

7467438-inflation-is-economic-reflection-of-portfolio-return-part-i-elementary-in-dex-adj

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POVZETEK: Avtor je raziskoval povezave med inflacijo in povratki portfelja, ki so običajno smiselno različne. Prikazuje, kako se lahko inflacijo modelira z uporabo teorije portfelja. Raziskava vključuje izračunaj elementarnega inflacijskega indeksa, ki je prilagajan samoumno inflacijo, varianco tržne inflacije in Sharpejev razmerje, ki je uporabljen kot merilnik nesporazumnosti.

METODA: Avtor je primerjaval inflacijo z povratki portfelja, ki so opisani v teoriji portfelja. Prva delitvena metoda je uporabljena za izračunaj elementarnega inflacijskega indeksa, ki je prilagajan samoumno inflacijo, varianco tržne inflacije in Sharpejev razmerje, ki je uporabljen kot merilnik nesporazumnosti.

KLJUČNE UGOTOVITVE:

- Elementarni inflacijski indeks bi moral biti izračunajen kot razmerje cijen, prilagajanih isti času s samoumno inflacijo, pred izvajanjem cijenovnega srednjičevanja.
- Varianca inflacije je merila nesporazumnosti inflacije, ki je utrujena nakupom.
- Sharpejev razmerje meri nesporazumnosti inflacije v trenutku in pri predviđanem ocenju inflacije.

POMEN ZA TRGOVANJE: Avtorjev model omogoča predviđanje inflacije z uporabo teorije portfelja, kar je koristan za finančne trgovce. Predviđanje inflacije je ključno za uspešno trgovino v inflacionih razmerjih.

Inflation is Economic Reflection of Portfolio Return.
Part I. Elementary Index Adjusted by Self-Inflation, Market-Based Variance,
and Sharpe Ratio of Elementary Inflation

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ABSTRACT

We reveal an economic identity of the modeling of an elementary index or inflation determined by a particular good or service and of a security's gross return traded at the exchange. We highlight that to be economically valid, a month-to-month elementary index should be calculated as the ratio of the prices adjusted to the same time by its elementary inflation before performing the price averaging. This is overlooked by the current theory and practice. We partially fill this flaw and derive the equation for the adjusted elementary inflation and obtain its successive approximations determined by the prices of trades adjusted by the elementary inflation to same time. The accuracy of these approximations is determined by the usage of unadjusted mean prices during the past periods. We use the likeness of the elementary index and security's gross return and introduce random elementary inflation and its market-based variance, which accounts for the impact of random trade volumes on the variance of inflation. We propose the well-known Sharpe (1966) Ratio as a measure of the uncertainty of the current and predicted assessments of the elementary inflation. To be economically reasonable, any forecast of elementary inflation should be accompanied by the forecast of its variance.

Keywords : adjusted elementary inflation, security's return, market-based variance, Sharpe Ratio

JEL: C00, E31, F3, G1, G11, G17

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1. Introduction

It is generally accepted that the economic meaning of inflation and portfolio returns has almost nothing in common. However, we show how one may use the description of portfolio returns to model and forecast inflation. That reveals strong latent economic ties between inflation and returns and gives a new look at the modeling uncertainty of inflation.

Inflation is among the issues that were studied for centuries from the very beginning of economic research. Probably, one of the first was Cantillon (1755): *“Everybody agrees that the abundance of money, or an increase in its use in exchange, raises the price of everything.”* (Part II, Ch. Six, p. 147). Not much has changed since then, and two centuries later Friedman (1953, p. 6) almost repeated Cantillon’s statement: *“..a substantial increase in the quantity of money within a relatively short period is accompanied by a substantial increase in prices.”*

The studies that describe origins, effects, and models of inflation are almost infinite, and our few references (Sargent, 1976; Friedman and Schwartz, 1982; Hall, 1982; McCallum, 1987; Cochrane, 2022) indicate our preferences only. The econometric assessment of inflation is a rather complex problem. Various indices, such as CPI (ILO et al., 2004; 2025), PPI (Eurostat, 2012), PCE (Fox et al., 2024), and others, are used to describe different features of inflation. One may find the rationale for the use of the CPI as the most widespread for inflation assessments (Reinsdorf and Triplett, 2009; Rippy, 2014). A perfect presentation of the history of the development of price indices is given by Diewert (1988).

Several citations from Diewert (1988; 1995) will help us to explain the parallels between the assessments of inflation via price indices and the methods of portfolio theory. The base notion used for the description of inflation is *“the consumer basket”*: *“The essence of the fixed basket approach or the tabular standard may be explained as follows. Suppose that there are N goods that consumers in a location can purchase during two periods, 0 and 1. Suppose that the quantities q_i purchased of the N goods are constant during the two periods”* (Diewert, 1988, p. 3).

We believe that the most impressive consumer basket was proposed by William Fleetwood, the Bishop of Ely, in 1707: *“The constant basket of goods he used to compare the value of money (or conversely, the level of prices) for an Oxford student of 1707 compared to an Oxford student of 1460 was 5 quarters of wheat, 4 hogsheads of beer, and 6 yards of cloth.”* (Diewert, 1988, p. 4). In the current measures, this student’s basket contains almost 1 metric ton of wheat, 1000 liters of beer, and a piece of cloth 1,5 meter wide and 5,5 meters long.

