{(N - 1)(1 - \rho) - N\rho_f} y_2 y_1 f_{10}(y_2) - \frac{N\rho_f}{(N - 1)(1 - \rho) - N\rho_f} y_2 y_1^2 f_{11}(y_2), \\ z_2 &= \frac{(2N - 1)(N - 1)(1 - \rho)}{(N - 1)(1 - \rho) - N\rho_f} y_1 f_{20}(y_2) - \frac{(2N - 1)N\rho_f}{(N - 1)(1 - \rho) - N\rho_f} y_2 y_1 f_{21}(y_2), \\ z_3 &= \frac{(2N - 1)^2(N - 1)(1 - \rho)}{(N - 1)(1 - \rho) - N\rho_f} f_{30}(y_2) - \frac{(2N - 1)^2 N\rho_f}{(N - 1)(1 - \rho) - N\rho_f} y_1 f_{31}(y_2), \end{aligned}$$

where

$$\begin{aligned} f_{10}(y_2) &= (16N^4 - 16N^3 + 8N^2) + (8N^4 - 36N^3 + 32N^2 - 12N)y_2 + (4N^3 + 2N^2 - 6N + 3)y_2^2, \\ f_{11}(y_2) &= (4N^3 - 6N^2 + 4N) + (12N^3 - 22N^2 + 16N - 5)y_2, \\ f_{20}(y_2) &= 2N^2(2N - 1) + 4N(N - 1)(2N - 3)y_2 + (4N^2 - 3)y_2^2, \\ f_{21}(y_2) &= (20N^3 - 30N^2 + 14N) - (8N^2 - 14N + 7)y_2, \end{aligned}$$

also,  $f_{30}(y_2) = (4N^3 - 6N^2) + 6Ny_2 - (2N + 1)y_2^2$  and  $f_{31}(y_2) = (4N^2 - 4N) - (2N - 3)y_2$ . Since  $\rho_f < 0$ , the coefficients multiplying each  $f(\cdot)$ -term in  $z_1, z_2, z_3$  are strictly positive. In addition, since all  $f(\cdot)$  are positive for  $0 < y_2 < 1 + (N - 1)\rho < N$ , we obtain  $z_1 > 0, z_2 > 0, z_3 > 0$ . Therefore,

$$\frac{\partial LHS}{\partial r}(1; 0, 0) > 0. \quad (70)$$

Since  $LHS(r; 0, 0)$  is  $C^2$  on  $\mathcal{D}_2$  that contains  $r = 1$ , there exists  $e_{mn} > 0$  such that

$$\left| \frac{\partial LHS}{\partial r}(r; 0, 0) - \frac{\partial LHS}{\partial r}(1; 0, 0) \right| \leq \eta \frac{\partial LHS}{\partial r}(1; 0, 0) \quad \forall |r - 1| \leq e_{mn}.$$

We define  $\alpha_3 = \min\{\alpha, e_{mn}\} \leq \alpha$  and redefine  $\underline{r} = \frac{1}{1 + \alpha_3}$  and  $\bar{r} = 1 + \alpha_3$ . A compact set

$$\mathcal{D}_3 = \{(r, \delta, \pi) : \underline{r} \leq r \leq \bar{r}, \quad \delta^2 + \pi^2 \leq \alpha_3^2\} \subseteq \mathcal{D}_2$$

Then, all the regularity conditions considered in constructing  $\mathcal{D}_2$  remain valid on  $\mathcal{D}_3$ , and

$$\frac{\partial LHS}{\partial r}(r; 0, 0) \geq (1 - \eta) \frac{\partial LHS}{\partial r}(1; 0, 0) > 0$$

holds for all  $r \in [\underline{r}, \bar{r}]$ . Therefore,  $LHS(r; 0, 0)$  is strictly increasing on  $r \in [\underline{r}, \bar{r}]$ .

Furthermore, at the symmetric benchmark, the group symmetry implies

$$LHS(\underline{r}; 0, 0) = -\frac{c_s^1(\bar{r}; 0, 0)}{\bar{r}c_s^2(\bar{r}; 0, 0)} LHS(\bar{r}; 0, 0),$$

because  $\bar{r} = 1 + \alpha_3 = \frac{1}{\underline{r}}$ . From [Corollary 4](#) and the strict monotonicity of  $LHS(r; 0, 0)$  on  $\mathcal{D}_3$ , we obtain  $LHS(\bar{r}; 0, 0) = LHS(1 + \alpha_3; 0, 0) \neq 0$ , and hence,  $LHS(\underline{r}; 0, 0)LHS(\bar{r}; 0, 0) < 0$ . To expand this to asymmetric environments near the symmetric benchmark, we define

$$\begin{aligned} m_1 &= \min\{|LHS(\underline{r}; 0, 0)|, |LHS(\bar{r}; 0, 0)|\} > 0, \\ M_1 &= \max_{\mathcal{D}_3} \|\nabla_{(\delta, \pi)} LHS(r; \delta, \pi)\|_2 < \infty. \end{aligned}$$

Here,  $M_1$  is well-defined because  $LHS(r; \delta, \pi)$  is  $\mathcal{C}^1$  on the compact set  $\mathcal{D}_3$ . We take  $\eta \frac{m_1}{M_1} = +\infty$  if  $M_1 = 0$ . When  $\|(\delta, \pi)\|_2 \leq \min\{\alpha_3, \eta \frac{m_1}{M_1}\}$ , by the mean-value theorem,

$$|LHS(r; \delta, \pi) - LHS(r; 0, 0)| \leq M_1 \sqrt{\delta^2 + \pi^2} \leq \eta m_1 \quad \forall r \in \{\underline{r}, \bar{r}\}.$$

Thus, the sign of  $LHS(r; \delta, \pi)$  at each endpoint is the same as the sign of  $LHS(r; 0, 0)$ , and consequently,  $LHS(\underline{r}; \delta, \pi)LHS(\bar{r}; \delta, \pi) < 0$  holds. We further define variables

$$\begin{aligned} m_2 &= \min_{r \in [\underline{r}, \bar{r}]} \frac{\partial LHS}{\partial r}(r; 0, 0), \\ M_2 &= \max_{\mathcal{D}_3} \left\| \nabla_{(\delta, \pi)} \frac{\partial LHS}{\partial r}(r; \delta, \pi) \right\|_2 < \infty. \end{aligned}$$

Here,  $M_2$  is well defined because  $LHS(r; \delta, \pi)$  is  $\mathcal{C}^2$  on the compact set  $\mathcal{D}_3$ . We take  $\eta \frac{m_2}{M_2} = +\infty$  if  $M_2 = 0$ . When  $\|(\delta, \pi)\|_2 \leq \min\{\alpha_3, \eta \frac{m_2}{M_2}\}$ , by the mean-value theorem,

$$\left| \frac{\partial LHS}{\partial r}(r; \delta, \pi) - \frac{\partial LHS}{\partial r}(r; 0, 0) \right| \leq M_2 \sqrt{\delta^2 + \pi^2} \leq \eta m_2 \quad \forall r \in [\underline{r}, \bar{r}].$$

Then,  $LHS(r; \delta, \pi)$  is strictly increasing on  $r \in [\underline{r}, \bar{r}]$ , i.e.,

$$\frac{\partial LHS}{\partial r}(r; \delta, \pi) \geq \frac{\partial LHS}{\partial r}(r; 0, 0) - \eta m_2 \geq (1 - \eta)m_2 > 0.$$

Lastly, we define

$$\epsilon^*(N, \rho, \rho_f, \sigma^2) = \min\{\alpha_3, \frac{\eta m_1}{M_1}, \frac{\eta m_2}{M_2}\} > 0.$$

For any  $(\delta, \pi)$  such that  $\|(\delta, \pi)\|_2 \leq \epsilon^*(N, \rho, \rho_f, \sigma^2)$ ,  $LHS(r; \delta, \pi)$  is continuous and strictly increasing for  $r \in [\underline{r}, \bar{r}]$ , and satisfies  $LHS(\underline{r}; \delta, \pi)LHS(\bar{r}; \delta, \pi) < 0$ . By the Intermediate Value Theorem, there exists a unique solution  $r^*(\delta, \pi) \in (\underline{r}, \bar{r})$  of Eq. (63). By the definition

of  $\mathcal{D}_2$ , price impact  $\lambda^i(r; \delta, \pi)$  evaluated at  $r^*(\delta, \pi)$  is positive, and thus, this is the equilibrium. The closed form functions  $\{c_s^i(r), c_{p,i}(r)\}_i$  uniquely determine equilibrium inference coefficients and complete the characterization.