The selection of the consumer basket generates the problem of appropriate measurements of its value as a result of trades during the past and the current periods and the possible predictions of the basket value for one or more periods in the future (ILO et al., 2004; 2025). We don't consider the entire diversity of indices that were developed and discuss here only two definitions, proposed by Laspeyres (1871) and Paasche (1874). Let us denote the current $Q(t)$ and past $Q(t_0)$ (1.1) values of the consumer basket determined by the weights or quantities $U_{\Sigma 0}=(U_{\Sigma I}(t_0),.. U_{\Sigma N}(t_0))$ of goods and services $n=I,..N$ sold during the past period:

$$Q(t) = \sum_{n=1}^N p_n(t) U_{\Sigma n}(t_0) \quad ; \quad Q(t_0) = \sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t_0) \quad (1.1)$$

$p_n(t)$ and $p_n(t_0)$ in (1.1) denote prices of $n=I,..N$ goods or services in the consumer basket that are averaged during the current and past periods. The Laspeyres index $In_L(t, t_0)$ (1.2) at time t of the current period with respect to time t_0 of the past period takes the form:

$$In_L(t, t_0) = 1 + i(t, t_0) = \frac{Q(t)}{Q(t_0)} = \frac{\sum_{n=1}^N p_n(t) U_{\Sigma n}(t_0)}{\sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t_0)} \quad (1.2)$$

The Laspeyres index $In_L(t, t_0)$ defines inflation $i(t, t_0)$ (1.2) related to the selected consumer basket. The Paasche index $In_P(t, t_0)$ (1.3) defines the same ratio of the current and past basket values, but the values are determined by the weights sold during the current period $U_{\Sigma I}=(U_{\Sigma I}(t),.. U_{\Sigma N}(t))$:

$$In_P(t, t_0) = 1 + i_p(t, t_0) = \frac{Q(t)}{Q(t_0)} = \frac{\sum_{n=1}^N p_n(t) U_{\Sigma n}(t)}{\sum_{n=1}^N p_n(t_0) U_{\Sigma n}(t)} \quad (1.3)$$

Diewert (1988, 1995) gave a review of many methods for averaging prices and price indices that were developed for almost two centuries. We consider here only a single definition: “..the prices should be weighted according to the relative mass-quantities that were sold at them.” (Walsh, 1901, p. 96; Diewert, 1995, p. 21) and “..the average price found by dividing the total sum (or the full value) by the total quantities.” (Walsh, 1921, p. 88). Actually, this definition decades later was reproduced by Berkowitz et al. (1988) as volume weighed average price (VWAP) $p(t)$ (1.4) at time t of the good, or service, traded at the market during the current averaging period:

$$p(t) = \frac{\sum_{j=1}^J p(t_j) U(t_j)}{\sum_{j=1}^J U(t_j)} = \frac{\sum_{j=1}^J p(t_j) U(t_j)}{U_{\Sigma}(t)} \quad (1.4)$$

In (1.4), $p(t_j)$ and $U(t_j)$ denote the prices and quantities or volumes of trades with the good or service made at time $t_j, j=I,..J$, during the current period, and $U_{\Sigma}(t)$ denotes the total quantity of the good sold during the current period. The expression of VWAP price $p(t_0)$ during the past period has the same form (1.4) but the trades made at time t_j during the past period.

Furthermore, Walsh (1901, p. 97) outlines that the expression of the average price $p(t)$ (1.4) was proposed by Segnitz (1870) and refers to Heinz (1876), who, “..says this method is employed in the statistical bureaus of several German states – Wurtemberg, Bavaria, Gotha, and partly in Prussia and Hannover.” (Walsh, 1901, p. 97). Thus, one should accept that the definition of VWAP $p(t)$ (1.4) was initially proposed not by Berkowitz et al. (1988), as it is generally accepted now, at least in financial economics, but more than a century before by Segnitz (1870), the researchers of “*statistical bureaus of several German states,*” and by Walsh (1901; 1921).

The assessments of the Laspeyres index $In_L(t, t_0)$ (1.2) are supported by *elementary indices* $In_e(t, t_0)$ (1.5) that describe the price changes of different goods or services: “.. the appropriate bilateral elementary price index between periods 0 and 1 for an outlet is the following ratio of unit values” (Diewert, 1995, p. 21).

$$In_e(t, t_0) = 1 + i_e(t, t_0) = p(t)/p(t_0) \quad (1.5)$$

“The unit value price index In_e was first proposed by Segnitz (1870, p. 184) for homogeneous commodities” (Diewert, 1995, p. 21). The *elementary index* $In_e(t, t_0)$ determines elementary inflation $i_e(t, t_0)$ (1.5) related to the change of prices of the particular good or service.

Modern portfolio theory has its roots in the famous article by H. Markowitz (1952). The number of studies in this field is almost infinite, and our few references only show a wide range of the extensions of Markowitz’s model with a lifetime portfolio selection (Merton, 1969), with the dependence of the portfolio on the transaction costs and taxes (Pogue, 1970), with the usage of higher statistical moments (Samuelson, 1970), and many others. Rubinstein (2002) and Boyd et al. (2024) gave 50- and 70-year reviews of the successful developments of Markowitz portfolio theory. We consider only Markowitz’s (1952) initial portfolio model and avoid all further complexifications. To emphasize the likeness with inflation, we simplify Markowitz’s model and consider the portfolio returns that occur only due to the prices of trades made at the exchange with portfolio securities and ignore the returns that result from the dividend payoffs. Indeed, the trades at the exchange with securities are made each trading day, and the dividend payoffs occur once a year. Such a different frequency of the per-day trades with securities at the exchange and a once-a-year payoff allows us to consider the resulting returns completely independently. Their mutual dependence near the dates of the payoffs is beyond our current study.

This article has two parts. In the first part we describe the likeness of the elementary index and the gross returns of a particular security. We reveal the latent problem of the adjustment of the trade

prices $p(t_j)$ before performing the summation in (1.4) by the elementary inflation and derive the expression of the mean price $p_a(t)$ and the elementary index adjusted by self-inflation to the same time. We emphasize the similarities between inflation and gross return and use the description of a security's returns to introduce random elementary inflation and its market-based variance. We suggest the Sharpe Ratio as a measure of the uncertainty of the current and future assessments of the elementary inflation.

In the second part of the article, we enhance our results for the case of the Laspeyres index $In_L(t, t_0)$ and the portfolio's returns. We show how our results obtained in portfolio modeling (Olkhov, 2025a-2026a) help fix up the century-old remark made by Walsh: *"There is a first period, and there is a second period which is compared with it. Price variations have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period. Which weights, then, are the right ones—those of the first period? Or those of the second?"* (Walsh, 1921, p. 90; ILO, et al, 2025, p. 16). In Part II, we show that the transformation that we derived for portfolio modeling may convert *"the weights of the commodities"* of the 2nd period into *"the weights of the commodities"* that exactly model the 1st period. That resolves Walsh's question: *"Which weights, then, are the right ones—those of the first period? Or those of the second?"* We show that our transformation establishes the identity between the description of inflation by the Laspeyres and elementary indices. We believe the methods developed for the description of the portfolio returns may be very useful for modeling inflation and may benefit the fiscal and monetary policies, when they aim at inflation management. The main text of Part I of our article is almost free from math. In Sec. 2, we consider the elementary index for a month-to-month period. We derive the adjustment of the trade prices $p(t_j)$ to the same time t during this month-long period by the elementary inflation before performing the summation and averaging of prices in (1.4). Then we derive the equation for the adjusted inflation, successive approximations of the adjusted inflation, and an elementary index adjusted by its inflation. In Sec. 3, we use similarities of elementary inflation and returns of a single security and introduce random inflation, its market-based variance, and the Sharpe Ratio of elementary inflation. Sec. 4. is Conclusion.