(Step 3: Equilibrium price impact  $\lambda^i(\delta, \pi)$  is continuous) To show the continuity of equilibrium price impacts in  $(\delta, \pi)$ , recall that for any  $(\delta, \pi)$  such that  $\|(\delta, \pi)\|_2 \leq \epsilon^*(N, \rho, \rho_f, \sigma^2)$  and for any  $r \in [\underline{r}, \bar{r}]$ ,  $LHS(r, \delta, \pi)$  is a  $C^2$  function, and  $\frac{\partial LHS}{\partial r}(r; \delta, \pi) \geq (1 - \eta)m_2 > 0$  so that  $\frac{\partial LHS}{\partial r}(r; \delta, \pi) \neq 0$ . Therefore, the Implicit Function Theorem shows that the unique solution  $r^*(\delta, \pi)$  is continuously differentiable. Thus, equilibrium price impact,

$$\lambda^i(\delta, \pi) = \lambda^i(r^*(\delta, \pi); \delta, \pi) = \mu \frac{S(r^*(\delta, \pi); \delta, \pi)}{E_i(r^*(\delta, \pi); \delta, \pi)},$$

is continuously differentiable, since  $S(r; \delta, \pi)$  and  $E_i(r; \delta, \pi)$  are  $C^2$  functions on  $\mathcal{D}_3$  and since  $E_i(r; \delta, \pi)$  is bounded away from zero. The proof is complete. ■

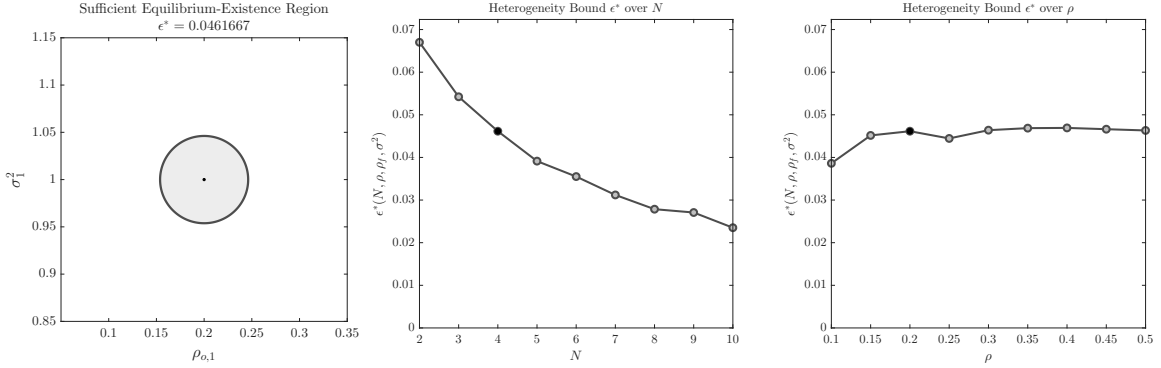


Figure 6: Sufficient condition for equilibrium existence in asymmetric two-group markets (Proposition 7). In the first panel, the gray circle represents a sufficient condition on  $(\rho_{0,1}, \sigma_1^2)$ , equivalent to  $\|(\delta, \pi)\|_2 \leq \epsilon^*$ , that equilibrium exists, is locally unique, and is continuous over  $(\delta, \pi)$ . The second and third panels show the heterogeneity bound  $\epsilon^*$  of the sufficient condition  $\|(\delta, \pi)\|_2 \leq \epsilon^*$  as the group size  $N$  and the within-group correlation  $\rho$  vary. We remark that  $\epsilon^*$  is maximized over auxiliary variables  $(\eta, \zeta) \in [0, 1]^2$  in the proof of Proposition 7; and  $N = 4, \rho = 0.2, \rho_f = -0.1, \sigma^2 = 1$ .

### Proposition 8 (Separating Liquidity and Learning Effects in Market Choices).

Consider a centralized market containing  $N_{cm} \geq 3$  symmetric traders after a marginal trader  $i$ 's entry and an OTC pair  $(i, j)$ . Correlations  $\bar{\rho}_{cm} = \frac{1}{N_{cm}-1} \sum_{k \neq i; k \in CM} \tilde{\rho}_{ik}$  and  $\tilde{\rho}_{ij}$  satisfy the equilibrium existence conditions (Corollaries 4 and 5).

- (i) The liquidity effect is larger in the OTC market if and only if  $\bar{\rho}_{cm} > \hat{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$ , where the range of  $\hat{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$  coincides with the equilibrium existence interval.
- (ii) The learning effect is larger in the OTC trade if and only if  $\bar{\rho}_{cm} > \underline{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$ , where  $\underline{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$  equals the lower equilibrium-existence bound for low  $\tilde{\rho}_{ij}$ .

- (iii) The OTC market provides better learning whenever it provides greater liquidity:  
 $\hat{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}) \geq \underline{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$  for all  $(N_{cm}, \sigma^2, \tilde{\rho}_{ij})$ . The equality holds if and only if  $\tilde{\rho}_{ij} = -1$ .

**Proof of Proposition 8 (Separating Liquidity and Learning Effects in Market Choices).** Using Corollaries 4 and 5, we compare the liquidity and learning effects between the two markets. Theorem 1 compares the net effects (expected utilities).

(Part (i): Liquidity effect) From Corollaries 4 and 5, the liquidity effect  $\frac{\mu(\mu+2\lambda^i)}{(\mu+\lambda^i)^2}$  is larger in the OTC market than in the centralized market if and only if

$$\begin{aligned} & (1 - \tilde{\rho}_{ij})(1 + \sigma^2 + \tilde{\rho}_{ij})(1 + (N_{cm} - 1)\bar{\rho}_{cm})(1 + \sigma^2 - \bar{\rho}_{cm}) \\ & > (N_{cm} - 1)(1 + \tilde{\rho}_{ij})(1 + \sigma^2 - \tilde{\rho}_{ij})(1 + \sigma^2 + (N_{cm} - 1)\bar{\rho}_{cm})(1 - \bar{\rho}_{cm}). \end{aligned} \quad (71)$$

Let us define  $\Gamma = \frac{(N_{cm}-1)(1+\tilde{\rho}_{ij})(1+\sigma^2-\tilde{\rho}_{ij})}{(1-\tilde{\rho}_{ij})(1+\sigma^2+\tilde{\rho}_{ij})}$ , which lies in  $[0, N_{cm} - 1)$  when  $-1 \leq \tilde{\rho}_{ij} < 0$ . An explicit derivation shows that  $\Gamma < 1$  if and only if  $\tilde{\rho}_{ij} < \hat{\rho} = \frac{N_{cm}\sigma^2 - \sqrt{N_{cm}^2\sigma^4 + 4(N_{cm}-2)^2(1+\sigma^2)}}{2(N_{cm}-2)}$ , when the OTC equilibrium exists. By definition,  $\hat{\rho}$  is strictly decreasing in  $N_{cm}$  and  $1/\sigma^2$ .

When  $\Gamma < 1$ , the leading coefficient is more negative in the l.h.s. of Eq. (71) than its r.h.s. Therefore, condition (71) holds if and only if  $\zeta_- < \bar{\rho}_{cm} < \zeta_+$ , where  $\zeta_- < 0$  and  $\zeta_+ > 0$  are the unique negative and positive solutions of

$$\Gamma = \frac{(1 + (N_{cm} - 1)\bar{\rho}_{cm})(1 + \sigma^2 - \bar{\rho}_{cm})}{(1 - \bar{\rho}_{cm})(1 + \sigma^2 + (N_{cm} - 1)\bar{\rho}_{cm})}. \quad (72)$$

The positive solution  $\zeta_+$  is strictly larger than 1, and thus, it is a redundant upper bound on the average correlation  $\bar{\rho}_{cm}$ . If  $\tilde{\rho}_{ij} = -1$ ,  $\zeta_- = -\frac{1}{N_{cm}-1}$  coincides with the lower bound of the equilibrium existence condition (Corollary 4); Otherwise,  $\zeta_- > -\frac{1}{N_{cm}-1}$  for all  $\tilde{\rho}_{ij} < 0$  satisfying  $\Gamma < 1$ . Moreover,  $\zeta_-$  is strictly increasing in  $\tilde{\rho}_{ij}$  because the right-hand side of Eq. (72) is strictly increasing in  $\bar{\rho}_{cm}$  and  $\frac{d\Gamma}{d\tilde{\rho}_{ij}} > 0$  so that  $\frac{d\zeta_-}{d\tilde{\rho}_{ij}} = \frac{d\Gamma}{d\tilde{\rho}_{ij}} \left( \frac{dRHS(72)}{d\bar{\rho}_{cm}} \right)^{-1} > 0$ .

When  $\Gamma > 1$ , the leading coefficient is higher in the l.h.s. of Eq. (71). The condition (71) holds if and only if  $\zeta_+ < \bar{\rho}_{cm}$ , subject to equilibrium existence. The corresponding negative solution  $\zeta_-$  is lower than  $-\frac{1}{N_{cm}-1}$  so that the alternative solution of quadratic inequality (71),  $\bar{\rho}_{cm} < \zeta_-$ , cannot support the equilibrium existence. Recall that the upper bound on  $\bar{\rho}_{cm}$  for the equilibrium existence (Corollary 4) is obtained as  $\lambda^i \rightarrow \infty$ . At this bound, the right-hand side of Eq. (72) equals  $N_{cm} - 1$ . Since that right-hand side is strictly increasing in  $\bar{\rho}_{cm}$  and  $\Gamma < N_{cm} - 1$ ,  $\zeta_+$  lies strictly inside the equilibrium-existence interval. The region  $\zeta_+ < \bar{\rho}_{cm}$  is therefore nonempty. Lastly, the implicit differentiation shows that the positive solution  $\zeta_+$  of Eq. (72) is strictly increasing in  $\tilde{\rho}_{ij}$ .

When  $\Gamma = 1$ , the inequality (71) is linear in  $\bar{\rho}_{cm}$ , and it holds if and only if  $\bar{\rho}_{cm} > 0$ . In fact, as  $\Gamma < 1$  increases to one, the negative cutoff  $\zeta_-$  converges to zero; As  $\Gamma > 1$  decreases

to one, the positive cutoff  $\zeta_+$  converges to zero. This completes the proof of part (i) by setting  $\hat{\rho}_{cm} = \zeta_-$  when  $\Gamma < 1$ ;  $\hat{\rho}_{cm} = \zeta_+$  when  $\Gamma > 1$ ; and  $\hat{\rho}_{cm} = 0$  when  $\Gamma = 1$ .

(Part (ii): *Learning effect*) From [Corollaries 4](#) and [5](#), the learning effect is larger in the OTC trade than in the centralized market if and only if

$$N_{cm}(1 - \tilde{\rho}_{ij})^2(1 + \sigma^2 - \bar{\rho}_{cm}) > 2(N_{cm} - 1)(1 + \sigma^2 - \tilde{\rho}_{ij})(1 - \bar{\rho}_{cm})^2. \quad (73)$$

Equalizing the left- and right-hand sides of Eq. (73) gives a quadratic equation for  $\bar{\rho}_{cm}$ . Letting  $\zeta_- < \zeta_+$  to be its two solutions, condition (73) holds if and only if  $\zeta_- < \bar{\rho}_{cm} < \zeta_+$ . Substituting  $\bar{\rho}_{cm} = 1$  into Eq. (73) gives  $N_{cm}(1 - \tilde{\rho}_{ij})^2\sigma^2 > 0$ , which implies  $\zeta_- < 1 < \zeta_+$ . Therefore,  $\zeta_+$  is a redundant upper bound on the average correlation  $\bar{\rho}_{cm}$ .

On the other hand,  $\zeta_- < \bar{\rho}_{cm}$  is a new condition, in addition to the equilibrium existence condition,  $\bar{\rho}_{cm}^- < \bar{\rho}_{cm} < \bar{\rho}_{cm}^+$ , in the centralized market ([Corollary 4](#)). First,  $\zeta_- < \bar{\rho}_{cm}^+$  holds for all  $-1 \leq \tilde{\rho}_{ij} < 0$ . To see this, we differentiate Eq. (73) with respect to  $\tilde{\rho}_{ij}$ , and find

$$\frac{\partial(LHS(73) - RHS(73))}{\partial \tilde{\rho}_{ij}} = -2[N_{cm}(1 - \tilde{\rho}_{ij})(1 + \sigma^2 - \bar{\rho}_{cm}) - (N_{cm} - 1)(1 - \bar{\rho}_{cm})^2] < 0. \quad (74)$$

In addition, when  $\tilde{\rho}_{ij} = 0$  and  $\bar{\rho}_{cm} = \bar{\rho}_{cm}^+$ , the l.h.s. minus the r.h.s. of Eq. (73) becomes

$$\frac{N_{cm} + 2\sigma^2}{N_{cm} - 2}[(N_{cm} - 2 + 2(N_{cm} - 1)\sigma^2)\bar{\rho}_{cm}^+ - (N_{cm} - 2)(1 + \sigma^2)],$$

which is positive if and only if  $\bar{\rho}_{cm}^+ > \frac{(N_{cm}-2)(1+\sigma^2)}{N_{cm}-2+2(N_{cm}-1)\sigma^2}$ . Recall that  $\bar{\rho}_{cm}^+$  is the positive solution of

$$\frac{(1 + (N_{cm} - 1)\bar{\rho}_{cm})(1 + \sigma^2 - \bar{\rho}_{cm})}{(1 - \bar{\rho}_{cm})(1 + \sigma^2 + (N_{cm} - 1)\bar{\rho}_{cm})} = (N_{cm} - 1). \quad (75)$$

Replacing  $\bar{\rho}_{cm}$  by  $\frac{(N_{cm}-2)(1+\sigma^2)}{N_{cm}-2+2(N_{cm}-1)\sigma^2}$  on the l.h.s. of Eq. (75) gives

$$\frac{(N_{cm} - 1)2((N_{cm} - 1)(1 + \sigma^2) - 1)}{(N_{cm} - 2)^2 + 2((N_{cm} - 1)(1 + \sigma^2) - 1)} < (N_{cm} - 1),$$

which shows that  $\bar{\rho}_{cm}^+ > \frac{(N_{cm}-2)(1+\sigma^2)}{N_{cm}-2+2(N_{cm}-1)\sigma^2}$  holds for all  $N_{cm} \geq 3$ ; Eq. (73) holds at  $\tilde{\rho}_{ij} = 0$  and  $\bar{\rho}_{cm} = \bar{\rho}_{cm}^+$ ; and combined with Eq. (74),  $\zeta_- < \bar{\rho}_{cm}^+$  holds for all  $-1 \leq \tilde{\rho}_{ij} < 0$ .