In App. A, we present derivation of the adjusted trade prices by elementary self-inflation during the month-long period. We derive the equation for the adjusted elementary inflation and successive approximations of the adjusted mean prices and elementary inflation. In Sec. B, we argue the

returns of a single security traded at the exchange, their means, market-based variance, and Sharpe Ratio. In App. B.2, we explain how these methods may be applied for the description of elementary inflation. We emphasize the dependence of the market-based variance of elementary inflation on the coefficients of variation of trade volumes. The references to our papers may help those who have enough will and skills to understand the results presented in this article in more math details.

2. Adjustment of Elementary Index by Elementary Inflation

To simplify our analysis of inflation, we neglect any existing econometric problems that hurdle the observations of market deals and prices and avoid accounting for possible mistakes, gaps in trade data, etc. We study the time series of values, volumes, and prices of consecutive trades made in the economy during the current and past periods with a single good or service. That reveals the substantive economic similarities between modeling the elementary index and the gross return of a single market security. Even for this simplified theoretical case, we show that there exist economic obstacles that limit the accuracy of possible measurements and predictions of inflation. It is evident that the inaccuracy of econometric measurements of the available economic data adds uncertainty and limitations for the empirical assessments and predictions of inflation.

We focus readers' attention on the definition of $p(t)$ (1.4) made by Walsh: “*..the prices should be weighted according to the relative mass-quantities that were sold at them.*” (Walsh, 1901, p. 96) or “*..the average price found by dividing the total sum (or the full value) by the total quantities.*” (Walsh, 1921, p. 88). His receipt remains unchanged till now (ILO et al., 2004; 2025). Meanwhile, it is generally accepted that to define an economically justified sum of the values of trades made at times t_j and t_k , one should adjust their trade prices $p(t_j)$ and $p(t_k)$ to the same time t . The economic meaning of the sum of two trade values with unadjusted prices is highly dubious.

The price adjustment by inflation while determining the mean price $p(t)$ (1.4) is not too obvious. Indeed, the definition of the elementary inflation $i_e(t, t_0)$ (1.5), which should be used for the adjustment of the prices $p(t_j)$ of trades made during the current interval Δ (A.2), depends on the mean price $p(t)$ (1.4), which in its turn depends on the adjustment of the prices $p(t_j)$ during interval Δ (A.2). These problems are interrelated, and the price adjustment must account for the value of elementary inflation $i_e(t, t_0)$ (A.1), determined by the price change of a single item. As we know up now, this mutual interdependence of the mean price $p(t)$ (1.4) and elementary inflation $i_e(t, t_0)$ (A.1) is overlooked in financial literature and the practice of statistical agencies.

In App. A, we consider *elementary inflation* $i_a(t, t_0)$ (A.5) determined by the *elementary* price index $In_a(t, t_0)$ (A.5), as the ratio of the current $p_a(t)$ (A.6) and past $p_a(t_0)$ (A.8) prices averaged by the adjusted trade prices $p(t_j)$ during the current Δ (A.2) and past Δ_0 (A.7) month-long periods.

The mean price $p_a(t)$ (A.10) is calculated over trade prices $p_a(t_j)$ (A.9) adjusted by inflation $i_a(t, t_0)$ during the current period Δ (A.2) to time t . The equation for adjusted *elementary* inflation $i_a(t, t_0)$ (A.5) takes the form (A.14). To derive the successive approximations of adjusted *elementary* inflation $i_a(t, t_0)$ (A.14), one may account for the past average unadjusted prices. In the first approximation one may take $p_a(t_0)=p(t_0)$ (A.15-A.17) and derive the first approximation of the adjusted inflation $i_a(t, t_0)$ (A.16) during the current period Δ . In the second approximation one may take $p_a(t_{-1})=p(t_{-1})$ (A.18) two periods in the past and obtain the adjusted inflation $i_a(t_0, t_{-1})$ (A.19) and adjusted price $p_a(t_0)$ (A.20). The substitution of the adjusted price $p_a(t_0)$ (A.20) into (A.14) defines the adjusted inflation $i_a(t, t_0)$ (A.14) in the second approximation.

3. Elementary random inflation, market-based variance, and Sharpe Ratio

The adjustment of trade prices during the current (A.2) and past (A.15) month-long periods by elementary month-to-month inflation to final moments t and t_0 of each period allows us, similar to the description of a security's random returns, to introduce elementary random inflation during the current period (A.2) and estimate its market-based variance.

We highlight that the stochastic factors that may impact price indices have been studied a lot since the end of the 19th century (Jevons, 1865; Edgeworth, 1888; ILO et al., 2004, *The Stochastic Approach to Price Indices*, p. 299; ILO et al., 2025, *Stochastic Approaches to Index Number Theory*, p. 61). However, Jevons, Edgeworth, and their followers studied price randomness as the result of random errors and inaccuracies of econometric observations of numerous items in the consumer basket, or due to random gaps of the observed data, random perturbations of observations, etc. The existing challenges and hurdles that prevent precise econometric observation of such a complex problem as the values of numerous goods and services of the consumer basket determine the main origin of these studies.

Meanwhile, we focus the readers' attention on the fact that the randomness of market trades made with the goods of the consumer basket during the averaging interval is an essential irremovable stochastic factor that randomly disturbs the measurements and forecasts of inflation. The irreducible stochastic nature of values and volumes of market trades determines the market-based

roots of the uncertainty of inflation's measurements and predictions. That is completely similar to portfolio modelling, where trade randomness determines market risks and variance of the portfolio's return. However, the current studies don't consider the randomness of market trades during the averaging period as a core source of inflation stochasticity. We partially fill this gap.