Second, substituting the lower bound of equilibrium existence  $\bar{\rho}_{cm}^- = -\frac{1}{N_{cm}-1}$  implies that  $-\frac{1}{N_{cm}-1} < \zeta_-$  holds if and only if

$$\tilde{\rho}_{ij} > \frac{\sigma^2 - \sqrt{(\frac{N_{cm}}{N_{cm}-1})^2 + (\sigma^2 + \frac{N_{cm}}{N_{cm}-1})2\sigma^2 \frac{N_{cm}}{N_{cm}-1}}}{(\sigma^2 + \frac{N_{cm}}{N_{cm}-1})}.$$

Here, the lower bound on  $\tilde{\rho}_{ij}$  is strictly between  $-1$  and  $0$ . Therefore, when  $\tilde{\rho}_{ij}$  is sufficiently low, the necessary and sufficient condition  $\zeta_- < \bar{\rho}_{cm}$  for Eq. (73) is replaced by  $-\frac{1}{N_{cm}-1} < \bar{\rho}_{cm}$  to account for the centralized-market equilibrium existence. This completes the proof

that condition (73) holds if and only if  $\underline{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}) = \max\{\zeta_-, -\frac{1}{N_{cm}-1}\} < \bar{\rho}_{cm}$ .

(Part (iii): *Liquidity- and learning-effect cutoffs*) We now show that condition (73) holds whenever condition (71) holds. To show this, let us define

$$A = \frac{N_{cm} - 2 + N_{cm}\tilde{\rho}_{ij}}{2(N_{cm} - 1)} = \lim_{\sigma^2 \downarrow 0} \underline{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}).$$

At  $\bar{\rho}_{cm} = A$ , condition (73) is simplified into

$$N_{cm}(1 - \tilde{\rho}_{ij})^2 \sigma^2 + \frac{N_{cm}^2(1 - \tilde{\rho}_{ij})^3}{2(N_{cm} - 1)} > \frac{N_{cm}}{2(N_{cm} - 1)} N_{cm}(1 - \tilde{\rho}_{ij})^2 \sigma^2 + \frac{N_{cm}^2(1 - \tilde{\rho}_{ij})^3}{2(N_{cm} - 1)},$$

which holds for all  $N_{cm} \geq 3$  and  $-1 \leq \tilde{\rho}_{ij} < 0$ . Thus,  $\zeta_- < A$ . Since  $-\frac{1}{N_{cm}-1} \leq A$ ,  $\underline{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}) = \max\{\zeta_-, -\frac{1}{N_{cm}-1}\} \leq A$ . The equality holds if and only if  $\tilde{\rho}_{ij} = -1$ .

On the other hand, substituting  $\bar{\rho}_{cm} = A$  into Eq. (71) gives an equivalent condition:

$$0 > N_{cm}(1 - \tilde{\rho}_{ij})(1 + \tilde{\rho}_{ij}) + \sigma^2(2 + (N_{cm} - 2)(1 + \tilde{\rho}_{ij})),$$

which fails to hold for all  $N_{cm} \geq 3$  and  $-1 \leq \tilde{\rho}_{ij} < 0$ . This implies that  $\hat{\rho}_{cm}(N_{cm}, \sigma^2, \tilde{\rho}_{ij}) \geq A$  whose equality holds if and only if  $\tilde{\rho}_{ij} = -1$ . This completes the proof. ■

**Example 4 (Heterogeneous Weights on Asset-Value Components).** This example studies trader asymmetry with heterogeneous weights on the asset-value components:  $(\omega_{cv}^i, \omega_{iv}^i)$  vary across traders  $i$ . Example 2 in Section 4.1 shows that heterogeneous weights and the resulting heterogeneous correlation profiles can make a trader with more extreme average exposures (i.e., a more negative average correlation) less attracted to the OTC market, contrary to the prediction of the symmetric benchmark (Theorem 1). This raises a natural question: how does weight heterogeneity alter the relative liquidity and learning effects across the two markets and, hence, traders' entry incentives? In particular, can one effect favor OTC trading while the other favors centralized trading?

To answer these questions, we consider a marginal trader who chooses between the centralized and OTC markets in a two-group environment extending Example 1. There are  $I = 4M$  traders, equally divided into buyers and sellers, and the idiosyncratic-value correlations satisfy  $\rho_o = -\frac{I}{I-2}\rho > 0$  within each group and  $\rho < 0$  across groups. Each buyer and seller group is further divided equally into a high- $\omega_{iv}^2$  type and a low- $\omega_{iv}^2$  type. Thus,  $\omega_{iv}^i = \omega_{iv}^H$  for  $i \in B_H \cup S_H$  and  $\omega_{iv}^i = \omega_{iv}^L$  for  $i \in B_L \cup S_L$ , where  $0 < \omega_{iv}^L < \omega_{iv}^H < 1$ .

Decomposing the marginal trader's market-choice decision into the liquidity and learning effects (Eq. (9)), weight heterogeneity changes the extent to which each effect motivates OTC trading relative to centralized-market trading. When traders are symmetric, the OTC market provides either better learning alone or both better learning and greater liquidity (Proposition 8, Figure 7A). With sufficiently heterogeneous weights,  $\omega_{iv}^H > \omega_{iv}^L$ , the OTC

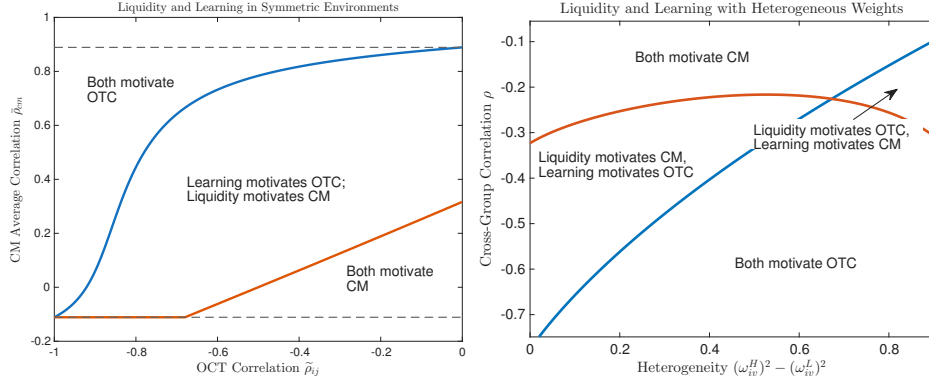


Figure 7: Liquidity and learning comparisons between the centralized and OTC markets. The blue lines equalize the liquidity effects across the two markets, while the red lines equalize the learning effects. Panel A considers the symmetric environment of [Example 1](#):  $N = 10$  and  $\sigma^2 = 1$ . The dashed lines indicate the equilibrium-existence bounds. Panel B considers the asymmetric environment with heterogeneous asset-value weights in [Example 4](#):  $I = 8$ ,  $\sigma^2 = 14$ , and  $\omega_{iv}^H = 0.95$ . The degree of heterogeneity is measured by  $(\omega_{iv}^H)^2 - (\omega_{iv}^L)^2$ .