In App. B1 we briefly describe random returns of a security traded at the exchange. We explain the market-based roots of the dependence of the variance of the returns on random trade volumes. We call the variance that accounts for random trade volumes as market-based variance. We show that in the approximation when trade volumes during averaging interval Δ (A.2) are assumed constant, VWAP $p(t)$ (1.4) and market-based variance $\Theta(t, t_0)$ (B.10) of returns take the form of the frequency-based mean price $\pi(t)$ (B.11) and variance $\Theta_\pi(t, t_0)$ (B.13).

In App. B2 we explain that the description of security's returns may be easily applied for the description of elementary indices and inflation. The trades with a security may be substituted by trades with a good or service. The definitions of the gross return of a security during the averaging period and of the elementary index are completely the same. The adjustments of the prices of all trades with a good or service by the elementary inflation define random inflation determined by random trade prices during the averaging period absolutely in the same way as one determines random returns by random prices of a security traded at the exchange.

We highlight that the practitioners should check up on the value of the coefficient of variation $\chi_I(t)$ (B.16) of trade volumes during the averaging period as a measure of their randomness. If $\chi_I(t) \ll 1$ one may use the approximation of constant trade volumes and simplified expressions for the frequency-based mean price $\pi(t)$ (B.11) and variance $\Theta_\pi(t, t_0)$ (B.13). Otherwise, one should consider the market-based variance $\Theta_I(t, t_0)$ (B.10) of adjusted random elementary index $In_a(t_j, t_0)$ and elementary inflation $i_a(t_j, t_0)$ (B.14).

Since Sharpe (1966) introduced his Sharpe Ratio $Sp(R)$ (B.17) that equals the ratio of the mean return $R(t, t_0)$ (B.5) of a security or of a portfolio to its volatility $\Theta^{1/2}(t, t_0)$ (B.10), this coefficient is used by almost all investors and portfolio managers. We propose that the similar Sharpe Ratio of inflation would be useful for the assessments of the uncertainty of inflation with respect to its market-based variance. For the case of elementary inflation $i_a(t, t_0)$ (A.14) determined by the price change of a single good or service, we propose the Sharpe Ratio $Sp(i)$ (3.1; B.19) of inflation:

$$Sp(i) = \frac{i_a(t, t_0)}{\Theta_I^{1/2}(t, t_0)} = \frac{i_a(t, t_0)}{In_a(t, t_0)} \left[\frac{1 + \chi_I^2(t)}{\psi_I^2(t) - 2\varphi_I(t) + \chi_I^2(t)} \right]^{1/2} \quad (3.1)$$

The Sharpe Ratio $Sp(i) > 1$ indicates that the assessment of inflation is rather definitive and its volatility is less than the mean inflation $i_a(t, t_0)$ (A.14). The Sharpe Ratio $Sp(i) < 1$ indicates high uncertainty of the mean inflation $i_a(t, t_0)$ with volatility $\Theta^{1/2}(t, t_0) > i_a(t, t_0)$. The predictions of the elementary inflation at horizon T with low Sharpe Ratio $Sp(i) < 1$ are highly dubious.

The economic meaning of the coefficients of variation $\psi_I(t)$, $\chi_I(t)$, and covariance $\rho_I(t)$ is discussed in App. B1 and (Olkhov, 2025a-2026a).

4. Conclusion

We show that one may describe the elementary index and inflation by the same methods that model the gross return of a security traded at the exchange. We argue that the economical validation of the elementary index requires the adjustment of the trade prices by the elementary inflation before performing the averaging of trade prices. The current theory and practice overlook this economic rule. We derive the successive approximations of the elementary index and inflation that account for the adjustment of trade prices by self-inflation. These approximations are determined by the usage of unadjusted prices during the past periods (A.14-A.20).

We show that the likeness of the trades with a good and a security traded at the exchange allows one to apply the description of the security's return for modeling elementary inflation. Based on trade prices adjusted by self-inflation during the averaging period, we introduce random elementary inflation. We apply the market-based variance of security's return to determine the market-based variance of elementary inflation. We propose the Sharpe Ratio of elementary inflation as a measure of the uncertainty of the current and predicted assessments of inflation.

We highlight that the current and future assessments of the elementary inflation as a mean value of a random variable should be complemented by the estimates of its uncertainty or variance. To be economically reasonable, any predictions of the elementary inflation should be complemented by the forecasts of its variance.

The description of the likeness between the modeling of the elementary inflation and security's return is the first step for the description of the similarities between the modelling of the Laspeyres index and inflation on one hand and the methods of portfolio return on the other hand. We describe that in the second part of this article.

Appendix A. Price Adjustment of Elementary Index and Inflation

Elementary indices $In_e(t, t_0)$ (1.5) were “*proposed by Segnitz (1870, p. 184) for homogeneous commodities*” (Diewert, 1995, p. 21). Elementary indices $In_e(t, t_0)$ (A.1; 1.5) define *elementary inflation* $i_e(t, t_0)$ (A.1) related to the change of prices of a single good or service:

$$In_e(t, t_0) = 1 + i_e(t, t_0) = \frac{p(t)}{p(t_0)} \quad (\text{A.1})$$

We focus readers’ attention on Walsh’s definition of mean $p(t)$ (1.4): “*..the prices should be weighted according to the relative mass-quantities that were sold at them.*” (Walsh, 1901, p. 96).

To define the economically justified sum of two values of trades made at time t_j and t_i , one should adjust trade prices $p(t_j)$ and $p(t_i)$ to the same time t . The economically justified averaging can be performed only over the prices adjusted to the same time. Let us consider the elementary indices $In_e(t, t_0)$, inflation $i_e(t, t_0)$ (A.1), and price averaging during a month-long interval Δ (A.2):

$$\Delta = [t_0; t] \quad ; \quad t_0 = t - \Delta \quad ; \quad t_j \in \Delta \quad ; \quad t = t_j \quad ; \quad j = 0, \dots, J \quad (\text{A.2})$$

The sum of two trade values with unadjusted prices has doubtful economic sense. The calculation of a month-to-month inflation requires that the prices of trades during the month-long period Δ (A.2) to be adjusted to the same time.

Let us start with the definition of the consecutive trades made with a single good or service at the market during a month-long interval Δ (A.2). The time series of values $C(t_j)$ and volumes $U(t_j)$ of trades made at time t_j during Δ are irregular or random. There is no need to account for each trade made with a good. To make a reliable assessment of the first two statistical moments of random trades during Δ , one should have enough trade terms. For example, if one measures total values and volumes of trades made with a good or service during half a day, one obtains a time series of 60 trades per month, which is enough for the assessments of the first two statistical moments. We denote the sums of values and volumes of all trades with a single good made in the market during the period δ_j (A.3) as values $C(t_j)$ and volumes $U(t_j)$ of consecutive trades made at time $t_j, j=1, \dots, J$. We assume that the duration δ between the consecutive trades at times t_j and t_{j+1} is constant and there are exactly J intervals $\delta J = \Delta$, for example, $J=60$ per a month:

$$\delta_j = [t_{j-1}; t_j] \quad ; \quad t_{j+1} = t_j + \delta \quad ; \quad j = 1, \dots, J \quad (\text{A.3})$$

The values $C(t_j)$ and volumes $U(t_j)$ determine the mean price $p(t_j)$ of collective trades during δ_j :

$$C(t_j) = p(t_j) \cdot U(t_j) \quad (\text{A.4})$$

Let us denote the adjusted elementary inflation $i_a(t, t_0)$ (A.5) during Δ (A.2) as the ratio of an adjusted mean price $p_a(t)$ (A.6) at time t at the end of Δ (A.2) and $p_a(t_0)$ (A.8) at time t_0 the beginning of Δ (A.2) or at the end of the past interval Δ_0 (A.7):

$$In_a(t, t_0) = 1 + i_a(t, t_0) = \frac{p_a(t)}{p_a(t_0)} \quad (\text{A.5})$$

$$p_a(t) = \frac{1}{\sum_{j=1}^J U(t_j)} \sum_{j=1}^J p_a(t_j) U(t_j) \quad ; \quad t_j \in \Delta \quad (\text{A.6})$$

$$\Delta_0 = [t_{-1}; t_0] \quad ; \quad t_{-1} = t_0 - \Delta \quad ; \quad t_j \in \Delta \quad ; \quad t = t_j \quad ; \quad j = 0, \dots, J \quad (\text{A.7})$$

$$p_a(t_0) = \frac{1}{\sum_{j=1}^J U(t_j)} \sum_{j=1}^J p_a(t_j) U(t_j) \quad ; \quad t_j \in \Delta_0 \quad (\text{A.8})$$

As $p_a(t_j)$ in (A.6), we denote prices of trades made at time t_j during Δ (A.2) that were adjusted by inflation $i_a(t, t_0)$ (A.5) to time t :

$$p_a(t_j) = p(t_j) \cdot \left[1 + \left(1 - \frac{j}{J} \right) i_a(t, t_0) \right] \quad (\text{A.9})$$

Indeed, the duration of the time interval $t-t_j$ equals $(1-j/J) \cdot \Delta$, and hence the inflation $i_a(t, t_0)$ (A.5) during Δ has the effect proportional to $(1-j/J)$. From (1.4) and (A.6; A.9) obtain for $p_a(t)$ (A.6):

$$p_a(t) = \frac{1}{\sum_{j=1}^J U(t_j)} \sum_{j=1}^J p(t_j) \left[1 + \left(1 - \frac{j}{J} \right) i_a(t, t_0) \right] U(t_j) = p(t) + z(t) i_a(t, t_0) \quad (\text{A.10})$$

$$z(t) = \frac{1}{\sum_{j=1}^J U(t_j)} \sum_{j=1}^J p(t_j) \left(1 - \frac{j}{J} \right) U(t_j) \quad (\text{A.11})$$

One may consider $p(t)$ (1.4) as the mean price $p_a(t)$ (A.10) adjusted for zero inflation $i_a(t, t_0)=0$, and $z(t)$ (A.11) describes the adjustment term of price $p_a(t)$ (A.10) for inflation $i_a(t, t_0)=I$:

$$p_a(t)|_{i_a=0} = p(t) \quad ; \quad p_a(t)|_{i_a=1} = p(t) + z(t) \quad (\text{A.12})$$

The adjusted price $p_a(t_0)$ in (A.8) has the similar form but for the past averaging period Δ_0 (A.7):

$$p_a(t_0) = p(t_0) + z(t_0) i_a(t_0, t_{-1}) \quad (\text{A.13})$$

In (A.13) we denote elementary adjusted inflation $i_a(t_0, t_{-1})$ during the past period Δ_0 (A.7). We emphasize that the trade values $C(t_j)$, volumes $U(t_j)$, and equation (A.4) permit us to calculate $z(t)$ (A.11) during Δ (A.2) and $z(t_0)$ during Δ_0 . From (A.5; A.10; A.13), obtain the equation (A.14) on adjusted elementary inflation $i_a(t, t_0)$ (A.5) during Δ :

$$\begin{aligned} In_a(t, t_0) = 1 + i_a(t, t_0) &= \frac{p_a(t)}{p_a(t_0)} = \frac{p(t) + z(t) i_a(t, t_0)}{p(t_0) + z(t_0) i_a(t_0, t_{-1})} \\ i_a(t, t_0) &= \frac{p(t) - p_a(t_0)}{p_a(t_0) - z(t)} \end{aligned} \quad (\text{A.14})$$

The expression of adjusted elementary inflation $i_a(t, t_0)$ (A.14) is determined by the adjusted price $p_a(t_0)$ in (A.13) during the past period Δ_0 (A.7). In the first approximation one may take:

$$p_a(t_0) = p(t_0) + z(t_0)i_a(t_0, t_{-1}) = p(t_0) \Leftrightarrow i_a(t_0, t_{-1}) = 0 \quad (\text{A.15})$$