market can instead provide greater liquidity even when it provides worse learning than the centralized market. This suggests that different dimensions of trader heterogeneity—pairwise correlations, individual correlation profiles (equivalently, asset-value weights), and individual information precision—may play different roles in determining the relative liquidity and learning effects across trading venues.  $\square$

### A.3 Proofs of Technical Lemmas

**Lemma 1.** *Suppose  $I \geq 3$  and  $-1 < \rho_{ij} < 0 \leq \bar{\rho} < 1$ . If  $a_n \geq 0$ , then  $a_{n-1} > 0$ , for all  $n = 1, \dots, 5$ . Moreover,  $a_0 > 0$ .*

**Proof of Lemma 1.** Eq. (21) defines a function  $f(\sigma^2) = a_5\sigma^{10} + a_4\sigma^8 + a_3\sigma^6 + a_2\sigma^4 + a_1\sigma^2 + a_0$ . The coefficients of this polynomial function can be expressed as follows: with common terms  $X = (I - 1)(1 - \bar{\rho}) - (1 + (I - 1)\bar{\rho})$ ,  $Y = (I - 1)((1 + \rho_{ij}) - (1 - \rho_{ij}))$ , and  $Z = (I - 2)(1 + (I - 1)\bar{\rho})(1 - \bar{\rho})$ , we get  $a_5 = X + Y$  and

$$\begin{aligned} a_4 = & X((1 + (I - 1)\bar{\rho})(1 - \bar{\rho}) + 2(1 + \rho_{ij}) + (1 - \rho_{ij})) \\ & + Y(1 - \rho_{ij}^2 + 2(1 + (I - 1)\bar{\rho}) + (1 - \bar{\rho})) + Z, \end{aligned} \quad (76)$$

$$\begin{aligned} a_3 = & X((1 + (I - 1)\bar{\rho})(1 - \bar{\rho})(2(1 + \rho_{ij}) + (1 - \rho_{ij})) + (1 + \rho_{ij})^2 + 2(1 - \rho_{ij}^2)) \\ & + Y((1 - \rho_{ij}^2)(2(1 + (I - 1)\bar{\rho}) + (1 - \bar{\rho})) + (1 + (I - 1)\bar{\rho})^2 + 2(1 + (I - 1)\bar{\rho})(1 - \bar{\rho})) \\ & + Z((1 + (I - 1)\bar{\rho})(1 - \bar{\rho}) + 2(1 + \rho_{ij}) + (1 - \rho_{ij})), \end{aligned} \quad (77)$$

$$\begin{aligned}
a_2 = & X((1 + (I - 1)\bar{\rho})(1 - \bar{\rho})((1 + \rho_{ij})^2 + 2(1 - \rho_{ij}^2)) + (1 + \rho_{ij})(1 - \rho_{ij}^2)) \\
& + Y((1 - \rho_{ij}^2)(2(1 + (I - 1)\bar{\rho})(1 - \bar{\rho}) + (1 + (I - 1)\bar{\rho})^2) + (1 + (I - 1)\bar{\rho})^2(1 - \bar{\rho})) \\
& + Z((1 + (I - 1)\bar{\rho})(1 - \bar{\rho})(2(1 + \rho_{ij}) + (1 - \rho_{ij})) + (1 + \rho_{ij})^2 + 2(1 - \rho_{ij}^2)), \quad (78)
\end{aligned}$$

$$\begin{aligned}
a_1 = & X(1 + (I - 1)\bar{\rho})(1 - \bar{\rho})(1 + \rho_{ij})(1 - \rho_{ij}^2) \\
& + Y(1 - \rho_{ij}^2)(1 + (I - 1)\bar{\rho})^2(1 - \bar{\rho}) \\
& + Z((1 + (I - 1)\bar{\rho})(1 - \bar{\rho})((1 + \rho_{ij})^2 + 2(1 - \rho_{ij}^2)) + (1 + \rho_{ij})(1 - \rho_{ij}^2)), \quad (79)
\end{aligned}$$

$$a_0 = (I - 2)(1 - \bar{\rho})^2(1 + (I - 1)\bar{\rho})^2(1 - \rho_{ij})(1 + \rho_{ij})^2 > 0.$$

The coefficient  $a_0$  is positive for any  $I \geq 3$  and  $-1 < \rho_{ij} < 0 \leq \bar{\rho} < 1$ .

( $a_5 \geq 0$  implies  $a_4 > 0$ ) Suppose  $a_5 \geq 0$ . Since  $a_5 = X + Y = I - 2 - 2(I - 1)(\bar{\rho} - \rho_{ij})$ , it is equivalent to assuming  $\bar{\rho} \leq \rho_{ij} + \frac{I-2}{2(I-1)} < \frac{I-2}{2(I-1)}$ . Recall the existence condition of symmetric centralized-market equilibrium (Corollary 4). Because

$$\frac{I - 2}{2(I - 1)} < \frac{(I - 2)^2 - 2(I - 1)\sigma^2 + \sqrt{I^2(I - 2)^2 + 4(I - 1)^2\sigma^4}}{2(I - 1)(I - 2)}$$

for any  $\sigma^2 > 0$ , it is feasible for an equilibrium to exhibit  $a_5 \geq 0$ . We first prove that  $a_4 > 0$  at the boundary  $a_5 = 0$  and then show that  $a_4$  increases as  $\rho_{ij}$  increases when  $a_5 \geq 0$ . Substituting  $\rho_{ij} = \bar{\rho} - \frac{I-2}{2(I-1)}$  into Eq. (76) gives

$$a_4 = \frac{I - 2}{4(I - 1)^2} [8(I - 1)^3\bar{\rho}^3 + 12(I - 1)^2\bar{\rho}^2 - 2(I - 1)(I - 4)\bar{\rho} + 3(I - 1)^2 + 1].$$

This shows that  $a_4 > 0$  when  $\rho_{ij} = \bar{\rho} - \frac{I-2}{2(I-1)}$  and  $I \leq 4$ . When  $I \geq 5$ ,  $\bar{\rho} < \frac{I-2}{2(I-1)}$  implies  $-2(I - 1)(I - 4)\bar{\rho} + 3(I - 1)^2 + 1 > -(I - 2)(I - 4) + 3(I - 1)^2 + 1 = 2I^2 - 4 > 0$ . Therefore, we get  $a_4 > 0$  when  $a_5 = 0$ . Differentiating  $a_4$  in Eq. (76) with respect to  $\rho_{ij}$  gives

$$\begin{aligned}
\frac{\partial a_4}{\partial \rho_{ij}} &= 3(I - 1)(1 - \bar{\rho}) - (1 + (I - 1)\bar{\rho}) + 4(I - 1)(1 + (I - 1)\bar{\rho}) + 2(I - 1)(1 - 3\rho_{ij}^2) \\
&> (3I - 4) + 4(I - 1)(I - 2)\bar{\rho} > 0.
\end{aligned}$$

This completes the proof that  $a_4 > 0$  when  $\rho_{ij} \geq \bar{\rho} - \frac{I-2}{2(I-1)}$ , or equivalently when  $a_5 \geq 0$ .