From (A.1; A.14; A.15), obtain adjusted elementary inflation $i_a(t, t_0)$ in the first approximation:

$$i_a(t, t_0) = i_e(t, t_0) \frac{p(t_0)}{p(t_0)-z(t)} = \frac{p(t)-p(t_0)}{p(t_0)-z(t)} \quad (\text{A.16})$$

The adjusted price $p_a(t)$ (A.10) in the first approximation takes the form:

$$p_a(t) = p(t) + i_a(t, t_0)z(t) = p(t) + i_e(t, t_0) \frac{p(t_0)z(t)}{p(t_0)-z(t)} = \frac{p(t)-z(t)}{p(t_0)-z(t)} p(t_0) \quad (\text{A.17})$$

In the second approximation one should account for the past price $p_a(t_{-1})$ similar to (A.15):

$$p_a(t_{-1}) = p(t_{-1}) + z(t_{-1})i_a(t_{-1}, t_{-2}) = p(t_{-1}) \Leftrightarrow i_a(t_{-1}, t_{-2}) = 0 \quad (\text{A.18})$$

The adjusted inflation $i_a(t_0, t_{-1})$ (A.19) during period Δ_0 (A.7) takes the form like (A.16):

$$i_a(t_0, t_{-1}) = i_e(t_0, t_{-1}) \frac{p(t_{-1})}{p(t_{-1})-z(t_0)} ; i_e(t_0, t_{-1}) = \frac{p(t_0)}{p(t_{-1})} - 1 \quad (\text{A.19})$$

The adjusted price $p_a(t_0)$ (A.13) during the past period Δ_0 (A.7) in the first approximation takes the same form as (A.17):

$$p_a(t_0) = p(t_0) + i_e(t_0, t_{-1}) \frac{p(t_{-1})z(t_0)}{p(t_{-1})-z(t_0)} = \frac{p(t_0)-z(t_0)}{p(t_{-1})-z(t_0)} p(t_{-1}) \quad (\text{A.20})$$

The substitution of the adjusted price $p_a(t_0)$ (A.20) into (A.14) defines the adjusted inflation $i_a(t, t_0)$ (A.14) in the second approximation. To derive the k -th successive approximation, one should account for the adjusted inflation $i_a(t_{1-k}, t_{-k})$ during k periods in the past:

$$\begin{aligned} \Delta_{-k} &= [t_{-k-1}; t_{-k}] ; t_{-k-1} = t_{-k} - \Delta ; t_j \in \Delta_{-k} ; t = t_j ; j = 0, \dots, J \\ p_a(t_{-k}) &= p(t_{-k}) + i_a(t_{-k}, t_{-k-1})z(t_{-k}) = p(t_{-k}) \Leftrightarrow i_a(t_{-k}, t_{-k-1}) = 0 \\ i_a(t_{1-k}, t_{-k}) &= i_e(t_{1-k}, t_{-k}) \frac{p(t_{-k})}{p(t_{-k})-z(t_{1-k})} ; i_e(t_{1-k}, t_{-k}) = \frac{p(t_{1-k})}{p(t_{-k})} - 1 \\ p_a(t_{1-k}) &= \frac{p(t_{1-k})-z(t_{1-k})}{p(t_{-k})-z(t_{1-k})} p(t_{-k}) \end{aligned}$$

Appendix B. Random Returns and Market-Based Variance of a Security

We consider an investor who at time t_0 in the past has collected $U_\Sigma(t_0)$ shares of a single security with the total value $C_\Sigma(t_0)$ at price $p(t_0)$ and since doesn't trade his shares. The value $C_\Sigma(t_0)$, the number of shares $U_\Sigma(t_0)$, and the price $p_\Sigma(t_0)$ obey the equation (B.1):

$$C_\Sigma(t_0) = p(t_0) \cdot U_\Sigma(t_0) \quad (\text{B.1})$$

Since time t_0 , the investor hasn't traded his shares. At the current time t during the averaging period Δ (A.2), the investor observes the trades made at the exchange with shares of his security and

models the current mean return and market-based variance of his portfolio composed of shares of one security. To emphasize the similarities between modeling returns and inflation, we consider the same month-long interval Δ (A.2) and assume that the collective trades with shares of the investor's security at the exchange made during constant aggregating intervals δ_j (A.3) twice a day so $J \sim 60$. We assume that all prices are adjusted to the current time t (A.2). We use results (Olkhov, 2022-2026b) to model the mean return, random returns, and variance of a single security traded at the exchange.

B.1 Mean return, random returns and market-based variance of a security

We describe consecutive trades made at time t_j at the exchange with shares of a single security with trade values $C(t_j)$, volumes $U(t_j)$, and prices $p(t_j)$ that obey trivial equation (A.4). We consider values $C(t_j)$ and volumes $U(t_j)$ of trades made at the exchange during the averaging period Δ (A.2) as random variables. Relations (B.2) define the m -th statistical moments of values $C(t; m)$ and volumes $U(t; m)$:

$$C(t; m) = \frac{1}{J} \sum_{j=1}^J C^m(t_j) \quad ; \quad U(t; m) = \frac{1}{J} \sum_{j=1}^J U^m(t_j) \quad ; \quad m = 1, 2 \quad (\text{B.2})$$

For the convenience, we define the sums of values $C_\Sigma(t; m)$ and volumes $U_\Sigma(t; m)$ (B.3) of the m -th power of values $C^m(t_j)$ and volumes $U^m(t_j)$

$$C_\Sigma(t; m) = J \cdot C(t; m) = \sum_{j=1}^J C^m(t_j) \quad ; \quad U_\Sigma(t; m) = J \cdot U(t; m) = \sum_{j=1}^J U^m(t_j) \quad (\text{B.3})$$

The time series of prices $p(t_j)$ during Δ (A.2) may be highly irregular or random. The equation (A.4) causes the statistical moments of prices $p(t_j)$ to be determined by the statistical moments and covariances of random values $C(t_j)$ and volumes $U(t_j)$. The average price $p(t; 1)$ has the form of VWAP (1.4; A.6; B.4) and is determined by the means of values $C(t; 1)$ and volumes $U(t; 1)$ (B.2) or by their sums $C_\Sigma(t; 1)$ and $U_\Sigma(t; 1)$ (B.3):

$$p(t) = \frac{C_\Sigma(t; 1)}{U_\Sigma(t; 1)} = \frac{C(t; 1)}{U(t; 1)} = \frac{1}{U_\Sigma(t; 1)} \sum_{j=1}^J p(t_j) U(t_j) \quad (\text{B.4})$$

The trades made at time t_j during Δ (A.2) and VWAP $p(t)$ (B.4) allow model mean $R(t, t_0)$ and random $R(t_j, t_0)$ (B.5) gross returns of the security with respect to its past price $p(t_0)$ at time t_0 :

$$R(t_j, t_0) = \frac{p(t_j)}{p(t_0)} \quad ; \quad R(t, t_0) = \frac{p(t)}{p(t_0)} \quad (\text{B.5})$$

We don't account for the dividend payoff that occurs once a year and describe only returns as the result of per-day trades during the month-long averaging period Δ (A.2).