( $a_4 \geq 0$  implies  $a_3 > 0$ ) Collecting terms by powers of  $(I - 3)$  rewrites Eq. (77) as

$$a_3 = (2 + \frac{I - 3}{2}\bar{\rho})a_4 + \frac{(I - 3)^2\bar{\rho}}{2}A_2 + \frac{I - 3}{2}A_1 + A_0,$$

where

$$\begin{aligned}
A_2 &= -4\rho_{ij}\bar{\rho}(1 - \bar{\rho}) + 2(1 - \bar{\rho}(1 - \bar{\rho}^2)) - 6\rho_{ij}^3 + 3\rho_{ij} + 1, \\
A_1 &= -4\rho_{ij}^3(1 + 6\bar{\rho}) - 16\rho_{ij}\bar{\rho}^2(1 - \bar{\rho}) + (4\bar{\rho} - 2)\rho_{ij}^2 + (15\bar{\rho} - 4)\rho_{ij} + 8\bar{\rho}^4 + 4\bar{\rho}^3 - 16\bar{\rho}^2 + 19\bar{\rho}, \\
A_0 &= -4\rho_{ij}(3\rho_{ij}^2\bar{\rho} + \rho_{ij}^2 + 2\bar{\rho}^2(1 - \bar{\rho})) + (4\bar{\rho} - 1)\rho_{ij}^2 + (10\bar{\rho} - 6)\rho_{ij} + 4\bar{\rho}^4 + 4\bar{\rho}^3 - 11\bar{\rho}^2 + 12\bar{\rho}.
\end{aligned}$$

Here,  $A_2 > 0$  because  $\rho_{ij} < 0$ ,  $0 \leq \bar{\rho} < 1$ , and  $-6\rho_{ij}^3 + 3\rho_{ij} + 1 \geq 1 - \frac{2}{\sqrt{6}} > 0$ . Since  $-2\rho_{ij}^2 > 2\rho_{ij}$ , when  $\bar{\rho} \leq \frac{2}{15}$ ,  $A_1 > \bar{\rho}(8\bar{\rho}^3 + 4\bar{\rho}^2 - 16\bar{\rho} + 19) \geq 0$ ; instead, when  $\bar{\rho} \geq \frac{2}{15}$ ,  $A_1 > 8\bar{\rho}^4 + 4\bar{\rho}^3 - 16\bar{\rho}^2 + 4\bar{\rho} + 2 = 2(2\bar{\rho}^2 - 1)^2 + 4\bar{\rho}(1 - \bar{\rho})^2 \geq 0$ . Lastly,  $A_0 > \bar{\rho}(4\bar{\rho}^3 + 4\bar{\rho}^2 - 11\bar{\rho} + 12) \geq 0$  when  $0 \leq \bar{\rho} < \frac{1}{2}$ , and  $A_0 \geq (2\bar{\rho}^2 + \bar{\rho} - 2)^2 + 3 + 2(2\bar{\rho} - 1)(1 - \bar{\rho}) > 0$  when  $1/2 \leq \bar{\rho} < 1$ . Since  $A_2, A_1, A_0 > 0$ , we conclude that  $a_4 \geq 0$  implies  $a_3 > 0$ .

( $a_3 \geq 0$  implies  $a_2 > 0$ ) Collecting the terms of Eq. (78) by powers of  $(I - 3)$  gives

$$a_2 = (1 + \bar{\rho})a_3 + (I - 3)^3\bar{\rho}^2 A_3 + (I - 3)^2\bar{\rho} A_2 + (I - 3)A_1 + A_0,$$

where

$$\begin{aligned} A_3 &= (1 - \bar{\rho})^2(2 - \bar{\rho}) - \rho_{ij}(6\bar{\rho} - \bar{\rho}^2 - 3) - 2\rho_{ij}^3, \\ A_2 &= (1 - \bar{\rho})^2(4 + 2\bar{\rho} - 5\bar{\rho}^2) + \rho_{ij}(3\bar{\rho}^3 - 22\bar{\rho}^2 - 9\bar{\rho} + 4) - 2(1 - \bar{\rho})^2\rho_{ij}^2 - 4(1 + \bar{\rho})\rho_{ij}^3, \\ A_1 &= \bar{\rho}(-8\bar{\rho}^4 + 2\bar{\rho}^3 + 19\bar{\rho}^2 - 10\bar{\rho} + 2) - \rho_{ij}(22\bar{\rho}^3 + 47\bar{\rho}^2 + 8\bar{\rho} + 2) \\ &\quad + \rho_{ij}^2(-8\bar{\rho}^3 + 10\bar{\rho}^2 - \bar{\rho} - 2) + \rho_{ij}^3(6\bar{\rho}^2 - 6\bar{\rho} - 1), \\ A_0 &= \bar{\rho}(-4\bar{\rho}^4 - 12\bar{\rho}^3 + 19\bar{\rho}^2 + 10\bar{\rho} + 2) - \rho_{ij}(4\bar{\rho}^4 + 4\bar{\rho}^3 + 37\bar{\rho}^2 + 24\bar{\rho} + 6) \\ &\quad + \rho_{ij}^2(-8\bar{\rho}^3 + 4\bar{\rho}^2 + 3\bar{\rho} - 2) - \rho_{ij}^3(1 - 4\bar{\rho} - 12\bar{\rho}^2). \end{aligned}$$

It is immediate that  $A_3 > 0$  if  $6\bar{\rho} - \bar{\rho}^2 - 3 \geq 0$ . Otherwise,  $(6\bar{\rho} - \bar{\rho}^2 - 3) \geq -3(1 - \bar{\rho})^2$ , and thus,  $A_3 \geq (1 - \bar{\rho})^2(2 - \bar{\rho}) - \sqrt{2}(1 - \bar{\rho})^3 \geq 0$ . Since  $\rho_{ij}^2 \leq -\rho_{ij}$ , we get

$$A_2 \geq (1 - \bar{\rho})^2(4 + 2\bar{\rho} - 5\bar{\rho}^2) + (-\rho_{ij})(-3\bar{\rho}^3 + 20\bar{\rho}^2 + 13\bar{\rho} - 6) + 4(1 + \bar{\rho})(-\rho_{ij})^3,$$

which, if  $-3\bar{\rho}^3 + 20\bar{\rho}^2 + 13\bar{\rho} - 6 \geq 0$ , is positive. Otherwise,  $\bar{\rho} < \frac{1}{2}$  and hence  $A_2 \geq (1 - \bar{\rho})^2(4 + 2\bar{\rho} - 5\bar{\rho}^2) - 2\sqrt{\frac{2}{1+\bar{\rho}}}(1 - \bar{\rho})^3 > 0$ . Also, since  $6\bar{\rho}^2 - 6\bar{\rho} - 1 < 0$ ,  $(22\bar{\rho}^3 + 47\bar{\rho}^2 + 8\bar{\rho} + 2) + (-8\bar{\rho}^3 + 10\bar{\rho}^2 - \bar{\rho} - 2) \geq 0$ , we get  $A_1 > \bar{\rho}(-8\bar{\rho}^4 + 2\bar{\rho}^3 + 19\bar{\rho}^2 - 10\bar{\rho} + 2) \geq 0$ . Since  $(-4\bar{\rho}^4 - 12\bar{\rho}^3 + 19\bar{\rho}^2 + 10\bar{\rho} + 2) > 0$ , if  $1 - 4\bar{\rho} - 12\bar{\rho}^2 \geq 0$ ,  $A_0 > -\rho_{ij}(4\bar{\rho}^4 - 4\bar{\rho}^3 + 41\bar{\rho}^2 + 27\bar{\rho} + 4) > 0$ ; otherwise,  $A_0 > -\rho_{ij}(4\bar{\rho}^4 - 4\bar{\rho}^3 + 29\bar{\rho}^2 + 23\bar{\rho} + 5) > 0$ . This shows  $A_3, A_2, A_1, A_0 > 0$  and thus  $a_2 - (1 + \bar{\rho})a_3 > 0$ . Consequently,  $a_3 \geq 0$  implies  $a_2 > 0$ .

( $a_2 \geq 0$  implies  $a_1 > 0$ ) We simplify Eq. (79) by defining  $x = \frac{\bar{\rho}}{1 - \bar{\rho}} \geq 0$  and  $y = \frac{-\rho_{ij}}{1 + \rho_{ij}} > 0$ :

$$a_1 - \frac{1}{2}a_2 = \frac{(1 - \bar{\rho})^4(1 + \rho_{ij})^3}{2}((I - 3)^3x^2 A_3 + (I - 3)^2x A_2 + (I - 3)A_1 + A_0),$$

where

$$\begin{aligned} A_3 &= (3 + 6y + y^2) + x(2y + 4y^2 + 2y^3) + x^2(2y + 4y^2), \\ A_2 &= (4 + 10y + 2y^2) + x(19 + 48y + 23y^2 + 6y^3) + x^2(24y + 48y^2 + 16y^3) + x^3(16y + 32y^2), \\ A_1 &= 2y + y^2 + x(12 + 44y + 28y^2 + 6y^3) + x^2(39 + 138y + 123y^2 + 32y^3) \\ &\quad + x^3(4 + 92y + 172y^2 + 42y^3) + x^4(1 + 44y + 84y^2), \end{aligned}$$

$$\begin{aligned}
A_0 &= 6y + 9y^2 + 2y^3 + x(12 + 62y + 70y^2 + 16y^3) + x^2(35 + 156y + 185y^2 + 42y^3) \\
&\quad + x^3(14 + 118y + 192y^2 + 36y^3) + x^4(3 + 42y + 72y^2).
\end{aligned}$$

Since  $x \geq 0$  and  $y > 0$ , it is immediate that  $A_3, A_2 \geq 0$  and  $A_1, A_0 > 0$ . This establishes that  $a_1 - \frac{1}{2}a_2 > 0$ , so that  $a_2 \geq 0$  implies  $a_1 > 0$ . This completes the proof of [Lemma 1](#). ■

**Lemma 2.** For any  $I \leq \hat{I} = 24$  and any  $N_{cm} \in [4, I]$ ,  $\hat{\sigma}_1^2(N_{cm}, -\frac{I-2}{I}) > \sigma_D^2(I, N_{cm})$  holds.

**Proof of Lemma 2.** The proof of [Lemma 1](#) gives closed-form expressions of all coefficients in  $f(\sigma^2) = \sum_{k=0}^5 a_k(\sigma^2)^k$  (Eq. (21)) with  $I = N_{cm}$  and arbitrary  $(\bar{\rho}, \rho_{ij})$ . Replacing  $I$  by  $N_{cm}$  and substituting  $\bar{\rho} = -\frac{I-N_{cm}}{I(N_{cm}-1)}$  and  $\rho_{ij} = -\frac{I-2}{I}$  ([Proposition 3](#) (ii)), we get

$$\begin{aligned}
a_5 &= \frac{-(I-2)}{I}, \\
a_4 &= \frac{-2[I^2(N_{cm}-1)(N_{cm}+2) - I(2N_{cm}^2 + 5N_{cm} - 6) - (N_{cm}^2 - 6N_{cm} + 4)]}{I^3(N_{cm}-1)}, \\
a_3 &= \frac{-N_{cm}[I^2(N_{cm}^3 + 6N_{cm}^2 - 22N_{cm} + 14) - 2I(N_{cm}^3 + 16N_{cm}^2 - 38N_{cm} + 20)]}{I^4(N_{cm}-1)^2} \\
&\quad + \frac{-N_{cm}(-N_{cm}^3 + 28N_{cm}^2 - 52N_{cm} + 24)}{I^4(N_{cm}-1)^2}, \\
a_2 &= \frac{2(I-1)N_{cm}[(3N_{cm}-2)I^2 - 2(N_{cm}-1)(N_{cm}^2 - 7N_{cm} + 2)I + N_{cm}(N_{cm}-2)(7N_{cm}-6)]}{I^5(N_{cm}-1)^2}, \\
a_1 &= \frac{4(I-1)^2N_{cm}^2(3N_{cm}^2 + 4(I-1)N_{cm} - 2I)}{I^6(N_{cm}-1)^2}, \\
a_0 &= \frac{8(I-1)^3N_{cm}^4}{I^7(N_{cm}-1)^2}.
\end{aligned}$$

Here, we normalize all coefficients by  $\frac{1}{N_{cm}-2}$  and abuse notation by using the same notation  $a_k$  and  $f(\sigma^2)$ . The normalization does not affect the root of  $f(\sigma^2) = 0$ . From the definition (36) of  $P(\sigma^2)$ , we obtain the following polynomial division:

$$\begin{aligned}
f(\sigma^2) - P(\sigma^2) &\left( \frac{\sigma^2}{I^6(N-1)^3} - \frac{I(I-2)(N-1)(N-4) - 2(N-2)(N^2-2+2-I)}{I^8(I-2)(N-1)^4} \right) \\
&= \frac{N(N-2)}{I^8(I-2)(N-1)^4} [r_3(\sigma^2)^3 + r_2(\sigma^2)^2 + r_1(\sigma^2) + r_0],
\end{aligned} \tag{80}$$

where we write  $N = N_{cm}$  for notational convenience, and

$$r_0 = 2N^3(I-1)^3[I^2(N^2 + N - 2) + I(-2N^2 + 4) + (-2N^3 + 4N^2 - 4N)], \tag{81}$$

$$\begin{aligned}
r_1 &= -N(I-1)^2I[I^3(2N^3 - 16N^2 + 18N - 4) + I^2(N^5 - 12N^4 + 17N^3 + 30N^2 - 48N + 8) \\
&\quad + I(-2N^5 + 26N^4 - 56N^3 + 16N^2 + 24N) + (-2N^6 + 22N^5 - 60N^4 + 76N^3 - 40N^2)], \tag{82}
\end{aligned}$$

$$\begin{aligned}
r_2 &= 2(I-1)I^2(N-1)[I^4(2N-1) + I^3(-N^4 + 2N^3 + 7N^2 - 16N + 4) \\
&\quad + I^2(6N^4 - 17N^3 - 7N^2 + 32N - 4) + I(3N^5 - 20N^4 + 45N^3 - 24N^2 - 16N) \\
&\quad + (-9N^5 + 32N^4 - 46N^3 + 28N^2)], \tag{83}
\end{aligned}$$

$$\begin{aligned}
r_3 &= -I^3(N-1)^2[I^4(N^2 - 4N - 1) + I^3(2N^2 + 2N + 12) + I^2(-8N^3 + 12N^2 - 4N - 28) \\
&\quad + I(16N^3 - 36N^2 + 28N + 16) + (4N^4 - 20N^3 + 32N^2 - 24N)]. \tag{84}
\end{aligned}$$

Recall that  $f(\sigma^2) = \sum_{k=0}^5 a_k(\sigma^2)^k$  and  $P(\sigma^2) = \sum_{k=0}^4 p_k(\sigma^2)^k$  have negative leading coefficients  $a_5, p_4 < 0$  and positive constant terms  $a_0, p_0 > 0$ . Therefore, if  $R(\sigma_D^2) > 0$ , then  $\hat{\sigma}^2 > \sigma_D^2$ . In the rest of the proof, we will focus on the residual function  $R(\sigma^2)$  instead of  $f(\sigma^2)$ , along with  $P(\sigma^2)$ .