As values $C(t_j)$, volumes $U(t_j)$, prices $p(t_j)$ (A.4), and returns $R(t_j, t_0)$ (B.4) of consecutive trades are random variables, their variances describe the uncertainty of their mean values. Markowitz (1952) proposed that investors should consider the forecast of the variance as a measure of market risk that in the future actual returns may be less than their forecast. To estimate the current uncertainty of mean return, the investor may use the observed values $C(t_j)$, volumes $U(t_j)$, and prices $p(t_j)$ of consecutive trades to model current variances of prices and returns during Δ (A.2). To derive the market-based variance $\Phi(t)$ (B.7) of random prices $p(t_j)$ that account for random volumes of the consecutive trades $U(t_j)$, one should consider the square of the equation (A.4):

$$C^2(t_j) = p^2(t_j) \cdot U^2(t_j) \quad (\text{B.6})$$

To derive price variance $\Phi(t)$ (B.7), one should use VWAP $p(t)$ (B.4) and follow relations determined by (B.6). We refer to Olkhov (2022; 2023; 2025a-2025c) for derivation, justifications, and validations of market-based variance $\Phi(t)$ (B.7) of random prices $p(t_j)$:

$$\Phi(t) = \frac{\psi^2(t) - 2\varphi(t) + \chi^2(t)}{1 + \chi^2(t)} p^2(t; 1) \quad (\text{B.7})$$

In (B.7), the functions $\psi(t)$ and $\chi(t)$ (B.8) denote the coefficients of variation of values $C(t_i)$ and volumes $U(t_i)$:

$$\psi^2(t) = \frac{\text{cov}\{C(t), C(t)\}}{C^2(t; 1)} = \frac{\Psi_C(t)}{C^2(t; 1)} \quad ; \quad \chi^2(t) = \frac{\text{cov}\{U(t), U(t)\}}{U^2(t; 1)} = \frac{\Psi_U(t)}{U^2(t; 1)} \quad (\text{B.8})$$

$$\text{cov}\{C(t), C(t)\} = \frac{1}{J} \sum_{j=1}^J [C(t_j) - C(t, 1)]^2 \quad ; \quad \text{cov}\{U(t), U(t)\} = \frac{1}{J} \sum_{j=1}^J [U(t_j) - U(t, 1)]^2$$

The function $\varphi(t)$ (B.9) denotes the ratio of the covariance of values $C(t_i)$ and volumes $U(t_i)$ to their mean values $C(t; 1)$ and $U(t; 1)$:

$$\varphi(t) = \frac{\text{cov}\{C(t), U(t)\}}{C(t; 1)U(t; 1)} \quad ; \quad U(t; 2) = U^2(t; 1)[1 + \chi^2(t)] \quad (\text{B.9})$$

$$\text{cov}\{C(t), U(t)\} = \frac{1}{J} \sum_{j=1}^J [C(t_j) - C(t, 1)] [U(t_j) - U(t, 1)]$$

Simple relation defines market-based variance $\Theta(t, t_0)$ (B.10) of returns during Δ :

$$\Theta(t, t_0) = \frac{\Phi(t)}{p^2(t_0)} = \frac{\psi^2(t) - 2\varphi(t) + \chi^2(t)}{1 + \chi^2(t)} R^2(t, t_0) \quad (\text{B.10})$$

We denote variance of price $\Phi(t)$ (B.7) and variance of returns $\Theta(t, t_0)$ (B.10) as market-based variances to emphasize that they, as well as VWAP $p(t)$ (B.4), account for the impact of random volumes $U(t_i)$ of consecutive trades. If one considers the approximation when all trade volumes $U(t_i) = U$ are constant during Δ (A.2), one may obtain (Olkhov, 2022; 2023):

$$\text{if } U(t_j) = U \text{ then VWAP } p(t) = \frac{1}{U_{\Sigma}(t; 1)} \sum_{j=1}^J p(t_j) U(t_j) \Rightarrow \pi(t) = \frac{1}{J} \sum_{j=1}^J p(t_j) \quad (\text{B.11})$$

$$\text{if } U(t_j) = U \text{ then } \chi^2(t) = \varphi(t) = 0; \Phi(t)(B.7) \Rightarrow \Phi_\pi(t) = \frac{1}{J} \sum_{j=1}^J [p(t_j) - \pi(t)]^2 \quad (B.12)$$

$$\text{if } U(t_j) = U \text{ then } \Theta(t, t_0)(B.10) \Rightarrow \Theta_\pi(t, t_0) = \frac{1}{J} \sum_{j=1}^J [R(t_j, t_0) - R(t, t_0)]^2 \quad (B.13)$$

The practitioners should care what market approximation they use. Do they account for random trade volumes or use the simplification with constant trade volumes? The simplified approximation of constant trade volumes is often used by default and may cause incorrect assessments of the current and future prices and variances, which are almost always determined by random volumes. One may find the enhancements of the above models to the description of portfolios composed of numerous securities traded at the exchange in Olkhov (2025a-2026a).

B.2 Random elementary index and random elementary inflation

In the main text and in Appendix A, we used the same notation to highlight the likeness of the elementary index and gross returns of a single security traded at the exchange. Indeed, one should accept that the definition of mean gross return $R(t, t_0)$ (B.5) and elementary index $In_e(t, t_0)$ (A.1) or adjusted elementary index $In_a(t, t_0)$ (A.5) are completely the same. We underline that the prices of trades with a single good, which define the elementary index during Δ (A.2), should be adjusted by elementary inflation to the current time t and to the past time t_0 during Δ_0 (A.7). The likeness of mean gross return $R(t, t_0)$ (B.5) and elementary index $In_e(t, t_0)$ (A.1) or elementary index $In_a(t, t_0)$ (A.5) adjusted to the current time t allows us to introduce adjusted random elementary index $In_a(t_j, t_0)$ and random inflation $i_a(t_j, t_0)$ (B.14) with respect to each trade at time t_j during Δ (A.2), similar to the definitions of the random gross returns $R(t_j, t_0)$ (B.5):

$$In_a(t_j, t_0) = 1 + i_a(t_j, t_0) = \frac{p_a(t_j)}{p_a(t_0)} \quad (B.14)$$

We recall that all prices $p_a(t_j)$ of trades made at time t_j during Δ (A.2) are adjusted to the current time t by elementary inflation $i_a(t_j, t_0)$ (B.14). The fluctuations of random elementary inflation $i_a(t_j, t_0)$ (B.14) describe its uncertainty during Δ . We indicate that the likeness of mean gross returns $R(t, t_0)$ (B.5) and elementary index $In_a(t, t_0)$ (A.5) results in the similarity of usual returns $r(t, t_0)$ (B.15) and elementary inflation $i_a(t, t_0)$:

$$r(t, t_0) = \frac{p(t) - p(t_0)}{p(t_0)} = R(t, t_0) - 1 \Leftrightarrow i_a(t_j, t_0) = \frac{p_a(t_j) - p_a(t_0)}{p_a(t_0)} = In_a(t, t_0) - 1 \quad (B.15)$$

Similar to the market-based variance $\Theta(t, t_0)$ (B.10) of random gross returns $R(t_j, t_0)$ (B.5), we define the market-based variance $\Theta_l(t, t_0)$ (B.16) of the adjusted random elementary index $In_a(t_j, t_0)$ (B.14):

$$\Theta_I(t, t_0) = E \left[\left(In_a(t_j, t_0) - In_a(t, t_0) \right)^2 \right] = \frac{\psi_I^2(t) - 2\varphi_I(t) + \chi_I^2(t)}{1 + \chi_I^2(t)} In_a^2(t, t_0) \quad (B.16)$$

The functions $\psi_I(t)$ and $\chi_I(t)$ (B.16) denote the coefficients of variation of values $C(t_i)$ and volumes $U(t_i)$ of trades with a single good that determine the elementary random index $In_a(t_j, t_0)$ (B.14) in the same form as (B.8). The function $\varphi_I(t)$ denotes the covariance of (B.9) values $C(t_i)$ and volumes $U(t_i)$ like (B.9).

It is evident that the market-based variance of random inflation $i_a(t_j, t_0)$ (B.14) equals $\Theta_I(t, t_0)$ (B.16):

$$\Theta_I(t, t_0) = E \left[\left(In_a(t_j, t_0) - In_a(t, t_0) \right)^2 \right] = E \left[\left(i_a(t_j, t_0) - i_a(t, t_0) \right)^2 \right]$$

We emphasize that the random volumes $U(t_i)$ of the consecutive trades during Δ (A.2) with the good of the random elementary index $In_a(t_j, t_0)$ or inflation $i_a(t_j, t_0)$ (B.14) determine the main impact of random market trades. The practitioners may use the coefficient of variation $\chi_I(t)$ (B.16) of random trade volumes as a measure of their randomness. When all trade volumes $U(t_i) = U$ are constant during Δ , the coefficient of variation $\chi_I(t) = 0$. In this case the variance $\Theta_I(t, t_0)$ (B.16) of adjusted random inflation $i_a(t_j, t_0)$ (B.14), as well as the variance of returns $\Theta(t, t_0)$ (B.10), is reduced to the frequency-based case $\Theta_\pi(t, t_0)$ (B.13). The constant trade volumes approximation is widespread, but this simplification may cause wrong assessments of current variance and incorrect forecasts (Olkhov, 2025b-2026a). Practitioners should check the applicability of constant trade volume approximation by the assessments of their coefficient of variation $\chi_I(t)$ (B.16).

Investors and portfolio managers use of the coefficient of variation at least since Sharpe (1966) introduced his Sharpe Ratio $Sp(R)$ (B.17):

$$Sp(R) = \frac{R(t, t_0)}{\Theta^{1/2}(t, t_0)} \quad (B.17)$$

The Sharpe Ratio $Sp(R)$ of portfolio returns is reciprocal to the coefficient of variation of returns. Actually, Sharpe (1966) considered the ratio $Sp(r)$ (B.18) of $r(t, t_0) = R(t, t_0) - I$ to the volatility:

$$Sp(r) = \frac{r(t, t_0)}{\Theta^{1/2}(t, t_0)} = \frac{r(t, t_0)}{R(t, t_0)} Sp(R) \quad ; \quad r(t, t_0) = R(t, t_0) - 1 \quad (B.18)$$

We recall that the square of the volatility equals the variance. It is obvious that the variance of $R(t, t_0)$ equals the variance of $r(t, t_0)$ (B.18). The relations between $Sp(R)$ (B.17) and $Sp(r)$ (B.18) are trivial, and we further consider $Sp(R)$. We propose that the use of the market-based Sharpe Ratio $Sp(i)$ (B.19) of inflation $i_a(t, t_0)$ (A.14):

$$Sp(i) = \frac{i_a(t, t_0)}{\Theta_I^{1/2}(t, t_0)} = \frac{i_a(t, t_0)}{In_a(t, t_0)} \left[\frac{1 + \chi_I^2(t)}{\psi_I^2(t) - 2\varphi_I(t) + \chi_I^2(t)} \right]^{1/2} \quad (B.19)$$

may be rather useful. In the approximation of constant trade volumes $U(t_j)=U$, the coefficients of variation and covariance $\chi_I(t) = \varphi_I(t) = 0$ and (B.19) takes a simple form:

$$Sp(i)_{|U(t_j)-const} = \frac{i_a(t, t_0)}{In_a(t, t_0)} \psi_I^{-1}(t) \quad (B.20)$$

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