Because the signs of coefficients  $\{r_k\}_{0 \leq k \leq 3}$  of  $R(\sigma^2)$  vary for  $I \geq N \geq 4$ , we consider a function  $R(\sigma^2) + \lambda P(\sigma^2) = \sum_{k=0}^4 (r_k + \lambda p_k)(\sigma^2)^k$  for some scalar  $\lambda \in \mathbb{R}$ , with  $r_4 = 0$ , and then show  $R(\sigma_D^2) + \lambda P(\sigma_D^2) = R(\sigma_D^2) > 0$ . A scalar  $\lambda$  and a cutoff  $\hat{I} > 4$  will be chosen such that  $r_k + \lambda p_k > 0$  for all  $0 \leq k \leq 4$  and  $4 \leq N \leq I \leq \hat{I}$ . We remark that the introduction of  $\lambda$  is to obtain a tractable sufficient condition  $I \leq \hat{I}$ . Yet, requiring all coefficients of  $R(\sigma^2) + \lambda P(\sigma^2)$  to be positive is stronger than the condition  $R(\sigma_D^2) > 0$  itself. Consequently, the resulting bound  $I < \hat{I}$  is restrictive and can give a lower  $\hat{I}$  than the maximal value for which  $R(\sigma_D^2) > 0$  holds. See ft. 38.

To choose  $\lambda$ , we first identify the signs of some coefficients of  $R(\sigma^2)$  and  $P(\sigma^2)$ . From Eq. (36), it is immediate that  $p_4 < 0$  and  $p_0 > 0$  for all  $I \geq N \geq 4$ . In addition, rewriting the expressions for  $p_2$  and  $p_3$  shows that

$$\begin{aligned}
p_3 &= -I^3 N(N-1)^2 [3(N-1)(I-N)^2 + (6(N-1)^2 - 4)(I-N) + N(N-1)(3N-8)] < 0, \\
p_2 &= -I^2 N^2 (I-1)(N-1) [(3(N-1)^2 - 5)(I-N) + 3N(N-1)(N-4)] \leq 0,
\end{aligned}$$

for all  $I \geq N \geq 4$ , where  $p_2 = 0$  if and only if  $I = N = 4$ . In addition,  $r_0 > 0$  for all  $I \geq N \geq 4$ . This can be seen by differentiating  $\frac{r_0}{2N^3(I-1)^3}$  from Eq. (81) with respect to  $I$ :

$$\frac{\partial}{\partial I} \frac{r_0}{2N^3(I-1)^3} = 2I(N^2 + N - 2) + (-2N^2 + 4) \geq 2N(N^2 - 4) + 4(N+1) > 0 \quad \forall N \geq 4.$$

The first inequality follows from  $I \geq N$ . This implies that, over  $N \geq 4$ ,  $\frac{r_0}{2N^3(I-1)^3}$  is minimized at  $I = N$ , at which

$$\left. \frac{r_0}{2N^3(I-1)^3} \right|_{I=N} = N^2(N-1)(N-2) > 0 \quad \forall N \geq 4.$$

This shows that  $r_0 > 0$  for all  $I \geq N \geq 4$ .

Given the signs of coefficients, we choose  $\lambda = -\frac{r_0}{p_0} < 0$ . Then, the leading coefficient of  $R(x) + \lambda P(x)$  is  $\lambda p_4 = -\frac{r_0}{p_0} p_4 > 0$ , and the constant term is  $r_0 + \lambda p_0 = 0$ . Therefore, it

remains to find  $\widehat{I} > 4$  such that

$$p_0 r_k - r_0 p_k > 0 \quad \text{for all } 4 \leq N \leq I \leq \widehat{I} \text{ and } k = 1, 2, 3.$$

First, for  $k = 1$ , the definitions of  $p_0, p_1, r_0, r_1$  give

$$p_0 r_1 - r_0 p_1 = -4I^2 N^6 (I - 2)(I - 1)^5 (N - 1) [I(N^2 - 6N + 4) - 2N^3 + 9N^2 - 6N].$$

We show that  $G_1(I, N) = I(N^2 - 6N + 4) - 2N^3 + 9N^2 - 6N < 0$  for all  $4 \leq N \leq I \leq 24$ . When  $N \leq 3 + \sqrt{5}$ ,  $G_1(I, N)$  is weakly decreasing in  $I$  and hence  $G_1(I, N) \leq G_1(N, N) = -N(N - 1)(N - 2) < 0$ . When  $N > 3 + \sqrt{5}$ ,  $G_1(I, N)$  is increasing in  $I$  and  $G_1(I, N) \leq G_1(24, N) = -2N^3 + 33N^2 - 150N + 96$ . Because  $\frac{d}{dN}G_1(24, N) = -6(N^2 - 11N + 25)$  has its unique root  $\frac{11+\sqrt{21}}{2}$  in  $(3 + \sqrt{5}, \infty)$ , we get

$$G_1(I, N) \leq G_1(24, N) = -2N^3 + 33N^2 - 150N + 96 \leq G_1(24, \frac{11 + \sqrt{21}}{2}) = \frac{-127 + 21\sqrt{21}}{2}.$$

This shows that  $G_1(I, N) < 0$ , and thus,  $p_0 r_1 - r_0 p_1 > 0$ , for all  $4 \leq N \leq I \leq 24$ . Second, for  $k = 2$ , the definitions of  $p_0, p_2, r_0, r_2$  give

$$\begin{aligned} p_0 r_2 - r_0 p_2 &= 2I^3 N^5 (I - 2)(I - 1)^4 (N - 1) [2(2N - 1)(I - N)^2 \\ &\quad + (N - 1)(N^3 + 2N^2 + 10N - 8)(I - N) + N(N - 2)(N - 1)^2(N + 4)]. \end{aligned}$$

Because  $N^3 + 2N^2 + 10N - 8$  is increasing in  $N > 0$  and equals 128 at  $N = 4$ , we obtain  $N^3 + 2N^2 + 10N - 8 > 0$  for all  $N \geq 4$ . This shows that  $p_0 r_2 - r_0 p_2 > 0$  for all  $I \geq N \geq 4$ . Lastly, for  $k = 3$ , the definitions of  $p_0, p_3, r_0, r_3$  give

$$\begin{aligned} p_0 r_3 - r_0 p_3 &= 4I^4 N^4 (I - 2)(I - 1)^3 (N - 1)^2 [(N^3 + 2N^2 - 4N + 3)(I - N)^2 \\ &\quad + (N - 1)(2N^3 + N^2 - 3N + 2)(I - N) + N(N - 2)(N - 1)^2(N + 1)]. \end{aligned}$$

Here,  $N^3 + 2N^2 - 4N + 3$  is increasing in  $N > 2$ , equals 83 at  $N = 4$ , and hence,  $N^3 + 2N^2 - 4N + 3 > 0$  for all  $N \geq 4$ . Also,  $2N^3 + N^2 - 3N + 2$  is increasing in  $N > (\sqrt{19} - 1)/3$ , equals 134 at  $N = 4$ , and hence,  $2N^3 + N^2 - 3N + 2 > 0$  for all  $N \geq 4$ . This shows that  $p_0 r_3 - r_0 p_3 > 0$  for all  $I \geq N \geq 4$ .

Consequently, for all  $4 \leq N \leq I \leq 24$  and all  $\sigma^2 > 0$ ,

$$R(\sigma^2) - \frac{r_0}{p_0} P(\sigma^2) = \frac{1}{p_0} \sum_{k=0}^4 (p_0 r_k - r_0 p_k) \sigma^2 > 0.$$

In particular, since  $\sigma_D^2 > 0$ , we get  $R(\sigma_D^2) - \frac{r_0}{p_0} P(\sigma_D^2) > 0$ . From Eq. (80), we conclude that  $f(\sigma_D^2) > 0$  and  $\widehat{\sigma}^2 > \sigma_D^2$  for all  $4 \leq N \leq I \leq 24$ . This completes the proof. ■